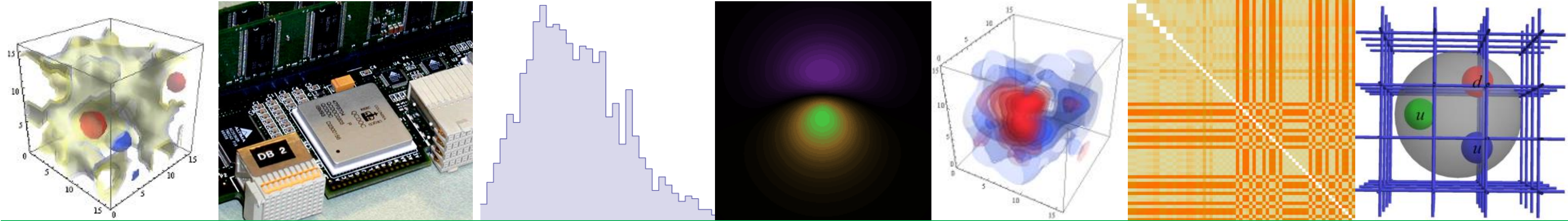


Gluon PDFs

Huey-Wen Lin

This work is supported by NSF under grant PHY 1653405
"CAREER: Constraining Parton Distribution Functions for New-Physics Searches"

Outline

§ Motivations

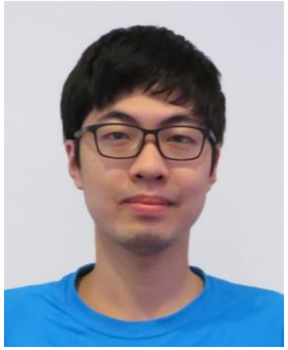
§ A first exploratory study

- In collaboration with Yi-Bo Yang, Zhoyou Fan, Adam Anthony and Keh-Fei Liu

Fan, Yang, et al, Phys.Rev.Lett. 121, 242001 (2018)

§ USQCD proposal & plans

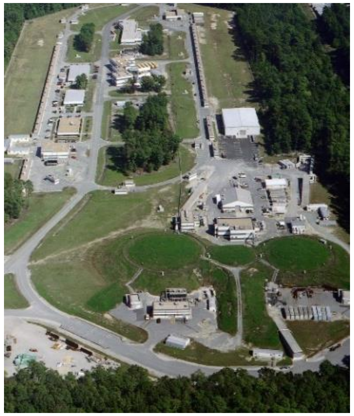
Some slides from Zhouyou Fan



Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of nucleon

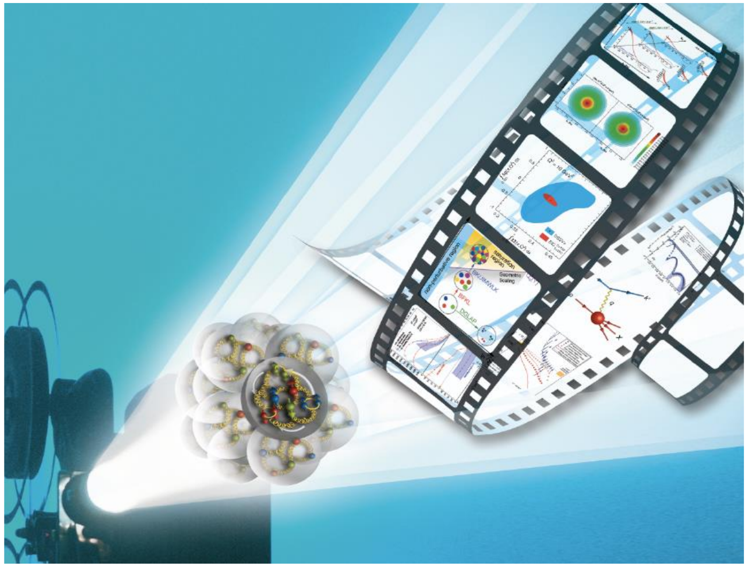
- Many ongoing/planned experiments
  (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, ...)



Imaging of the proton

How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?

EIC White Paper, 1212.1701



Global Analysis

§ Experiments cover diverse kinematics of parton variables

- Global analysis takes advantage of all data sets

Theory Input

Exp't Input

Global Analysis of PDFs

§ Some choices made for the analysis

- Choice of data sets and kinematic cuts
- Strong coupling constant $\alpha_s(M_Z)$
- How to parametrize the distribution

