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A complete set of splitting functions in nuclear matter to any order in opacity



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Outline



A lot of credit for this work goes to my collaborators Matt Sievert and Boram Yoon

M. Sievert et al. (2018)

M. Sievert et al. (2019)

- Introduction and motivation
- Calculation of gluon emission off quarks beyond the soft approximation and to any order in opacity
- Calculation of all in-medium splitting functions / numerical evaluation
- Conclusions

Introduction & Motivation

- In heavy ion collisions medium-induced energy loss and more generally in-medium parton showers are the cornerstone of high-p_T physics
- The associated phenomena were dubbed jet quenching and established in a myriad of observables

M. Gyulassy et al . (1992)



Heavy flavor suppression, b jets,di-b jets



dijet asymmetries, y,Z°-tagged jets



Inclusive hadron suppression, hadron correlations



Jet substructure modification, shapes, frag. Functions, splitting functions

Full in medium splitting

 Full massless and massive in-medium splitting functions now available to first order in opacity

G. Ovanesyan et al . (2011)

 SCET-based effective theories created to solve this problem
 F. Ringer et al. (2016)

Representative example

$$\begin{split} & \left(\frac{dN^{\text{med}}}{dxd^{2}k_{\perp}}\right)_{Q \to Qg} = \frac{\alpha_{s}}{2\pi^{2}}C_{F}\int \frac{d\Delta z}{\lambda_{g}(z)}\int d^{2}q_{\perp}\frac{1}{\sigma_{el}}\frac{d\sigma_{el}^{\text{med}}}{d^{2}q_{\perp}} \left\{ \left(\frac{1+(1-x)^{2}}{x}\right) \left[\frac{B_{\perp}}{B_{\perp}^{2}+\nu^{2}} \times \left(\frac{B_{\perp}}{B_{\perp}^{2}+\nu^{2}} - \frac{C_{\perp}}{C_{\perp}^{2}+\nu^{2}}\right) \left(1 - \cos[(\Omega_{1} - \Omega_{2})\Delta z]\right) + \frac{C_{\perp}}{C_{\perp}^{2}+\nu^{2}} \cdot \left(2\frac{C_{\perp}}{C_{\perp}^{2}+\nu^{2}} - \frac{A_{\perp}}{A_{\perp}^{2}+\nu^{2}} - \frac{B_{\perp}}{B_{\perp}^{2}+\nu^{2}}\right) \left(1 - \cos[(\Omega_{1} - \Omega_{3})\Delta z]\right) + \frac{B_{\perp}}{B_{\perp}^{2}+\nu^{2}} \cdot \frac{C_{\perp}}{C_{\perp}^{2}+\nu^{2}} \left(1 - \cos[(\Omega_{2} - \Omega_{3})\Delta z]\right) \\ &+ \frac{A_{\perp}}{A_{\perp}^{2}+\nu^{2}} \cdot \left(\frac{D_{\perp}}{D_{\perp}^{2}+\nu^{2}} - \frac{A_{\perp}}{A_{\perp}^{2}+\nu^{2}}\right) \left(1 - \cos[\Omega_{4}\Delta z]\right) - \frac{A_{\perp}}{A_{\perp}^{2}+\nu^{2}} \cdot \frac{D_{\perp}}{D_{\perp}^{2}+\nu^{2}} \left(1 - \cos[\Omega_{5}\Delta z]\right) \\ &+ \frac{1}{N_{c}^{2}}\frac{B_{\perp}}{B_{\perp}^{2}+\nu^{2}} \cdot \left(\frac{A_{\perp}}{A_{\perp}^{2}+\nu^{2}} - \frac{B_{\perp}}{B_{\perp}^{2}+\nu^{2}}\right) \left(1 - \cos[(\Omega_{1} - \Omega_{2})\Delta z]\right) \right] \\ &+ x^{3}m^{2} \left[\frac{1}{B_{\perp}^{2}+\nu^{2}} \cdot \left(\frac{1}{B_{\perp}^{2}+\nu^{2}} - \frac{1}{C_{\perp}^{2}+\nu^{2}}\right) \left(1 - \cos[(\Omega_{1} - \Omega_{2})\Delta z]\right) + \dots\right] \right\} \end{split}$$

