

# Sextet BSM model 2019

with the Lattice Higgs Collaboration (LatHC)

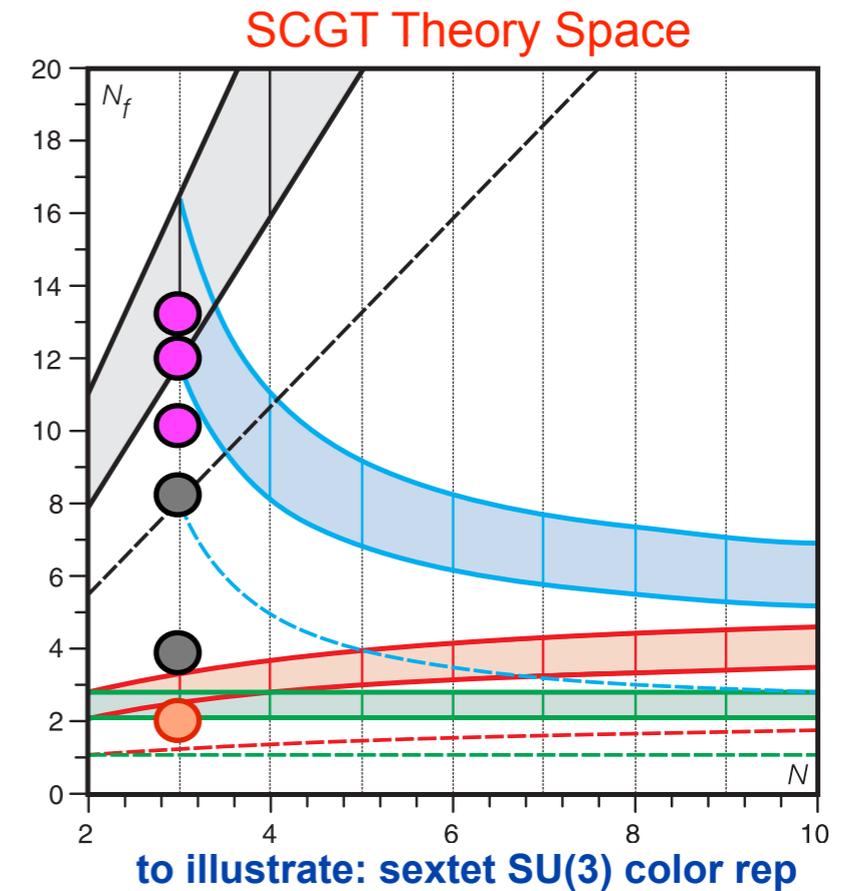
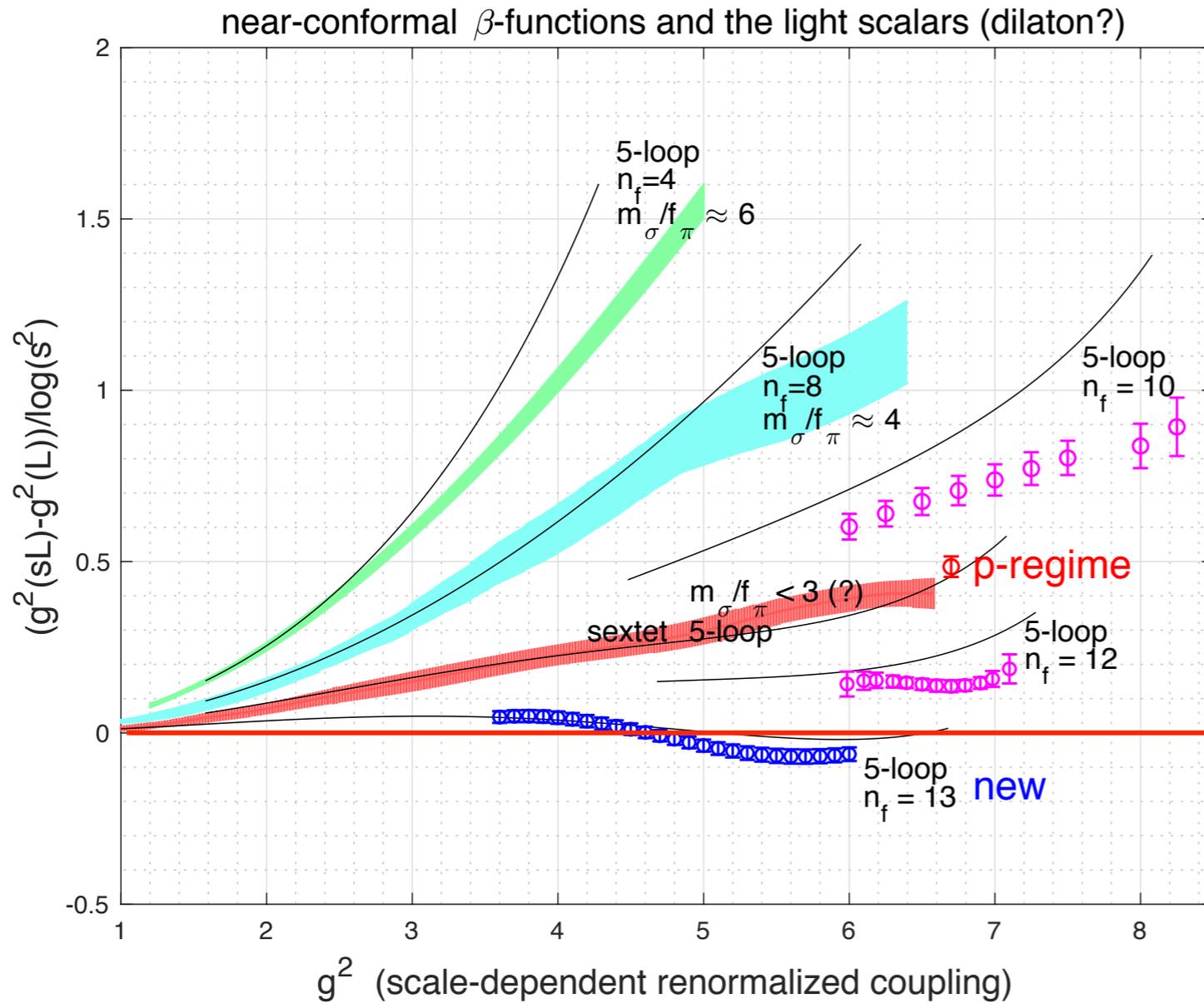
Julius Kuti

