Anisotropic Hydrodynamics with a Scalar Collisional Kernel[1,2]

Dekrayat Almaalol

Collaborators: Michael Strickland, Mubarak Alqahtani

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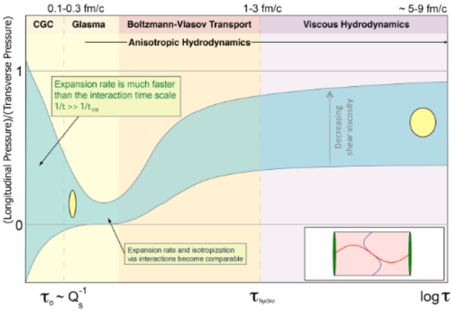
[1] Dekrayat Almaalol, Michael Strickland Phys. Rev. C 97, 044911

[2] Dekrayat Almaalol, Mubarak Alqahtani, Michael Strickland Phys. Rev. C 99, 014903

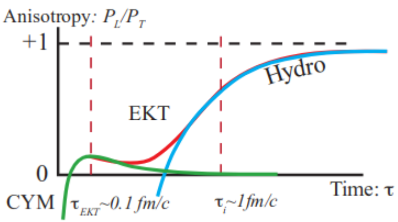
Motivation 1

big picture/ultimate Goal:
understanding the equilibration and isotropization processes in the QGP system

weak coupling approximation:
$\sqrt{s} \to \infty \Rightarrow$ weak coupling
$f \gg 1$: QYM $\Rightarrow$ classical approximation
$g^2 N_c f \ll 1$ perturbative $\Rightarrow$ weakly coupled classical theory $\Rightarrow$ EKT..



(M. Strickland arXiv:1410.5786)



(Aleksi Kurkela Nucl.Phys. A956 (2016))

Motivation 2

- Effective kinetic theory:

$$p^\mu \partial_\mu f_p = C[f_p]$$

- $C[f_p]$ describes interactions/collisions that drives the system back to equilibrium
- Choice of $C[f]$ impacts transport coefficients
- Prior aHydro studies $\Rightarrow$ RTA (Anderson-Witting)

  (Dekrayat Almaalol, Mubarak Alqahtani, and Michael Strickland Phys. Rev. C 99, 044902 - Apr 2019)
- More realistic collisional kernel $\Rightarrow$ better access to EKT
- How dynamics depend on the choice of $C[f]$?

immediate goal:

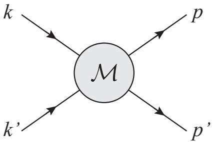
- explore sensitivity of the aHydro equations to the non-linear dynamics of the effective kinetic theory collisional kernel

- set the stage for working with the QCD collisional kernel $\Rightarrow$ (preliminary results today).

LO Scalar Collisional Kernel

Leading order $\lambda\phi^4$ conformal theory

$2 \leftrightarrow 2$ scattering



$$C[f_p] = \frac{1}{32}\int dK dK' dP' \, |\mathcal{M}|^2 \, (2\pi)^4 \delta^{(4)}(k^\alpha + k'^\alpha - p^\alpha - p'^\alpha) \, \mathcal{F}(k,k',p,p')$$

$$\mathcal{F}(k,k',p,p') \equiv f_k f_{k'}(1+af_p)(1+af_{p'}) - (1+af_k)(1+af_{k'})f_p f_{p'}$$

" Romatschke-Strickland "

$$f(x,p) = f_{\rm eq}\left(\frac{1}{\Lambda}\sqrt{\mathbf{p}^2 + \xi(\mathbf{n}\cdot\mathbf{p})^2} + \frac{\mu}{\Lambda}\right)$$

$$f_{\rm eq}(x) = 1/[\exp(x) - a]$$

ξ anisotropy parameter

Λ scale parameter

$\gamma \equiv \exp(\mu/\Lambda)$ particle fugacity

$$\int dP \equiv \int \frac{d^4p}{(2\pi)^4} \, 2\pi\delta(p^\mu p_\mu - m^2) \, 2\theta(E_p) = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{E_p}$$

