Quantum Computing for QCD?

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Content of the talk

- Quantum Computing (QC) for QCD: what do we want to do?
- Strategy:
  - big goals with enough intermediate steps
  - explore as many paths as possible
  - leave room for serendipity
- Tensor tools: QC friends and competitors (RG)
- Quantum simulations experiments (analog): cold atoms, ions ...
- Quantum computations (digital): IBM, IonQ, Rigetti, ...
- Abelian Higgs model with cold atom ladders
- Benchmark for real time scattering (arXiv:1901.05944, PRD in press with Erik Gustafson and Judah Unmuth-Yockey)
- Symmetry preserving truncations (YM, arxiv:1903.01918)
- Quantum Joule experiments (arXiv:1903.01414, with Jin Zhang and Shan-Wen Tsai)
- Pitch for a quantum center for theoretical physics (HEP, NP, ...)
- Conclusions

Computing with quantum devices (Feynman 82)?

- The number of transistors on a chip doubled almost every two years for more than 30 years
- At some point, the miniaturization involves quantum mechanics
- Capacitors are smaller but they are still on (charged) or off (uncharged)
- qubits: $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$ is a superposition of the two possibilities.
- Can we use quantum devices to explore large Hilbert spaces?
- Yes, if the interactions are localized (generalization of Trotter product formula, Lloyd 96)

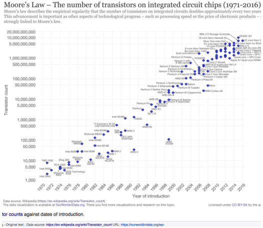


Figure: Moore's law, source: Wikipedia

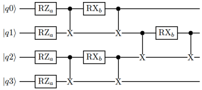


Figure 1: Circuit for 4 qubits with open boundary conditions

Figure: Quantum circuit for the quantum Ising model (E. Gustafson)

Golden dreams of robust quantum hardware

QIS and QC: what do we want to do?

Problems where perturbation theory and classical sampling fail:

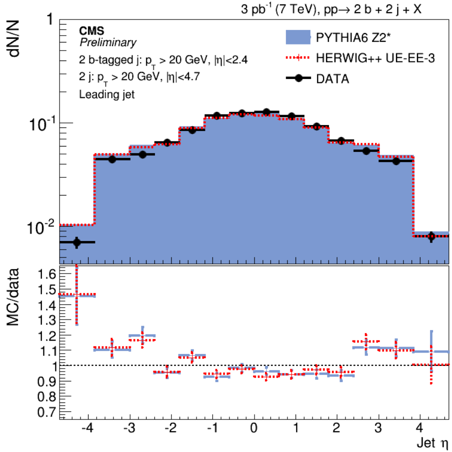
- Real-time evolution for QCD
- Jet Physics (crucial for the LHC program)
- Finite density QCD (sign problem)
- Near conformal systems (BSM, needs very large lattices)
- Early cosmology
- Strong gravity (and error correction)

Strategy: many intermediate steps towards big goals

- High expectations for QC: new materials, fast optimization, security, ...
- Risk management: theoretical physics is a multifaceted landscape
- Lattice gauge theory lesson: big goals can be achieved with small steps
- Example of a big goal: ab-initio jet physics
- Examples of small steps: real-time evolution in 1+1 Ising model, 1+1 Abelian Higgs model, Schwinger model, 2+1 $U(1)$ gauge theory ,....
- Many possible paths: quantum simulations (trapped ions, cold atoms,...), quantum computations (IBM, Rigetti,...)
- Small systems are interesting: use Finite Size Scaling (data collapse, Luscher's formula,....)

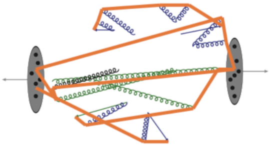
Jet Physics ab initio: a realistic long term QC goal?

Pythia, Herwig, and other jet simulation models encapsulate QCD ideas, empirical observations and experimental data. It is crucial for the interpretation of collider physics experiments. Could we recover this understanding from scratch (ab-initio lattice QCD)?



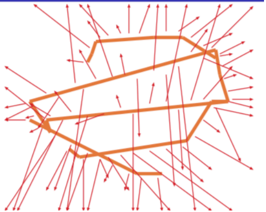
Jet Physics (from T. Sjostrand's talk at TRIUMF, 2013)

The structure of an event – 9

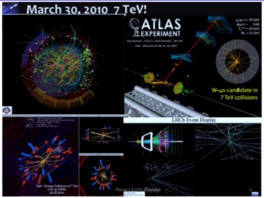


Everything is connected by colour confinement strings
Recall! Not to scale: strings are of hadronic widths

The structure of an event – 10



The structure of an event – 12



These are the particles that hit the detector

The Monte Carlo method

Want to generate events in as much detail as Mother Nature
$\Longrightarrow$ get average *and* fluctutations right
$\Longrightarrow$ make random choices, $\sim$ as in nature

$$\sigma_{\text{final state}} = \sigma_{\text{hard process}} \mathcal{P}_{\text{tot,hard process}\to\text{final state}}$$

(appropriately summed & integrated over non-distinguished final states)

where $\mathcal{P}_{\text{tot}} = \mathcal{P}_{\text{res}} \mathcal{P}_{\text{ISR}} \mathcal{P}_{\text{FSR}} \mathcal{P}_{\text{MPI}} \mathcal{P}_{\text{remnants}} \mathcal{P}_{\text{hadronization}} \mathcal{P}_{\text{decays}}$

with $\mathcal{P}_i = \prod_j \mathcal{P}_{ij} = \prod_j \prod_k \mathcal{P}_{ijk} = \ldots$ in its turn

$\Longrightarrow$ **divide and conquer**

an event with n particles involves $\mathcal{O}(10n)$ random choices,
(flavour, mass, momentum, spin, production vertex, lifetime, ...)
LHC: $\sim$ 100 charged and $\sim$ 200 neutral (+ intermediate stages)
$\Longrightarrow$ several thousand choices
(of $\mathcal{O}(100)$ different kinds)

We need to start with something simple!

Figure: Mike Creutz's calculator used for a Z_2 gauge theory on a 3^4 lattice (circa 1979).

Clear goals and good fundamentals (and some help from Moore's law) led to major accomplishments in HEP and NP. This is not always recognized by physics departments for instance for flavor physics.