University of California, San Diego

USQCD All Hands Meeting

BNL April 26-27, 2019

# road map of near-conformal $\beta$ -functions and emergent light scalars...



light  $0^{++}$  scalar emerging

one massless fermion doublet  $\begin{bmatrix} u \\ d \end{bmatrix}$

$\chi$ SB on  $\Lambda \sim \text{TeV}$  scale

three Goldstone pions

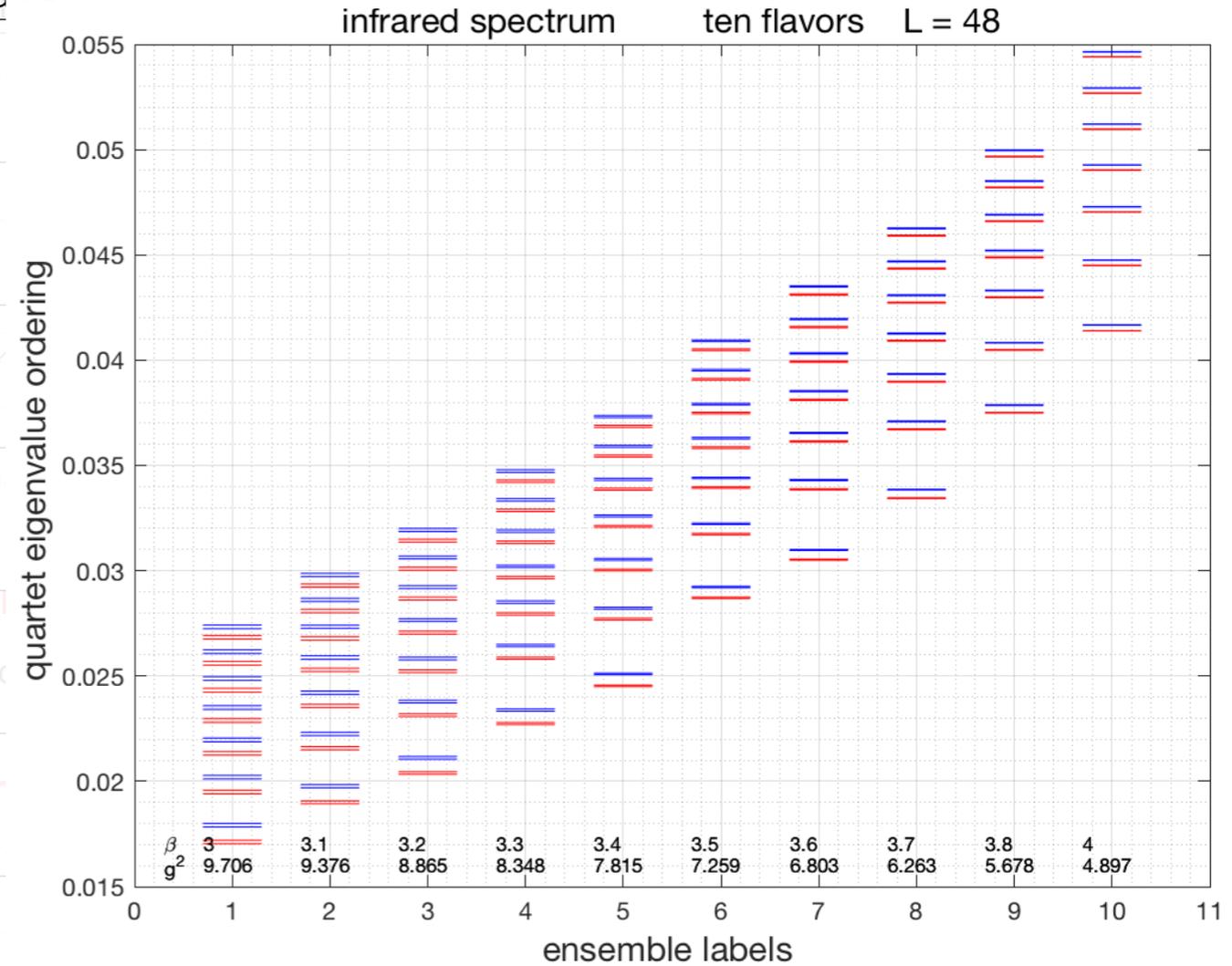
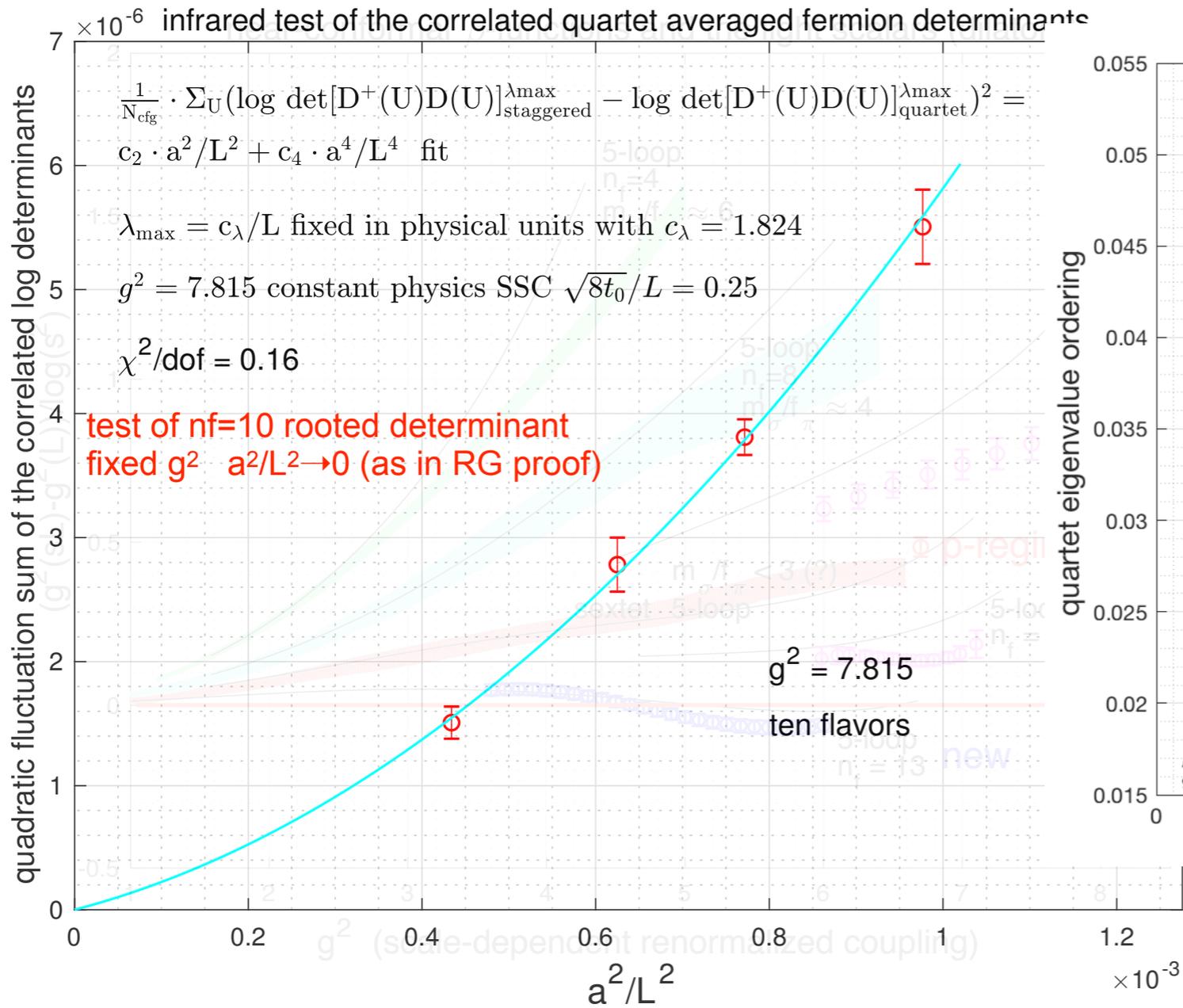
become longitudinal components of weak bosons

if Susskind and Weinberg only knew ...

composite Higgs mechanism

- spoiler alert: controversies around the  $n_f=10$  and  $n_f=12$   $\beta$ -functions
- LatHC reports  $n_f=13$  conformal IRFP — journal publications in preparation

# road map of near-conformal $\beta$ -functions and emergent light scalars...



- two zeros of 5-loop  $\beta$ -function collide between nf=12 and nf=13 and turn into complex conjugate pair of zeros
- sextet model closest to CW among near-conformal (except perhaps nf=12?) complex pair of CFT?
- what are the tantalizing BSM field theory questions?

## tantalizing big questions in the sextet case study:

How light is the emergent  $0^{++}$  scalar?

Needs the origin of near-conformal walking?

Needs IR EFT like  $\sigma$ -model, or dilaton, or ...?

IR EFT needs new strategy to reach massless chiral limit

new walking paradigm?

with this motivation, review of our recent results and the proposed plan, but:

Tantalus, a king of ancient Phrygia in Greek mythology, made the mistake of gravely offending the gods. As a punishment, once dead the king was forced to stand in a pool of water, with fruit hanging just over his head. The water would recede every time the king tried to take a sip, and the fruit would lift away every time he reached to take a bite.

# origin of the emergent light 0++ scalar $\sigma$ -particle?

if Higgs-like particle of linear  $\sigma$ -model:

$SU(2) \otimes SU(2) \sim O(4)$  for sextet model

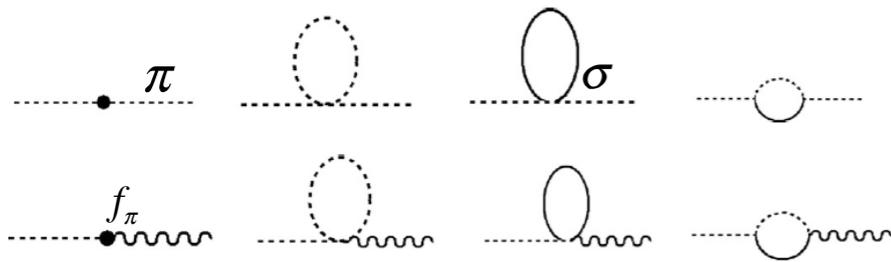
$$L = \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}\mu^2(\sigma^2 + \vec{\pi}^2) + \frac{1}{4}g(\sigma^2 + \vec{\pi}^2)^2 - \varepsilon\sigma$$

triviality analysis in  $m_\sigma/f_\pi < 3$  range  
 $m_\pi = 0$  circa 1987-1988

$m_\sigma^2 \geq 3m_\pi^2$  tree level relation

$m_\sigma^2 \geq 2m_\pi^2$  with loop corrections

non-linear  $\sigma$ -model  $\rightarrow$  dilaton EFT:



p-regime data: 0++ is tracking the Goldstone pion with  $m_\pi^2 \geq m_\sigma^2$ , not like linear  $\sigma$ -model  
 New EFT is needed to extrapolate data to massless chiral limit

$$L = \frac{1}{2}\partial_\mu \sigma \partial_\mu \sigma - V(\sigma) + \frac{f_\pi^2}{4}(D_\mu \Sigma^\dagger D_\mu \Sigma) \cdot \left(1 + 2a \frac{\sigma}{f_\pi} + b \frac{\sigma^2}{f_\pi^2} + b_3 \frac{\sigma^3}{f_\pi^3} + \dots\right)$$

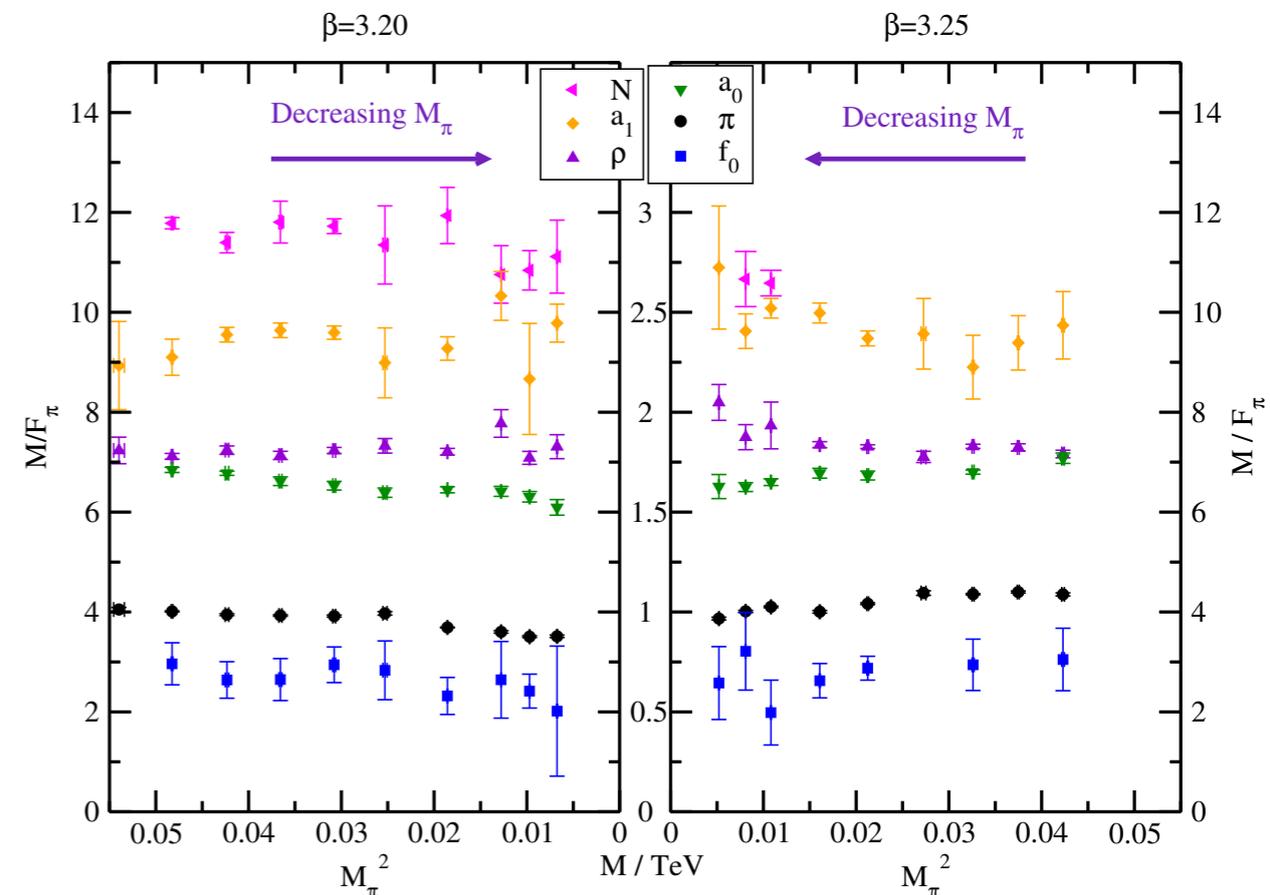
$\Sigma = e^{i\pi^a \tau^a / f_\pi}$  with  $\tau^a$  Pauli matrices

$$V(\sigma) = \frac{1}{2}m_\sigma^2 \cdot \sigma^2 + d_3 \left(\frac{m_\sigma^2}{2f_\pi}\right) \cdot \sigma^3 + d_4 \left(\frac{m_\sigma^2}{8f_\pi^2}\right) \cdot \sigma^4 + \dots$$

linear  $\sigma$ -model limit (SM):  $a = b = d_3 = d_4 = 1$

dilaton EFT will require  $a = b^2$ ,  $b_3 = 0$  and special  $y(\mu)$  in  $L$

analyze dilaton EFT first and test if non-linear  $\sigma$ -model limit exists without special anomalous dimension  $y(M)$  in  $L$



