Self-similarity and spectral functions of non-Abelian plasmas in 2+1D

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In preparation
- In high energy nuclear collisions high density of gluons at the saturation scale $Q_s$. 
- If $Q_s$ large $\rightarrow \alpha_s(Q_s) \ll 1$, $\rightarrow$ gluons at $Q_s$ overoccupied classical color fields.
- Many systems (non-Abelian gauge theories and scalars in 3D and scalars in 2D etc.) with $f \gg 1$ $\rightarrow$ universal self-similar attractors.
- At high energy the initial color fields are boost invariant $\rightarrow$ effectively 2D.
- Gauge systems in 3D exhibit self-similarity. Less is known about 2D.
Aims of this talk:

Questions:
- 3D gauge and scalar systems exhibit self-similarity. How about 2D gauge theory?
- Teaser: there are quasiparticles in the self-similar regime in 3D (PRD 98 (2018) no.1, 014006). How about 2D?

Methods - two different theories:
- 2+1D gauge theory.
- Dimensionally reduced 3+1D theory = 2+1D gauge + adjoint scalar. Mimics the boost invariant system.

Answers:
- 2+1D and eff. 2+1D systems exhibit self-similarity.
- Quasiparticle excitations for large $p$. Small $p \rightarrow$ inverse lifetime $\approx \omega$, which makes interpretation more difficult.
Introduction: self-similarity in 3D

Gauge and scalar systems exhibit self-similar behavior at late times. Dynamics governed by universal scaling exponents.

\[ f(t, p) = (Qt)^\alpha f_S((Qt)^\beta p), \]  \hspace{1cm} (1)

**Figure:** Occupation number distribution. Phys.Rev. D89 (2014) no.11, 114007

**Figure:** Third moment of the occupation number distribution, also with rescaling. Phys.Rev. D89 (2014) no.11, 114007
Self-similarity in 2+1D

Figure: With rescaling

- Self-similar evolution also in 2+1D!

In 2D results:

\[ \alpha = -\frac{3}{5} \quad (2) \]
\[ \beta = -\frac{1}{5}. \quad (3) \]

In 3D we had:

\[ \alpha = -\frac{4}{7} \quad (4) \]
\[ \beta = -\frac{1}{7}. \quad (5) \]

- \( \alpha = (d + 1)\beta \) → energy conservation.
- \( \beta = -\frac{1}{5} \): kinetic theory + small angle approximation.
- \( \beta \rightarrow \) energy cascade to UV.
Self-similarity in eff. 2+1D

The same exponents also in eff. 2+1D simulations.

For scalar distribution $f_\pi$, the self-similar scaling is violated for $p < m_D$. One observes enhancement in the IR.

Figure: Gauge distribution

Figure: Scalar distribution
Gauge-invariant hard scale: self-similar evolution

\[ \Lambda_E^2(t) = \frac{g^2}{d_A Q^4} \sum_{k,l,i=1,2} \left\langle (D_k^{ab} F_{ki}^b(t, x)) (D_l^{ad} F_{li}^d(t, x)) \right\rangle \] (6)

\[ \Lambda_\pi^2(t) = \frac{g^2}{d_A Q^4} \sum_{k,l=1,2} \left\langle (D_k^{ab} D_k^{bc} \phi^c(t, x)) (D_l^{ad} D_l^{de} \phi^e(t, x)) \right\rangle. \] (7)

- Gauge invariant hard scale follows self-similar evolution in both theories for both gauge and scalar excitations.
Attractor in 2+1D

- $Q \approx \sqrt[4]{g^2 \epsilon}$,
  
  \[ f(t = 0, p_\perp) = \frac{Q}{g^2} n_0 e^{-\frac{p_\perp^2}{2p_0^2}}, \quad g^2 \epsilon \sim n_0 p_0^4 \]

- Dashed lines: initial condition, full lines: 2D theory, dash-dotted lines: 2D + scalar theory.