Following the “Kogut sequence"

An introduction to lattice gauge theory and spin systems*

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This article is an interdisciplinary review of lattice gauge theory and spin systems. It discusses the fundamentals, both physics and formalism, of these related subjects. Spin systems are models of magnetism and phase transitions. Lattice gauge theories are cutoff formulations of gauge theories of strongly interacting particles. Statistical mechanics and field theory are closely related subjects, and the connections between them are developed here by using the transfer matrix. Phase diagrams and critical points of continuous transitions are stressed as the keys to understanding the character and continuum limits of lattice theories. Concepts such as duality, kink condensation, and the existence of a local, relativistic field theory at a critical point of a lattice theory are illustrated in a thorough discussion of the two-dimensional Ising model. Theories with exact local (gauge) symmetries are introduced following Wegner's Ising lattice gauge theory. Its gauge-invariant “loop” correlation function is discussed in detail. Three – dimensional Ising gauge theory is studied thoroughly. The renormalization group of the two dimensional planar model is presented as an illustration of a phase transition driven by the condensation of topological excitations. Parallels are drawn to Abelian lattice gauge theory in four dimensions. Non-Abelian gauge theories are introduced and the possibility of quark confinement is discussed. Asymptotic freedom of O(n) Heisenberg spin systems in two dimensions is verified for $n \geq$ and is explained in . simple terms. The direction of present-day research is briefly reviewed.

CONTENTS

I. INTRODUCTION—AN OVERVIEW OF THIS ARTICLE

This article consists of a series of introductory lectures on lattice gauge theory and spin systems. It is intended to explain some of the essentials of these subjects to students interested in the field and research physicists whose expertise lies in other domains. The expert in lattice gauge theory will find little new in the following pages aside from the author’s personal perspective and overview. The style of this presentation is informal. The article grew out of a half-semester graduate course on lattice physics presented at the University of Illinois during the fall semester of 1978.

Lattice spin systems are familiar to most physicists because they model solids that are studied in the laboratory. These systems are of considerable interest in these lectures, but we shall also be interested in more abstract questions. In particular, we shall be using space–time lattices as a technical device to define cutoff field theories. The eventual goal of these studies is to construct solutions of cutoff theories so that field theories defined in real continuum Minkowski space–time can be understood. The lattice is mere scaffolding—an intermediate step used to analyze a difficult nonlinear system of an infinite number of degrees of freedom. Different lattice formulations of the same

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Figure: Cover page of J. Kogut RMP 51 (1979).

The Fermilab cluster, accurate estimates of $|V_{ub}|$, $g-2$, nuclear form factors for neutrino experiments, exploration of the boundary of the conformal window ...

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$B_s \to K\ell\nu$ decay from lattice QCD

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(Dated: January 10, 2019)

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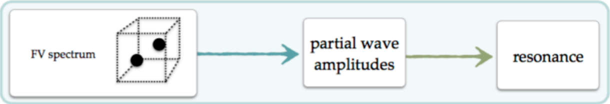
30 years later: scattering amplitudes and resonances

Scattering amplitudes



$$\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$$

E_L = finite volume spec.
L = finite volume
F = known function
$\mathcal{M}$ = scattering amp.

not an extrapolation!

- Lüscher (1986, 1991) [elastic scalar bosons]
- Rummukainen & Gottlieb (1995) [moving elastic scalar bosons]
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005) [QFT derivation]
- Feng, Li, & Liu (2004) [inelastic scalar bosons]
- Hansen & Sharpe / RB & Davoudi (2012) [moving inelastic scalar bosons]
- RB (2014) [general 2-body result]

Figure: From Raul Briceno talk at Lattice 2017

Discretization for classically intractable problems

QC requires a complete discretization of QFT

- Discretization of space: lattice gauge theory formulation
- Discretization of field integration: tensor methods for compact fields (as in Wilson lattice gauge theory and nonlinear sigma models, the option followed here)
- Quantum computing (QC) methods for scattering in ϕ^4 (non-compact) theories are discussed by JLP (Jordan Lee Preskill)
- JLP argue that QC is necessary because of the asymptotic nature of perturbation theory (PT) in λ for ϕ^4 and propose to introduce a field cut (but this makes PT convergent! YM PRL 88 (2002))
- Non compact fields methods ($\lambda\phi^4$) see: Macridin, Spentzouris, Amundson, Harnik, PRA 98 042312 (2018) and Klco and Savage arXiv:1808.10378 ...

Tensor Renormalization Group (TRG)

- TRG: first implementation of Wilson program for lattice models with controllable approximations; no sign problems; truncation methods need to be optimized
- Models we considered: Ising model, $O(2)$, $O(3)$, principal chiral models, gauge models (Ising, $U(1)$ and $SU(2)$))
- Used for quantum simulators, measurements of entanglement entropy, central charge, Polyakov's loop ...
- Our group: PRB 87 064422 (2013), PRD 88 056005 (2013), PRD 89 016008 (2014), PRA90 063603 (2014), PRD 92 076003 (2015), PRE 93 012138 (2016) , PRA 96 023603 (2017), PRD 96 034514 (2017), PRL 121 223201 (2018), PRD 98 094511 (2018)
- Basic references for tensor methods for Lagrangian models: Levin and Nave, PRL 99 120601 (2007), Z.C. Gu et al. PRB 79 085118 (2009), Z. Y. Xie et al., PRB 86 045139 (2012)
- Schwinger model/fermions/CP(N): Yuya Shimizu, Yoshinobu Kuramashi; Ryo Sakai, Shinji Takeda; Hikaru Kawauchi.

Important ideas of the tensor reformulation

- In most lattice simulations, the variables of integration are compact and character expansions (such as Fourier series) can be used to rewrite the partition function and average observables as discrete sums of contracted tensors.
- Example: the $O(2)$ model $\mathrm{e}^{\beta\cos(\theta_i-\theta_j)} = \sum_{n_{ij}=-\infty}^{+\infty} \mathrm{e}^{in_{ij}(\theta_i-\theta_j)} I_{n_{ij}}(\beta)$
- This reformulations have been used for RG blocking but they are also suitable for quantum computations/simulations when combined with truncations.
- Important features:
  - Truncations do not break global symmetries
  - Standard boundary conditions can be implemented
  - Matrix Product State ansatzs are exact

TRG blocking: simple and exact!

Character expansion for each link (Ising example):

$$\exp(\beta\sigma_1\sigma_2) = \cosh(\beta)(1 + \sqrt{\tanh(\beta)}\sigma_1\sqrt{\tanh(\beta)}\sigma_2) =$$
$$cosh(\beta) \sum_{n_{12}=0,1} (\sqrt{\tanh(\beta)}\sigma_1\sqrt{\tanh(\beta)}\sigma_2)^{n_{12}}.$$

Regroup the four terms involving a given spin σ_i and sum over its two values ± 1. The results can be expressed in terms of a tensor: $T^{(i)}_{xx'yy'}$ which can be visualized as a cross attached to the site i with the four legs covering half of the four links attached to i. The horizontal indices $x, \ x'$ and vertical indices $y, \ y'$ take the values 0 and 1 as the index n_{12}.

$$T^{(i)}_{xx'yy'} = f_x f_{x'} f_y f_{y'} \delta\left(\text{mod}[x + x' + y + y', 2]\right) \ ,$$

where $f_0 = 1$ and $f_1 = \sqrt{\tanh(\beta)}$. The delta symbol is 1 if $x + x' + y + y'$ is zero modulo 2 and zero otherwise.