 For the first time we were able to do is higher order and resummed calculations





Higher order opacity corrections

- The very thick medium approaches they simplify the problem to resum the interactions
 R. Baier et al . (1996)
- Finite opacity approaches builds expansion in the correlation between the scattering centers
 M. Gyulassy et al. (2000)

Clearly the next logical steps are - Derive all in-medium splittings to any order in opacity with no approximations other than eikonal

- Perform numerical evaluation to find the first correction to jet and heavy flavor observables

It was also done to higher orders in opacity (~9) in the soft gluon emission limit



M. Gyulassy et al . (2000)

S. Wicks (2009)

Theoretical framework

- The theoretical framework is completely general – it is applicable for both cold nuclear matter and the QGP.
- This is achieved by isolating the medium in transport parameters and universal gluon-mediated interactions



- The limit we are interested in
- We neglect collisional energy losses

$$\mathcal{L}_{opac.} = \mathcal{L}_{QCD} + \mathcal{L}_{ext}^{qG} + \mathcal{L}_{ext}^{gG} + \mathcal{L}_{G.F.} + \cdots$$
$$v(q_T^2) \rightarrow \frac{-g_{eff}^2}{q_T^2 + \mu^2} \qquad \frac{d\sigma^{el}}{d^2q} = \frac{1}{(2\pi)^2} \frac{C_F}{2N_c} [v(q_T^2)]^2$$

In deep inelastic scattering (DIS) the lowest order processes involve prompt quark. Even at NLO the prompt gluon jet contribution is small

F. Ringer et al . (2018)

$$\frac{1}{p_N^-} \ll l_f^+ \sim \lambda^+ \sim L^+$$
$$\mathcal{O}\left(\frac{\perp^2}{Q^2}\right)$$

Lightcone wave functions and parton branchings

$$xp^{+} \left. \frac{dN}{d^{2}k \, dx \, d^{2}p \, dp^{+}} \right|_{\mathcal{O}(\chi^{0})} = \frac{\alpha_{s} \, C_{F}}{2\pi^{2}} \, \frac{(k-xp)_{T}^{2} \left[1+(1-x)^{2}\right]+x^{4}m^{2}}{[(k-xp)_{T}^{2}+x^{2}m^{2}]^{2}} \, \times \left(p^{+} \frac{dN_{0}}{d^{2}p \, dp^{+}}\right)$$

- Certain advantages can provide in "one shot" both massive and massless splitting functions
- Have checked that results agree for massless and massive DGLAP

Opacity expansion building blocks – direct terms



 Interaction in the amplitude and the conjugate amplitude (Direct or single Born diagrams)

Representative forward cut diagram

$$D_{4} = \left[\frac{-1}{2N_{c}C_{F}}e^{+i[\Delta E^{-}(\underline{k}-x\underline{p})-\Delta E^{-}(\underline{k}-x\underline{p}+x\underline{q})]z^{+}}\right]\psi(x,\underline{k}-x\underline{p})\left[0-e^{-i\Delta E^{-}(\underline{k}-x\underline{p})z^{+}}\right]$$
$$\times \left[e^{+i\Delta E^{-}(\underline{k}-x\underline{p}+x\underline{q})z^{+}}-e^{+i\Delta E^{-}(\underline{k}-x\underline{p}+x\underline{q})x_{0}^{+}}\right]\psi^{*}(x,\underline{k}-x\underline{p}+x\underline{q}),$$

 Vitruallity changes enter the interference phases and are related to the propagators