$$xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2}P(x)$$

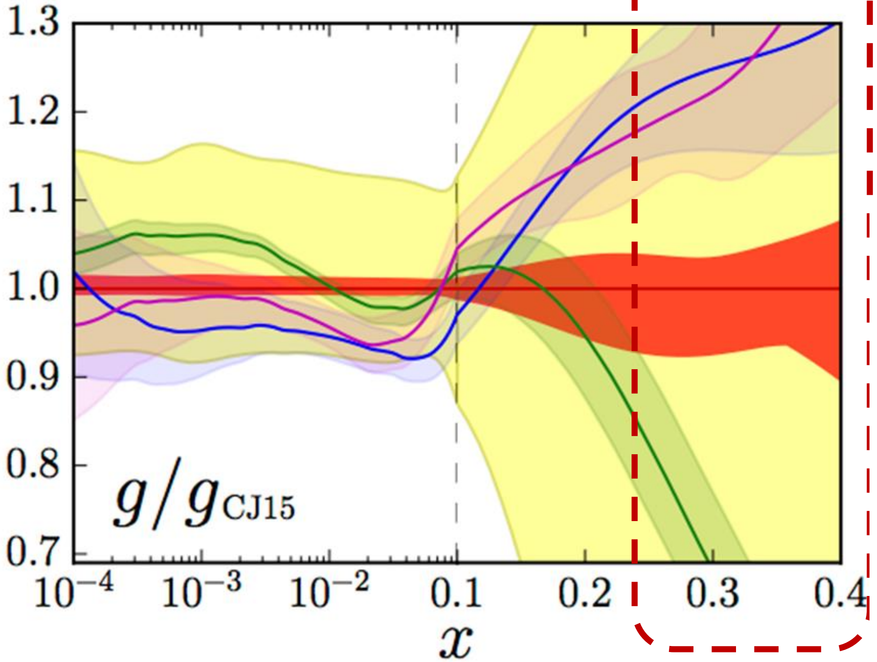
- Assumptions imposed

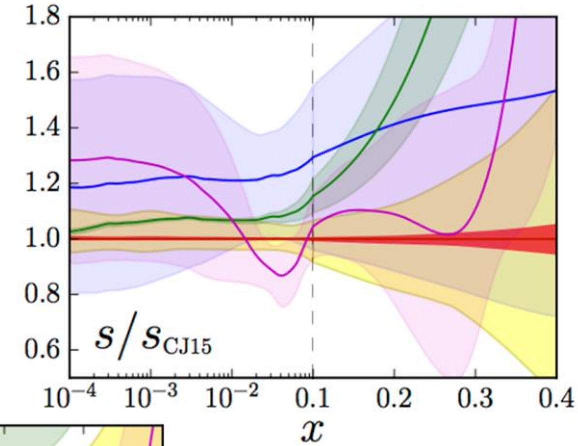
SU(3) flavor symmetry, charge symmetry, strange and sea distributions

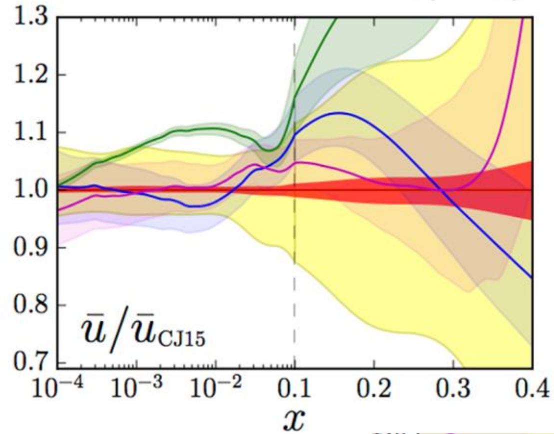
$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

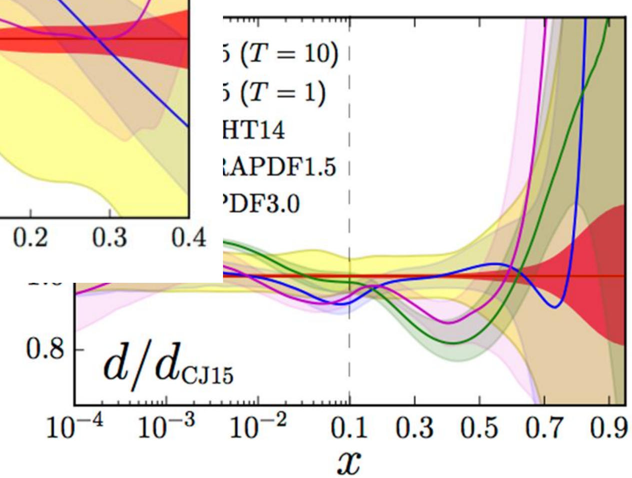
Global Analysis

- Discrepancies appear when data is scarce
- Many groups have tackled the analysis
  - CTEQ, MSTW, ABM, JR, NNPDF, etc.