## dilaton EFT with $\sigma(x)$ dilaton field and $\pi^a(x)$ Goldstone bosons of sextet flavor doublet:

$$L = \frac{1}{2} \partial_\mu \chi \partial_\mu \chi - V_d(\chi) + \frac{f_\pi^2}{4} \left( \frac{\chi}{f_d} \right)^2 \text{tr} \left[ \partial_\mu \Sigma^\dagger \partial_\mu \Sigma \right] - \frac{f_\pi^2 m_\pi^2}{4} \left( \frac{\chi}{f_d} \right)^y \text{tr} (\Sigma + \Sigma^\dagger)$$

$y = 3 - \gamma$  where  $\gamma$  is the mass anomalous dimension

$\chi(x) = f_d e^{\sigma(x)/f_d}$  describes the dilaton field  $\sigma(x)$

pion field  $\Sigma = e^{i\pi^a \tau^a / f_\pi}$  with  $\tau^a$  Pauli matrices, tree level pion mass  $m_\pi^2 = 2Bm$

literature has long history  
Golterman-Shamir

Appelquist et al.

we do our own analysis

Appelquist et al.

notation for comparison

$$V_\sigma = \frac{m_d^2}{2f_d^2} \left( \frac{\chi^2}{2} - \frac{f_d^2}{2} \right)^2 \quad \text{relevant deformation of IRFP theory}$$

$$V_d = \frac{m_d^2}{16f_d^2} \chi^4 \left( 4 \ln \frac{\chi}{f_d} - 1 \right) \quad \text{nearly marginal deformation}$$

}	$f_\pi$	Goldstone decay constant
}	$m_\pi = 2mB$	Goldstone pions
}	$f_d$	dilaton decay constant
}	$m_d$	dilaton mass
}	$F_\pi, M_\pi, F_d, M_d$ with mass deformation	

$$M_\pi^2 \cdot F_\pi^{2-y} - 2B_\pi \cdot f_\pi^{(2-y)} \cdot m = 0 \quad \text{general V indep. scaling law}$$

$$F_\pi^{(4-y)} \cdot (1 - f_\pi^2 / F_\pi^2) - 2y \cdot n_f f_\pi^{(6-y)} B_\pi / m_d^2 f_d^2 \cdot m = 0 \quad \boxed{V'_\sigma(\chi = F_d)}$$

$$3F_\pi^2 / M_\pi^2 - f_\pi^2 / M_\pi^2 - 2M_d^2 / m_d^2 \cdot f_\pi^2 / M_\pi^2 - y(y-1) n_f f_\pi^4 / m_d^2 f_d^2 = 0 \quad \boxed{V''_\sigma(\chi = F_d)}$$

$$F_\pi^{(4-y)} \cdot \log(F_\pi / f_\pi) - y \cdot n_f f_\pi^{(6-y)} B_\pi \cdot m / m_d^2 f_d^2 = 0 \quad \boxed{V'_d(\chi = F_d)}$$

$$(F_\pi^2 / M_\pi^2) \cdot (3 \log(F_\pi / f_\pi) + 1) - (M_d^2 / m_d^2) \cdot (f_\pi^2 / M_\pi^2) - y(y-1) n_f f_\pi^4 / 2m_d^2 f_d^2 = 0 \quad \boxed{V''_d(\chi = F_d)}$$

$M_\pi, F_\pi, M_d$  input data at each  $m$

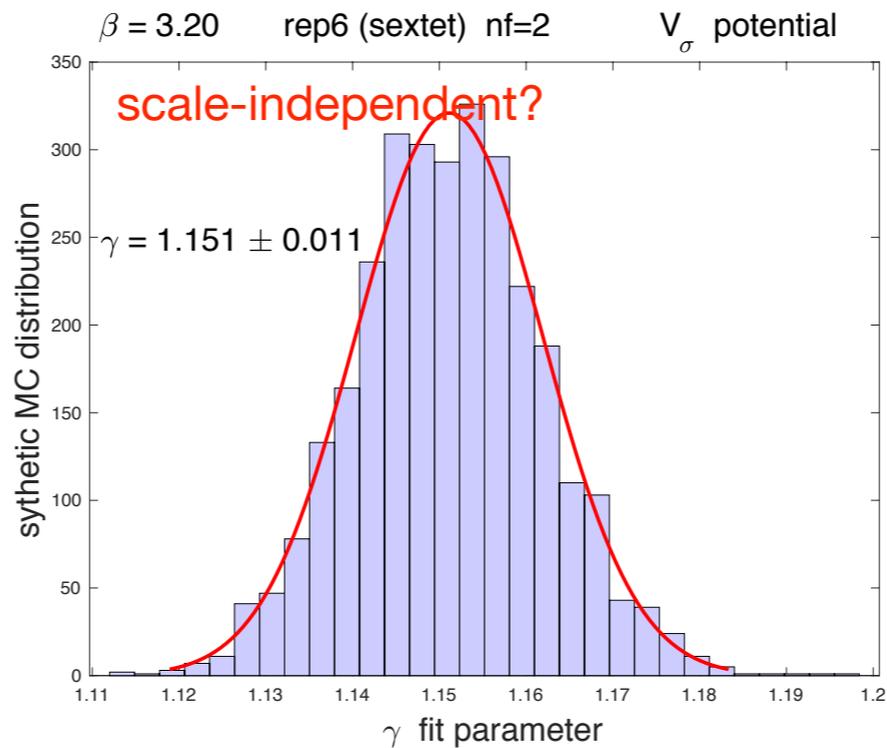
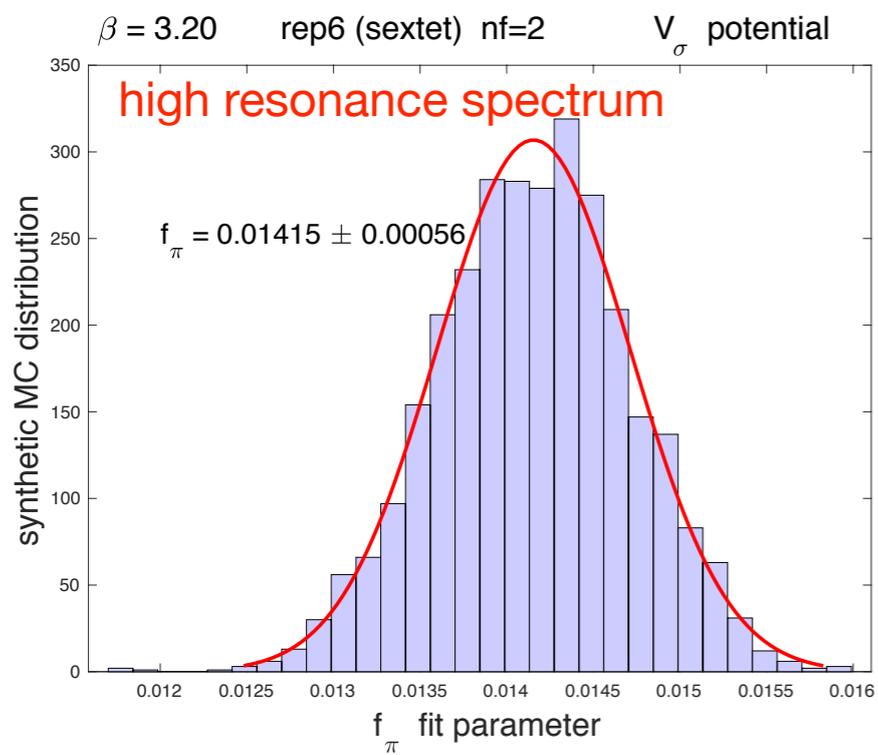
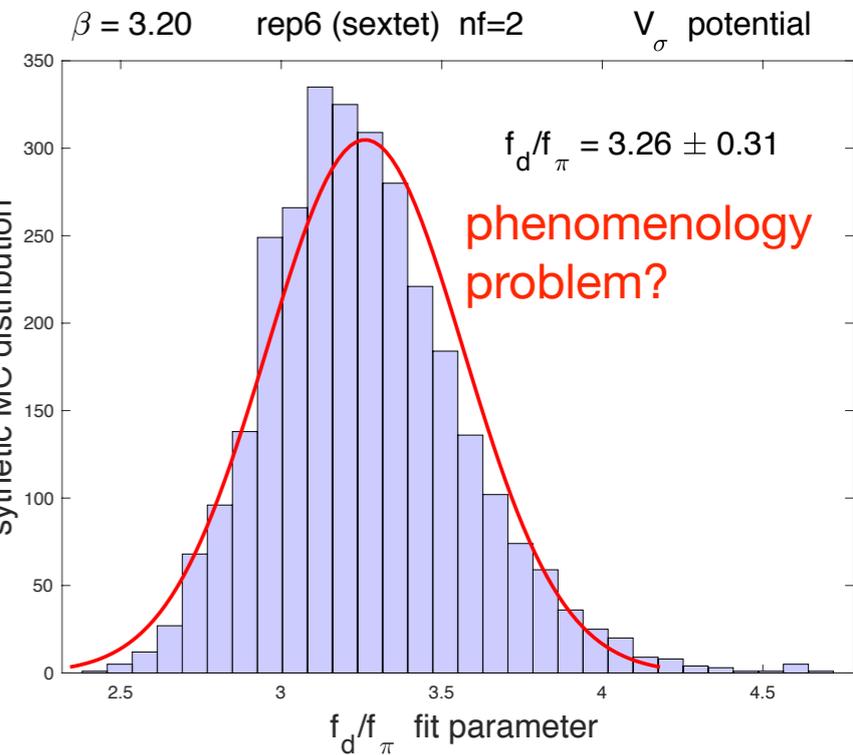
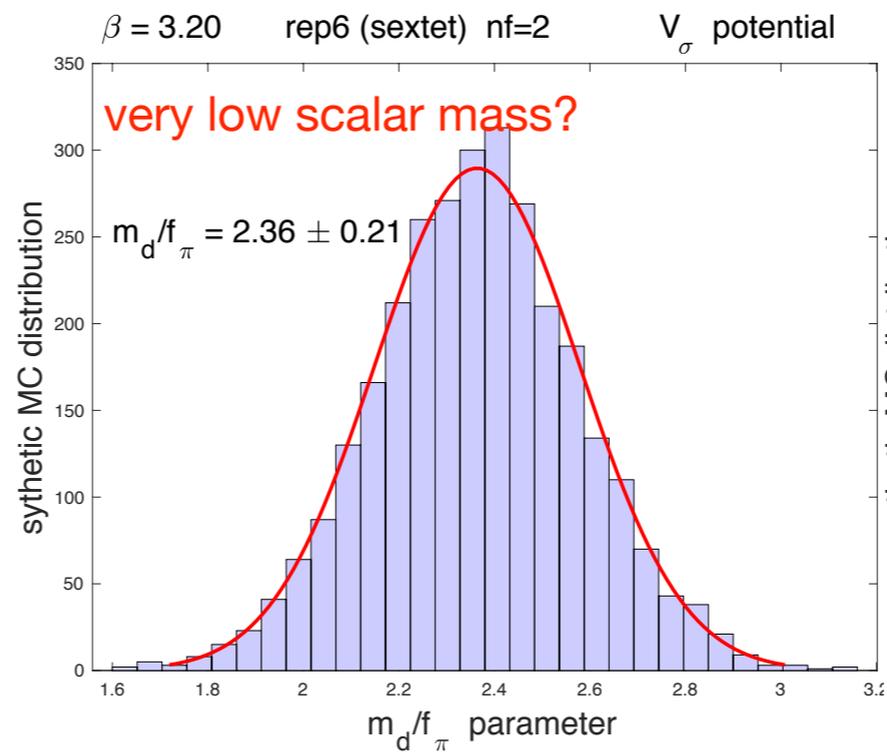
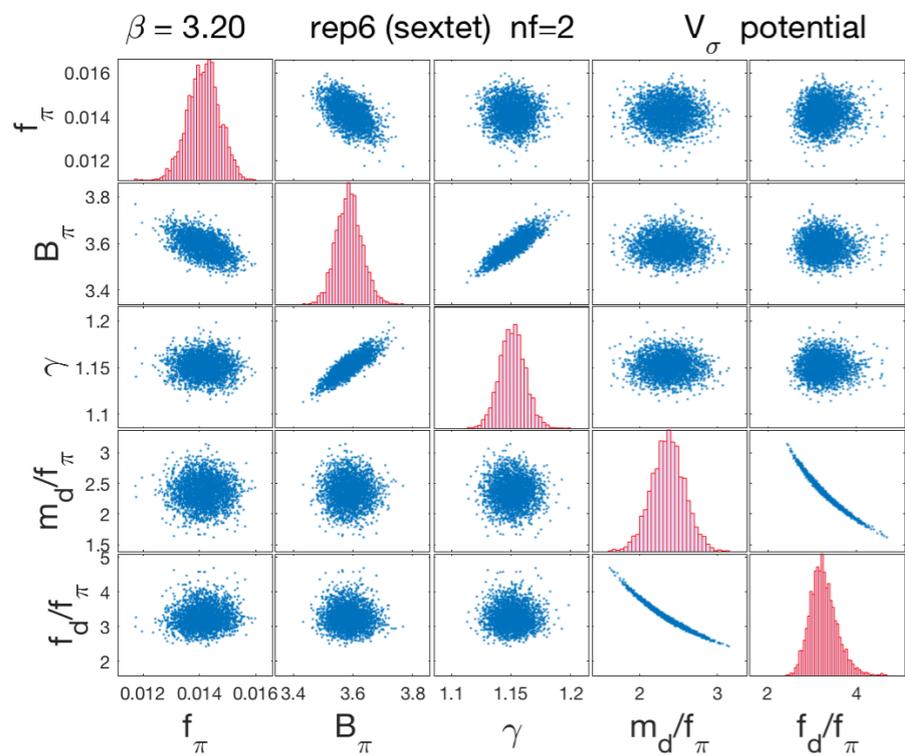
$f_\pi, B, f_d, m_d, y$  fitted for all  $m$

