Quantum Kinetic Theory of Spin Polarization of Massive Quarks in Perturbative QCD: Leading Log

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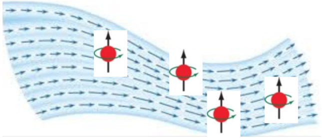
Overview

Experimentally observation of Λ polarization in heavy ion collisions Nature 548, 62 (2017)

Spin in a relativistic fluid.



Theoretical progress:

• **Spin-orbital coupling in HIC from global OAM**
Liang and Wang 2005 Becattini et al 2008

• **Relativistic Hydrodynamics with spin**
W. Florkowski et al.2018 K. Hattori et al 2019

•**Free streaming Boltzmann equation for massive spin-$\frac{1}{2}$ fermions**
Mueller and Venugopalan arXiv:1901:10492
N. Weickgenannt et al.,arXiv:1902.06513
Gao and Liang arXiv:1902.06510
K. Hattori et al arXiv:1903.01653

Our objective: Formulate quantum kinetic equations for massive spin-$\frac{1}{2}$ quarks with collision term in pQCD

Setup

The time evolution of density matrix $\hat{\rho}$ of massive quarks satisfy Lindblad like equation:

$$\frac{\partial \hat{\rho}}{\partial t} = -\frac{i}{\hbar}[H_{eff}, \hat{\rho}] - \Gamma \cdot \hat{\rho}$$

where:

$$H_{eff} = -\frac{\hbar}{2}\boldsymbol{\sigma} \cdot (\boldsymbol{\omega} + e\mathbf{B})$$

$$\hat{\rho}_{equilibruim} = e^{-H_{eff}/T} \sim 1 - H_{eff}/T + O(\boldsymbol{\omega}^2, \mathbf{B}^2) + \cdots$$

$$\Gamma = \Gamma_0 + O(\boldsymbol{\omega}, \mathbf{B}) + \cdots$$

We begin by Γ_0, which describes how the spin density matrix, initially polarized relaxes to un-polarized one with vanishing of vorticity and external magnetic field.

Density matrix

Density matrix $\hat{\rho}$ in phase space$(\mathbf{x}, \mathbf{p})$ in Schwinger-Keldysh formalism in "ra" basis are

$$[\mathbf{x}_1^i, \mathbf{p}_1^j] = i\hbar\delta^{ij}, \quad [\mathbf{x}_2^i, \mathbf{p}_2^j] = -i\hbar\delta^{ij}, \quad [\mathbf{x}_{r/a}^i, \mathbf{p}_{a/r}^j] = i\hbar\delta^{ij}$$

$$\mathbf{x}_r = \frac{\mathbf{x}_1 + \mathbf{x}_2}{2}, \quad \mathbf{x}_a = \mathbf{x}_1 - \mathbf{x}_2, \quad \mathbf{p}_r = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2}, \quad \mathbf{p}_a = \mathbf{p}_1 - \mathbf{p}_2$$

$$\hat{\rho}(\mathbf{x}_r, \mathbf{p}_r) = \int \frac{d^3\mathbf{p}_a}{(2\pi)^3} e^{i\mathbf{p}_a\cdot\mathbf{x}_r} \hat{\rho}(\mathbf{p}_r, \mathbf{p}_a)$$

$$\hat{\rho}(\mathbf{p}_r, \mathbf{p}_a) = \hat{\rho}(\mathbf{p}_1, \mathbf{p}_2) = \hat{\rho}(\mathbf{p})(2\pi)^3\delta(\mathbf{p}_1 - \mathbf{p}_2)$$

- $\boldsymbol{p}_a \sim \boldsymbol{\partial}_{\boldsymbol{x}_r}$ Note: Assuming the background is spatially homogeneous!!!!

$$\hat{\rho}(\boldsymbol{p}) = \sum_{s,s'=\pm} |\boldsymbol{p}, s\rangle \rho_{s,s'}(\boldsymbol{p}) \langle \boldsymbol{p}, s'| = \frac{1}{2} f(\mathbf{p}) + \mathbf{S}(\mathbf{p}) \cdot \boldsymbol{\sigma}$$

Total particle number, $n = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} f(\boldsymbol{p})$

Total spin polarization, $\boldsymbol{S} = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \boldsymbol{S}(\boldsymbol{p})$

Time evolution of density matrix

$$\hat{\rho}(t) = \left\langle U_1(t,t_0)\hat{\rho}(t_0)U_2^\dagger(t,t_0)\right\rangle_A$$