Propagators hide in the wavefunctions

 $p^-\!-\!k^-\!-\!(p\!-\!k)^-=\Delta E^-(\underline{k}\!-\!x\underline{p})$

Opacity expansion building blocks –virtual terms



A more interesting diagram- Double born can contribute to virtuality changes

$$V_8 = \left[\frac{N_c}{2C_F} e^{i[\Delta E^-(\underline{k}-x\underline{p})-\Delta E^-(\underline{k}-x\underline{p}-\underline{q})]z^+}\right] \psi(x,\underline{k}-x\underline{p}) \left[0-e^{-i\Delta E^-(\underline{k}-x\underline{p})x_0^+}\right] \\ \times \left[e^{+i\Delta E^-(\underline{k}-x\underline{p}-\underline{q})z^+}-e^{+i\Delta E^-(\underline{k}-x\underline{p}-\underline{q})x_0^+}\right] \psi^*(x,\underline{k}-x\underline{p}-\underline{q}).$$

Parton branching to any order in opacity

Treating color (one complication in QCD).



- Color is not entangled, homogeneous structure and multiplicative factors that can be algebraically treated
- Finally, relative to the splitting vertex we classify the as
- Initial/Initial, Initial/Final, Final/Initial and Final/Final

M. Sievert et al . (2018) $C_3 =$



 $\mathcal{C}_1 = \frac{1}{N_c C_F} \operatorname{tr}[t^b t^b t^a M^a] = \mathcal{C}_0 \,,$







Master equation – matrix form

 Upper triangular structure. Suggests specific strategy how to solve it. Calculated: initial conditions, kernels, and wrote a Mathematica code to solve it

$$\begin{bmatrix} f_{F/F}^{(N)}(\underline{k}, \underline{k}', \underline{p}; x^+, y^+) \\ f_{I/F}^{(N)}(\underline{k}, \underline{p}; x^+, y^+) \\ f_{F/I}^{(N)}(\underline{p}; x^+, y^+) \end{bmatrix} = \int_{x_0^+}^{\min[x^+, y^+, R^+]} \frac{dz^+}{\lambda^+} \int \frac{d^2q}{\sigma_{el}} \frac{d\sigma^{el}}{d^2q} \begin{bmatrix} \mathcal{K}_1 \ \mathcal{K}_2 \ \mathcal{K}_3 \ \mathcal{K}_4 \\ 0 \ \mathcal{K}_5 \ 0 \ \mathcal{K}_6 \\ 0 \ 0 \ \mathcal{K}_7 \ \mathcal{K}_8 \\ 0 \ 0 \ 0 \ \mathcal{K}_9 \end{bmatrix} \begin{bmatrix} f_{F/F}^{(N-1)}(\underline{k}, \underline{k}', \underline{p}; x^+, y^+) \\ f_{I/F}^{(N-1)}(\underline{k}, \underline{p}; x^+, y^+) \\ f_{F/I}^{(N-1)}(\underline{k}, \underline{p}; x^+, y^+) \\ f_{I/I}^{(N-1)}(\underline{p}; x^+, y^+) \end{bmatrix}$$

Simplest kernel

$$\mathcal{K}_{9} = \left[e^{-\underline{q} \cdot \underline{\nabla}_{p}} e^{+(z^{+}-x^{+})\partial_{x^{+}}} e^{+(z^{+}-y^{+})\partial_{y^{+}}} \right] + \left[-\frac{1}{2} \right] \left[e^{+(z^{+}-x^{+})\partial_{x^{+}}} + e^{+(z^{+}-y^{+})\partial_{y^{+}}} \right]$$

Most complicated kernel

$$\begin{split} \mathcal{K}_{1} &= \left[e^{i[\Delta E^{-}(\underline{k}-x\underline{p}+x\underline{q})-\Delta E^{-}(\underline{k}-x\underline{p})]z^{+}} e^{i[\Delta E^{-}(\underline{k}'-x\underline{p})-\Delta E^{-}(\underline{k}'-x\underline{p}+x\underline{q})]z^{+}} \right] \left[e^{-\underline{q}\cdot\underline{\nabla}_{p}} e^{+(z^{+}-x^{+})\partial_{x^{+}}} e^{+(z^{+}-y^{+})\partial_{y^{+}}} \right] \\ &+ \left[\frac{N_{c}}{C_{F}} e^{i[\Delta E^{-}(\underline{k}-x\underline{p}-(1-x)\underline{q})-\Delta E^{-}(\underline{k}-x\underline{p})]z^{+}} e^{i[\Delta E^{-}(\underline{k}'-x\underline{p})-\Delta E^{-}(\underline{k}'-x\underline{p}-(1-x)\underline{q})]z^{+}} \right] \\ &\times \left[e^{-\underline{q}\cdot\underline{\nabla}_{k}} e^{-\underline{q}\cdot\underline{\nabla}_{k'}} e^{-\underline{q}\cdot\underline{\nabla}_{p}} e^{+(z^{+}-x^{+})\partial_{x^{+}}} e^{+(z^{+}-y^{+})\partial_{y^{+}}} \right] \\ &+ 8 \text{ more lines} \end{split}$$

