Centrality dependence of collectivity in kinetic theory

26.06.2019

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Our Long Range Plan (as a community)

European Strategy Update

NSAC Long Range Plan 2015

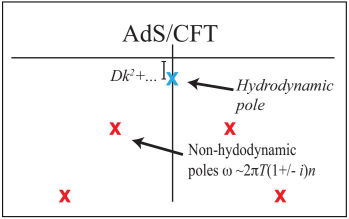
HL-LHC WG5 report, arXiv:1812.06772

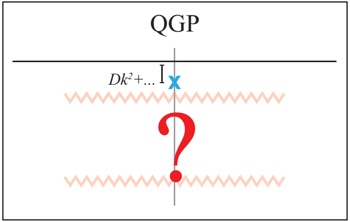
Our stated goal:

Probe the inner workings of QGP

All QFTs are more than hydrodynamics

Non-fluid modes are part of the inner workings of the QGPs of all QFTs.





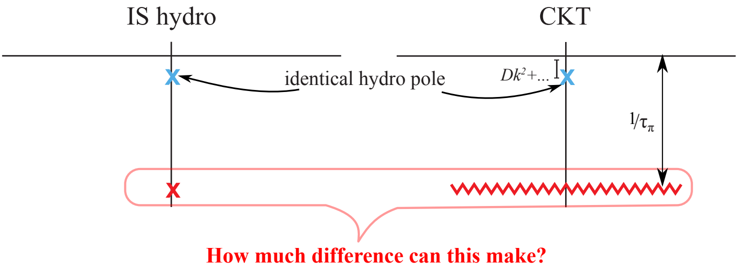
To understand the QGP beyond hydrodynamics, we need models that go beyond hydrodynamics.

$\Longrightarrow$ plenary by Bin Wu, Thursday 27

A kinetic theory exactly as hydro as Israel-Stewart

Israel-Stewart combines hydro poles with an ad hoc non-hydro mode

$$\tau_\pi \left(D\Pi^{\mu\nu} + \frac{4}{3}\Pi^{\mu\nu}\nabla_\alpha u^\alpha \right) = -\left(\Pi^{\mu\nu} + 2\eta\sigma^{\mu\nu}\right)$$

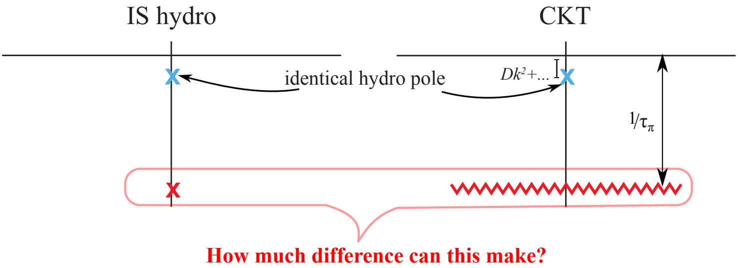


Are experiments sensitive not only to the presence, but also to the nature of excitations that relax on time scale τ_π?

A kinetic theory exactly as hydro as Israel-Stewart

Israel-Stewart combines hydro poles with an ad hoc non-hydro mode

$$\tau_\pi \left(D\Pi^{\mu\nu} + \frac{4}{3}\Pi^{\mu\nu}\nabla_\alpha u^\alpha \right) = -\left(\Pi^{\mu\nu} + 2\eta\sigma^{\mu\nu}\right)$$



We construct a conformal kinetic theory with identical hydro poles and different non-hydro excitations of same relaxation time τ_π.

Boost-invariant conformal kinetic transport (CKT)

Isotropization time approximation $\tau_{\mathrm{iso}} = \frac{1}{\gamma\, \epsilon^{1/4}}$

($v_\mu = \frac{p_\mu}{p}$, $p = |\vec{p}|$, invariance under boosts with $u_z = \frac{z}{t}$) (Kurkela, Wiedemann & Wu, arXiv:1905.05139)

