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Hydrodynamics far from equilibrium: a concrete example

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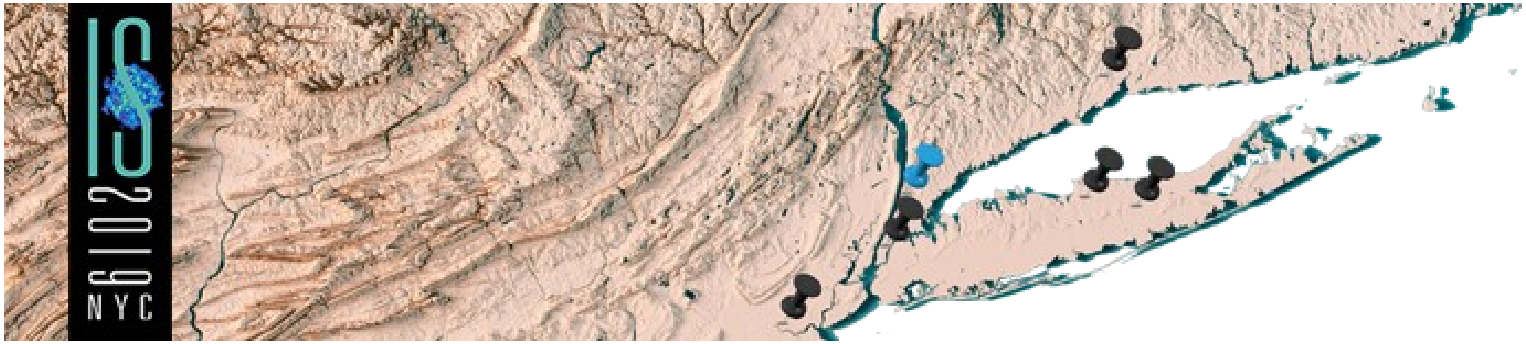
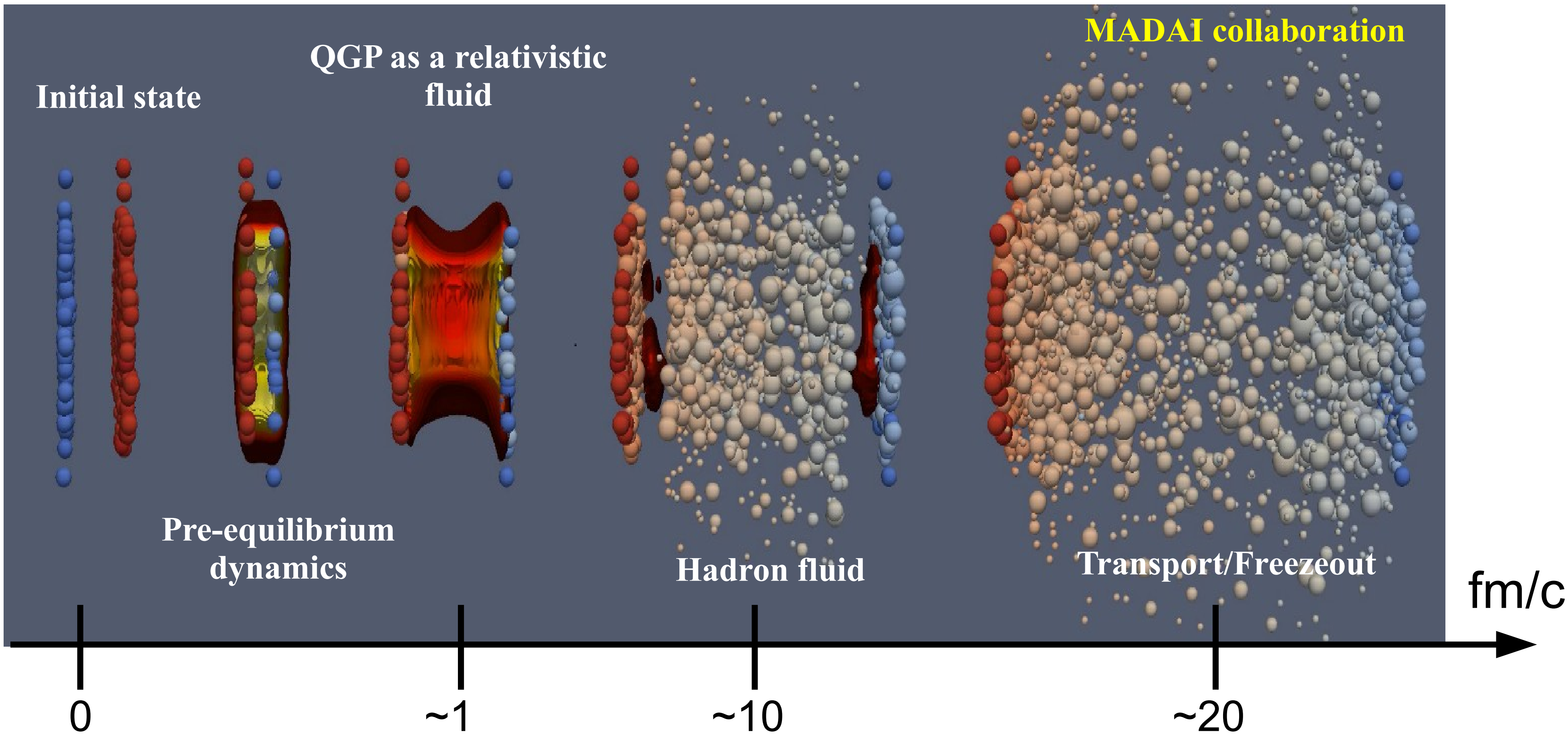


