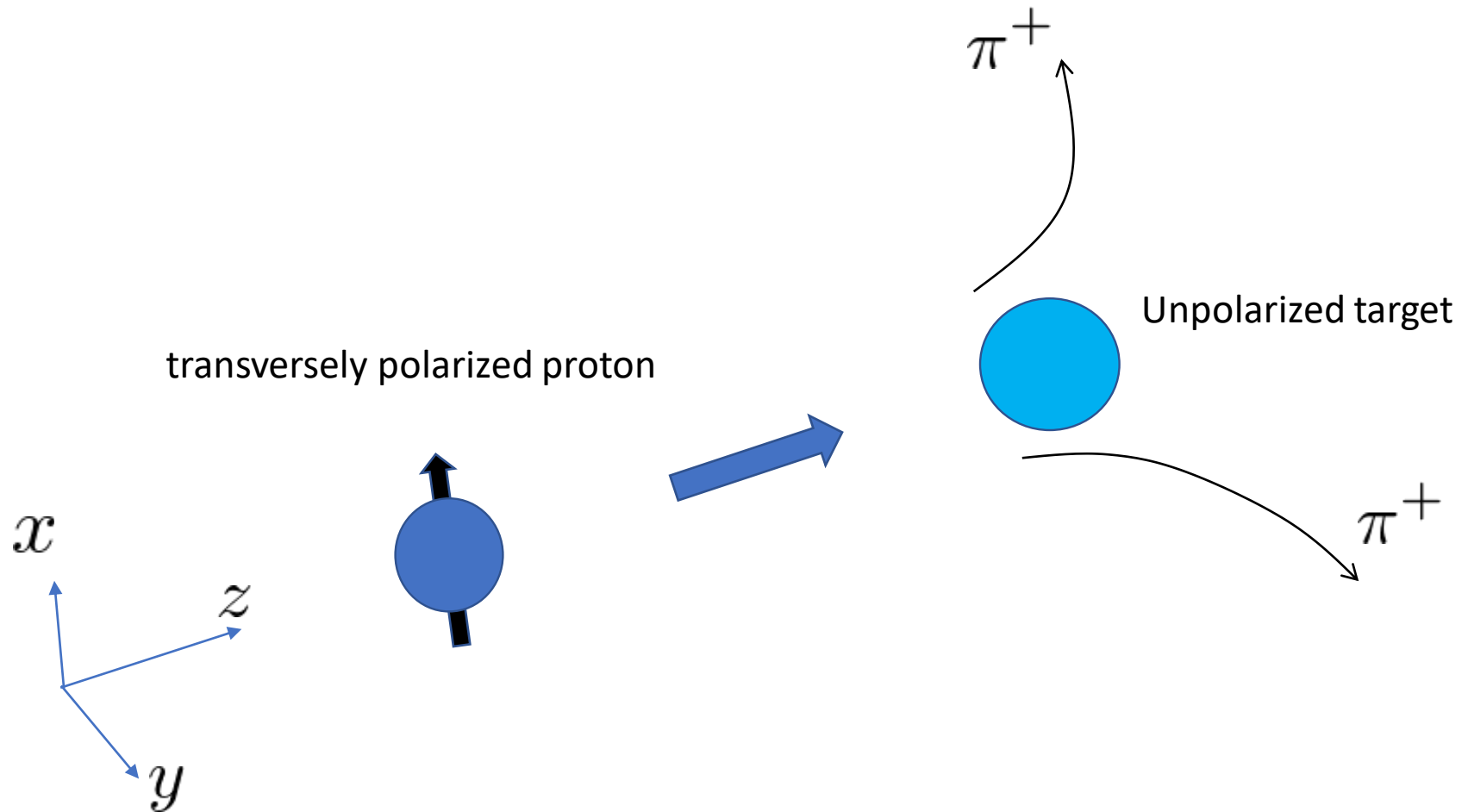


Computing the gluon Sivers function at small- x

Yoshitaka Hatta
(BNL)

Based on [Phys. Lett. B790 \(2019\) 361](#), with [Xiaojun Yao](#) and [Yoshikazu Hagiwara](#)