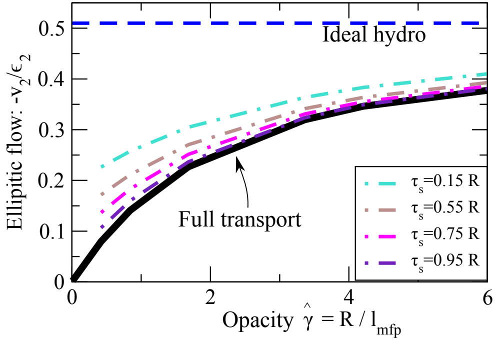
$$\boxed{\frac{1}{p} p^\mu \partial_\mu f = -C[f] = -\frac{[-v_\mu u^\mu]}{\tau_{\mathrm{iso}}}\, \left(f - f_{\mathrm{iso}}(p^\mu u_\mu)\right)}$$

- Equations of motion close for $T^{\mu\nu}(t, r, \theta)$
- Physics depends on only one dimensionless parameter: opacity $\hat{\gamma} \equiv \gamma R^{3/4}\, (\varepsilon_0\, \tau_0)^{\frac{1}{4}}$
- Hydrodynamic properties known, e.g., $\frac{\eta}{s} = \frac{1}{5\gamma} \frac{T}{\varepsilon^{1/4}}\Big|_{\mathrm{QCD}} = \frac{0.11}{\gamma}$
- Hydro sector of CKT matches that of (conformal) IS theory.
- CKT interpolates between free-streaming particles ($\hat{\gamma} \to 0$) and ideal hydrodynamics ($\hat{\gamma} \to \infty$).

v_2 sensitive to nature of non-hydro modes

- Keep hydrodynamic excitations fixed throughout evolution.
- Switch non-hydro structure from CKT to IS at time τ_s.

(Kurkela, Wiedemann, Wu, arXiv:1805.04081) **CKT vs IS Hydro**



Non-hydro modes matter more for small opacity. Talk by Bin Wu.

How “fluid” is this kinetic theory?

Fluid dynamics is a gradient expansion, determined by **transport coefficients** (that are known as function of γ). Up to 2nd order:

$$
\begin{aligned}
T^{\mu\nu}_{\rm hyd} &= (\varepsilon + p)\, u^\mu\, u^\nu + p\, g^{\mu\nu} + \Pi^{\mu\nu}_{\rm hyd} \\
\Pi^{\mu\nu} &= -2\eta_s \sigma^{\mu\nu} + 2\tau_\Pi\, \eta_s \left[{}^{<}D\sigma^{\mu\nu>} + \frac{4}{3}\sigma^{\mu\nu}\nabla_\alpha u^\alpha \right] + 4\lambda_1 \sigma^{<\mu}_{\alpha}\, \sigma^{\nu>\lambda} \\
\sigma^{\mu\nu} &= \left\{ \frac{1}{2}\left[\Delta^{\mu\alpha}\nabla_\alpha u^\nu + \Delta^{\nu\alpha}\nabla_\alpha u^\mu\right] - \frac{1}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha \right\}.
\end{aligned}
$$

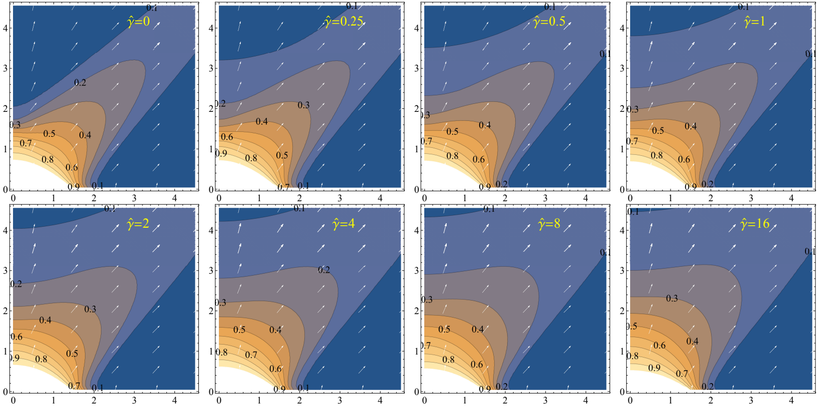
“Fluid quality” quantifies how “fluid” the kinetic dynamics is:

$$
\boxed{Q(t,r) = \frac{\sqrt{\left(T_{\rm kin} - T_{\rm hyd}\right)^{\mu\nu}\left(T_{\rm kin} - T_{\rm hyd}\right)_{\mu\nu}}}{\left(u_\mu\, T^{\mu\nu}_{\rm kin} u_\nu\right)}}
$$