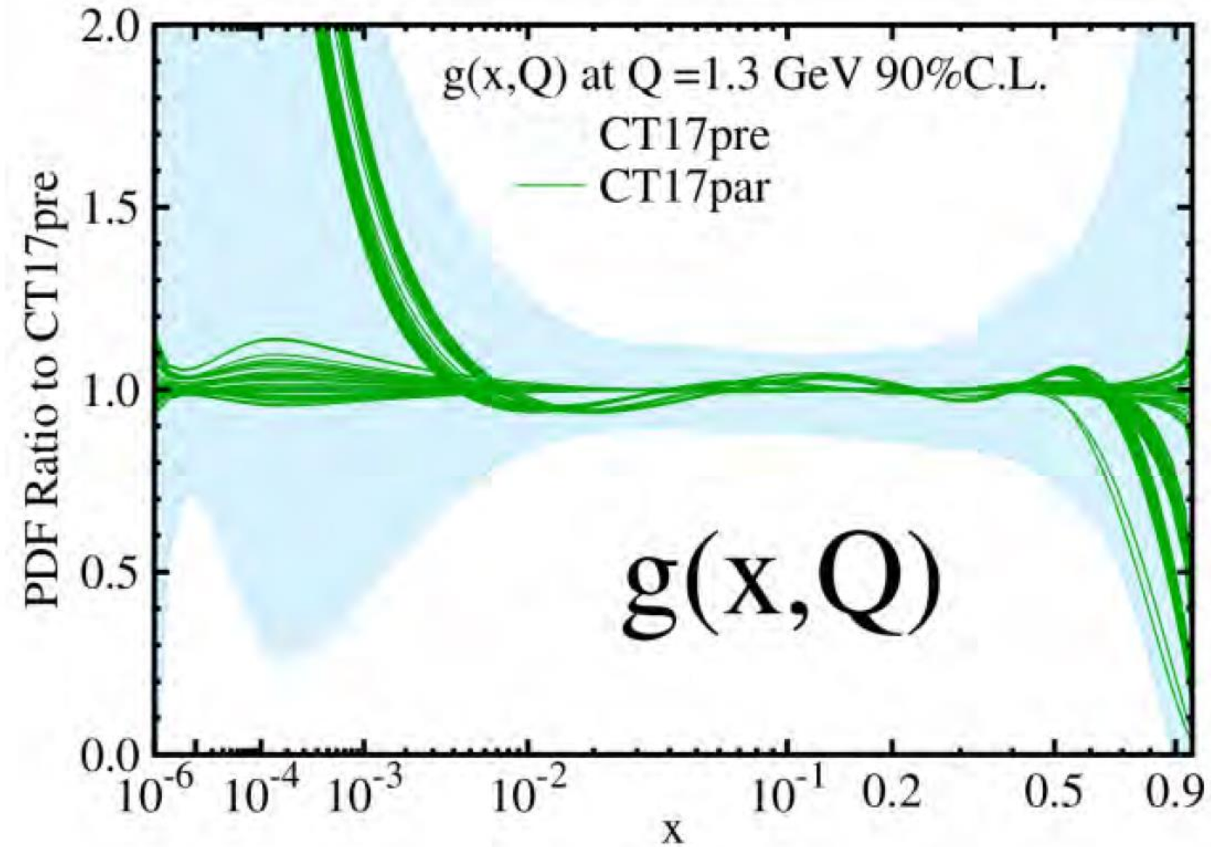




CTEQ-JLAB
https://www.jlab.org/theory/cj/

Global Analysis

- CT18: sample of various nonperturbative parametrizations
- No data to constrain very large or very small *x*



C T E Q

CTEQ-TEA (Slide by C. P. Yuan @DIS2019)

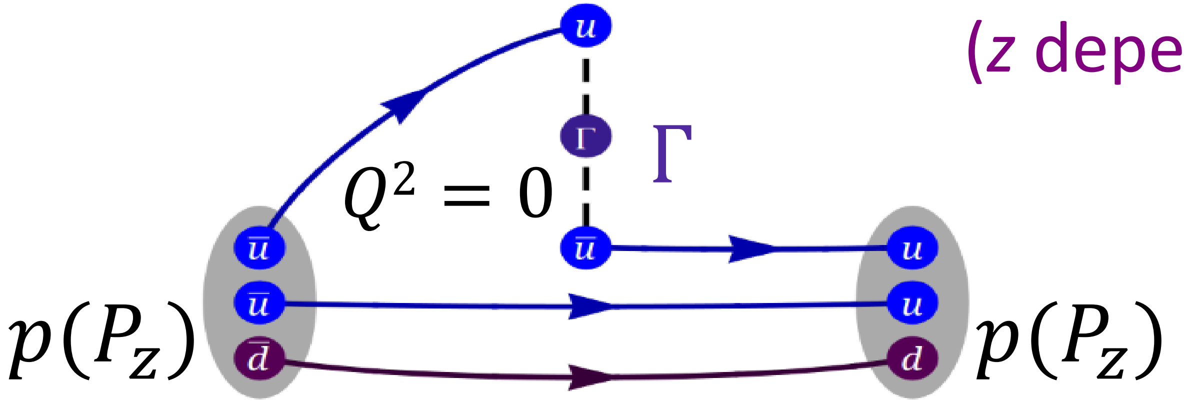
What can we do on the lattice?