Lorentz-invariant integration measure

AHydro equations of motion

$$p^\mu \partial_\mu f_p = C[f_p] \Rightarrow \quad \begin{array}{c} n^{th} \text{ momentum integral operator} \\ \hat{\mathcal{O}}_n \, g \equiv \int dP \, p^{\mu_1} p^{\mu_2} \cdots p^{\mu_n} \, g(p) \end{array} \quad \Rightarrow \partial_\mu I^{\mu\nu_1\nu_2\cdots\nu_n} = \mathcal{C}^{\nu_1\nu_2\cdots\nu_n}$$

0+1d Dynamical Equations for a General Kernel in a Conformal System

- **0th moment:** $\partial_\mu n^\mu = 0 \Rightarrow$ (number conservation)

  u_μ projection $\Rightarrow$ $\quad Dn + n\theta = 0$

  $$\boxed{\partial_\tau n = -\frac{n}{\tau}}$$

- **1st moment:** $\partial_\mu T^{\mu\nu} = 0 \Rightarrow$ (Energy-momentum conservation)

  $u_\mu u_\nu$ projection $\Rightarrow$ $\quad D\varepsilon + \varepsilon\theta - \sum_I P_I = 0$

  ; With $\varepsilon \equiv u^\mu u^\nu T_{\mu\nu} \qquad P_L \equiv X_i^\mu X_i^\nu T_{\mu\nu}$

  $$\boxed{\partial_\tau \varepsilon = -\frac{\varepsilon + P_L}{\tau}}$$

- **2nd moment:** $\partial_\lambda I^{\lambda\mu\nu} = \mathcal{C}^{\mu\nu} \Rightarrow$ (Dissipative dynamics)

  $zz - \frac{1}{3}(xx + yy + zz)$ projection$\Rightarrow$

  ; With $I_i \equiv u^\mu X_i^\nu X_i^\lambda I_{\mu\nu\lambda} \qquad \mathcal{C}^{ii} \equiv X_i^\mu X_i^\nu \mathcal{C}_{\mu\nu}$

  $$\boxed{\partial_\tau I_i + (\theta - 2\theta_i) = \mathcal{C}^{ii}}$$

(Bjorken flow $\Rightarrow$ Milne coordinates $\theta \equiv u^\mu \partial_\mu = \partial\tau$, $\theta_x = \theta_y = 0$, and $\theta_z \equiv \partial_z u^z = -1/\tau$.)

Moments of the Scalar Kernel $\partial_\lambda I^{\lambda\mu\nu} = \mathcal{C}^{\mu\nu}$

$$\mathcal{C}_{\rm sc} = 0 \qquad\qquad \mathcal{C}^{\mu}{}_{\rm sc} = 0$$

$$\mathcal{C}^{\mu\nu} = \frac{1}{32}\int dK dK' dP dP' \, |\mathcal{M}|^2 (2\pi)^4 \delta^4(k^\alpha + k'^\alpha - p^\alpha - p'^\alpha)\mathcal{F}(k,k',p,p')p^\mu p^\nu$$

$$\boxed{\mathcal{C}^{\mu\nu} = \frac{1}{128\pi^2}\int dK dK' d\Omega_p \frac{p|\mathcal{M}|^2}{E_{p'}}\mathcal{F}(k,k',p,p')p^\mu p^\nu \bigg|_{p\to\tilde{p}}}$$

(8d integral)

$$\begin{aligned} p \to \tilde{p} \quad &\equiv \quad \frac{kk'(1-\cos\theta_{kk'})}{k(1-\cos\theta_{kp}) + k'(1-\cos\theta_{k'p})} \\ &= \quad \frac{kk' - \mathbf{k}\cdot\mathbf{k}'}{k + k' - \mathbf{k}\cdot\hat{\mathbf{p}} - \mathbf{k}'\cdot\hat{\mathbf{p}}} \end{aligned}$$

To evaluate this integral, we used Monte Carlo integration. GNU Scientific Library (GSL) VEGAS algorithm was used with 10^7 evaluations per iteration.)