What you will see:

- ✔ Motivation: why fluid-dynamical descriptions work?
- ✔ Derivation of fluid dynamics using method of moments
- ✔ Can we have hydrodynamic behavior far from equilibrium?

Empirical: fluid-dynamical models of heavy ion collisions work well at RHIC and LHC energies



Main assumption: fluid dynamics is applied on *very small* time scales ~1 fm

Does this make sense?

Validity of fluid dynamics *traditionally* associated with:

- *proximity to (local) equilibrium*
- *"small" gradients*

Separation of scales $\longrightarrow$ **<u>macroscopic:</u>** L **<u>microscopic:</u>** ℓ

Knudsen number: $K_N \sim \dfrac{\ell}{L} \ll 1$

Do these things occur early in Heavy Ion Collisions?

No reason to believe that they do.

Then why does hydro work? What assumptions are really required?

We can study this problem in Kinetic theory

$$k^\mu \partial_\mu f_{\mathbf{k}} = C[f]$$

Boltzmann eq.

?????

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta + \quad ...$$
$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \quad ...$$

2nd- order hydro

In particular, we can use Israel-Stewart´s approach

Israel-Stewart theory: basic ideas

Israel-Stewart theory: *14-moment approximation*

$$\boxed{f_{\mathbf{k}} = f_{0\mathbf{k}} + f_{0\mathbf{k}}\left(1 - a f_{0\mathbf{k}}\right)\phi_{\mathbf{k}}}$$

equilibrium *non-equilibrium*

1 – ***Truncated*** Taylor series in momentum

$$\phi_{\mathbf{k}} = \varepsilon + \varepsilon_{\mu} k^{\mu} + \varepsilon_{\mu\nu} k^{\mu} k^{\nu}$$

- degrees of freedom *reduced* by the **explicit truncation** of expansion!
- *14* fields left

W. Israel & J M.Stewart, Ann. Phys. (N.Y.) 118, 341 (1979).

Israel-Stewart theory: *14-moment approximation*

$$\boxed{f_{\mathbf{k}} = f_{0\mathbf{k}} + f_{0\mathbf{k}}\left(1 - a f_{0\mathbf{k}}\right)\phi_{\mathbf{k}}}$$

equilibrium *non-equilibrium*

2 – Expansion coefficients mapped to conserved currents via ***matching conditions***

4 eqs.

$$\begin{aligned} u_\mu N^\mu &= n_0 \\ u_\mu T^{\mu\nu} &= \varepsilon_0 u^\nu \end{aligned}$$

10 eqs.

$$\begin{aligned} \pi^{\mu\nu} &= \Delta^{\mu\nu}_{\alpha\beta} T^{\alpha\beta} \\ n^\mu &= \Delta^\mu_\alpha N^\alpha \\ \Pi &= -\frac{1}{3}\Delta_{\mu\nu} T^{\mu\nu} \end{aligned}$$