LaMET in a Nutshell

Steps for LaMET

1) Calculate nucleon matrix elements on the lattice

(*z* dependence)



2) Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi}\, e^{-izk_z} \left\langle P \,\middle|\, \overline{\psi}(z)\, \Gamma \exp\!\left(-ig\int_0^z dz'\, A_z(z')\right) \psi(0) \,\middle|\, P \right\rangle$$

3) Recover true distribution (take $P_z \to \infty$ limit)

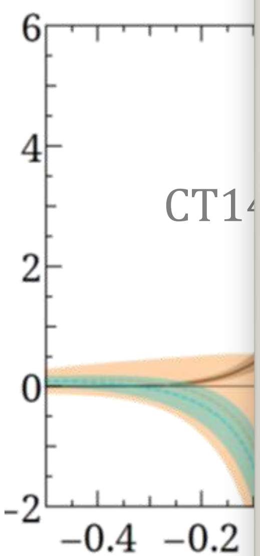
$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu) + \mathcal{O}\left(M_N^2/P_z^2\right) + \left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$$

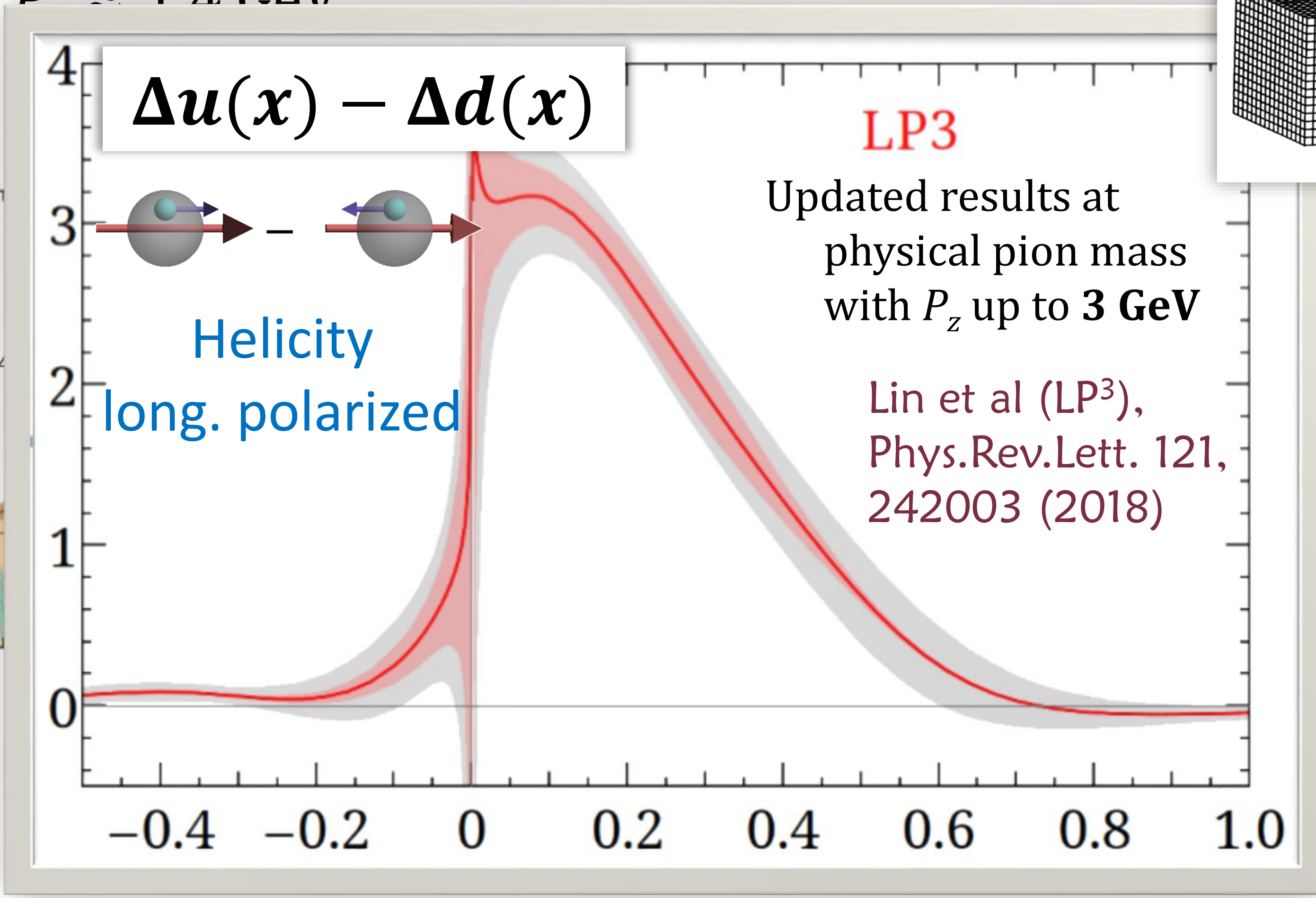
X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