Explicit solution to second order in opacity

 Present the first exact result to this order (including the ability to discuss broad or narrow sources)

$$\begin{split} xp^{+} \frac{dN}{d^{2}k \, dx \, d^{2}p \, dp^{+}} \Big|_{\mathcal{O}(\chi^{2})} &= \frac{C_{F}}{2(2\pi)^{3}(1-x)} \int_{x_{0}^{+}}^{R^{+}} \frac{dz_{2}^{+}}{\lambda^{+}} \int_{x_{0}^{+}}^{z_{2}^{+}} \frac{dz_{1}^{+}}{\lambda^{+}} \int \frac{d^{2}q_{1}}{\sigma_{el}} \frac{d\sigma^{el}}{d^{2}q_{1}} \frac{d\sigma^{el}}{d^{2}q_{2}} \times \left\{ \left(p^{+} \frac{dN_{0}}{d^{2}p \, dp^{+}} \right) \mathcal{N}_{1} \right. \\ &+ \left(p^{+} \frac{dN_{0}}{d^{2}(p-q_{1}) \, dp^{+}} \right) \mathcal{N}_{2} + \left(p^{+} \frac{dN_{0}}{d^{2}(p-q_{2}) \, dp^{+}} \right) \mathcal{N}_{3} + \left(p^{+} \frac{dN_{0}}{d^{2}(p-q_{1}-q_{2}) \, dp^{+}} \right) \mathcal{N}_{4} \right\} \\ \mathcal{N}_{1} = \end{split}$$

$$\begin{aligned} \left|\psi(\underline{k}-x\underline{p})\right|^{2} \left[\frac{(C_{F}+N_{c})^{2}}{C_{F}^{2}} - \frac{N_{c}(C_{F}+N_{c})}{C_{F}^{2}}\cos(\delta z_{1}\Delta E^{-}(\underline{k}-x\underline{p})) + \frac{N_{c}^{2}}{2C_{F}^{2}}\cos(\delta z_{2}\Delta E^{-}(\underline{k}-x\underline{p})) \\ - \frac{N_{c}(2C_{F}+N_{c})}{2C_{F}^{2}}\cos((\delta z_{1}+\delta z_{2})\Delta E^{-}(\underline{k}-x\underline{p}))\right] + 9 \text{ more pages} \end{aligned}$$

 For broad sources and in the soft gluon limit we have checked that the result reduces to the GLV second order in opacity

Generalizing the result to all in-medium splittings (4)

Note – all splittings have the same topology.
 Same - structure, interference phases,
 propagators

Different - mass dependence, wavefunctions, color (which also affects transport coefficients)



$$\langle \psi(x,\underline{\kappa})\,\psi^*(x,\underline{\kappa}')\rangle = \frac{8\pi\alpha_s\,f(x)}{[\kappa_T^2 + \nu^2 m^2]\,[\kappa_T'^2 + \nu^2 m^2]} \left[g(x)\,(\underline{\kappa}\cdot\underline{\kappa}') + \nu^4 m^2\right] \ \Delta E^-(\underline{\kappa}) = -\frac{\kappa_T^2 + \nu^2 m^2}{2x(1-x)p^+}$$

Master table that gives all ingredients

We have now solved the problem for all splitting functions

M. Sievert et al . (2019)

Improvements in physics & code



Refactoring

➤ Code is restructured (in C++) and shortened (24K → 8K lines). 20x speed improvement