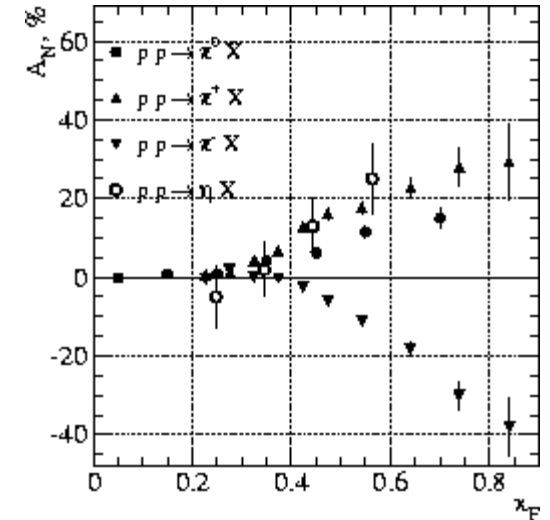
Transverse Single Spin Asymmetry (SSA)



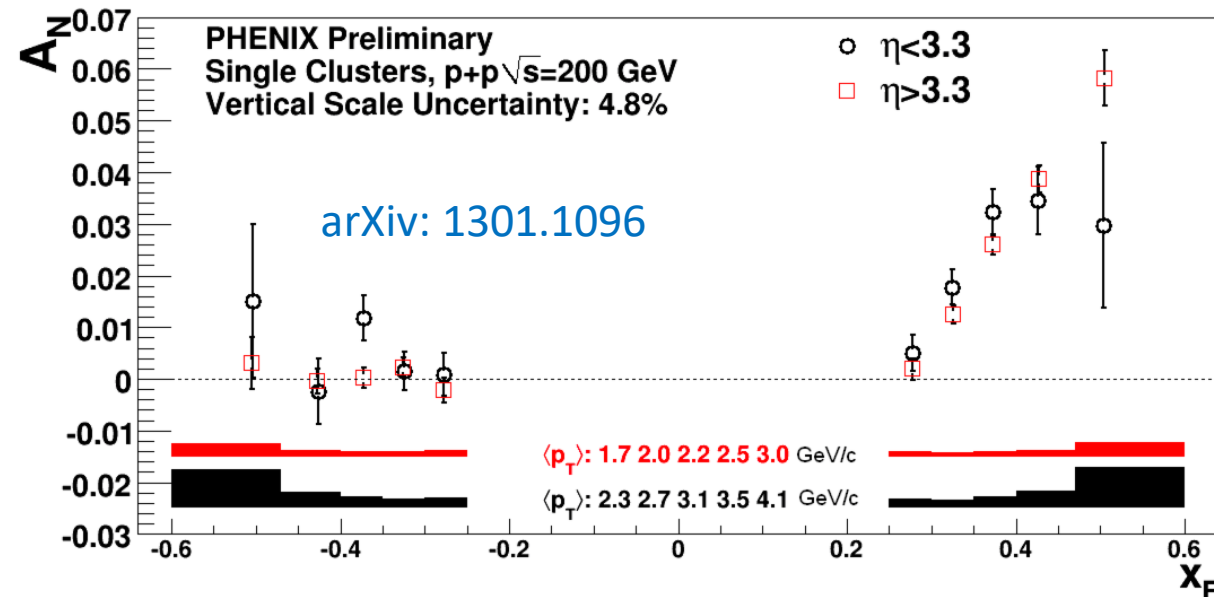
Production of hadrons is left-right asymmetric!

SSA in $p^\uparrow p$: experiments

$$A_N = \frac{d\sigma^L - d\sigma^R}{d\sigma^L + d\sigma^R}$$



E704 (Fermilab, 1991)



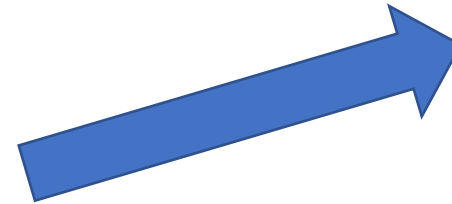
$$x_F = \frac{2p_z}{\sqrt{s}}$$

Enhanced in the forward region

$x_F \rightarrow 1$

Sivers function

Sivers (1990)