W. Israel & J M.Stewart, Ann. Phys. (N.Y.) 118, 341 (1979).

Israel-Stewart theory: *14-moment approximation*

$$\boxed{f_{\mathbf{k}} \;=\; f_{0\mathbf{k}} + f_{0\mathbf{k}}\left(1 - a f_{0\mathbf{k}}\right)\phi_{\mathbf{k}}}$$

equilibrium *non-equilibrium*

3 – Equations of motion taken from the ***second moment*** of the Boltzmann equation

$$\Delta^{\lambda\rho}_{\mu\nu}\left(\partial_\alpha \int_K k^\alpha k^\mu k^\nu f_{\mathbf{k}} = \int_K C\left[f\right] k^\mu k^\nu\right) \longleftrightarrow \text{shear}$$

$$u_\nu \Delta^{\lambda}_{\mu}\left(\partial_\alpha \int_K k^\alpha k^\mu k^\nu f_{\mathbf{k}} = \int_K C\left[f\right] k^\mu k^\nu\right) \longleftrightarrow \text{diffusion}$$

$$u_\mu u_\nu\left(\partial_\alpha \int_K k^\alpha k^\mu k^\nu f_{\mathbf{k}} = \int_K C\left[f\right] k^\mu k^\nu\right) \longleftrightarrow \text{bulk}$$

W. Israel & J M.Stewart, Ann. Phys. (N.Y.) 118, 341 (1979).

Final Equations of motion

GSD et al, PRD 85, 114047 (2012)

$$\dot{\Pi} = -\frac{\Pi}{\tau_\Pi} - \beta_\Pi\theta - \ell_{\Pi n}\partial\cdot n - \tau_{\Pi n} n\cdot\dot{u} - \delta_{\Pi\Pi}\Pi\theta - \lambda_{\Pi n} n\cdot\nabla\alpha_0 + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} \,, \tag{20}$$

$$\dot{n}^{\langle\mu\rangle} = -\frac{n^\mu}{\tau_n} + \beta_n\nabla^\mu\alpha_0 - n_\nu\omega^{\nu\mu} - \delta_{nn}n^\mu\theta - \ell_{n\Pi}\nabla^\mu\Pi + \ell_{n\pi}\Delta^{\mu\nu}\partial_\lambda\pi^\lambda_\nu + \tau_{n\Pi}\Pi\dot{u}^\mu - \tau_{n\pi}\pi^\mu_\nu\dot{u}^\nu - \lambda_{nn}n^\nu\sigma^\mu_\nu + \lambda_{n\Pi}\Pi\nabla^\mu\alpha_0 - \lambda_{n\pi}\pi^{\mu\nu}\nabla_\nu\alpha_0 \,, \tag{21}$$

$$\dot{\pi}^{\langle\mu\nu\rangle} = -\frac{\pi^{\mu\nu}}{\tau_\pi} + 2\beta_\pi\sigma^{\mu\nu} + 2\pi_\alpha^{\langle\mu}\omega^{\nu\rangle\alpha} - \tau_{\pi n}n^{\langle\mu}\dot{u}^{\nu\rangle} + \ell_{\pi n}\nabla^{\langle\mu}n^{\nu\rangle} - \delta_{\pi\pi}\pi^{\mu\nu}\theta - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi n}n^{\langle\mu}\nabla^{\nu\rangle}\alpha_0 + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} \,. \tag{22}$$

Many terms originally omitted by Israel and Stewart.

W. Israel & J M.Stewart, Ann. Phys. (N.Y.) 118, 341 (1979).

- Nontrivial assumption: application of matching conditions

- This step does *not* require proximity to eq.

- *All previous steps can be applied* ***assuming the form:*** $f_{0\mathbf{k}}\left(\lambda, u_\mu k^\mu / \Lambda\right)$

scalars

$$\boxed{f_{\mathbf{k}} = f_{0\mathbf{k}} + \delta f_{\mathbf{k}}}$$

isotropic, non-equilibrium

correction, anisotropic

Matching conditions

$$f_{\mathbf{k}} = f_{0\mathbf{k}}\left(\lambda, u_\mu k^\mu/\Lambda\right) + \delta f_{\mathbf{k}}$$