Calculate $T^{\mu\nu}_{\mathrm{kin}}$ as function of opacity $\left(\hat{\gamma} \equiv \gamma R^{3/4}\,(\varepsilon_0\,\tau_0)^{\frac{1}{4}}\right)$

Initialize $F(\tau_0, r) = \varepsilon_0 \delta(v_z) P_{\mathrm{Woods-Saxon}}\left(\frac{r}{R}\right)\left(1 + \cancel{\sum_m \delta_m \#}\right)$
Plot $\varepsilon(r/R, t/R)$

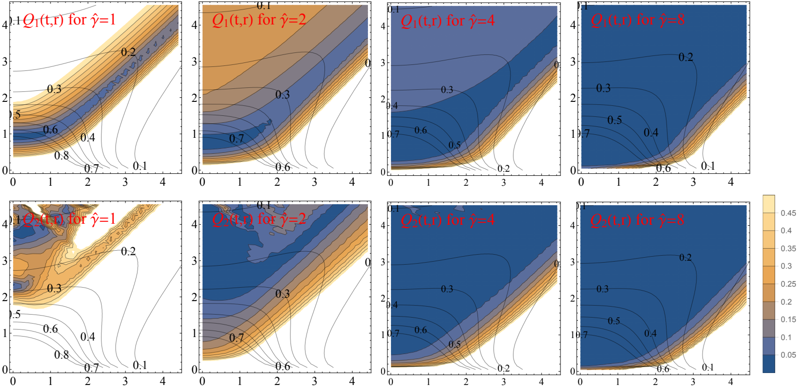
(Kurkela, Wiedemann, Wu, arXiv:1905.05139)



How “fluid” is this kinetic theory?

particle-like: $\hat{\gamma} \lesssim 2$ | transition: $2 \lesssim \hat{\gamma} \lesssim 4$ | hydro-like: $4 \lesssim \hat{\gamma}$

(Kurkela, Wiedemann & Wu, arXiv:1905.05139)



Upper (lower) panels: Q_1 (Q_2) measured up to 1st (2nd) order.

How "fluid" is PbPb @ LHC?

1. Work done in kinetic theory

$$f_{\text{work}}(\hat{\gamma})\frac{dE_{\perp,\text{free}}}{d\eta_s} = \frac{dE_\perp(t\to\infty)}{d\eta_s}$$

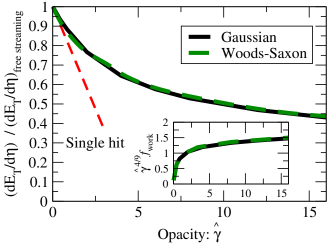
2. Initial energy density

$$\varepsilon_0 = \frac{\frac{dE_{\perp,\text{free}}}{d\eta_s}}{\tau_0\,\pi\;R^2} = \frac{\frac{dE_\perp}{d\eta_s}}{f_{\text{work}}(\hat{\gamma})\tau_0\,\pi\;R^2}$$

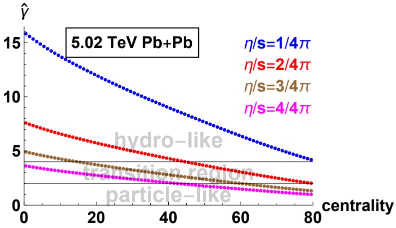
$$\Longrightarrow \qquad \hat{\gamma} \equiv \gamma R^{\frac{3}{4}}\;(\varepsilon_0\,\tau_0)^{\frac{1}{4}}$$

$$\boxed{= \frac{0.11}{\frac{\eta}{s}}\left(\frac{R\,\frac{dE_\perp}{d\eta}}{\pi f_{\text{work}}(\hat{\gamma})}\right)^{1/4}}$$

from data!



(Kurkela, Wiedemann & Wu, arXiv:1905.05139)



$$\frac{\eta}{s} = \frac{1}{5}\tau_R T = \frac{1}{5\gamma}\frac{T}{\varepsilon^{1/4}}\Big|_{\text{QCD}} = \frac{0.11}{\gamma}$$

Input for data comparison (1)

- In non-viscous fluid dynamics,

$$\frac{\partial v_n}{\partial \epsilon_n}\bigg|_{\epsilon_n=0}\left(c_s^2\right) = \mathbf{c}_{\text{eos}}\,\frac{\partial v_n}{\partial \epsilon_n}\bigg|_{\epsilon_n=0}\left(c_s^2=\frac{1}{3}\right)$$

$\mathbf{c}_{\text{eos}}$ applied to kinetic theory.