Unitary time evolution operator $U_{1,2}(t,t_0) = \mathcal{P}e^{-i\int_{t_0}^{t} dt'\, H_{1,2}(t')}$

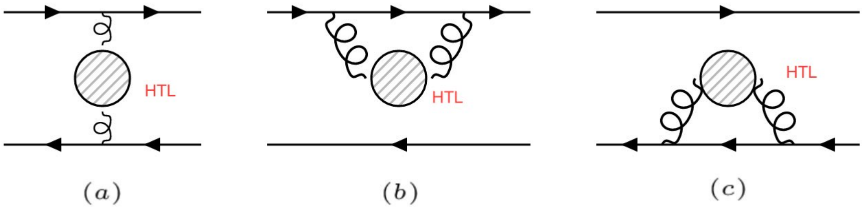
QCD: $H_I = g\int d^3\boldsymbol{x}\,\bar{\psi}(\boldsymbol{x})\gamma^\mu t^a\psi(\boldsymbol{x})A_\mu^a(\boldsymbol{x})$

$$\psi(\boldsymbol{x}) = \frac{1}{\sqrt{V}}\sum_{\boldsymbol{n},s}\frac{1}{\sqrt{2E_{p_{\boldsymbol{n}}}}}u(\boldsymbol{p_n},s)e^{i\boldsymbol{p_n}\cdot\boldsymbol{x}}a_{\boldsymbol{p_n},s}$$

$$A_\mu(\boldsymbol{x},t) = \frac{1}{V}\sum_{\boldsymbol{n}}A_\mu(\boldsymbol{p_n},t)e^{i\boldsymbol{p_n}\cdot\boldsymbol{x}} + \text{h.c.}$$

$$H_I(t) = \frac{g}{V}\sum_{\boldsymbol{n},s;\boldsymbol{n}',s'}\frac{1}{\sqrt{2E_{p_{\boldsymbol{n}}}}\sqrt{2E_{p_{\boldsymbol{n}'}}}}\bar{u}(\boldsymbol{p_n},s)\gamma^\mu u(\boldsymbol{p_{n'}},s')A_\mu(\boldsymbol{p_n}-\boldsymbol{p_{n'}},t)a^{\dagger\boldsymbol{s}}_{\boldsymbol{p_n}}a^{\boldsymbol{s}'}_{\boldsymbol{p_{n'}}}$$

Time evolution of density matrix



$$U_0^\dagger(\Delta t)\hat{\rho}(\Delta t)U_0(\Delta t) = \hat{\rho}(0) + \int_0^{\Delta t} dt_1 \int_0^{\Delta t} dt_2 \langle H_I^{\mathrm{int}(1)}(t_1)\hat{\rho}(0)H_I^{\mathrm{int}(2)}(t_2)\rangle_A$$

$$+(-i)^2 \int_0^{\Delta t} dt_1 \int_0^{t_1} dt_1' \langle H_I^{\mathrm{int}(1)}(t_1)H_I^{\mathrm{int}(1)}(t_1')\rangle_A \hat{\rho}(0)$$

$$+(+i)^2 \hat{\rho}(0) \int_0^{\Delta t} dt_2 \int_0^{t_2} dt_2' \langle H_I^{\mathrm{int}(2)}(t_2')H_I^{\mathrm{int}(2)}(t_2)\rangle_A$$

Time evolution of density matrix

$$\frac{d}{dt}\rho_{s,s'}(\boldsymbol{p},t) = g^2 C_2(F)(\Gamma_{\rm cross} + \Gamma_{\rm self\,energy})\,, \quad C_2(F) = \frac{N_c^2-1}{2N_c}$$

$$\Gamma_{\rm cross} = \int \frac{d^3\boldsymbol{p}'}{(2\pi)^3}\frac{1}{4E_pE_{p'}}\sum_{s'',s'''}[\bar{u}(\boldsymbol{p},s)\gamma^\mu u(\boldsymbol{p}',s'')]\rho_{s'',s'''}(\boldsymbol{p}')$$

$$[\bar{u}(\boldsymbol{p}',s''')\gamma^\nu u(\boldsymbol{p},s')]G^{(12)}_{\mu\nu}(E_p - E_{p'},\boldsymbol{p}-\boldsymbol{p}')$$