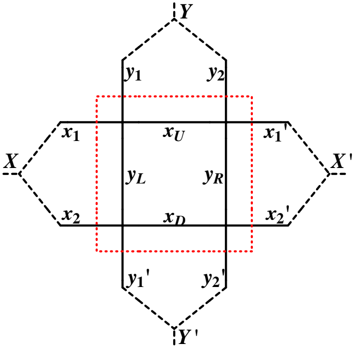
TRG blocking (graphically)

Exact form of the partition function: $Z = (\cosh(\beta))^{2V} \mathrm{Tr} \prod_i T^{(i)}_{xx'yy'}$.

Tr mean contractions (sums over 0 and 1) over the links.

Reproduces the closed paths ("worms") of the HT expansion.

TRG blocking separates the degrees of freedom inside the block which are integrated over, from those kept to communicate with the neighboring blocks. Graphically :



TRG Blocking (formulas)

Blocking defines a new rank-4 tensor $T'_{XX'YY'}$ where each index now takes four values.

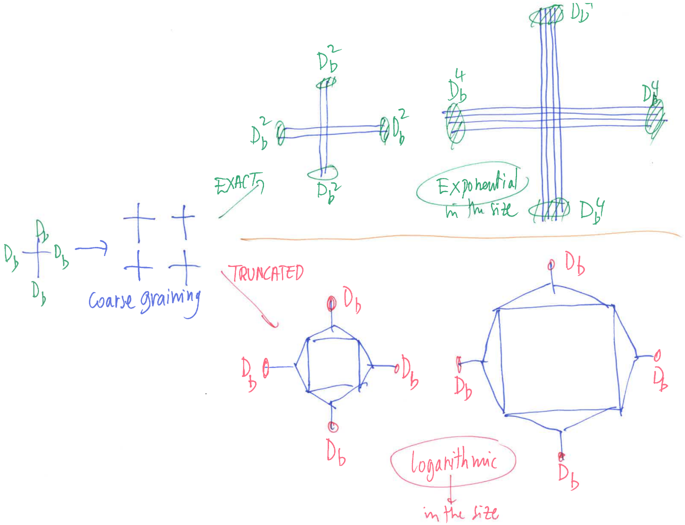
$$T'_{X(x_1,x_2)X'(x'_1,x'_2)Y(y_1,y_2)Y'(y'_1,y'_2)} = \sum_{x_U,x_D,x_R,x_L} T_{x_1 x_U y_1 y_L} T_{x_U x'_1 y_2 y_R} T_{x_D x'_2 y_R y'_2} T_{x_2 x_D y_L y'_1} \ ,$$

where $X(x_2, x_2)$ is a notation for the product states e. g. , $X(0,0) = 1,\ X(1,1) = 2,\ X(1,0) = 3,\ X(0,1) = 4$. The partition function can be written exactly as

$$Z = (\cosh(\beta))^{2V} \mathrm{Tr} \prod_{2i} T'^{(2i)}_{XX'YY'} \ ,$$

where $2i$ denotes the sites of the coarser lattice with twice the lattice spacing of the original lattice. Using a truncation in the number of "states" carried by the indices, we can write a fixed point equation.

TRG is a competitor for QC: CPU time $\propto \log(V)$ with no sign problems (both sides will benefit!)



FAQ: Do truncation break global symmetries? No (Y.M. arXiv 1903.01918)

- Truncations of the tensorial sums are necessary, but do they break the symmetries of the model?
- In arXiv 1903.01918, we consider the tensor formulation of the non-linear O(2) sigma model and its gauged version (the compact Abelian Higgs model), on a D-dimensional cubic lattice, and show that tensorial truncations are compatible with the general identities derived from the symmetries of these models (selection rule ...).
- This selection rule is due to the quantum number selection rules at the sites and is independent of the particular values taken by the tensors. So if we set some of the tensor elements to zero as we do in a truncation, this does not affect the selection rule.
- The universal properties of these models can be reproduced with highly simplified formulations desirable for implementations with quantum computers or for quantum simulations experiments. Truncations are compatible with universality.

The O(2) model (Ising model with spin on a circle)

Integration measure: $\int \mathcal{D}\Phi = \prod_x \int_{-\pi}^{\pi} \frac{d\varphi_x}{2\pi}$

Action: $S[\Phi] = -\beta \sum_{x,i} \cos(\varphi_{x+\hat{i}} - \varphi_x)$ are invariant under

$$\varphi'_x = \varphi_x + \alpha$$

This implies that for a function f of N variables

$$\langle f(\varphi_{x_1}, \ldots, \varphi_{x_N}) \rangle = \langle f(\varphi_{x_1} + \alpha, \ldots, \varphi_{x_N} + \alpha) \rangle$$

Since f is 2π-periodic and can be expressed in terms Fourier modes

$$\langle \mathrm{e}^{(i(n_1\varphi_{x_1} + \ldots n_N\varphi_{x_N}))} \rangle = \mathrm{e}^{((n_1 + \ldots n_N)\alpha)} \langle \mathrm{e}^{(i(n_1\varphi_{x_1} + \ldots n_N\varphi_{x_N}))} \rangle$$

This implies that if

$$\sum_{n=1}^{N} n_i \neq 0, \quad \text{then} \quad \langle \mathrm{e}^{(i(n_1\varphi_{x_1} + \cdots + n_N\varphi_{x_N}))} \rangle = 0.$$

The tensor formulation

At each link, we use the Fourier expansion

$$e^{\beta \cos(\varphi_{x+\hat{i}}-\varphi_x)} = \sum_{n_{x,i}=-\infty}^{+\infty} e^{i n_{x,i}(\varphi_{x+\hat{i}}-\varphi_x)} I_{n_{x,i}}(\beta)$$

where the I_n are the modified Bessel functions of the first kind. After inegrating over the φ:

$$Z = I_0^V(\beta) \mathrm{Tr} \prod_x T^x_{(n_{x-\hat{1},1}, n_{x,1}, \dots, n_{x,D})}.$$

The local tensor T^x has $2D$ indices. The explicit form is

$$T^x_{(n_{x-\hat{1},1}, n_{x,1}, \dots, n_{x-\hat{D},D}, n_{x,D})} = \sqrt{t_{n_{x-\hat{1},1}} t_{n_{x,1}}, \dots, t_{n_{x-\hat{D},D}} t_{n_{x,D}}} \times \delta_{n_{x,out}, n_{x,in}}$$

with $t_n \equiv I_n(\beta)/I_0(\beta)$ and

$$n_{x,in} = \sum_i n_{x-\hat{i},i} \text{ and } n_{x,out} = \sum_i n_{x,i}$$

Current conservation from $\delta_{n_{x,out},n_{x,in}}$

If we interpret the tensor indices $n_{x,i}$ with $i < D$ as spatial current densities and $n_{x,D}$ as a charge density, the Kronecker delta $\delta_{n_{x,out},n_{x,in}}$ in the tensor is a discrete version of Noether current conservation

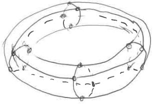
$$\sum_i (n_{x,i} - n_{x-\hat{i},i}) = 0,$$

If we enclose a site x in a small D-dimensional cube, the sum of indices corresponding to positive directions ($n_{x,out}$) is the same as the sum of indices corresponding to negative directions ($n_{x,in}$).