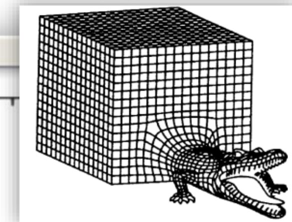
Some Reasonable Results

§ Exciting! Two collaborations' results at physical pion mass

- Boost $P_z \approx 1.4$ GeV
- Study







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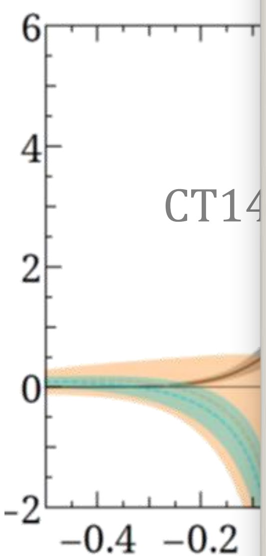
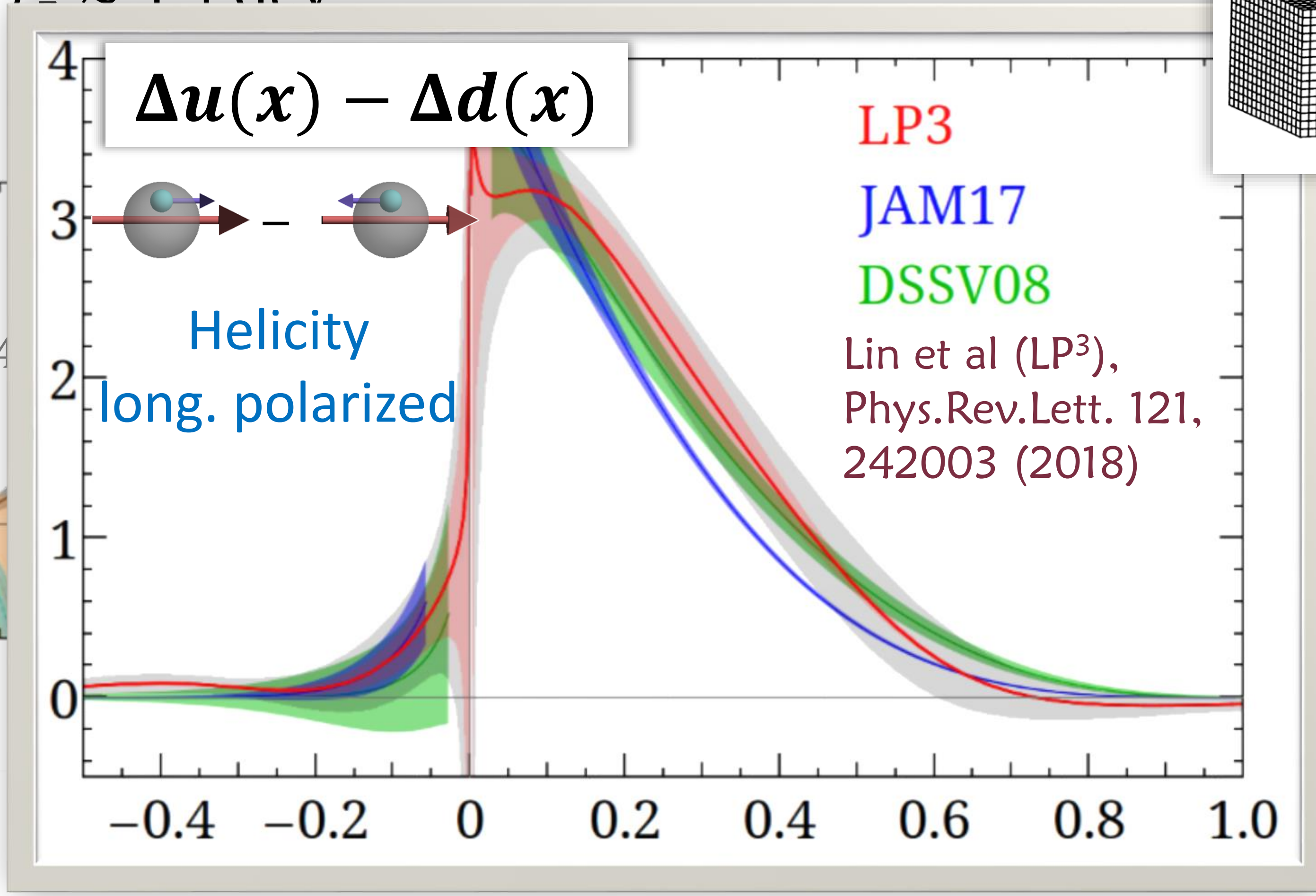
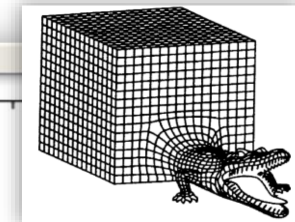






Numerical Setup

- Calculations carried out with valence overlap fermions on RBC N_f = 2+1 DWF gauge configurations with
  - $L^3 \times T = 24^3 \times 64$
  - $a = 0.1105(3)$ fm
  - $m_\pi^{sea} = 330$ MeV
- For the nucleon two-point function:
  - Valence quark mass about the strange quark mass (the corresponding pion mass is 678 MeV)
  - Counting independent smeared point sources, the statistics used for these grid-source measurements is
    - $203 \times 4 \times 32 \times 8 = 207,872$

Gluon Operators

- Lattice MEs of unpolarized gluon quasi-PDF

$$\widetilde{H}(z,P_z)=\langle P|O_3(z)|P\rangle$$
$$O_3(z)=\frac{1}{P_0}O(F_\mu^z,F^{z\mu};z)$$

where the operator $O(F_1,F_2;z)=F_1(z)U(z,0)F_2(0)$.