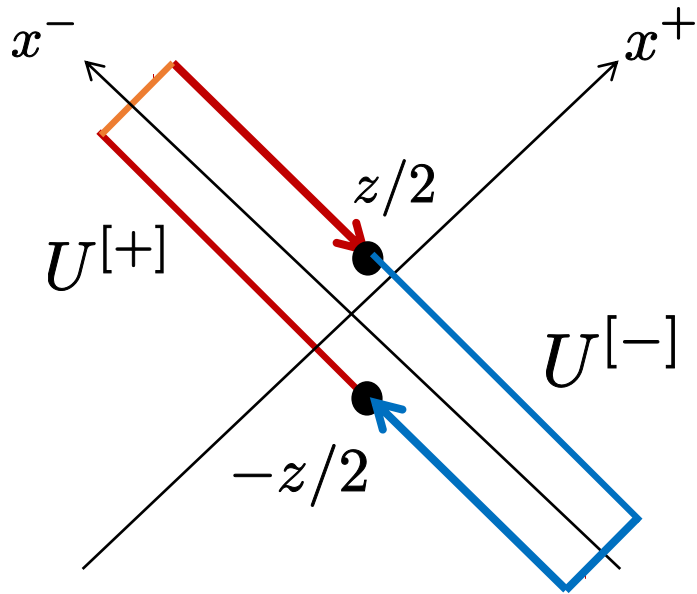
Distribution of partons can be left-right asymmetric

$$f(x, \vec{k}_\perp) = f_1(x, |k_\perp|) + (\vec{k}_\perp \times \vec{S}_\perp)^z f_{1T}^\perp(x, |k_\perp|)$$

One of the possible origins of single spin asymmetry

Universality up to a sign

$$f_{1T}^\perp(x, k_\perp) \sim F.T. \langle PS_\perp | \bar{\psi}(z/2) \gamma^+ U^{[\pm]} \psi(-z/2) | PS_\perp \rangle$$



$U^{[+]}$ or $U^{[-]}$,
Sivers function flips signs ([Collins 1993](#))

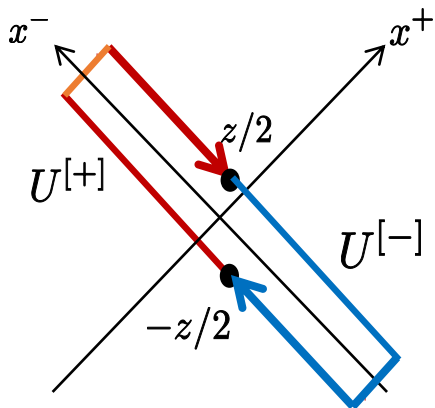
→ Opposite signs when used in SIDIS (initial state interaction)
and Drell-Yan (final state interaction)

Gluon Sivers function

There are **two** ways to make it gauge invariant

Bomhof, Mulders, Pijlman (2006)

Dominguez, Marquet, Xiao, Yuan (2011)



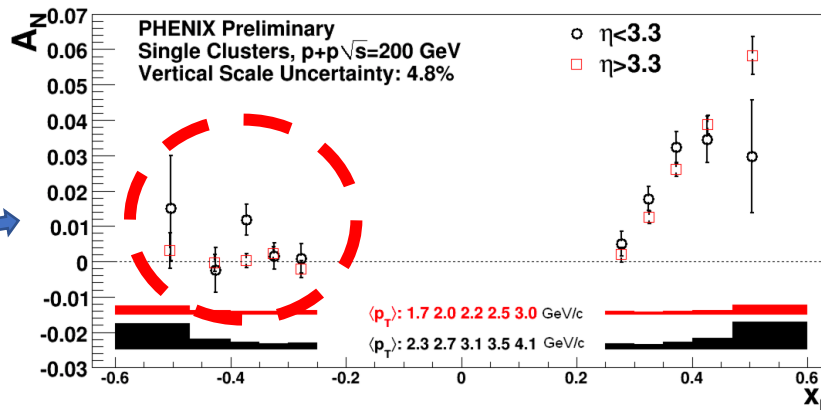
Weizsacker-Williams distribution

$$\text{Tr}[F(-z/2)U^{[+]}F(z/2)U^{[+]}]$$

Dipole distribution

$$\text{Tr}[F(-z/2)U^{[+]}F(z/2)U^{[-]}]$$

Expected to be dominant in
 { SSA of high-mass states
 SSA in the backward region