IML: Implicit Maximum Likelihood test

IML is very different from ML fitting

Perfect fits for  $V_\sigma$  !

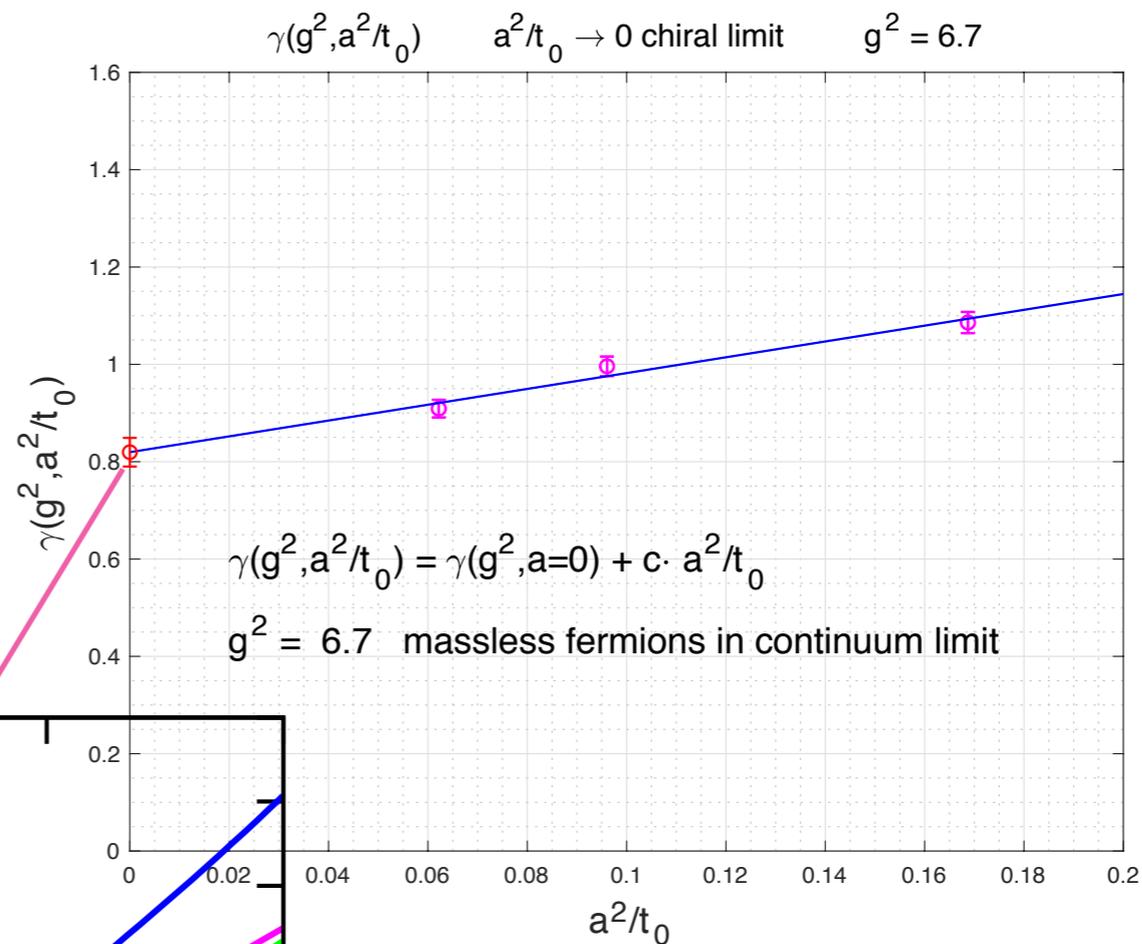
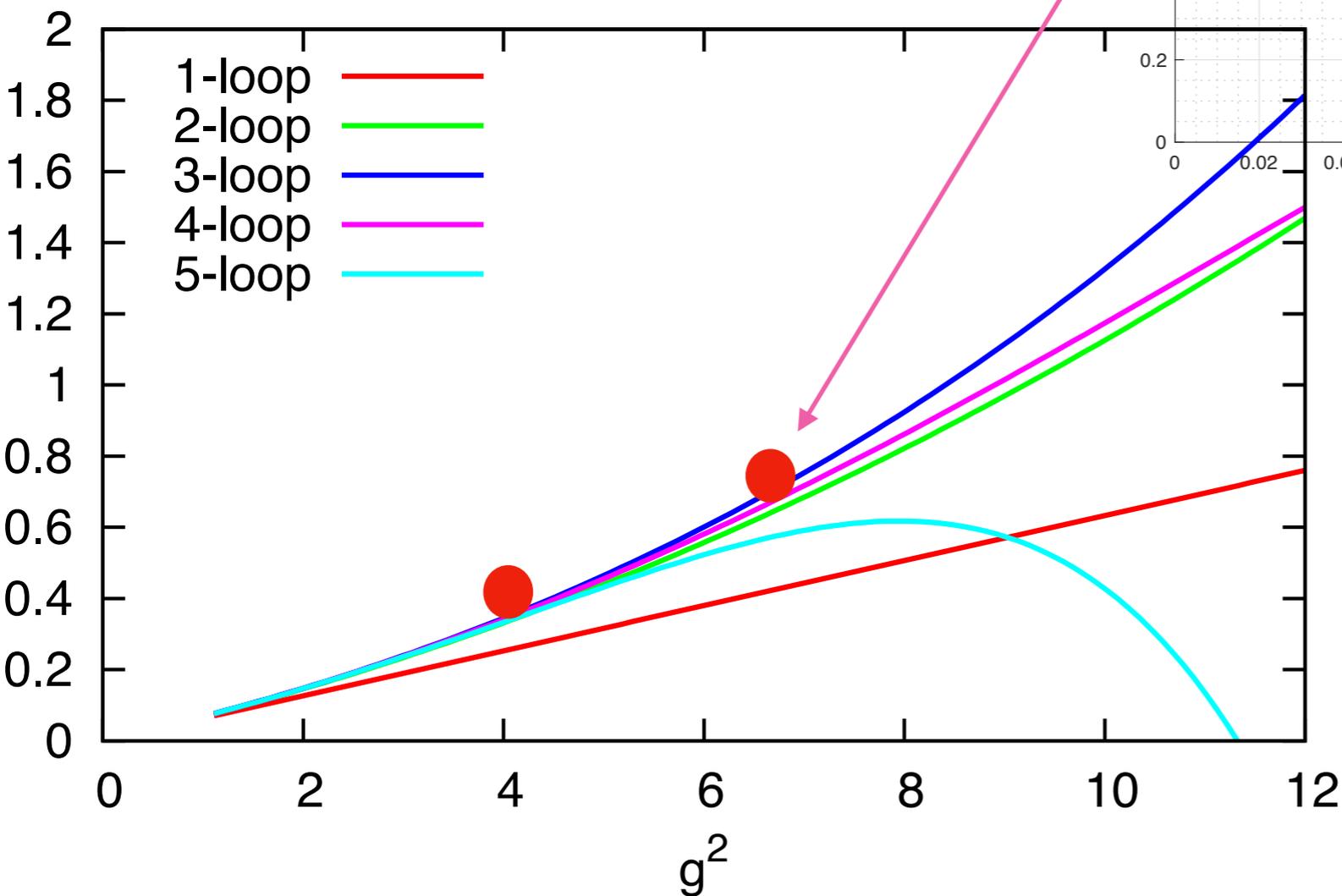
$V_d$  fails!