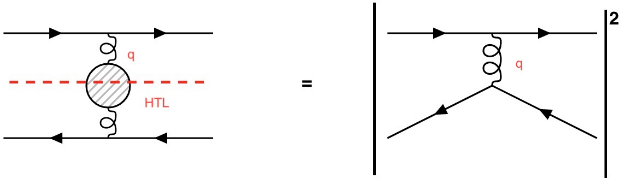
$$\Gamma_{\rm self\,energy} = \int \frac{d^3\boldsymbol{p}'}{(2\pi)^3}\frac{1}{4E_pE_{p'}}\sum_{s''}[\bar{u}(\boldsymbol{p},s)\gamma^\mu u(\boldsymbol{p}',s'')][\bar{u}(\boldsymbol{p}',s'')\gamma^\nu u(\boldsymbol{p},s)]$$

$$\rho_{s,s'}(\boldsymbol{p})G^{(21)}_{\mu\nu}(E_p - E_{p'},\boldsymbol{p}-\boldsymbol{p}')$$

$$G^{(ij)}_{\mu\nu}(q^0,\boldsymbol{q}) = \int d^4x\, e^{i(q^0t-\boldsymbol{q}\cdot\boldsymbol{x})}\,\langle A^{(i)}_\mu(\boldsymbol{x},t)A^{(j)}_\nu(\boldsymbol{0},0)\rangle_A\,, i,j = 1,2\,(\text{SK contour}$$

$$G^{(12)}_{\mu\nu}(q^0,\boldsymbol{q}) = n_B(q^0)\rho_{\mu\nu}(q^0,\boldsymbol{q}) \quad G^{(21)}_{\mu\nu}(q^0,\boldsymbol{q}) = (n_B(q^0)+1)\rho_{\mu\nu}(q^0,\boldsymbol{q})$$

Leading log contribution from t channel soft gluon exchange $gT \ll q \ll T$



The log arises from

$$\int_{m_D}^{T} \frac{dq}{q} \sim \log(T/m_D) \sim \log(1/g)$$

$$m_D^2 = \frac{g^2 T^2}{6}(2N_c + N_F)$$

Quantum kinetic equation for spin polarization of massive quarks

$\hat{\rho}(\boldsymbol{p}) = \frac{1}{2} f(\boldsymbol{p}) + \boldsymbol{S}(\boldsymbol{p}) \cdot \boldsymbol{\sigma}$, in leading log order of $g^4 \log(1/g)$

$$\frac{\partial f(\boldsymbol{p}, t)}{\partial t} = C_2(F) \frac{m_D^2 g^2 \log(1/g)}{(4\pi)} \frac{1}{2pE_p} \Gamma_f$$

$$\Gamma_f = 2pf(\boldsymbol{p}) + (\frac{3}{2} TE_p p - \frac{TE_p^3}{2p} + \frac{\eta T m^4}{2p^2}) \boldsymbol{\nabla}_p^2 f(\boldsymbol{p})$$

$$+ \frac{Tm^2}{2p^2} (\eta + \frac{3E_p}{p} - \eta \frac{3E_p^2}{p^2}) (\boldsymbol{p} \cdot \boldsymbol{\nabla}_p)^2 f(\boldsymbol{p})$$

$$+ \frac{1}{p^2} (pE_p^2 - \frac{\eta T m^2}{2} - \eta E_p m^2 - \frac{3TE_p m^2}{2p} + \frac{3\eta T m^2 E_p^2}{2p^2}) (\boldsymbol{p} \cdot \boldsymbol{\nabla}_p) f(\boldsymbol{p})$$

m: quark mass $\quad E_p = \sqrt{m^2 + p^2} \quad \eta = \frac{1}{2} ln \frac{E_p + p}{E_p - p}$ *:rapidity.*

•Detailed balance $\Gamma_f = 0$ when $f(\boldsymbol{p}) = f^{\mathrm{eq}}(\boldsymbol{p}) = ze^{-E_p/T}$

$$\frac{\partial \boldsymbol{S}(\boldsymbol{p},t)}{\partial t} = C_2(F)\frac{m_D^2 g^2 \log(1/g)}{(4\pi)}\,\frac{1}{2pE_p}\,\boldsymbol{\Gamma}_S$$

$$\boldsymbol{\Gamma}_S^i = (2p + \frac{TE_p}{p} - \frac{\eta m^2 T}{p^2})\boldsymbol{S}^i(p) + (pTE_p - \frac{m^2 TE_p}{2p} + \frac{\eta m^4 T}{2p^2})\nabla_p^2 \boldsymbol{S}^i(\boldsymbol{p})$$