Moments of the RTA kernel

In the relaxation-time approximation

$$C_{\mathrm{RTA}}[f_p] = \frac{E_p}{\tau_{\mathrm{eq}}}\left[f_{\mathrm{eq}}(p/T) - f_p\right]$$

$\tau_{\mathrm{eq}} = \frac{5\bar{\eta}}{T}$ (independent of momenta)

Landau matching;

$$\partial_\mu n^\mu = 0 \qquad \mathcal{C}_{\mathrm{RTA}} = \frac{1}{\tau_{\mathrm{eq}}}[n - n_{\mathrm{eq}}] = 0$$

$$\partial_\mu T^{\mu\nu} = 0 \qquad \mathcal{C}^{\nu}_{\mathrm{RTA}} = \frac{1}{\tau_{\mathrm{eq}}}[\varepsilon - \varepsilon_{\mathrm{eq}}] = 0$$

$$\boxed{T = R^{\frac{1}{4}}(\xi)\Lambda \qquad (\mu = 0)}$$

$$\boxed{\begin{aligned} &T = \mathcal{R}(\xi)\sqrt{1+\xi}\,\Lambda \\ &\Gamma = \frac{\gamma}{(1+\xi)^2\mathcal{R}^3(\xi)} \qquad (\mu \neq 0) \end{aligned}}$$

$$\partial_\lambda I^{\lambda\mu\nu} = \mathcal{C}^{\mu\nu}_{\mathrm{RTA}}$$

$$\boxed{\mathcal{C}^{\mu\nu}_{\mathrm{RTA}} = \frac{1}{\tau_{\mathrm{eq}}}\int \frac{d^3p}{(2\pi)^3}\left[f_{\mathrm{eq}}(p/T) - f_p\right] p^\mu p^\nu}$$

Matching: Small Anisotropy Expansion

$$\partial_\tau I_i + (\theta - 2\theta_i) = \mathcal{C}^{ii}$$

$$\partial_\tau \xi - \frac{2(1+\xi)}{\tau} = \frac{\Lambda}{\kappa_a}\left[(1+\xi)^{3/2}\bar{\mathcal{C}}^{xx}(\xi) - (1+\xi)^{5/2}\bar{\mathcal{C}}^{zz}(\xi)\right]$$

$$\lim_{\xi\to 0} \bar{\mathcal{C}}^{zz}_{\mathrm{sc}} = \alpha\lambda^2\xi + \mathcal{O}(\xi^2)$$

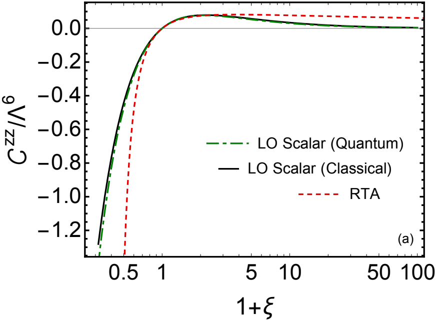
$$\alpha = 0.4394 \pm 0.0002$$

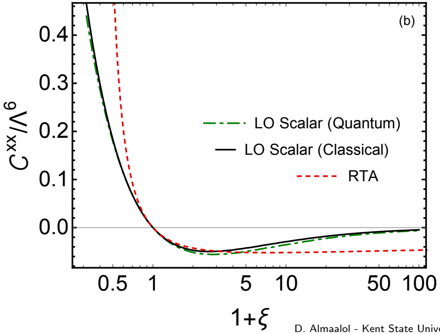
$$\lim_{\xi\to 0} \bar{\mathcal{C}}^{zz}_{\mathrm{RTA}} = \frac{8\gamma}{15\pi^2\bar{\eta}}\xi + \mathcal{O}(\xi^2)$$

$$\boxed{\lambda^2 = \frac{8}{15\pi^2\alpha\gamma\bar{\eta}}}$$

$T = R^{\frac{1}{4}(\xi)}\Lambda;\qquad \tau_{\mathrm{eq}} = \frac{5\bar{\eta}}{T}$