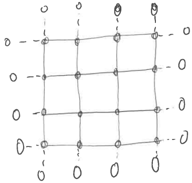
We can “assemble" such elementary objects by tracing over indices corresponding to their interface and construct an arbitrary domain. Each tracing automatically cancels an in index with an out index and consequently, at the boundary of the domain, the sum of the in indices remains the same as the sum of the out indices.

Boundary conditions

- Periodic boundary conditions (PBC) allow us to keep a discrete translational invariance. As a consequence the tensors themselves are translation invariant and assembled in the same way at every site, link etc.
- Open boundary conditions (OBC) can be implemented by introducing new tensors that can be placed at the boundary. The only difference is that there are missing links at sites or missing plaquettes a links (zero index on “missing" links")



PBC



OBC

Explanation of the selection rule (YM 1903.01918)

$$\text{If } \sum_{n=1}^{N} n_i \neq 0, \quad \text{then} \quad \langle \mathrm{e}^{(i(n_1\varphi_{x_1}+\cdots+n_N\varphi_{x_N}))}\rangle = 0.$$

The insertion of various $\mathrm{e}^{in_Q\varphi_x}$ is required in order to calculate the averages function $\langle \mathrm{e}^{(i(n_1\varphi_{x_1}+\cdots+n_N\varphi_{x_N}))}\rangle$ This can be done by inserting an "impure" tensor which differs from the "pure" tensor by the Kronecker symbol replacement $\delta_{n_{x,out},n_{x,in}} \rightarrow \delta_{n_{x,out},n_{x,in}+n_Q}$.

In absence of insertions of $\mathrm{e}^{in_Q\varphi_x}$, the Kronecker delta at the sites leads to a global conservation (sum in = sum out).

We can now repeat this procedure with insertions of $\mathrm{e}^{in_Q\varphi_x}$. Each insertion adds n_Q, which can be positive or negative, to the sum of the out indices. We can apply this bookkeeping on an existing tensor configuration until we have gathered all the insertions and we reach the boundary of the system.

For PBC, this means that all the in and out indices get traced in pairs at the boundary. This is only possible if the sum of the inserted charges is zero which is the content of Eq. (25). For OBC, all the boundary indices are zero and the same conclusion apply.

In summary we have shown that the selection rule is a consequence of the Kronecker delta appearing in the tensor and is independent of the particular values taken by the tensors. So if we set some of the tensor elements to zero as we do in a truncation, this does not affect the selection rule.

Algebraic aspects (in one dimension)

In the Hamiltonian formalism, we introduce the angular momentum eigenstates which are also energy eigenstates

$$\hat{L}|n\rangle = n|n\rangle, \hat{H}|n\rangle = \frac{n^2}{2}|n\rangle$$

We assume that n can take any integer value from $-\infty$ to $+\infty$. As $\hat{H} = (1/2)\hat{L}^2$, it is obvious that $[\hat{L}, \hat{H}] = 0$.

The insertion of $\mathrm{e}^{i\varphi_x}$ in the path integral, translates into as operator $\widehat{\mathrm{e}^{i\varphi}}$ which raises the charge $\widehat{\mathrm{e}^{i\varphi}}|n\rangle = |n+1\rangle$, while its Hermitean conjugate lowers it $(\widehat{\mathrm{e}^{i\varphi}})^\dagger|n\rangle = |n-1\rangle$.

This implies the commutation relations

$$[L, \widehat{\mathrm{e}^{i\varphi}}] = \widehat{\mathrm{e}^{i\varphi}},\ [L, \widehat{\mathrm{e}^{i\varphi}}^\dagger] = -\widehat{\mathrm{e}^{i\varphi}}^\dagger, [\widehat{\mathrm{e}^{i\varphi}}, \widehat{\mathrm{e}^{i\varphi}}^\dagger] = 0.$$

Truncation effects on algebra

By truncation we mean that there exists some n_{max} for which

$$\widehat{e^{i\varphi}}|n_{max}\rangle = 0, \text{and } (\widehat{e^{i\varphi}})^\dagger| - n_{max}\rangle = 0.$$

The only changes the commutation relations are

$$\begin{aligned} \langle n_{max}|[\widehat{e^{i\varphi}}, \widehat{e^{i\varphi}}^\dagger]|n_{max}\rangle &= 1, \\ \langle -n_{max}|[\widehat{e^{i\varphi}}, \widehat{e^{i\varphi}}^\dagger]| - n_{max}\rangle &= -1, \end{aligned} \quad (1)$$

instead of 0. The truncation only affects matrix elements involving the $\widehat{e^{i\varphi}}$ operators but does not contradict that: If $\sum_{n=1}^{N} n_i \neq 0$, then $\langle 0|(\widehat{e^{i\varphi}})^{n_1} \dots (\widehat{e^{i\varphi}})^{n_N}|0\rangle = 0$ (with $(\widehat{e^{i\varphi}})^{-n} \equiv (\widehat{e^{i\varphi}}^\dagger)^n$ for $n > 0$))
Note: similar questions appear in quantum links formulations (see R. Brower, The QCD Abacus, hep-lat/9711027)

Example 2: the compact Abelian Higgs model

This is a gauged $O(2)$ model with gauge fields on the links $A_{x,\hat{i}}$.

$$\int \mathcal{D}\Phi = \prod_x \int_{-\pi}^{\pi} \frac{d\varphi_x}{2\pi} \prod_{x,i} \int_{-\pi}^{\pi} \frac{dA_{x,i}}{2\pi}.$$

$$S = -\beta \sum_{x,i} \cos(\varphi_{x+\hat{i}} - \varphi_x + A_{x,i}) - \beta_p \sum_{x,i<j} \cos(A_{x,i} + A_{x+\hat{i},j} - A_{x+\hat{i}+\hat{j},i} - A_{x,j}).$$

The symmetry of the $O(2)$ model becomes local

$$\varphi'_x = \varphi_x + \alpha_x \text{ and } A'_{x,i} = A_{x,i} - (\alpha_{x+\hat{i}} - \alpha_x),$$

Truncations do not break these symmetries (Y. M. arXiv 1903.01918).
For Hamiltonian and optical lattice implementations see: Phys. Rev. D 92, 076003 (2015), Phys. Rev. Lett. 121, 223201 (2018)

TRG Formulation of 3D Z_2 Gauge Theory

$$Z = \sum_{\{\sigma\}} \exp\left(\beta \sum_{P} \sigma_{12}\sigma_{23}\sigma_{34}\sigma_{41}\right),$$