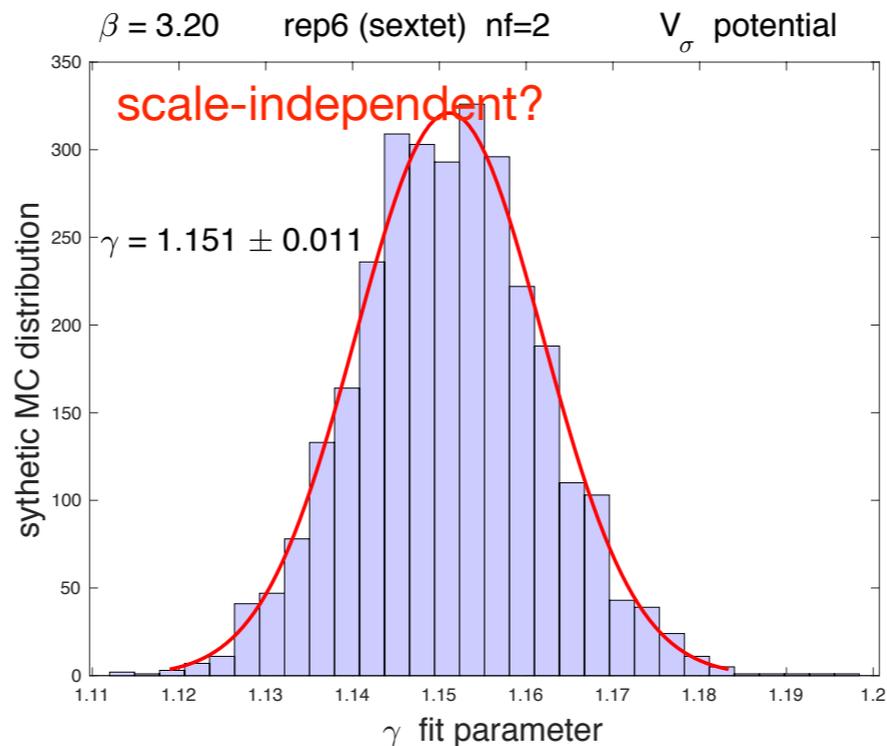
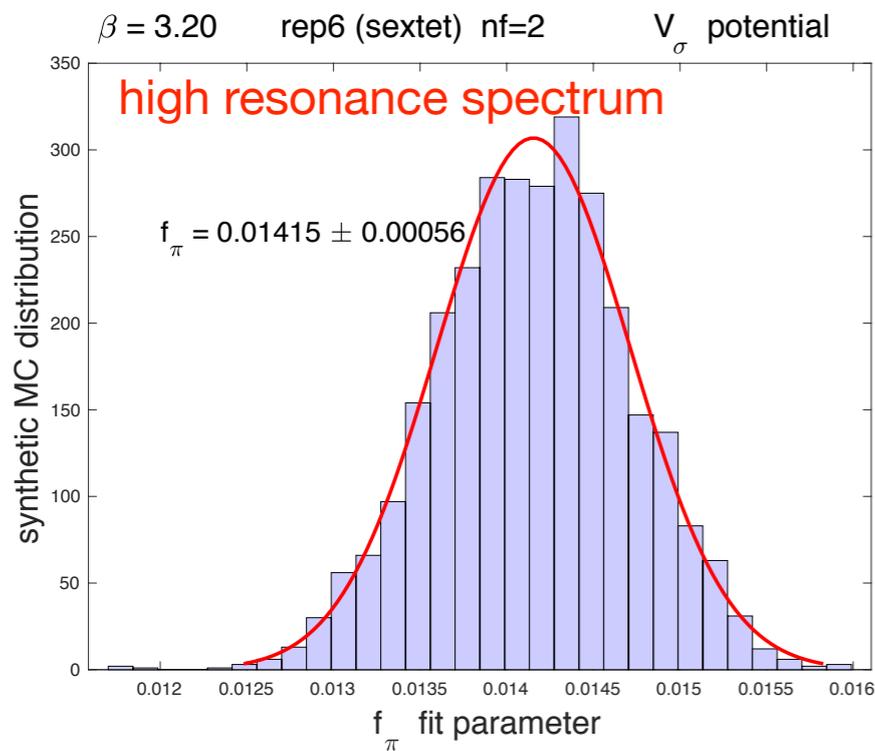
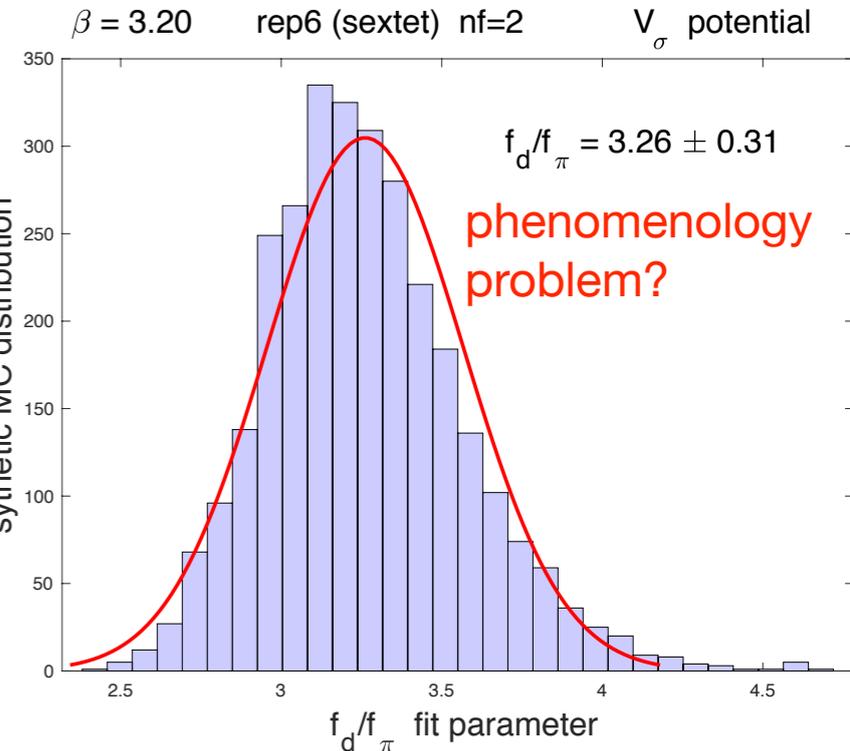
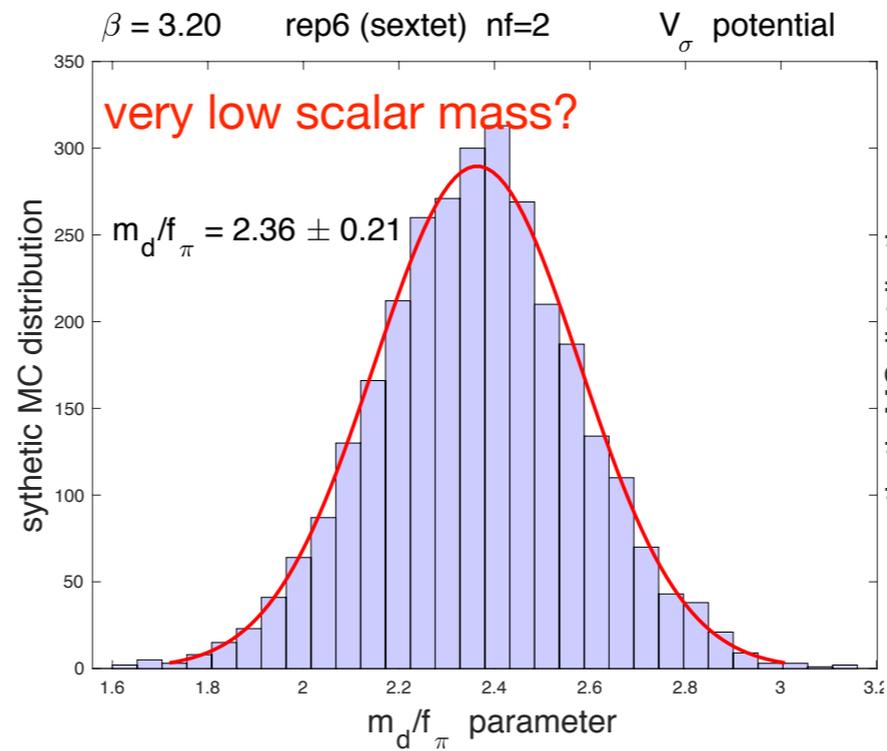
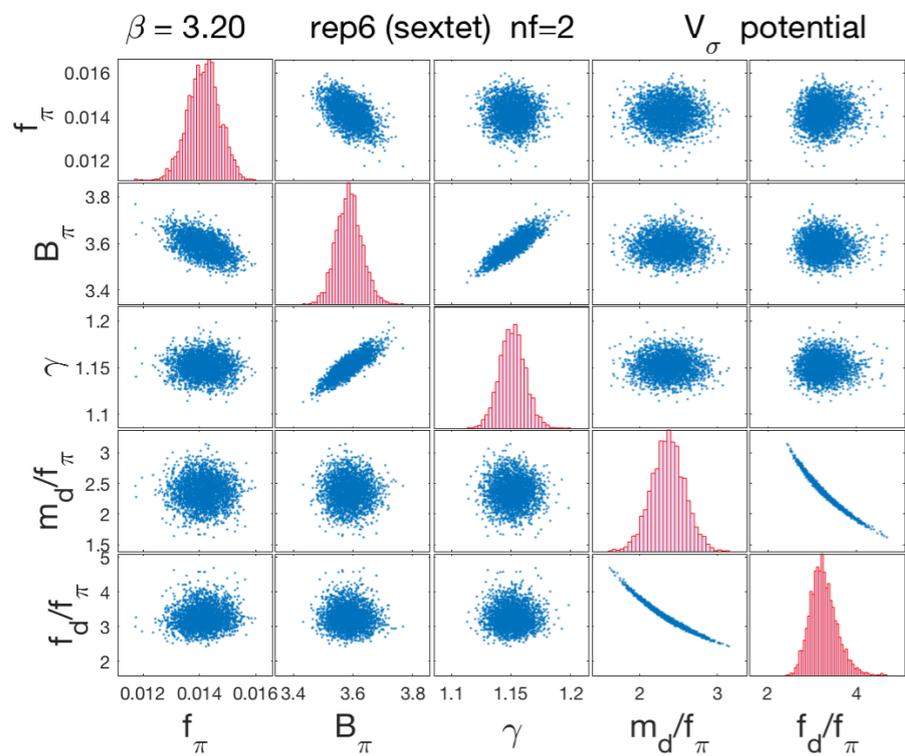
# scale dep. anomalous dimension $\gamma$ from Dirac spectrum: sextet data