- 3 different initial conditions. At later times they fall on top of each other. → Dynamics not sensitive to such initial conditions.
Use linear response theory, extract retarded propagator as in (PRD 98 (2018) no.1, 014006) for 3D gauge system. Split the gauge field into background field and a fluctuation

\[ A_\mu(t, x) \rightarrow A_\mu(t, x) + a_\mu(t, x). \]  

(8)

\( a \) evolves according to linearized EOMS. Use

\[ \langle \hat{a}_i^b(t, p) \rangle = \int \Delta t \, G_{R,ik}(t, t', p) j_c^k(t', p). \]  

(9)

Source \( j \) can be chosen such that \( G_R \) can be obtained from \( \langle ja \rangle \). Finally obtain the spectral function as

\[ \rho = 2 \Im G_R. \]  

(10)
Spectral function in 2+1D: Preliminary

Figure: Numerically extracted spectral function

- Curves correspond to $\omega \rho$
- Peak position $\leftrightarrow \omega(p)$ dispersion relation.
- Peak width $\leftrightarrow \gamma(p)$ damping rate (inverse lifetime)
- At small $p$: $\omega \approx \gamma$ i.e. quasiparticle interpretation problematic.
  At large $p$: we see quasiparticle peaks.
Conclusions

We have

- Observed self-similar behavior in 2+1D gauge theories. In both theories both gauge and scalar fields approach a universal attractor that is the same for both.
- The scaling exponents are $\alpha = -3/5$ and $\beta = -1/5$. Different from 3D.
- Scalar distribution IR enhanced.
- Extracted spectral function from 2+1D simulations. We find that quasiparticles exist for large $p$. However for small $p$ quasiparticle description becomes problematic.

Outlook

- Work in progress also in terms of transport coefficients ($\kappa$, poster by JP). We also want to look at plasma instabilities etc.
Correlation functions

Figure: 2D

Scalar correlator is enhanced in the infrared.

Figure: 2D+Scalar
The Debye mass extracted from the longitudinal $<EE>$ correlator follows self-similar evolution.

$$\langle E_L E_L^* \rangle \approx \frac{A}{1 + (p^2/m_D^2)^{1+\delta}},$$  \hspace{1cm} (11)$$

for momenta $p \lesssim \Lambda$. At early times $\delta \approx 0.2 - 0.3$. At $Qt = 2000$ $\delta \approx 0.08 - 0.12$ for both theories.
Scaling exponents

Extract the scaling exponents $\alpha$ and $\beta$. Define a rescaled distribution

$$f_{\text{test}}(t,p) = (t/t_r)^{-\alpha} f\left(t, (t/t_r)^{-\beta} p\right).$$

(12)

- $f_{\text{test}}(t_r,p) \equiv f(t_r,p)$ for the reference time $Q t_r = 500$.
- Self-similarity $\rightarrow f_{\text{test}}(t,p)$ time-independent.

Quantify the deviation by computing

$$\chi_m^2(\tilde{\alpha}, \beta) = \frac{1}{N_t} \sum_i \frac{\int d\log p \ (p^m \Delta f(t_i,p))^2}{\int d\log p \ (p^m f(t_r,p))^2},$$

(13)

- $\Delta f(t_i,p) = f_{\text{test}}(t_i,p) - f(t_r,p)$
- $\tilde{\alpha} \equiv \alpha - 3\beta$.
- Momentum integrals are performed in the interval $0.2 \leq p/Q \leq 5$. 
The deviations $\chi^2_m$ are averaged over the test times $Q t_i = 75, 200, 1500, 4000, 16000$ for different moments with $m = 2, \ldots, 5$.

Define a likelihood function

$$W(\tilde{\alpha}, \beta) = \frac{1}{N} \exp \left( 1 - \frac{\chi^2(\tilde{\alpha}, \beta)}{\chi^2_{\text{min}}} \right).$$ (14)

- $\chi^2(\tilde{\alpha}_0, \beta_0) \equiv \chi^2_{\text{min}}$ takes its minimal value.
- The normalization $N$ satisfies $\int d\tilde{\alpha} \, d\beta \, W(\tilde{\alpha}, \beta) = 1$,
  $W(\beta) = \int d\tilde{\alpha} \, W(\tilde{\alpha}, \beta)$.
- The uncertainty $\sigma_\beta$ for every $m$, fit $\propto \exp[-(\beta - \beta_0)^2/(2\sigma^2_\beta)]$.
- The statistical error $\sigma_{\chi^2_\beta}$ of the $\chi^2$ fit is the maximum $\sigma_\beta$ among the different $m$. 
Scaling exponents

Best fit values

\[ \alpha_{\text{fit}} - 3 \beta_{\text{fit}} = 0.01 \pm 0.02 \]  \hspace{1cm} (15)

\[ \beta_{\text{fit}} = -0.19 \pm 0.015. \]  \hspace{1cm} (16)