For each plaquette the weight is

$$\sum_{n=0,1} (\sqrt[4]{\tanh(\beta)}\sigma_{12}\sqrt[4]{\tanh(\beta)}\sigma_{23}\sqrt[4]{\tanh(\beta)}\sigma_{34}\sqrt[4]{\tanh(\beta)}\sigma_{41})^n.$$

Regrouping the factors with a given σ_l and summing over ± 1 we obtain a tensor attached to this link

$$A^{(l)}_{n_1 n_2 n_3 n_4} = \left(\sqrt[4]{\tanh\beta}\right)^{n_1+n_2+n_3+n_4} \times \delta\left(\mathrm{mod}[n_1+n_2+n_3+n_4, 2]\right).$$

A and B tensors

The four links attached to a given plaquette p must carry the same index 0 or 1. For this purpose we introduce a new tensor

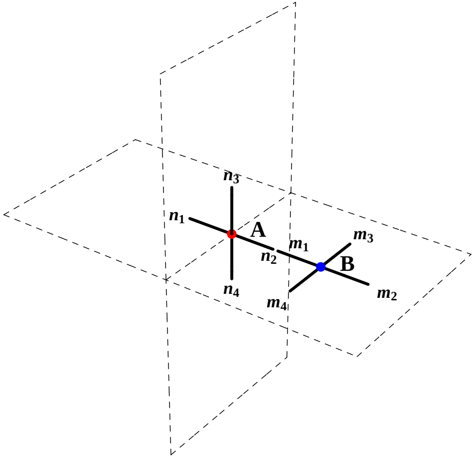
$$
\begin{aligned}
B^{(p)}_{m_1 m_2 m_3 m_4} &= \delta(m_1, m_2, m_3, m_4) \\
&= \begin{cases} 1, & \text{all } m_i \text{ are the same} \\ 0, & \text{otherwise.} \end{cases}
\end{aligned}
$$

The partition function can now be written as

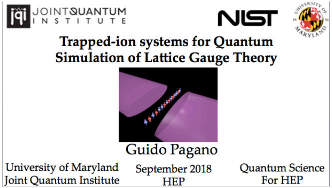
$$
Z = (2\cosh\beta)^{3V} \mathrm{Tr} \prod_l A^{(l)}_{n_1 n_2 n_3 n_4} \prod_p B^{(p)}_{m_1 m_2 m_3 m_4},
$$

Note: one can move the $\tanh(\beta)$ from links to plaquettes

A and B tensors graphically



Schwinger model with trapped ions



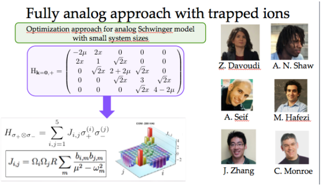
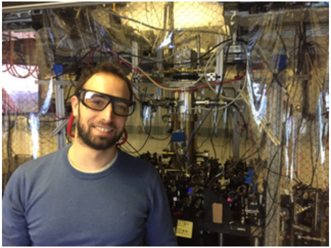


Figure: From Guido Pagano talk at Fermilab.

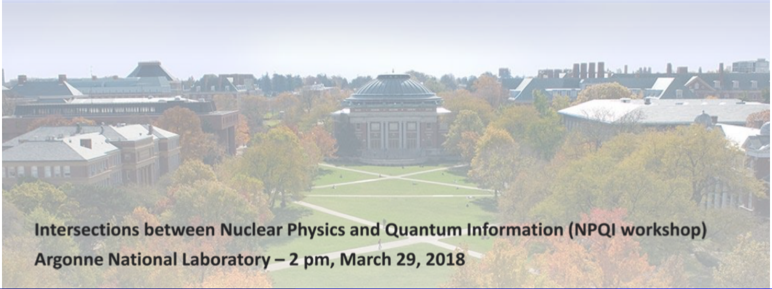


Dynamical gauge fields with cold atoms and molecules at UIUC

Overview of QIS and Quantum Emulation studies at University of Illinois

>> ***Prospects for studying dynamical gauge fields with cold atoms and molecules***

Bryce Gadway [Paul Kwiat, Virginia Lorenz, Brian DeMarco]
University of Illinois at Urbana-Champaign



The Abelian Higgs model on a 1+1 space-time lattice

a.k.a. lattice scalar electrodynamics. Field content:

- Complex (charged) scalar field $\phi_x = |\phi_x| e^{i\theta_x}$ on space-time sites x
- Abelian gauge fields $U_{x,\mu} = \exp iA_\mu(x)$ on the links from x to $x + \hat{\mu}$
- $F_{\mu\nu}$ appears when taking products of U's around an elementary square (plaquette) in the $\mu\nu$ plane
- Notation for the plaquette: $U_{x,\mu\nu} = e^{i(A(x)_\mu + A(x+\hat{\mu})_\nu - A(x+\hat{\nu})_\mu - A(x)_\nu)}$
- $\beta_{pl.} = 1/e^2$ and κ is the hopping coefficient

$$\begin{aligned} S &= -\beta_{pl.} \sum_x \sum_{\nu<\mu} \mathrm{Re}\mathrm{Tr}\left[U_{x,\mu\nu}\right] + \lambda \sum_x \left(\phi_x^\dagger \phi_x - 1\right)^2 + \sum_x \phi_x^\dagger \phi_x \\ &- \kappa \sum_x \sum_{\nu=1}^{d} \left[e^{\mu_{ch.}\delta(\nu,t)} \phi_x^\dagger U_{x,\nu} \phi_{x+\hat{\nu}} + e^{-\mu_{ch.}\delta(\nu,t)} \phi_{x+\hat{\nu}}^\dagger U_{x,\nu}^\dagger \phi_x \right]. \end{aligned}$$

$$Z = \int D\phi^\dagger D\phi D U e^{-S}$$

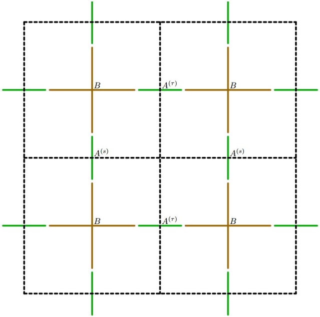
Unlike other approaches (Reznik, Zohar, Cirac, Lewenstein, Kuno,....) we will not try to implement the gauge field on the optical lattice.

Gauge-invariant tensor form: $Z = Tr[\prod T]$

(see PRD.88.056005 and PRD.92.076003)

$$Z =\propto \mathrm{Tr}\left[\prod_{h,v,\Box} A^{(s)}_{m_{up} m_{down}} A^{(\tau)}_{m_{right} m_{left}} B^{(\Box)}_{m_1 m_2 m_3 m_4}\right].$$

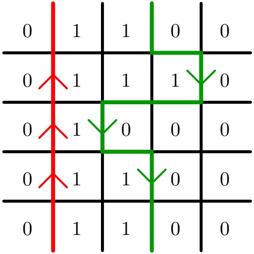
The traces are performed by contracting the indices as shown

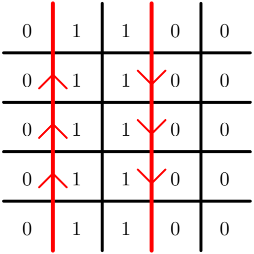


Polyakov loop: definition

Polyakov loop, a Wilson line wrapping around the Euclidean time direction: $\langle P_i \rangle = \langle \prod_j U_{(i,j),\tau} \rangle$ =exp($-F(\text{single charge})/kT$); the order parameter for deconfinement.