- Chebyshev expansion of mode number
- infinite volume limit from FSS
- $m \rightarrow 0$  chiral limit at fixed  $a$
- $a \rightarrow 0$  continuum limit

SU(3)  $N_f = 2$  sextet



scale-dependent  $y=3-\gamma$  in Lagrangian (and in fitting) requires theory for walking !



- the dilaton potential of Shamir-Golterman as tree level theory expanding around IRFP is inconsistent
- similar conclusion at nf=8 and one lattice spacing
- consistent LatHC analysis for two lattice spacing, with third under construction
- only  $m_d/f_\pi$  is sensitive to scalar mass input

**beyond tree-level analysis requires two orders of magnitude drop in fermion mass: switch from p-regime to epsilon regime and related RMT**

# epsilon regime and RMT

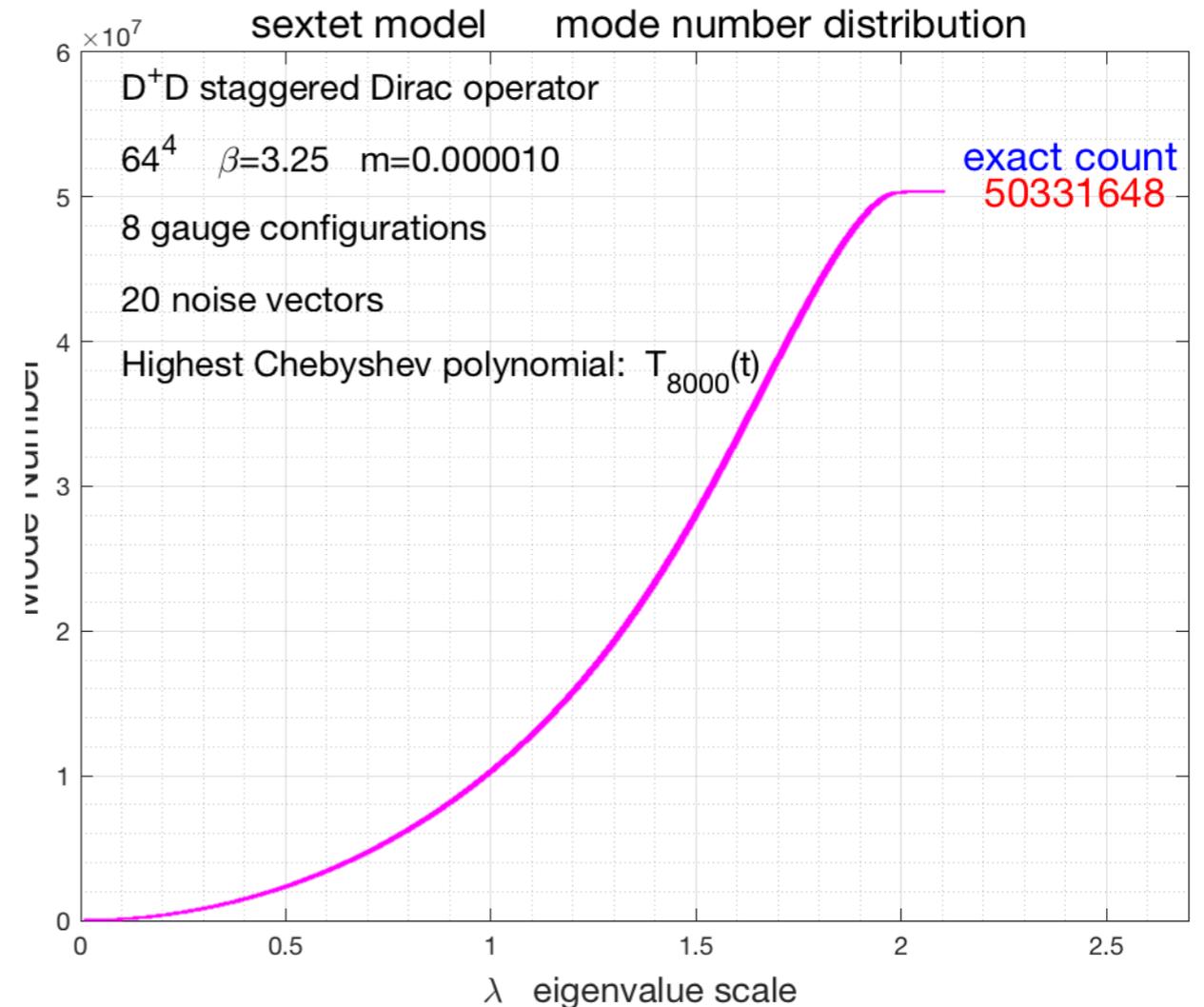
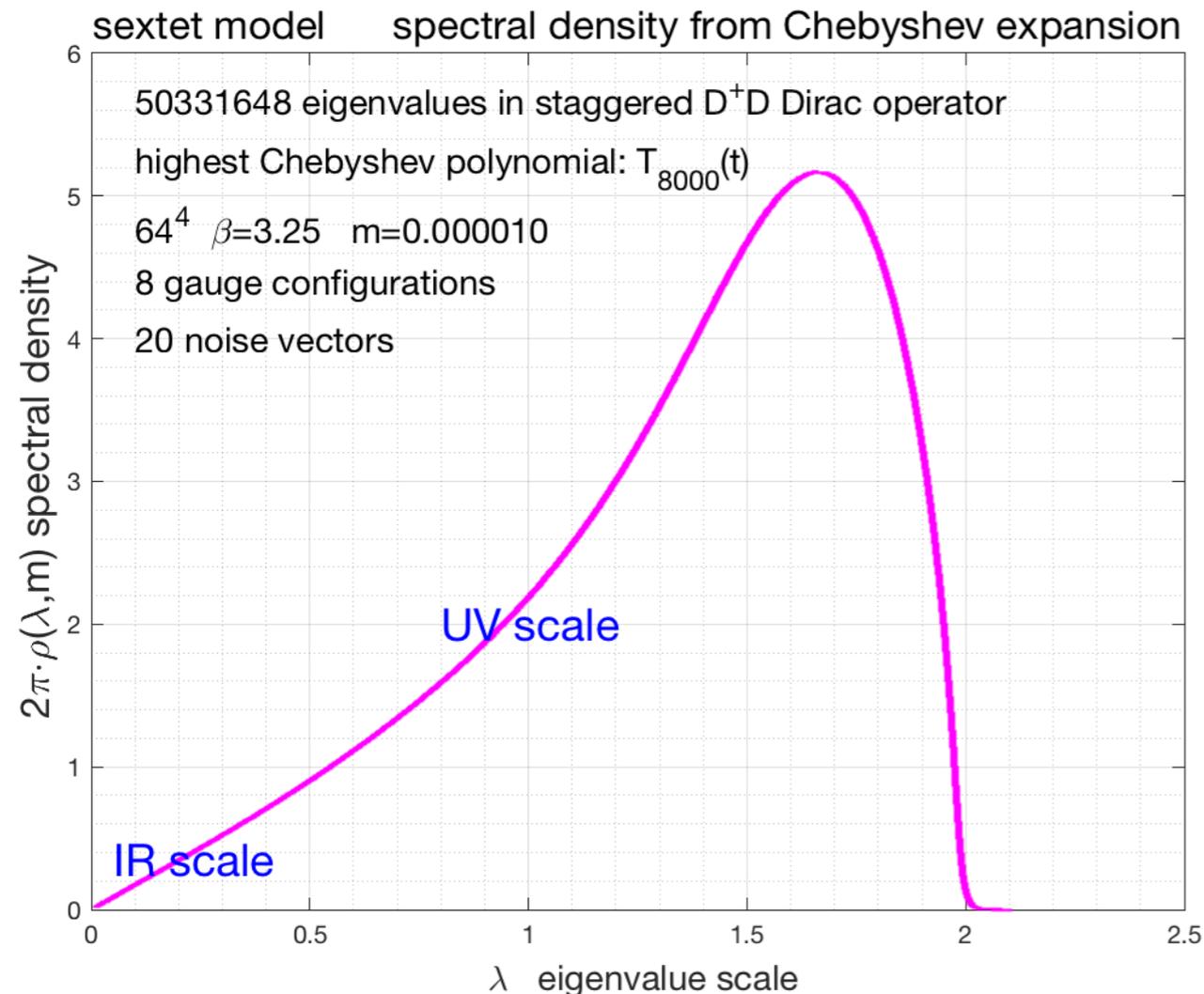
$$\mathcal{L}_\varepsilon = \frac{1}{2} \partial_\mu \chi \partial_\mu \chi - V_d(\chi) + \frac{m_\pi^2 f_\pi^2}{4} \left( \frac{\chi}{f_d} \right)^y \text{tr} [\Sigma_0 + \Sigma_0^\dagger]$$