Proposals to study at EIC, RHIC, AFTER@LHC

e.g., Zheng, Aschenauer, Lee, Xiao, Yin (2018)

Little constraints from theory. **Factorization** often assumed. $f_{1T}^\perp(x, k_\perp) \approx A(x)B(k_\perp)$

Evolution of the Sivers function $f_{1T}^\perp(x, k_\perp)$

Q^2 evolution \rightarrow usual TMD evolution

Small-x evolution \rightarrow ??

$$f(x, \vec{k}_\perp) = f_1(x, |k_\perp|) + (\vec{k}_\perp \times \vec{S}_\perp)^z f_{1T}^\perp(x, |k_\perp|)$$

BFKL, BK ??

In general, spin effects are suppressed at small-x.

\rightarrow sub-eikonal effects, double logarithmic resummation $(\alpha_s \ln^2 1/x)^n$

\rightarrow Talk by Matt Sievert on Friday

Spin-dependent Odderon

Jian Zhou (2013)

BFKL and BK \rightarrow Evolution of the **dipole S-matrix**

$$\langle PS | \text{tr} U(r_{\perp}) U^{\dagger}(0_{\perp}) | PS \rangle$$

This cannot depend on spin S^{μ} if longitudinally polarized (parallel to P^{μ})

However, it can depend on S^{μ} if transversely polarized!

$$\langle PS | \text{tr} U(r_{\perp}) U^{\dagger}(0_{\perp}) | PS \rangle = P(r_{\perp}) + i S_{\perp} \times r_{\perp} Q(r_{\perp})$$

real
Pomeron

imaginary
Odderon

Dipole gluon Sivers and Odderon

Boer, Echevarria, Mulders, Zhou (2015)

$$f_{1T}^\perp(x, k_\perp) \propto \frac{k_\perp^2}{x} \tilde{Q}(x, k_\perp) \quad \text{Fourier transform of Odderon}$$

Small-x evolution of (dipole) gluon Sivers \rightarrow evolution of Odderon in QCD

Single logarithmic, well understood in the literature including saturation effects

Bartels (1979); Kwiecinski, Praszalowicz (1980)

Kovchegov, Szymanowski, Wallon (2003)

YH, Iancu, Itakura, McLerran (2004)

$$\frac{\partial}{\partial Y} O(x_\perp, y_\perp) = \frac{\bar{\alpha}_s}{2\pi} \int d^2 z_\perp \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2} \left[O(x_\perp, z_\perp) + O(z_\perp, y_\perp) - O(x_\perp, y_\perp) \right. \\ \left. - O(x_\perp, z_\perp) N(z_\perp, y_\perp) - N(x_\perp, z_\perp) O(z_\perp, y_\perp) \right],$$

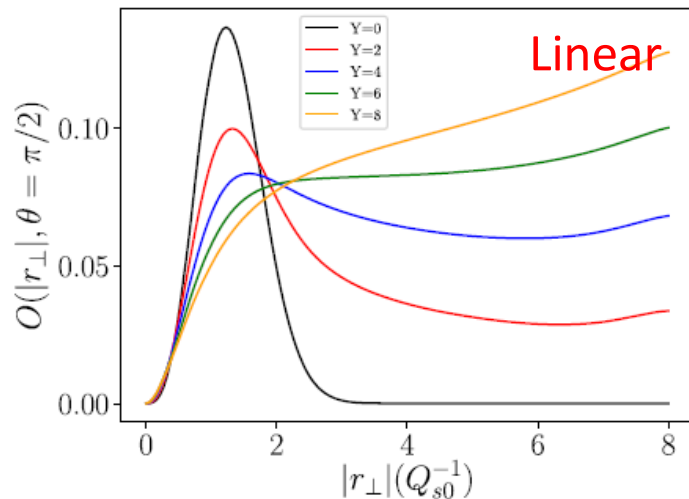
Initial condition anti-symmetric $O(x_\perp, y_\perp) = -O(y_\perp, x_\perp)$

Bartels-Vacca-Lipatov solution in the linear regime. Regge intercept exactly unity.