Landau matching

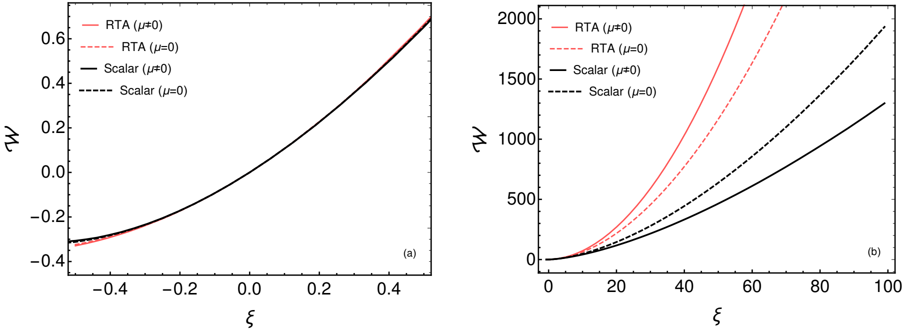




Small / Large Values of ξ

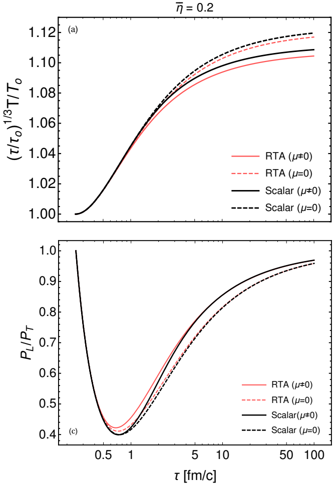
$$\partial_\tau \xi - \frac{2(1+\xi)}{\tau} = \frac{\Lambda}{\kappa_a}\left[(1+\xi)^{3/2}\bar{\mathcal{C}}^{xx}(\xi) - (1+\xi)^{5/2}\bar{\mathcal{C}}^{zz}(\xi)\right]$$

$$\partial_\tau \xi - \frac{2(1+\xi)}{\tau} + \frac{\mathcal{W}(\xi)}{\tau_{\rm eq}} = 0 \quad ; \mathcal{W}(\xi) \equiv \begin{cases} \xi(1+\xi)^2\,\mathcal{R}^2(\xi) & \text{(RTA)} \\ \frac{2}{3\alpha\mathcal{R}(\xi)}\left[(1+\xi)^2\tilde{\mathcal{C}}^{zz}_{\rm sc}(\xi) - (1+\xi)\tilde{\mathcal{C}}^{xx}_{\rm sc}(\xi)\right] & \text{(LO Scalar)} \end{cases}$$

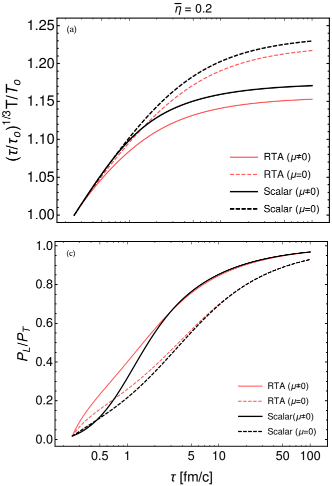


Isotropic vs Highly Oblate Initial Conditions

Initial vlaues: $T_0 = 500\text{MeV}$ $\tau_0 = 0.25$ fm/c



Isotropic $\Rightarrow \mathcal{P}_L(\tau_0)/\mathcal{P}_T(\tau_0) = 1$



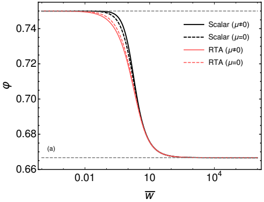
anisotropic $\Rightarrow \mathcal{P}_L(\tau_0)/\mathcal{P}_T(\tau_0) = 10^{-8}$