epsilon regime with very small fermion mass deformation

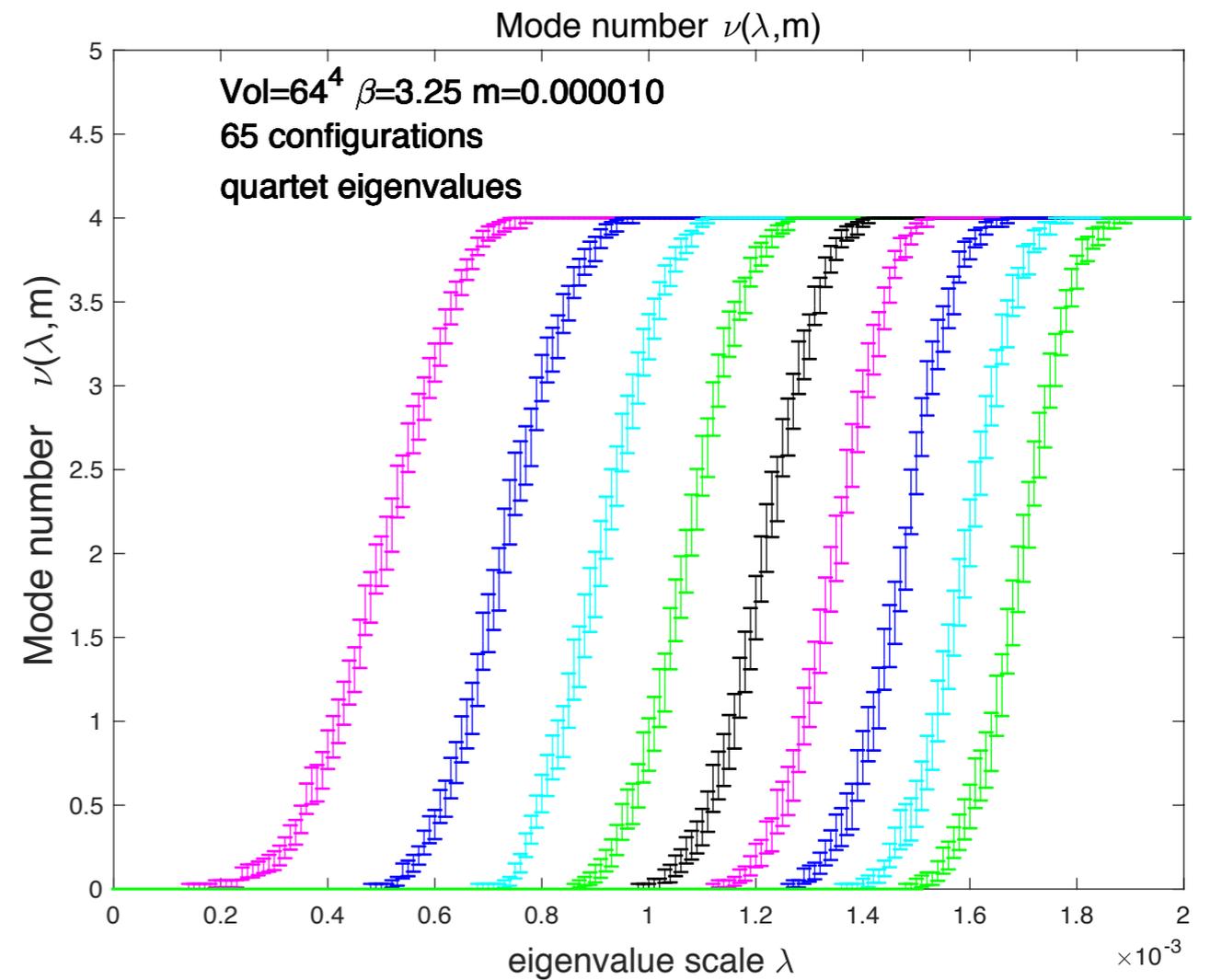
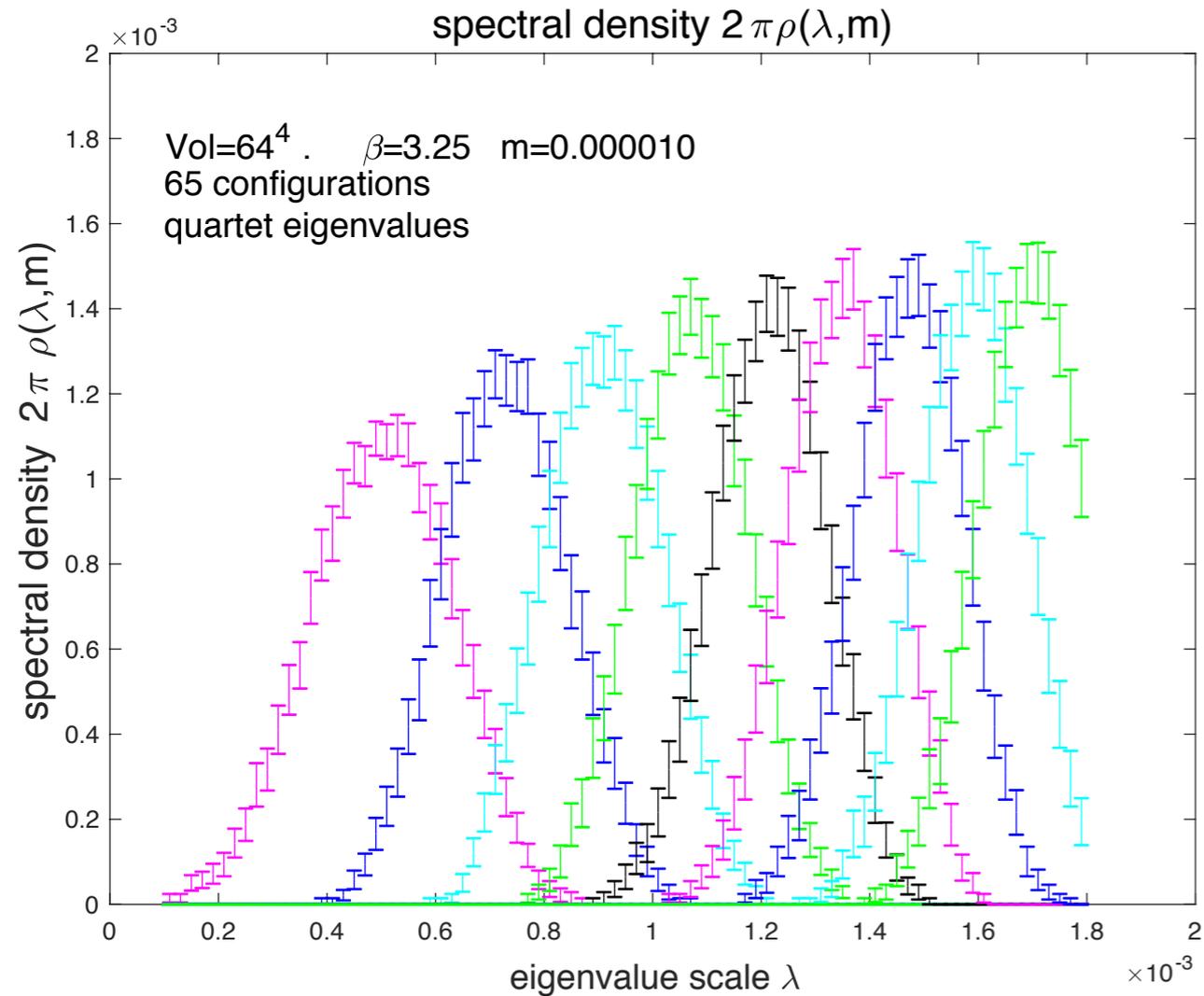
$$\mathcal{L}_\delta = \frac{1}{2} \partial_\mu \chi \partial_\mu \chi - V(\chi) + \frac{f_\pi^2}{4} \left( \frac{\chi}{f_d} \right)^2 \text{tr} [\partial_t \Sigma_0 \partial_t \Sigma_0^\dagger]$$

delta regime  $m=0$   
very small fermion mass deformation can be added

new ensembles at equivalent p-regime pion mass  $M_{\pi} \sim 100$  and size volume  $64^4$



# epsilon regime and RMT



successful testing

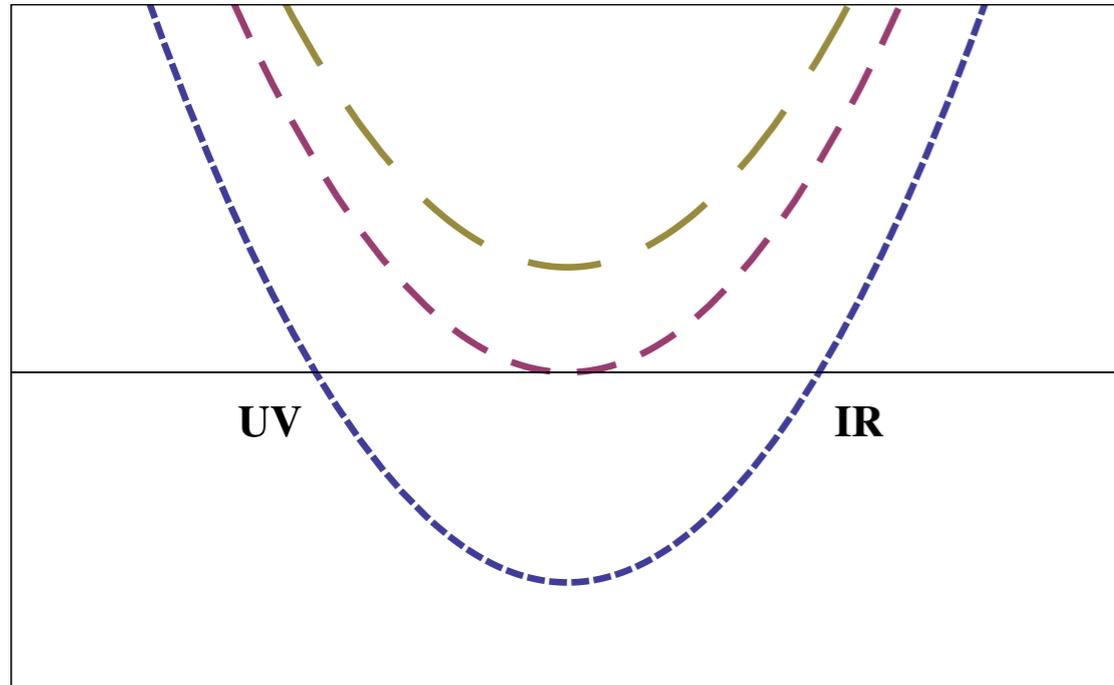
ongoing analysis (preliminary results not shown)

we exhausted our current gpu allocation

# walking and complex CFT

PHYSICAL REVIEW D **82**, 045013 (2010)