Numerical results (1)

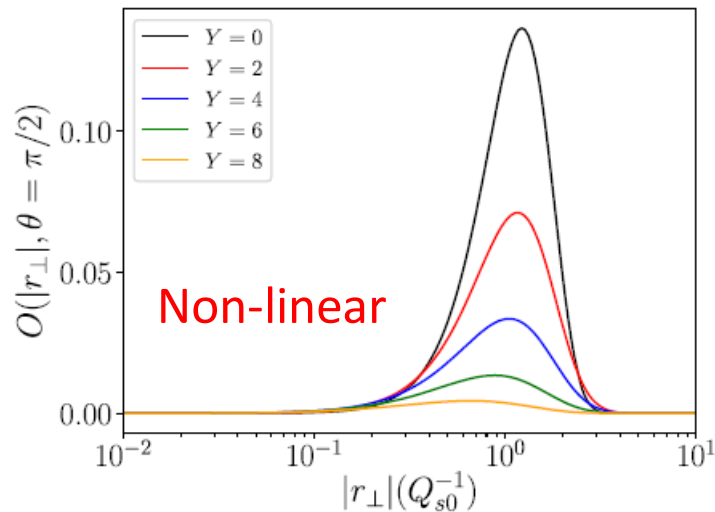
Initial condition

$$O(|r_{\perp}|, \theta) = \kappa Q_{s0}^3 |r_{\perp}|^3 \sin \theta e^{-r_{\perp}^2 Q_{s0}^2},$$



BLV solution \rightarrow Odderon amplitude constant?
Strongly dependent on r_{\perp}

Sudakov-like suppression at small- r_{\perp}

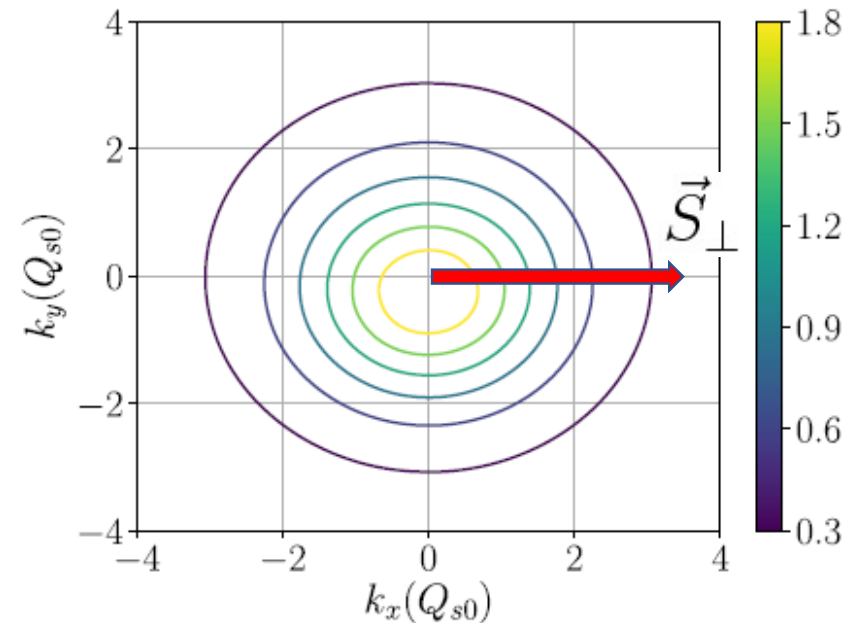
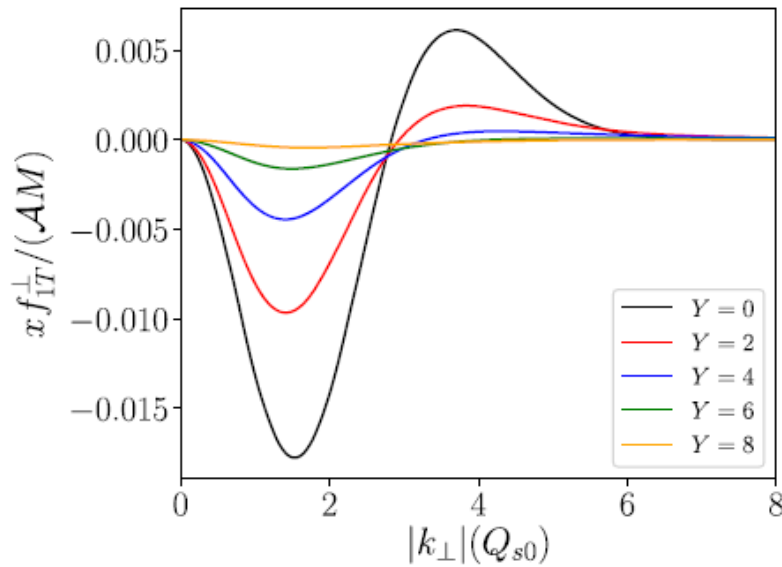