- Better definitions should subtract the trace term

$$O_0(z)=\frac{P_0}{\frac{3}{4}P_0^2+\frac{1}{4}P_z^2}\left(O\left(F_\mu^t,F^{\mu t};z\right)-\frac{1}{4}g^{tt}O(F_\nu^\mu,F_\mu^\nu;z)\right)$$
$$O_1(z)=\frac{1}{P_z}O(F_\mu^t,F^{z\mu};z))$$
$$O_2(z)=\frac{P_0}{\frac{1}{4}P_0^2+\frac{3}{4}P_z^2}\left(O\left(F_\mu^z,F^{\mu z};z\right)-\frac{1}{4}g^{zz}O(F_\nu^\mu,F_\mu^\nu;z)\right)$$

- Gluon momentum fraction

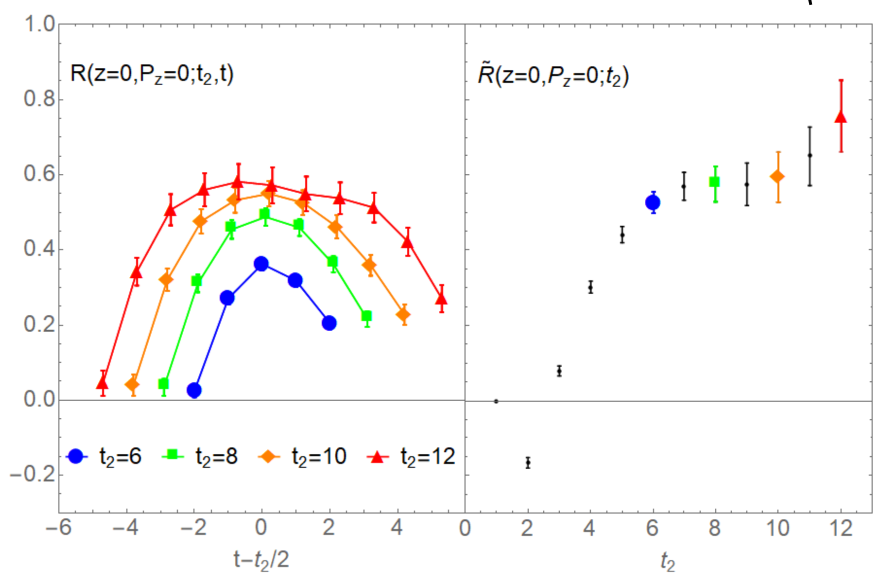
$$\langle x\rangle_g=\widetilde{H}(z=0,P_z)$$

Getting the Ground-State ME

- Bare gluon nucleon matrix element can be obtained from the derivative of the summed ratio as

$$\sum_{0<t<t_{\text{sep}}} R(z,P_z;t_{\text{sep}},t) - \sum_{0<t<t_{\text{sep}}-1} R\big(z,P_z;t_{\text{sep}}-1,t\big) = \widetilde{H}_g(z,P_z) + \mathcal{O}(e^{\Delta m t_{\text{sep}}})$$

$$\text{where } R(z,P_z;t_2,t) \equiv \frac{P_0 \left\langle 0 \left| \Gamma^e \int d^3y\, e^{-iyP} \chi(\vec{y},t_2) O_0(z)\, \chi\left(\vec{0},0\right) \right| 0 \right\rangle}{\left(\frac{3}{4}P_0^2 + \frac{1}{4}P_z^2\right) \left\langle 0 \left| \Gamma^e \int d^3y e^{-iy_3P_3} \chi(\vec{y},t_2)\, \chi\left(\vec{0},0\right) \right| 0 \right\rangle}$$