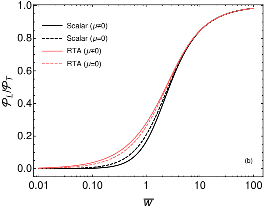
Anisotropic Attractor

$$w \equiv \tau T(\tau)\,, \qquad \bar{\pi} \equiv \frac{\pi}{\varepsilon} = 4\left(\varphi - \frac{2}{3}\right)$$

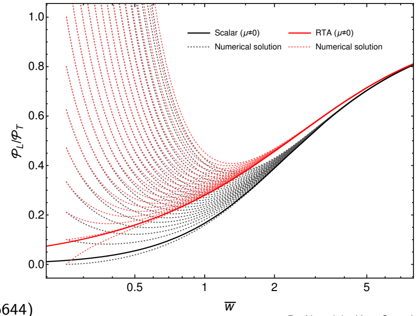
$$\varphi \equiv \tau \frac{\dot{w}}{w}$$

$$\overline{w}\varphi \frac{\partial \varphi}{\partial \overline{w}} = \left[\frac{1}{2}(1+\xi) - \frac{\overline{w}}{4}\mathcal{W}(\xi)\right]\bar{\pi}'$$





- higher degree of momentum anisotropy
- slower attractor behaviour



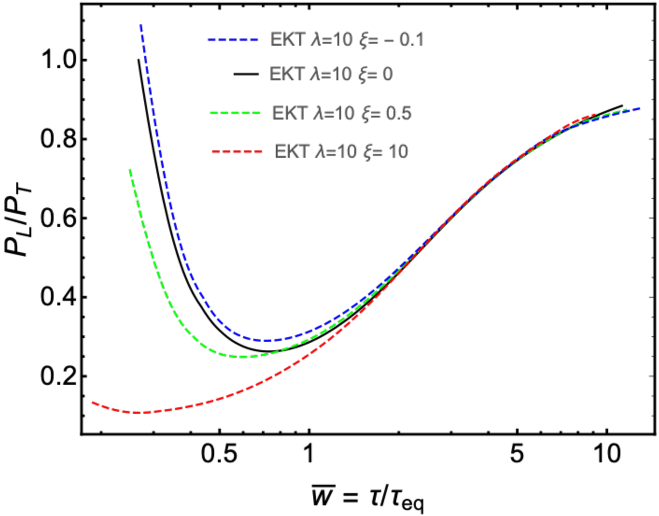
(M. Strickland , J. Noronha , G. Denicol arXiv:1709.06644)

QCD attractor "preliminary"

- Effective kinetic theory for QCD (AMY)

$$p^\mu \partial_\mu f_p = C_{22}[f_p] + C_{12}[f_p]$$

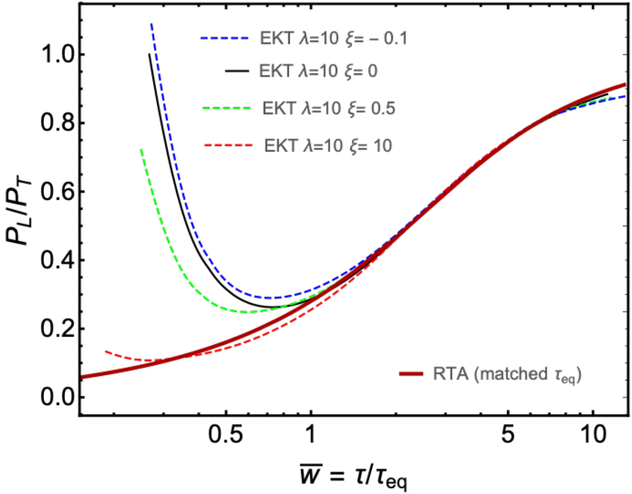
- numerical method (kurkela: 1506.06647)



$(\lambda = 10$
$\mathrm{T}_0 = 500\,\mathrm{MeV}$
$\bar{\eta} = 0.624$
$\tau_0 = 0.25\,\mathrm{fm/c})$

QCD attractor: Comparison to RTA

- attractor behavior at very early times.