5 parameters – can be associated with velocity, energy density and particle density

$$\left\{ \begin{array}{rcl} \lambda & = & \lambda\left(n, \varepsilon\right) \\ \Lambda & = & \Lambda\left(n, \varepsilon\right) \end{array} \right.$$

14-moment approx.: shear term only

$$f_{\mathbf{k}} = f_{0\mathbf{k}} + \frac{1}{2I_{42}}\pi^{\mu\nu}k_\mu k_\nu$$

$$I_{42} = \frac{1}{15}\int \frac{d^3k}{(2\pi)^3 k^0}\left|\mathbf{k}\right|^4 f_{0\mathbf{k}}$$

Equations of motion: *ultrarelativistic* gas of *hard spheres*

We recover the usual equation for the shear stress:

$$\boxed{\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} - \frac{2I_{40}}{3I_{50}}\sigma\pi^{\lambda\langle\mu}\pi_\lambda^{\nu\rangle} = 2\frac{\eta}{\tau_\pi}\sigma^{\mu\nu} - 2\sigma_\lambda^{\langle\mu}\pi^{\nu\rangle\lambda} - \frac{4}{3}\pi^{\mu\nu}\theta}$$

Transport coefficients:

functional dependence on $f_{0\mathbf{k}}$

$$\frac{1}{\tau_\pi} = \left(1 + 4\frac{P_0 I_{40}}{n_0 I_{50}}\right)\frac{1}{3\ell_{\mathrm{mfp}}}$$

$$\eta = \frac{4I_{40}\varepsilon^2}{3n_0 I_{50} + 12 P_0 I_{40}}\ell_{\mathrm{mfp}}$$

Thermodynamic integrals: $I_{nq} = \frac{(-1)^q}{(2q+1)!!}\int dK E_{\mathbf{k}}^{n-2q}\left(\Delta^{\alpha\beta}k_\alpha k_\beta\right)^q f_{0\mathbf{k}}$

Equations of motion: *ultrarelativistic* gas of *hard spheres*

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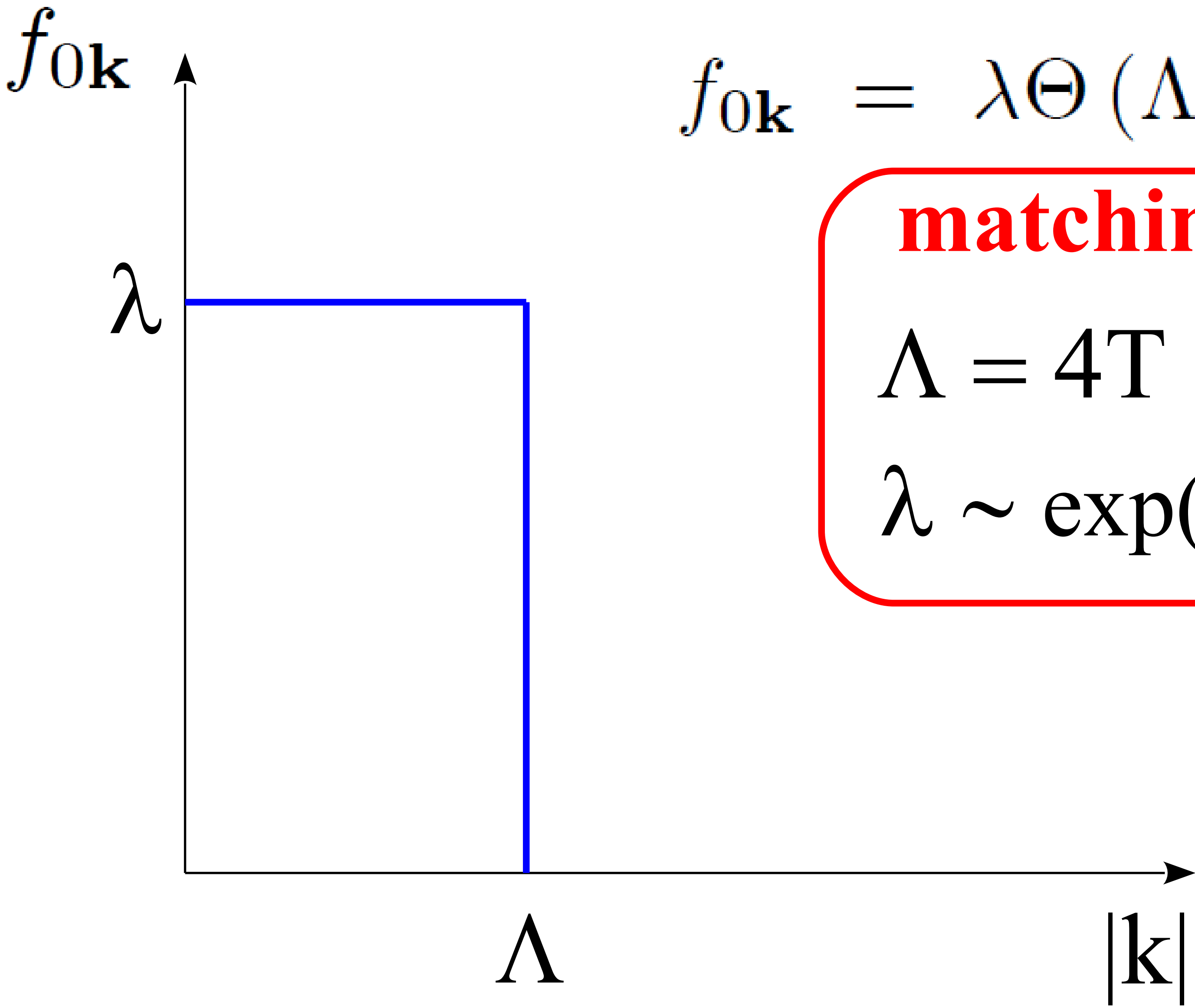
"Equilibrium" Transport coefficients:

$$\begin{aligned} \tau_\pi &= \frac{9}{5}\ell_{\mathrm{mfp}} \\ \eta &= \frac{6}{5}\frac{T}{\sigma} \end{aligned}$$

Coefficients derived by Israel-Stewart

Example of non-equilibrium state: "over-occupied" state



$$f_{0\mathbf{k}} = \lambda\Theta\left(\Lambda - E_{\mathbf{k}}\right)$$

matching

$\Lambda = 4T$

$\lambda \sim \exp(\alpha)$

Equations of motion: *ultrarelativistic* gas of *hard spheres*

We recover the usual equation for the shear stress:

$$\boxed{\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} - \frac{2I_{40}}{3I_{50}}\sigma\pi^{\lambda\langle\mu}\pi^{\nu\rangle}_{\lambda} = 2\frac{\eta}{\tau_\pi}\sigma^{\mu\nu} - 2\sigma^{\langle\mu}_{\lambda}\pi^{\nu\rangle\lambda} - \frac{4}{3}\pi^{\mu\nu}\theta}$$

Over-occupied Transport coefficients:

$$\tau_\pi = \frac{18}{13}\ell_{\mathrm{mfp}}$$

$$\eta = \frac{84}{65}\frac{T}{\sigma_T}$$

- qualitatively the same
- appears to be slightly more viscous

Equations of motion: *ultrarelativistic* gas of *hard spheres*

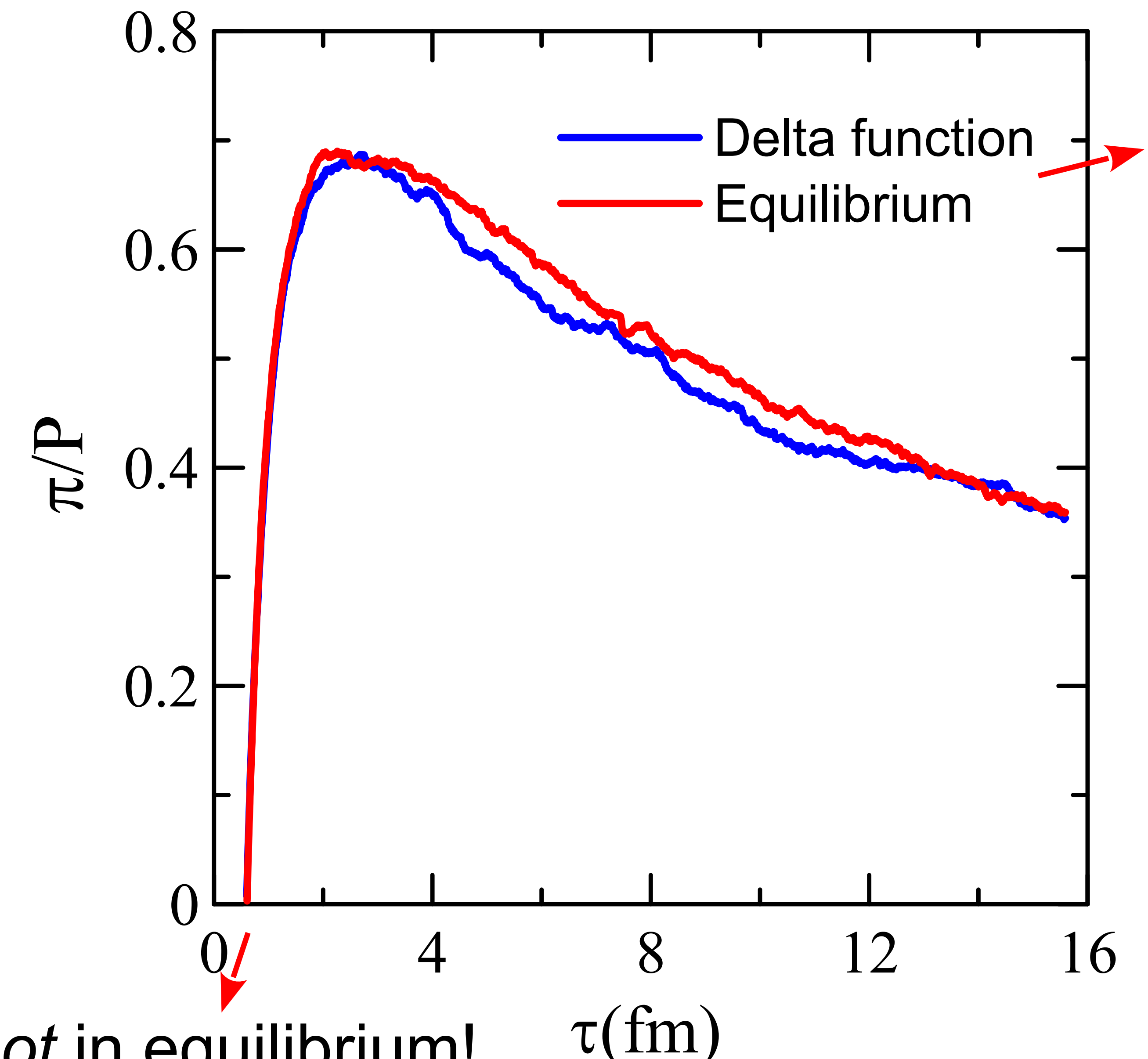
We recover the usual equation for the shear stress:

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} - \frac{2I_{40}}{3I_{50}}\sigma\pi^{\lambda\langle\mu}\pi^{\nu\rangle}_{\lambda} = 2\frac{\eta}{\tau_\pi}\sigma^{\mu\nu} - 2\sigma^{\langle\mu}_{\lambda}\pi^{\nu\rangle\lambda} - \frac{4}{3}\pi^{\mu\nu}\theta$$

$f_{0\mathbf{k}}$	$\lambda\exp\left(-E_\mathbf{k}/\Lambda\right)$	$\lambda\Theta\left(\Lambda - E_\mathbf{k}\right)$	$\lambda\delta\left(E_\mathbf{k} - \Lambda\right)$
τ_π	$\frac{9}{5}\ell_{\text{mfp}}$	$\frac{18}{13}\ell_{\text{mfp}}$	$\frac{9}{7}\ell_{\text{mfp}}$
η	$\frac{6}{5}\frac{T}{\sigma_T}$	$\frac{84}{65}\frac{T}{\sigma_T}$	$\frac{9}{7}\frac{T}{\sigma_T}$

Coefficients do not change much with f_{0k}. Can we see this?

Boltzmann eq. + Bjorken flow:
ultrarelativistic gas of *hard spheres*



Initial conditions, fixed energy

shear viscosity

$$\frac{\eta}{n} \approx 6$$

Evolution of shear stress does not see this non-equilibrium effect

<u>*not*</u> in equilibrium!

Conclusions

- The applicability of fluid-dynamical models of heavy ion collisions cannot be easily justified

- The derivation of hydrodynamics using the method of moments is more general than previously considered: **hydrodynamic equations can be obtained even far from equilibrium.**