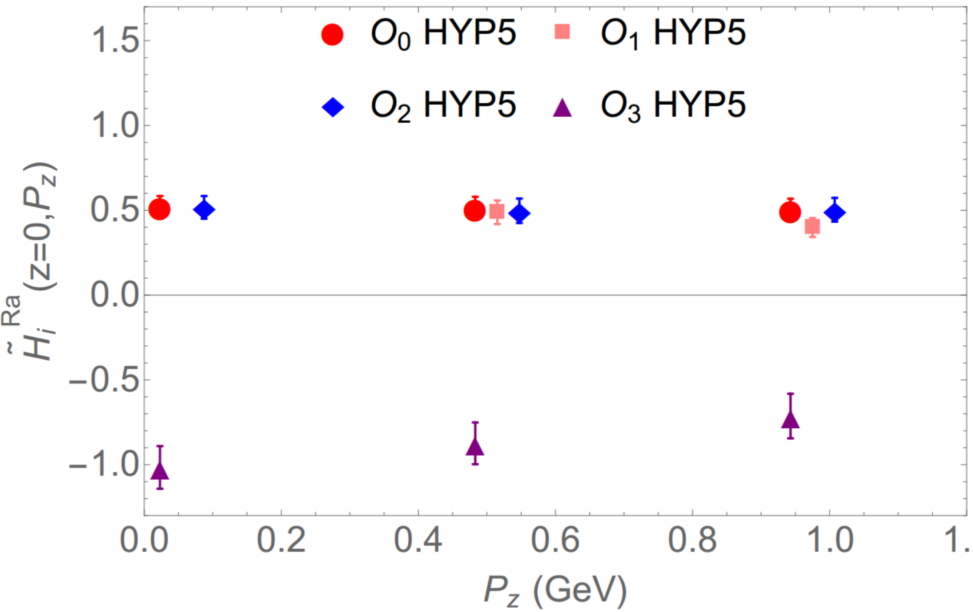


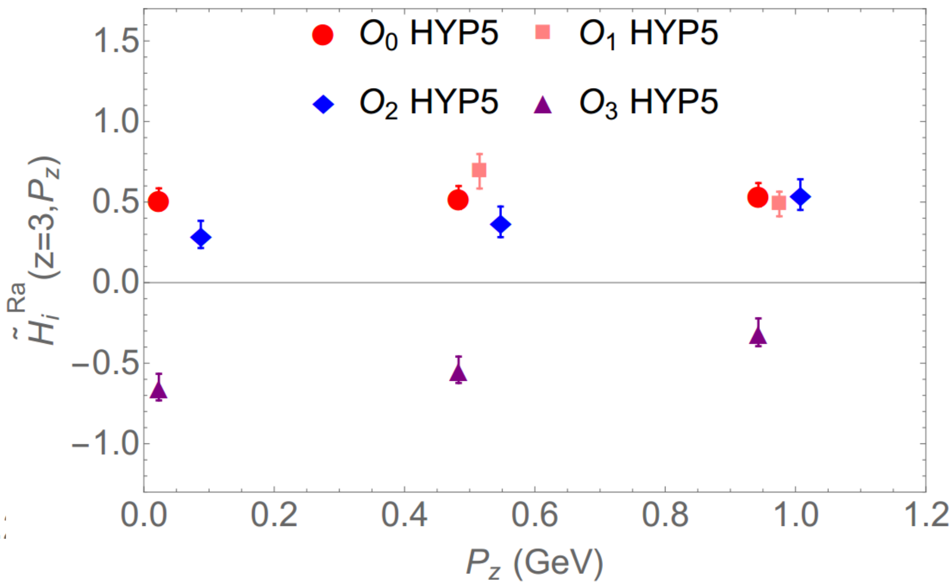
Ratio $R\big(z,P_z;t_{\text{sep}},t\big)$ and derivative of the summed ratio $\tilde{R}\big(z,P_z;t_{\text{sep}}\big)$ for the glue operator $O_0(z)$

Numerical Results

§ The "ratio renormalized" gluon quasi-PDF matrix element

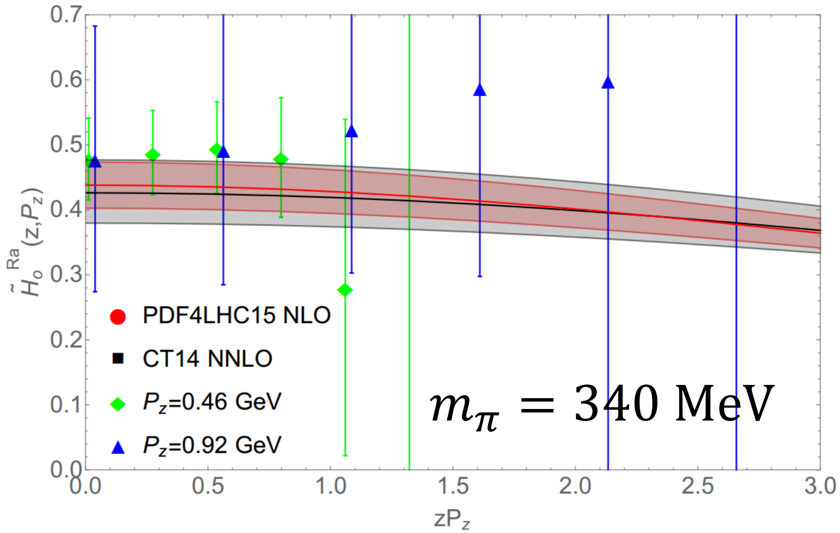
$$\tilde{H}_0^{Ra}(z, P_z, \mu) = \frac{\tilde{H}_0^{\overline{\mathrm{MS}}}(0, 0, \mu)}{\tilde{H}_0(z, 0)} \tilde{H}_0(z, P_z)$$

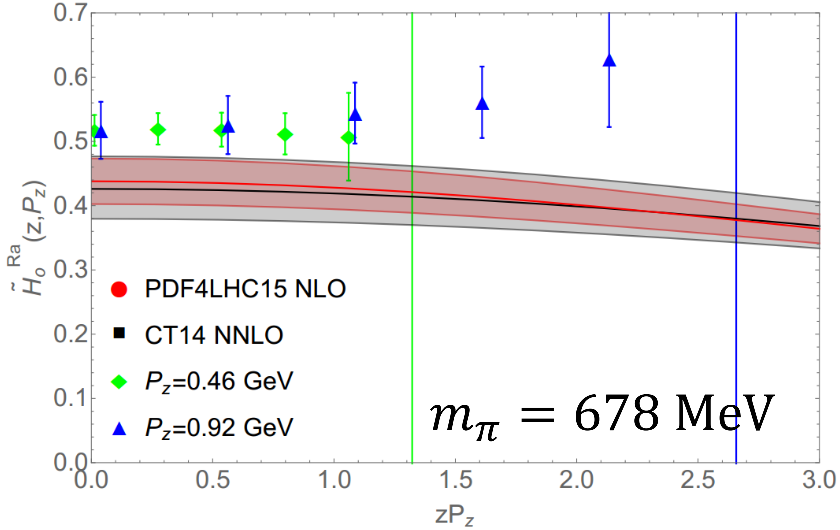




$\widetilde{H}^{Ra}(z = 0, P_z, \mu) = \langle x \rangle_g$ in $\overline{\mathrm{MS}}$ at 2 GeV