Conclusions and Outlook

- 1st study of the realistic scalar collisional kernel in aHydro
- Development of larger momentum anisotropy
- Determined the non-equilibrium attractor for the LO scalar kernel
- Preliminary: implement realistic QCD scattering kernel in Ahydro equations
  (D. Almaalol, A. Kurkela, M. Strickland: **In progress**)

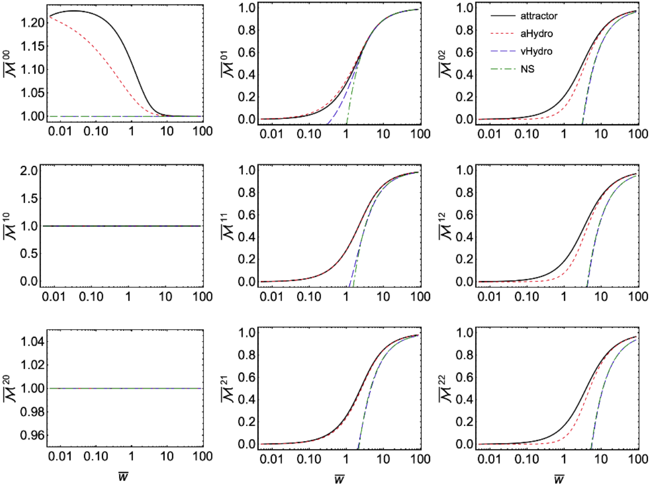
Thank you

Back up: Exact solution for the Attractor

$$\frac{\partial f(\tau, w, p_T)}{\partial \tau} = \frac{f_{\rm eq}(\tau, w, p_T) - f(\tau, w, p_T)}{\tau_{\rm eq}(\tau)}\,,$$

$$\mathcal{M}^{nm}[f] \equiv \int dP\, (p\cdot u)^n\, (p\cdot z)^{2m}\, f(\tau, w, p_T)\,.$$

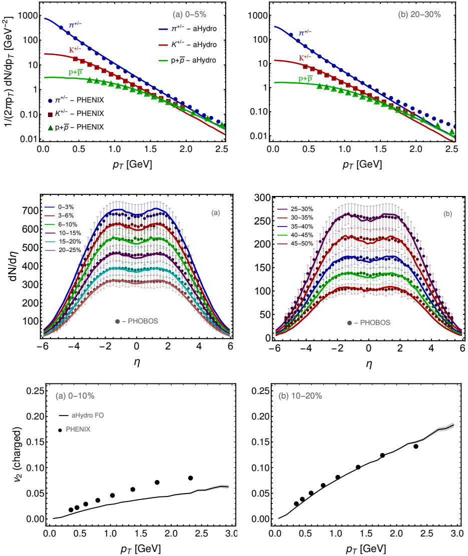
$$f(\tau, w, p_T) = D(\tau, \tau_0) f_0(w, p_T) + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\rm eq}(\tau')} D(\tau, \tau') f_{\rm eq}(\tau', w, p_T)\,.$$



(Michael Strickland, Ubaid Tantary arXiv:1903.03145)

Motivation2: Choice of $C[f]$ - previous work

- Prior aHydro studies $\Rightarrow$ RTA (Anderson-Witting)



QP-aHydro

(RHIC 200 GeV Au-Au)

$T_0 = 455\,\mathrm{MeV}\quad \bar{\eta} = 0.179$

$\tau_0 = 0.25$

(Dekrayat Almaalol, Mubarak Alqahtani, and Michael Strickland Phys. Rev. C 99, 044902 - Apr 2019)