With periodic boundary condition, the insertion of the Polyakov loop (red) forces the presence of a scalar current (green) in the opposite direction (left) or another Polyakov loop (right).





In the Hamiltonian formulation, we add $-\frac{\tilde{Y}}{2}(2(\bar{L}^z_{i^\star} - \bar{L}^z_{(i^\star+1)}) - 1)$ to H.

Universal functions (FSS): the Polyakov loop

arXiv:1803.11166 (Phys. Rev. Lett. 121, 223201) and
arXiv:1807.09186 (Phys. Rev. D 98, 094511)

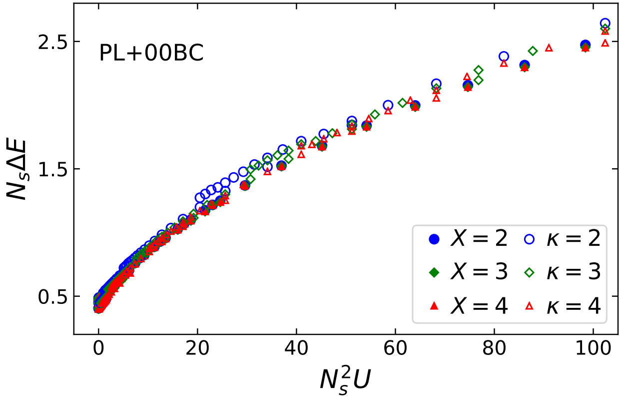


Figure: Data collapse of $N_s \Delta E$ defined from the insertion of the Polyakov loop, as a function of $N_s^2 U$, or $(N_s g)^2$ (collapse of 24 datasets). Numerical work by Judah Unmuth-Yockey and Jin Zhang.

Optical lattice implementation with a ladder

After taking the time continuum limit:

$$\bar{H} = \frac{\tilde{U}_g}{2} \sum_i \left(\bar{L}^z_{(i)}\right)^2 + \frac{\tilde{Y}}{2} \sum_i (\bar{L}^z_{(i)} - \bar{L}^z_{(i+1)})^2 - \tilde{X} \sum_i \bar{L}^x_{(i)}$$

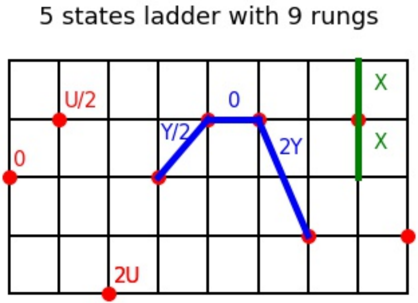


Figure: Ladder with one atom per rung: tunneling along the vertical direction, no tunneling in the the horizontal direction but short range attractive interactions. A parabolic potential is applied in the spin (vertical) direction.

A first quantum simulator for the abelian Higgs model?

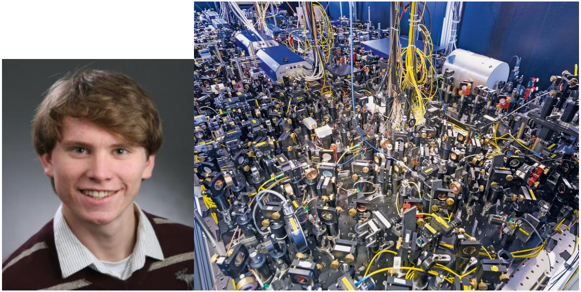


Figure: Left: Johannes Zeiher, a recent graduate from Immanuel Bloch's group can design ladder shaped optical lattices with nearest neighbor interactions. Right: an optical lattice experiment of Bloch's group.

Concrete Proposal

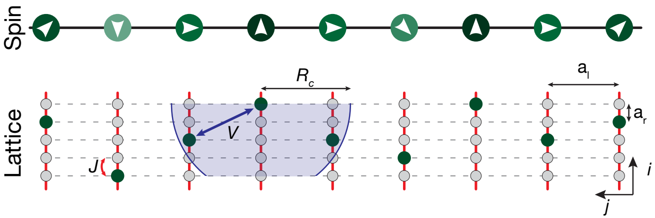
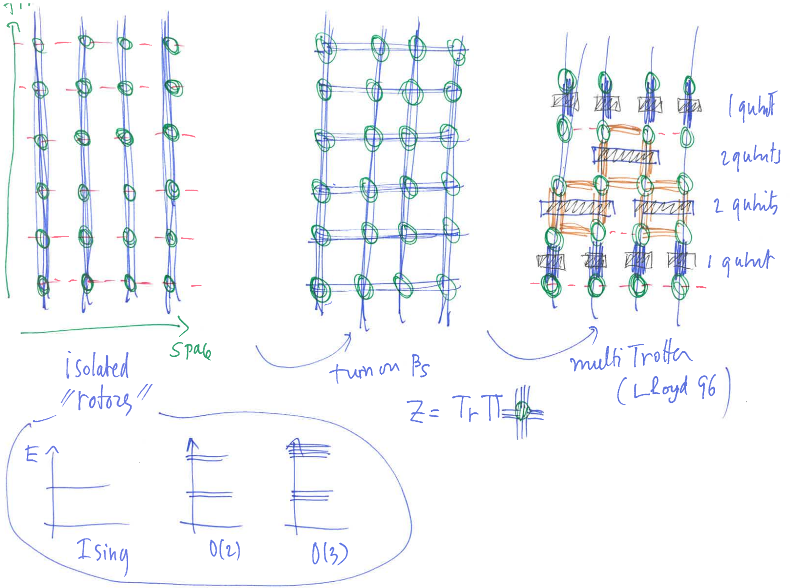


Figure: Multi-leg ladder implementation for spin-2. The upper part shows the possible m_z-projections. Below, we show the corresponding realization in a ladder within an optical lattice. The atoms (green disks) are allowed to hop within a rung with a strength J, while no hopping is allowed along the legs. The lattice constants along rung and legs are a_r and a_l respectively. Coupling between atoms in different rungs is implemented via an isotropic Rydberg-dressed interaction V with a cutoff distance R_c (marked by blue shading).