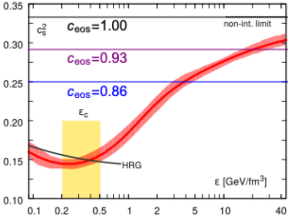
- Nonlinear eccentricity dependence

$$v_n(\epsilon_n) \propto \epsilon_n\left(1+\mathbf{c}_{\text{nl}}\,\epsilon_n^2\right)$$

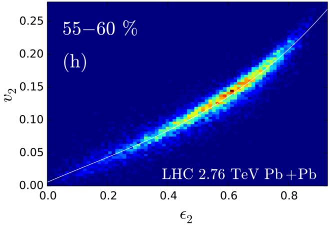
fit to hydro simulations, $\mathbf{c}_{\text{nl}} = \mathbf{0.75}$ applied to kinetic theory.

Both corrections exact for $\hat{\gamma} \to \infty$.

(HotQCD, PRD 90 (2014) 094503)



(Niemi, Eskola, Paatelainen, arXiv:1505.02677)



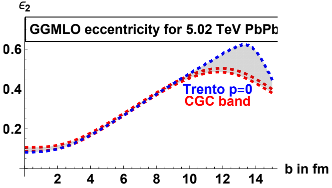
Input for data comparison (2)

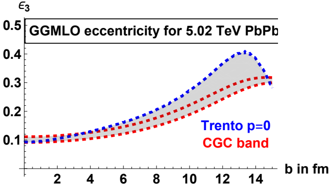
- From data on particle flow $v_n(p_\perp)$ and spectra $\frac{dN}{dp_\perp d\eta}$

$$v_n^{energy} = \frac{\int dp_\perp v_n(p_\perp) p_\perp \frac{dN}{dp_\perp d\eta}}{\int dp_\perp p_\perp \frac{dN}{dp_\perp d\eta}} = \mathbf{c}_{\mathrm{energy}} v_n\{2, \Delta\eta = 1\}$$

  We determine $\mathbf{c}_{\mathrm{energy}}$ from data. ($c_{\mathrm{energy}} = 1.34$ for ALICE 5 TeV PbPb)

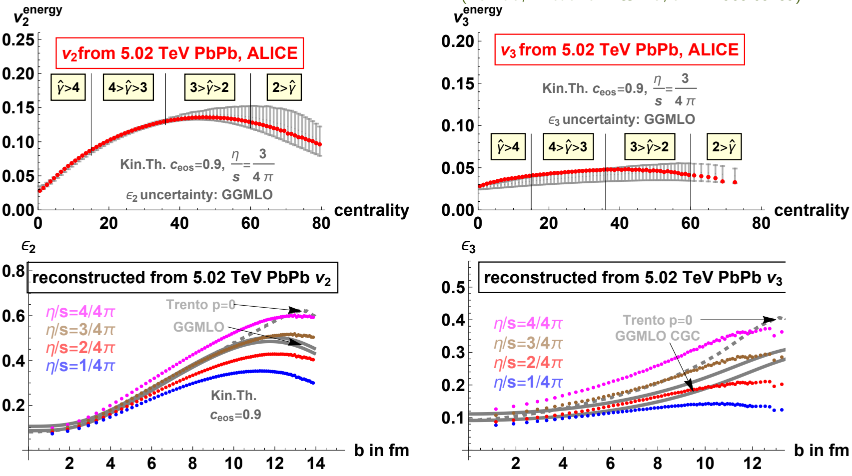
- Centrality dependence of initial geometry
  1. rms radius R from optical Glauber (negligible uncertainty ...)
  2. spatial eccentricities (either GGMLO (arXiv:1902.07168), or "reconstructed")





Compare $\frac{v_m}{\epsilon_m}(\hat{\gamma})$ to data / reconstruct ϵ_m from data

(Kurkela, Wiedemann & Wu, arXiv:1905.05139)



($c_{\rm eos}$: results corrected for non-ideal equation of state; GGMLO arXiv:1902.07168)

Reconstructing $\hat{\gamma}$ and ϵ_2 for pPb

(Kurkela, Wiedemann & Wu, arXiv:1905.05139)