Suppression due to the nonlinear terms

Peak position does not move.
(No geometric scaling)

See, also, [Lappi, Ramnath, Rummukainen, Weigert \(2016\)](#)

Numerical results (2)



Always there is a node in k_\perp

Factorization $f_{IT}^\perp(x, k_\perp) \approx A(x)B(k_\perp)$ holds approximately, unlike in unpolarized TMDs
(Good news for phenomenology)

SSA in open charm production

SIDIS $ep^\uparrow \rightarrow DX$

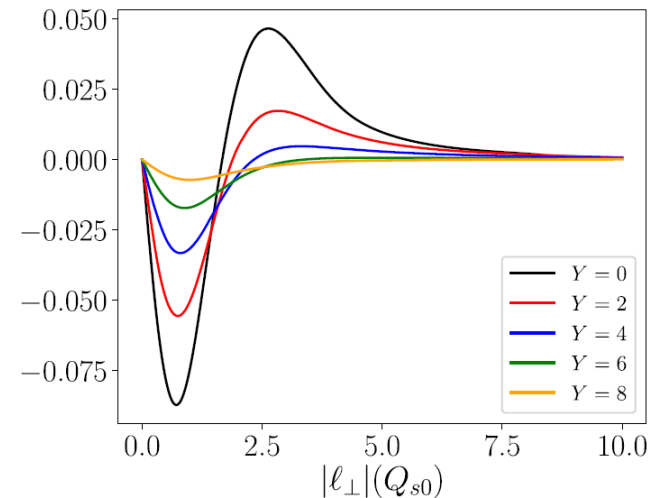
$$\frac{d\sigma}{dx_B dz dQ^2 dy d^2\ell_\perp} = \frac{\alpha_{\text{em}}^2 e_c^2}{2\pi^4 x_B Q^2} \mathcal{A} \int \frac{d^2k_\perp}{(2\pi)^2} H(k_\perp, \ell_\perp, Q^2) \epsilon^{ij} S_{\perp i} k_{\perp j} \tilde{Q}(|k_\perp|),$$

$H(k_\perp, \ell_\perp, Q^2)$

$$= \left[1 - y + \frac{y^2}{2}\right] \left\{ (z^2 + (1-z)^2) \left[\frac{\ell_\perp - k_\perp}{\rho + (\ell_\perp - k_\perp)^2} - \frac{\ell_\perp}{\rho + \ell_\perp^2} \right]^2 + m_c^2 \left(\frac{1}{\rho + (\ell_\perp - k_\perp)^2} - \frac{1}{\rho + \ell_\perp^2} \right)^2 \right\} \\ + 4(1-y)z^2(1-z)^2 Q^2 \left(\frac{1}{\rho + (\ell_\perp - k_\perp)^2} - \frac{1}{\rho + \ell_\perp^2} \right)^2$$

Integral dominated by $k_\perp \sim \ell_\perp$

Node in $\tilde{Q}(k_\perp) \rightarrow$ node in $\frac{d\sigma}{d^2\ell_\perp}$



Summary

- Quark Sivers → most well-studied TMD, rich phenomenology
Gluon Sivers → relatively unexplored, interesting at EIC.
- Related to Odderon at small-x, single-logarithmic evolution
- Suppressed at very small-x due to saturation effects
- Factorization of x and k_{\perp} looks reasonably OK
- SSA of open charm at EIC. Predict a node (nodes) in $P_{h\perp}$

Different signs in SIDIS and Drell-Yan ← initial/final state effects
Different signs for D^0 and \bar{D}^0 ← C-oddness
Different signs in different $P_{h\perp}$ regions ← Node in k_{\perp}