From tensors to circuits



Time evolution with QC: IBM and Rigetti

PHYSICAL REVIEW A **98**, 032331 (2018)

Quantum-classical computation of Schwinger model dynamics using quantum computers

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(Received 22 March 2018; published 28 September 2018)

We present a quantum-classical algorithm to study the dynamics of the two-spatial-site Schwinger model on IBM's quantum computers. Using rotational symmetries, total charge, and parity, the number of qubits needed to perform computation is reduced by a factor of $\sim$5, removing exponentially large unphysical sectors from the Hilbert space. Our work opens an avenue for exploration of other lattice quantum field theories, such as quantum chromodynamics, where classical computation is used to find symmetry sectors in which the quantum computer evaluates the dynamics of quantum fluctuations.

PHYSICAL REVIEW LETTERS **121**, 170501 (2018)

Simulation of Nonequilibrium Dynamics on a Quantum Computer

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(Received 21 June 2018; revised manuscript received 6 September 2018; published 22 October 2018)

We present a hybrid quantum-classical algorithm for the time evolution of out-of-equilibrium thermal states. The method depends on classically computing a sparse approximation to the density matrix and, then, time-evolving each matrix element via the quantum computer. For this exploratory study, we investigate a time-dependent Ising model with five spins on the Rigetti Forest quantum virtual machine and a one spin system on the Rigetti 8Q-Agave quantum processor.

Quantum circuit for the quantum Ising model

Quantum circuit with 3 Trotter steps (arXiv:1901.05944 E. Gustafson, YM and J. Unmuth-Yockey)

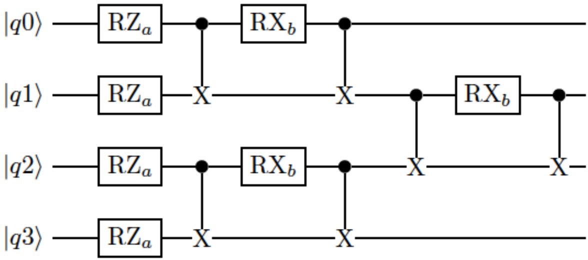


Figure 1: Circuit for 4 qubits with open boundary conditions

Trotter Fidelity

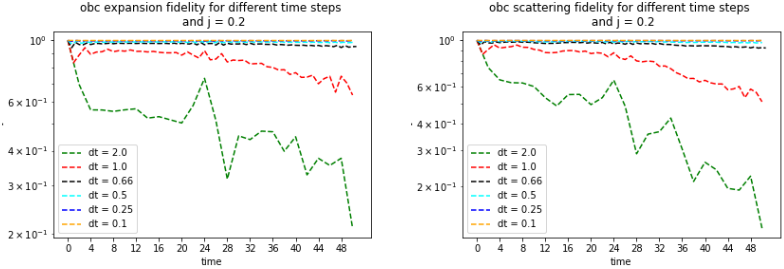


Figure: fidelity of Trotter operator at multiple different Trotter steps for (left to right) expansion and scattering with open boundary conditions (E. Gustafson, YM and J. Unmuth-Yockey arXiv:1901.05944)

Approximate quantum mechanical picture

- $J = 0$: energy is the sum of the “on-site" energies.
- unique ground state $E_0 = -N_s h_T$.
- degenerate “one-particle" states where one on-site state with energy $+h_T$ can be placed at N_s locations.
- If the $+h_T$ energy is located at the site j, we call this state $|j\rangle$.
- nearest neighbor interactions can be included perturbatively. At order J in the one-particle sector, we have a hopping that stays in the one-particle sector.
- If periodic boundary conditions are imposed, Fourier modes diagonalize the perturbation.
- This lift the degeneracy by a term $-2J\cos(m2\pi/N_s)$ (kinetic energy).
- The perturbation also contains operators that connect to the 3-particles states. This leads to energy shifts of order J^2/h_T.
- If we neglect these second order effects (pair creation), we have a simple approximate quantum mechanical behavior.

Approximate quantum mechanical picture

We can then prepare the system in an initial state $|\psi\rangle$ and calculate $\langle\Psi(t)|n_l|\Psi(t)\rangle$ with $n_l = (1-\sigma^x_l)/2$ (the particle occupation) in the original basis.

For instance, for $|\Psi(0)\rangle = |j\rangle = |0,0,0,\ldots,0,1_j,0,\ldots\rangle$, we obtain

$$\langle\Psi(t)|n_l|\Psi(t)\rangle \simeq |J_{l-j}(2Jt)|^2,$$

where the (discrete) Bessel functions are defined as

$$J_n(2Jt) = (1/N_s)(-i)^n \sum_{m=0}^{N_s-1} \exp(i((2\pi/N_s)mn + 2Jt\cos((2\pi/N_s)m)),$$

which corresponds to the usual definition in the limit of large N_s.

Perturbation theory vs. exact diagonalization

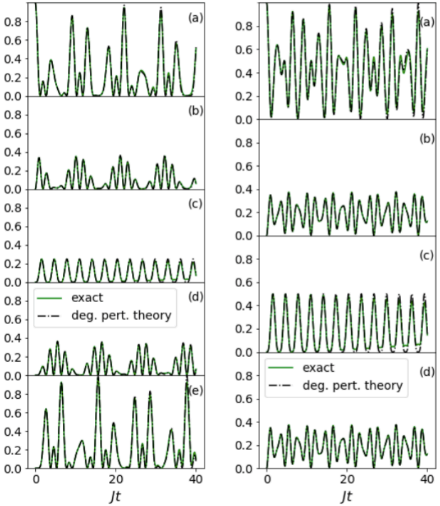
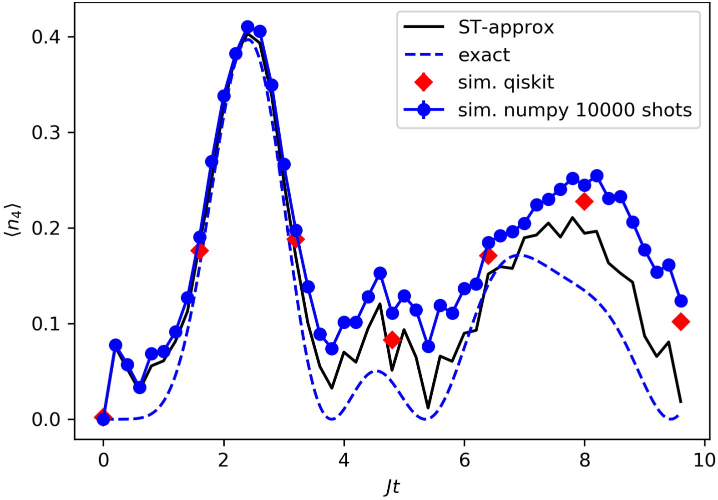


Figure: Comparison of exact diagonalization and degenerate perturbation theory at various sites for one and two particle processes (arXiv:1901.05944))

Syst. and stat. errors (1901.05944, PRD in press)



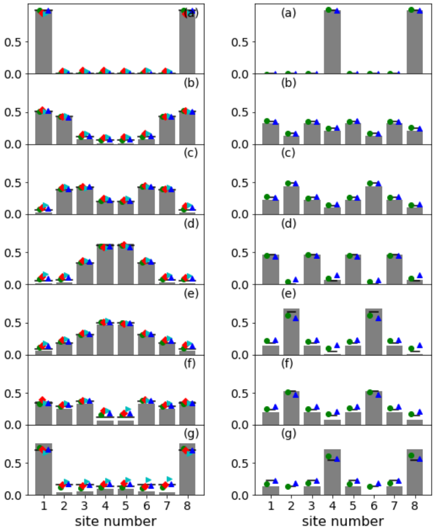


Figure: Evolution of two-particle initial states with OBC (Left) and PBC (Right). Simulations with QISKIT and numpy for current trapped ions or near future superconducting qubits (arXiv:1901.05944).