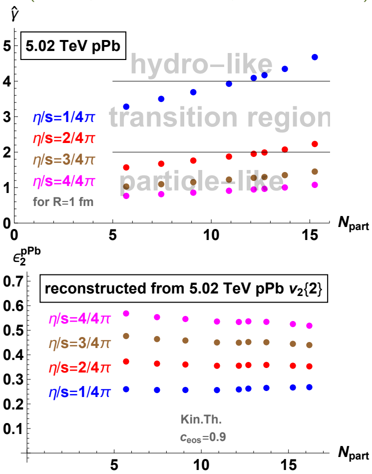
- Hydro-interpretation disfavored.

  $\hat{\gamma}$-values indicate dominance of non-hydro modes.

- Negligible N_{part}-dependence of ϵ_2 reconstructed from v_2.

  event activity $\neq$ centrality

  (Dispersion of ϵ_2 at fixed N_{part} could be comparable to width of N_{part}-integrated ϵ_2-distribution.)



$\hat{\gamma}$-scaling as test of conformality

- Scaling in **non-conformal kinetic theory** $(\sigma = \text{fixed})$ small system, $\tau_R = \frac{1}{n(\tau)\sigma}$, $n(\tau) = \frac{1}{\tau A_\perp}\frac{dN}{d\eta_s}$ (Heiselberg & Levy 9812034, Voloshin & Poskanzer 9906075)

$$\boxed{\frac{v_2}{\epsilon_2} \propto \frac{R}{\tau_R(\tau = R)} \sim \frac{\sigma}{A_\perp}\frac{dN}{d\eta_s}}$$

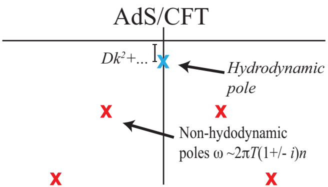
- **Conformal scaling variable** $\hat{\gamma} = \gamma \left(\frac{R \langle p_\perp \rangle \frac{dN}{d\eta_s}}{\pi f_{\text{work}}(\hat{\gamma})} \right)^{1/4}$

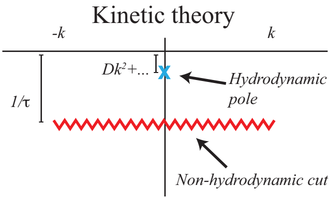
$$\boxed{\left.\frac{v_2}{\epsilon_2}\right|_{\hat{\gamma}<1} \propto \left(\langle p_\perp \rangle R \frac{dN}{d\eta_s} \right)^{1/4}} \qquad \boxed{\left.\hat{\gamma}\right|_{\hat{\gamma}\gg 1} \propto \left(\frac{dN}{d\eta_s} \right)^{1/3}}$$

Whether flow exhibits conformal scaling could inform us about the microscopic dynamics underlying collectivity.

Summary

- v_m's are sensitive to nature of non-hydro excitations.
- 1-parameter CKT captures system-size and $\sqrt{s}$-dependence
- Could a strongly coupled QFT be similarly successful? Definitively worth testing! But note parametric differences:
  - $\tau_{non-hyd} \propto \frac{1}{T}$ in AdS/CFT, $\tau_{\rm iso} \propto \frac{1}{\gamma \epsilon^{1/4}}$ in kinetic theory
  - and one has to explain why particles reach the detector





Back-up

Small systems as test of inner workings

Decreasing the transverse system size R

- increases the smallest wavenumber $k \propto 1/R$
- time $t \sim R$ of in-medium propagation decreases
- ε decreases $\Longrightarrow \tau_R = \frac{1}{\gamma \varepsilon^{1/4}}$ increases

$$G_R(t,k) = \underbrace{c_{\mathrm{hyd}} \exp\left[-D\,k^2\,t\right]}_{\text{reduced for smaller R}} + \underbrace{c_{\mathrm{non-hyd}} \exp\left[-t/\tau_R\right]}_{\text{enhanced for smaller R}}$$

Reducing system size is one tool to enhance and characterize non-hydrodynamic modes.

$\Longrightarrow$ plenary of **Bin Wu**

based on Kurkela, Wiedemann & Wu, arXiv:1905.05139

Compare $\frac{v_m}{\epsilon_m}(\hat{\gamma})$ to data