Numerical Results





§ Unpolarized gluon quasi-PDF

$$\widetilde{H}^{Ra}(z, P_z, \mu) = \int dx\ e^{ixzP^z} x\tilde{g}(x, P_z, \mu)$$

§ For unpolarized gluon PDF,

$$H(\omega, \mu) = \int dx\ e^{ix\omega} xg(x, \mu)$$

§ With renorm., quasi-PDF ME agrees with FT of pheno. PDF

§ Further improvement needs larger P_z with mom. smearing

Key Issues

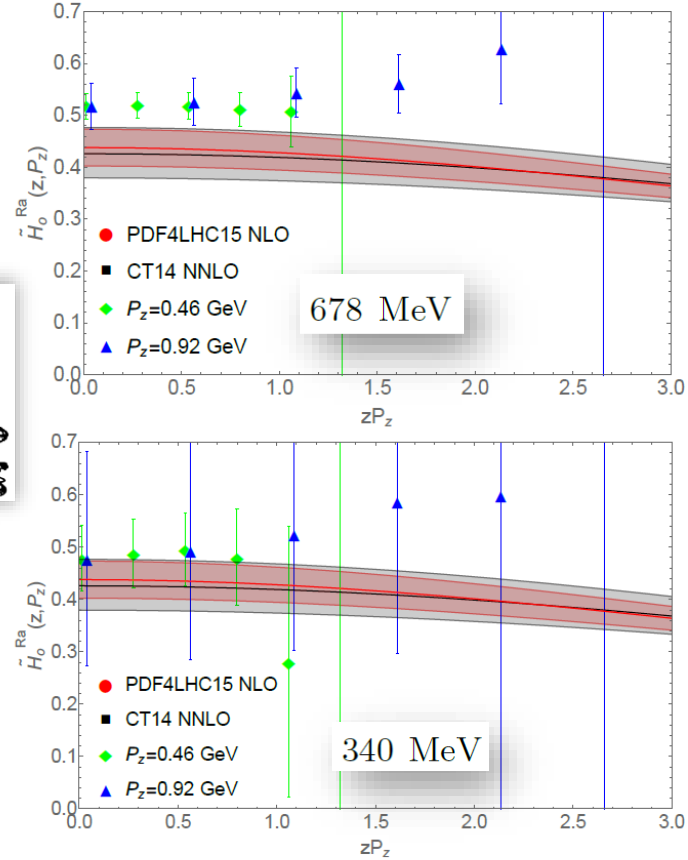
- Noise is the main problem here at larger zP_z
- Larger P_z needs smaller lattice spacing
  - Minimization of potential $(aP_z)^n$ error
  - Finer time resolution to extract ground-state signal
- We propose calculation using a = 0.06 fm ($a^{-1} \approx 3.4$ GeV) HISQ 310-MeV pion lattice
  - Thanks to MILC collaboration for sharing these configurations
  - Will stay at light/strange unitary points (with a PQ point around 450 MeV)
  - Ask time to generate tons of 2pt correlators and gluon loops
- Explore improvements in operators and other techniques, such as *cluster-decomposition error reduction (CDER) technique* K. Liu, et al PRD96, 114504(2017)

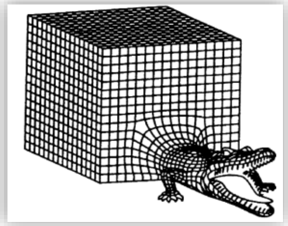
Gluon PDF

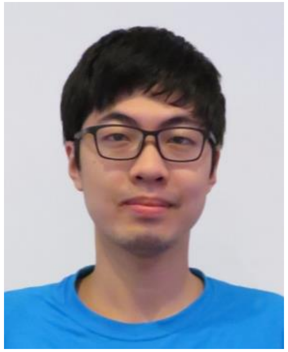
§ Pioneering first glimpse into gluon PDF using LaMET

- Lattice details: overlap/2+1DWF, 0.16fm, 340-MeV sea pion mass
- Study strange/light-quark
- Promising results using coordinate-space comparison, but signal does not go far in z
- Hard numerical problem to be solved

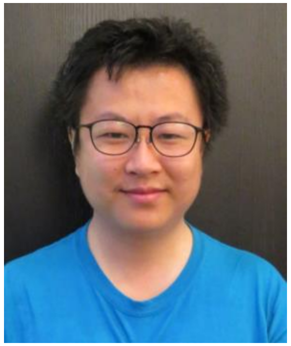
Fan, Yang et al, Phys.Rev.Lett. 121, 242001 (2018)







Zhouyou Fan



Yi-Bo Yang