Quantum Joule expansions (arXiv:1903.01414)

Quantum Joule Expansion of One-dimensional Systems

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[2] *Department of Physics and Astronomy, University of Iowa, Iowa City, IA 52242, USA*

(Dated: March 5, 2019)

We investigate the Joule expansion of nonintegrable quantum systems that contain bosons or fermions in one-dimensional lattices. A barrier initially confines the particles to be in half of the system in a thermal state described by the canonical ensemble. At long times after the barrier is removed, few-body observables can be approximated by a thermal expectation of another canonical ensemble with an effective temperature. The weights for the diagonal ensemble and the canonical ensemble match well for high initial temperatures that correspond to negative effective final temperatures after the expansion. The negative effective temperatures for finite systems go to positive inverse temperatures in the thermodynamic limit for bosons, but is a true thermodynamic effect for fermions. We compare the thermal entanglement entropy and density distribution in momentum space for the canonical ensemble, diagonal ensemble and instantaneous long-time states calculated by exact diagonalization. We propose the Joule expansion as a way to dynamically create negative temperature states for fermion systems.

I. INTRODUCTION

With the remarkable advances in efficient computing algorithms and cold atom experiments, nonequilibrium dynamics has been extensively studied both theoretically and experimentally in recent years. Quantum thermalization is one of the most important topics in this area. In 1929, the classical theory of statistical mechanics was reformulated quantum-mechanically by von Neumann [1], which opens the door to the study of quantum thermalization through the unitary dynamics of quantum sys-

states [18, 23].

A celebrated experiment in the context of classical statistical mechanics is the Joule expansion. The Joule expansion (free expansion) of an ideal gas from an initial volume V to a final volume $2V$ does not change the temperature of the gas and the increase in entropy is $nR\ln 2$. For interacting gases, the temperature decreases for attractive interactions, such as for the van der Waals gas, and increases for repulsive interactions. But what happens for quantum systems? The Joule expansion of an isolated perfect quantum gas is discussed in [24], where

Temperature before and after expansion (graphs by Jin Zhang)

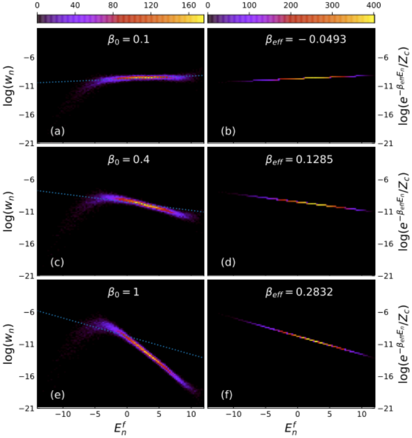

FIG. 1. (Color online) Two-dimensional histograms for the weights of eigenstates in the DE, W_n, (a,c,e) and those in the corresponding CE $e^{-\beta_{eff} E_n}/Z(\beta_{eff})$ (b,d,f). Results are for spinless fermions with 20 sites, 5 particles and $J_2 = V_2 = 1$.

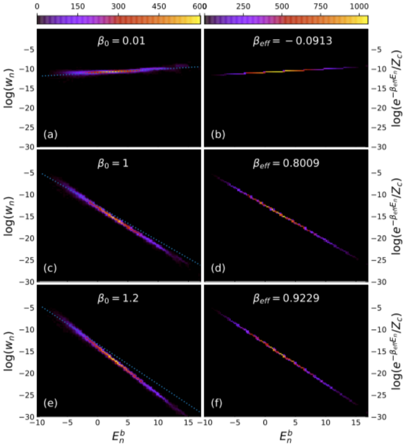

FIG. 2. (Color online) Same as Fig. 1, but for bosons with 20 sites, 5 particles and $U = 3$.

Pitch for a Center dedicated to Quantum Simulations and Computations motivated by Theoretical Physics

This results from discussions with PIs T. Bhattacharya, M. Carena, B. Gadway, Y. Meurice, J. Thaler and QMAP/UC Davis (A. Albrecht).

- Goals: Foster a US Quantum Computation community focused on theoretical physics and provide dedicated facilities for quantum simulations
- Fields of research: high-energy, nuclear and condensed matter physics, gravitation and cosmology
- Resources needed: A dedicated quantum simulation facility hosted by a national lab with administrative support for educational and research needs. Support for visits, workshops and internships
- Student education:
  - Courses in different institutions attended remotely
  - Weekly seminars given by junior scientists and attended remotely
  - Summer schools
  - Fellowships for visits and residencies at national labs
  - Internships in quantum computing industry

Quantum Simulations and Computations for Theoretical Physics

- Community of Faculty and Scientists in QC
  - Weekly research seminars (attended remotely)
  - Frequent workshops (attended in person)
  - Funded Visits/sabbaticals at labs
- Experimental component: quantum simulation experiments focused on specific theoretical physics problems (lattice gauge theory, models of superconductivity, many-body problems ...)
- Broad interest for the concept expressed by individuals from:
  - Labs : Fermilab, BNL, Argonne, Los Alamos, ...
  - Universities : Boston U., U. Chicago, U. Illinois, U. Iowa, U. Maryland, MIT, MSU, Stanford, Syracuse U., QMAP/UC Davis , UC Riverside, UCSB, INT, U. Washington, ...
- Labs are already involved with the QC industry (IBM, Google,..)
- Milestones: proof of principle in 5 years, new results in 10 years, having longer term objectives helps balance the risks

Conclusions

- QC/QIS in HEP and NP: we need big goals with many intermediate steps
- Tensor Field Theory is a generic tool to discretize path integral formulations of lattice model with compact variables
- TRG: exact blocking, a friendly competitor to QCD; we have all the basic blocks for QCD; sampling is also possible (Gattringer ...)
- Truncations respect symmetries
- TRG: gauge-invariant approach for the quantum simulation of gauge models.
- Finite size scaling: small systems are interesting
- Real time scattering can be calculated with digital or analog methods (comparison is possible)
- Need for quantum simulations and computations dedicated to theoretical physics
- Thanks!

Thanks for listening

Acknowledgements:

This research was supported in part by the Dept. of Energy under Award Numbers DOE grants DE-SC0010113, and DE-SC0019139