Effective incorporation of simulated QGP medium

Reduced overhead for calling QGP medium grid function. 2x speed improvement

Efficient on-node parallelization

New parallelization shows much better scaling 10x speed improvement

Overall improvement: **18 days** → **1 hour**

Medium-induced splitting intensity

Porting to code

 Results are directly exported from Mathematica to C++

Challenges

Arise from larger number of evaluations



$$\mathcal{I}_{x_{\min}}^{x_{\max}} = \int_{x_{\min}}^{x_{\max}} dx \int d^2k \ x \frac{dN}{d^2k \ dx}$$

Energy loss – not a well defined concept for parton shower processes - define splitting intensity

 The main result is a change in the energy dependence of the splitting intensity – smother, or more slowly varying with E (understand jet modification with p_T)

Differential branching spectra



- Broder angular enhancement region
- Oscillating series the average of 1st and 1^{st+}2nd order- candidate for pheno.

- Reduction of small-x and large-x probabilities (assymptotics modulated by thermal mass)
- Enhancement of democratic branching (x~0.5)



Application of the new formalism & code to jets



- First application to inclusive heavy flavor jets using the technique of semiinclusive jet functions
- Application to jet fragmentation inclusive and photon tagged jets captures the qualitative differences needs improvement (higher orders)

Conclusions

- Consistent progress over the past 2 decades, from energy loss to in medium parton showers to first order in opacity.
- For the first time consistent NLO and resummed calculations in the QGP environment - same language, techniques as in HEP
- Calculated the quark splitting function to any order in opacity and presented explicit results to 2nd order. Cross checked the result against the known soft gluon emission limit
- We have now extended the derivation to all orders in opacity for all 4 splitting functions. Demonstrated numerical evaluation in realistic media
- Applications of new code/grids to jet physics problems, applications to second order in opacity in the near future

Motivation for numerical evaluation

Approaches to the theoretical problem



R. Baier et al . (1996)

The very thick medium approaches – they simplify the problem to resum the interactions

M. Gyulassy et al. (2000)

Coherent

Coherent

BDMPS Heuristic

IC Inner (G AC Inner IC Full

Finite opacity approaches – builds expansion in the correlation between the scattering centers

2.5

1.5

0.5

Rediation intensity

 $\omega dI/d\omega$

X. Feal et al. (2018)

Realistic QGP - q ~ 2.5 scatterings, g ~ 5. Also problems with BDMPS not recovering known assymptotic limits

Any order in opacity result

• Any order in opacity result

$$x \frac{dN^{(n)}}{dx d^{2}\mathbf{k}} = \frac{C_{R}\alpha_{s}}{\pi^{2}} \frac{1}{n!} \left(\frac{L}{\lambda_{g}(1)}\right)^{n} \int \prod_{i=1}^{n} \left\{ d\mathbf{q}_{i} \left(\frac{\lambda_{g}(1)}{\lambda_{g}(i)}\right) \left(\bar{v}_{i}^{2}(\mathbf{q}_{i}) - \delta^{2}(\mathbf{q}_{i})\right) \right\}$$
Medium properties

$$\times \left[-2 \mathbf{C}_{(1,\dots,n)} \cdot \sum_{m=1}^{n} \mathbf{B}_{(m+1,\dots,n)(m,\dots,n)} \left(\cos\left(\sum_{k=2}^{m} \omega_{(k,\dots,n)} \Delta z_{k}\right) - \cos\left(\sum_{k=1}^{m} \omega_{(k,\dots,n)} \Delta z_{k}\right) \right) \right]$$

propagators

Interference phases

Numerical implementation

- Numerics can be challenging due to lengthy equations and mulit-dimensional integration
- Implementation for the case of QGP (simplified Bjorken expansion)

Lashoff-Regas et al . (2014)

 Still, code needed 3 days for a set of splittings



iEBE-VISHNU package

- Hydro + hadron cascade simulator for relativistic heavy-ion collisions
- Developed by Chun Shen and collaborators