Luca Vecchi 2010



walking and complex CFT new paradigm?  
flavor symmetry group is the same for walker and the CFT!

we started work earlier on the realization of walking based on this idea

to distinguish near-conformal and conformal finite volume correlators (drifting scaling exponents distinguished from fixed conformal exponents).  
we ran into difficulty not knowing the conformal exponents of the complex theory.

Rychkov et al. turned to a two-dimensional example (Potts model) for very detailed realization without apparently knowing about Vecchi

We ask now: can we realize on the lattice Potts conformal field theories for continuous Q (flavor)

$$\mathcal{L}_{\text{CFT}} + \frac{f}{2} \mathcal{O}_{ij}^\dagger \mathcal{O}^{ij}$$

$$\Lambda \frac{d\bar{f}}{d\Lambda} = v\bar{f}^2 + (2\Delta - d)\bar{f} + a$$

$$\beta'_{\bar{f}}|_{\pm} = \pm 2\sqrt{D}$$

$$\Delta_{\pm} = \frac{d}{2} \pm \sqrt{D}$$

# walking and complex CFT

Potts model  $Q$  potts spin  $\sim$  flavor  
described by CFT

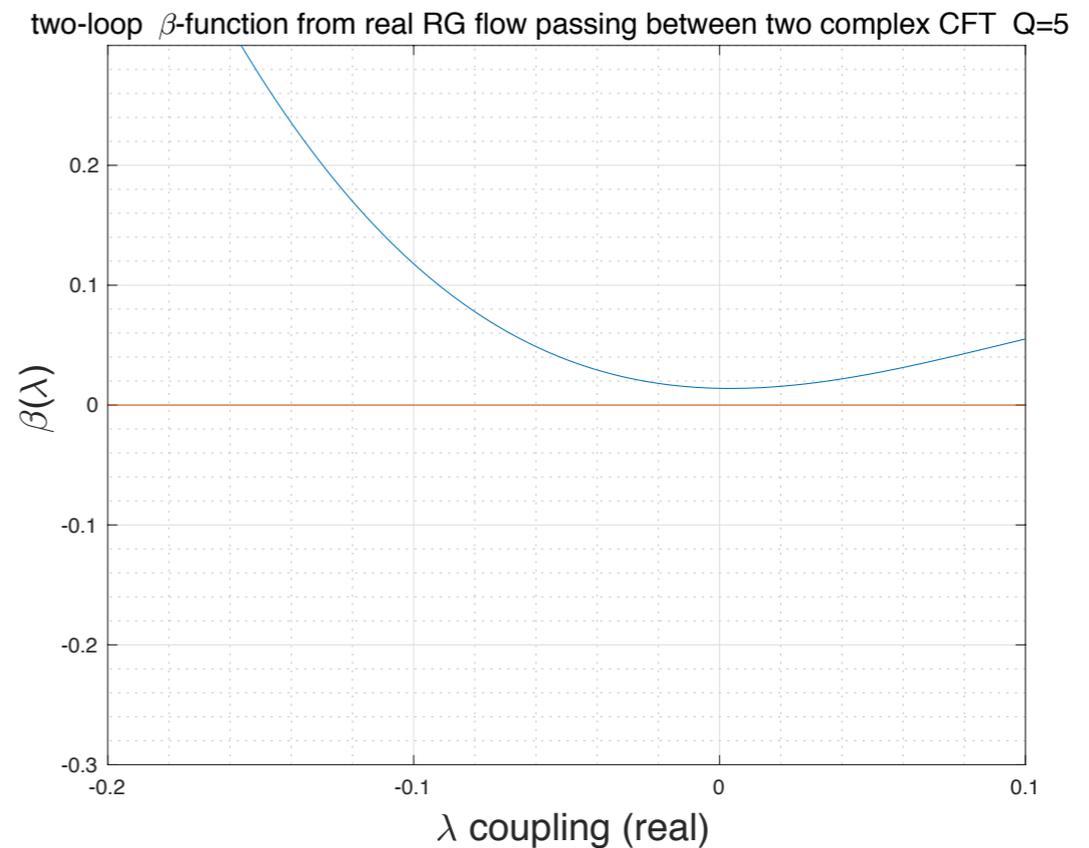
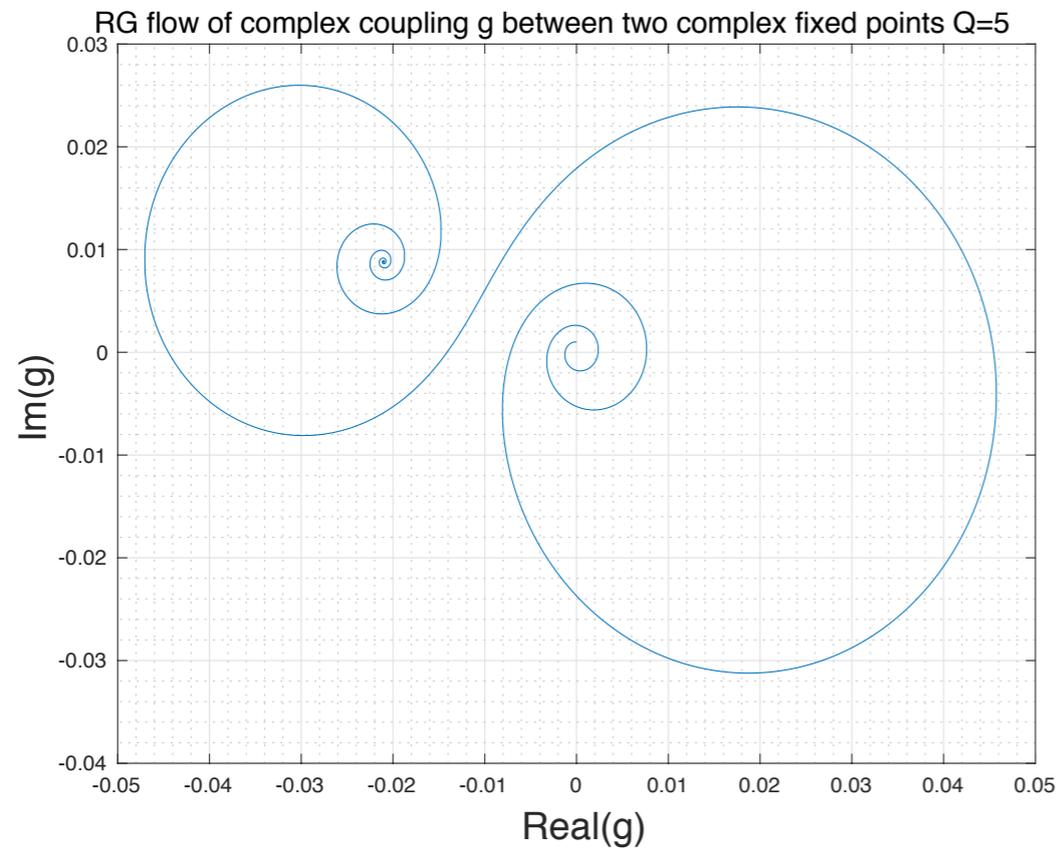
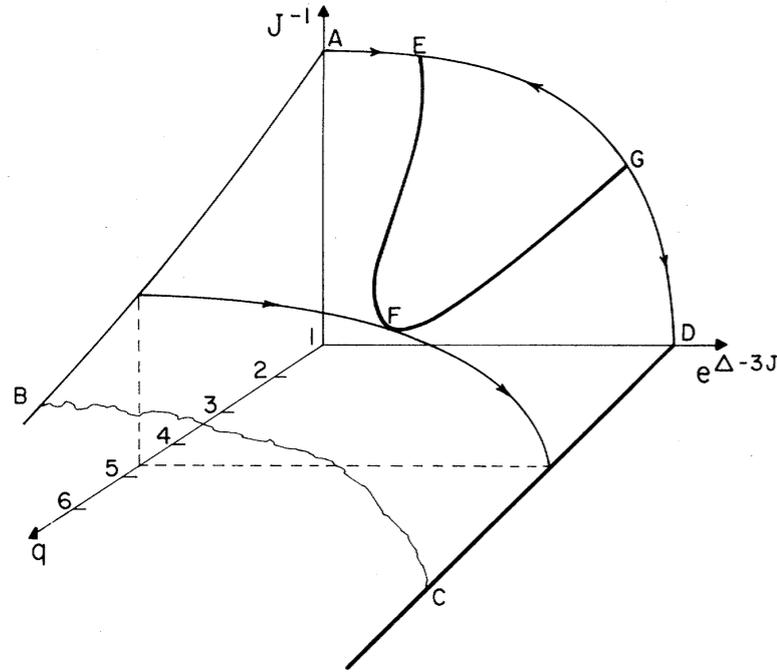
$Q=2-4$  pair of real CFT with pair of zeros of the beta function works for continuous  $Q$  in cluster rep

$Q > 4$  complex CFT, like  $Q=5, 6, 7 \dots$

What was identified before as  $Q=5$  Potts is near-conformal and walking, supported by the complex pair, not critical in very well defined sense

We have very efficient CUDA code working on lattice realization of the complex CFT paradigm

more in my BSM workshop talk



Thank you