$$+[\frac{\eta m^2 T}{2p^2}(1 - \frac{3E_p^2}{p^2}) + \frac{3m^2 TE_p}{2p^3}](\boldsymbol{p}\cdot\boldsymbol{\nabla}_p)^2 \boldsymbol{S}^i(\boldsymbol{p})$$

$$+\frac{1}{p^2}[pE_p^2 - \frac{3m^2 TE_p}{2p} + \eta m^2(-E_p - \frac{T}{2} + \frac{3TE_p^2}{2p^2})](\boldsymbol{p}\cdot\boldsymbol{\nabla}_p)\boldsymbol{S}^i(\boldsymbol{p})$$

$$+2T[\eta(\frac{1}{2} - \frac{E_p^2}{p^2} + \frac{mE_p}{2p^2} + \frac{E_p^3}{2p^2(E_p+m)}) + (\frac{E_p}{p} - \frac{m}{2p} - \frac{m^2}{2p(E_p+m)})]\boldsymbol{p}^i(\boldsymbol{\nabla}_p\cdot\boldsymbol{S}(\boldsymbol{p}))$$

$$-2T[\eta(\frac{1}{2} - \frac{E_p^2}{p^2} + \frac{mE_p}{2p^2} + \frac{E_p^3}{2p^2(E_p+m)}) + (\frac{E_p}{p} - \frac{m}{2p} - \frac{m^2}{2p(E_p+m)})]\boldsymbol{\nabla}_p^i(\boldsymbol{p}\cdot\boldsymbol{S}(\boldsymbol{p}))$$

$$-\frac{T}{p^2}[\frac{E_p(E_p+2m)}{p(E_p+m)} + \frac{\eta m E_p}{E_p+m}(-\frac{3E_p}{p^2} + \frac{1}{E_p+m})]\boldsymbol{p}^i(\boldsymbol{p}\cdot\boldsymbol{S}(\boldsymbol{p}))$$

In massless limit

$$\hat{\rho}(\boldsymbol{p}) = f_+(\boldsymbol{p})\mathcal{P}_+(\boldsymbol{p}) + f_-(\boldsymbol{p})\mathcal{P}_-(\boldsymbol{p}) \,, \quad \mathcal{P}_\pm(\boldsymbol{p}) = \frac{1}{2}\left(\mathbf{1} \pm \hat{\boldsymbol{p}} \cdot \boldsymbol{\sigma}\right)$$

$$\hat{\rho}(\boldsymbol{p}) = \frac{1}{2}\left(f_+(\boldsymbol{p}) + f_-(\boldsymbol{p})\right) + \frac{1}{2}\left(f_+(\boldsymbol{p}) - f_-(\boldsymbol{p})\right)\hat{\boldsymbol{p}} \cdot \boldsymbol{\sigma}$$

$$f(\boldsymbol{p})^{equilibrium} = f_+(\boldsymbol{p}) + f_-(\boldsymbol{p}) = e^{-p/T}(e^{\mu_+/T} + e^{\mu_-/T})$$

$$\boldsymbol{S}^{equilibrium} = \frac{1}{2}\left(f_+(\boldsymbol{p}) - f_-(\boldsymbol{p})\right)\hat{\boldsymbol{p}} \equiv \frac{1}{2}e^{-p/T}(e^{\mu_+/T} - e^{\mu_-/T})\hat{\boldsymbol{p}}$$

•Non trivial check of detailed balance in equilibrium

$$\Gamma_f(\boldsymbol{p}) = \Gamma_{\boldsymbol{S}} = 0$$

Discussion and Outlook

•Go beyond spatial homogeneity limit to include the advective term **x**

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}_p \cdot \frac{\partial}{\partial \boldsymbol{x}}\right) \hat{\rho}(\boldsymbol{x}, \boldsymbol{p}) = \Gamma \cdot \hat{\rho}(\boldsymbol{x}, \boldsymbol{p}), \quad \boldsymbol{v}_p \equiv \frac{\boldsymbol{p}}{E_p}$$

•To include the spin coupling to vorticity $\boldsymbol{\omega}$ and external magnetic field $\boldsymbol{B}$ in both the free streaming and collision terms

•To calculate the transport coefficients in the recent developed spin hydrodynamics.