Lattice QCD and current tensions in the Standard Model: $|V_{cb}|$ from $\bar{B} \to D^* \ell \bar{\nu}$ at non-zero recoil

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On behalf of the Fermilab/MILC collaborations

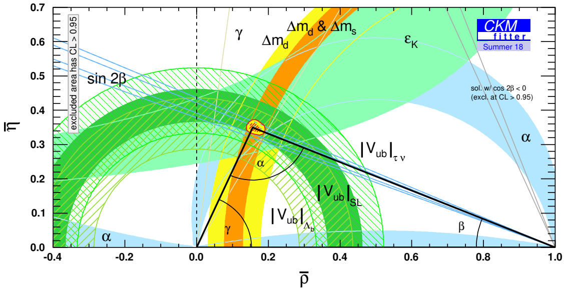
The V_{cb} matrix element: Tensions

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Determination	$\lvert V_{cb}\rvert\ (\cdot 10^{-3})$
Exclusive	39.2 ± 0.7
Inclusive	42.2 ± 0.8

PDG 2016

- Matrix must be unitary (preserve the norm)
- Based on CLN, motivated this work



The V_{cb} matrix element: Measurement from exclusive processes

$$\underbrace{\frac{d\Gamma}{dw}\left(\bar{B}\to D^*\ell\bar{\nu}_\ell\right)}_{\text{Experiment}} = \underbrace{\frac{G_F^2 m_B^5}{48\pi^2}(w^2-1)^{\frac{1}{2}} P(w)\left|\eta_{ew}\right|^2}_{\text{Known factors}} \underbrace{\left|\mathcal{F}(w)\right|^2}_{\text{Theory}} \left|V_{cb}\right|^2$$

- The amplitude $\mathcal{F}$ must be calculated in the theory
  - Extremely difficult task, QCD is non-perturbative
- Can use effective theories (HQET) to say something about $\mathcal{F}$
  - Separate light (non-perturbative) and heavy degrees of freedom as $m_Q \to \infty$
  - $\lim_{m_Q\to\infty} \mathcal{F}(w) = \xi(w)$, which is the Isgur-Wise function
  - **We don't know how $\xi(w)$ looks like, but we know $\xi(1) = 1$**
  - At large (but finite) mass $\mathcal{F}(w)$ receives corrections $O\left(\alpha_s, \frac{\Lambda_{QCD}}{m_Q}\right)$
- Reduction in the phase space $(w^2-1)^{\frac{1}{2}}$ limits experimental results at $w \approx 1$
  - Need to extrapolate $\left|V_{cb}\right|^2 \left|\eta_{ew}\mathcal{F}(w)\right|^2$ to $w = 1$
  - This extrapolation is done using well established parametrizations

The V_{cb} matrix element: The parametrization issue

All the parametrizations perform an expansion on the z parameter

$$z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

- Boyd-Grinstein-Lebed (BGL)

Phys. Rev. Lett. 74 (1995) 4603-4606

Phys.Rev. D56 (1997) 6895-6911

Nucl.Phys. B461 (1996) 493-511

$$f_X(w) = \frac{1}{B_{f_X}(z)\phi_{f_X}(z)} \sum_{n=0}^{\infty} a_n z^n$$

  - B_{f_X} Blaschke factors, includes contributions from the poles
  - ϕ_{f_X} is called *outer function* and must be computed for each form factor
  - Unitarity constrains $\sum_n |a_n|^2 \leq 1$

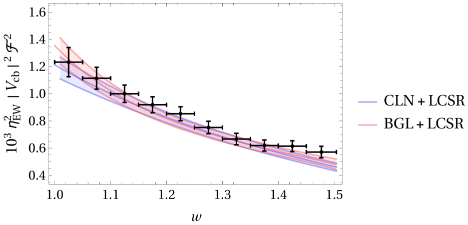
- Caprini-Lellouch-Neubert (CLN)

Nucl. Phys. B530 (1998) 153-181

$$\mathcal{F}(w) \propto 1 - \rho^2 z + cz^2 - dz^3, \quad \text{with } c = f_c(\rho),\, d = f_d(\rho)$$

  - Relies strongly on HQET, spin symmetry and (old) inputs
  - Tightly constrains $\mathcal{F}(w)$: four independent parameters, one relevant at $w = 1$

The V_{cb} matrix element: The parametrization issue



From *Phys. Lett. B769 (2017) 441-445* using Belle data at non-zero recoil and lattice data at zero recoil

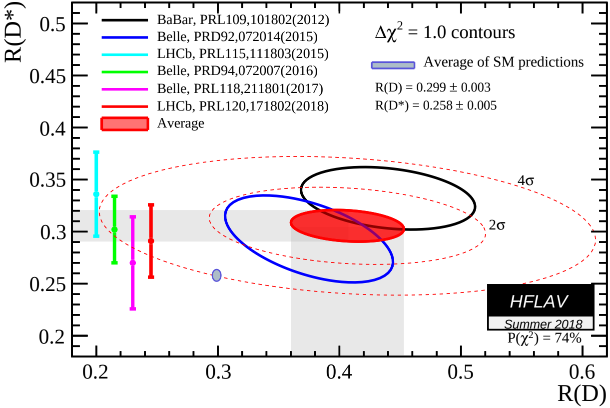
- CLN seems to underestimate the slope at low recoil
- The BGL value of $|V_{cb}|$ is compatible with the inclusive one

$$|V_{cb}| = 41.7 \pm 2.0 (\times 10^{-3})$$

- Current discrepancy might be an artifact
- Data at $w \gtrsim 1$ is urgently needed to settle the issue
- Experimental measurements perform badly at low recoil

We would benefit enormously from a high precision lattice calculation at $w \gtrsim 1$

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_\tau\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_\ell\right)}$$



- Current 4σ tension with the SM

Calculating V_{cb} on the lattice: Formalism

- Form factors

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{V}^\mu\,\left|\bar{B}(p_B)\right\rangle}{2\sqrt{m_B\,m_{D^*}}} = \frac{1}{2}\epsilon^{\nu *}\varepsilon^{\mu\nu}_{\rho\sigma}v^\rho_B v^\sigma_{D^*} h_V(w)$$

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{A}^\mu\,\left|\bar{B}(p_B)\right\rangle}{2\sqrt{m_B\,m_{D^*}}} =$$

$$\frac{i}{2}\epsilon^{\nu *}\left[g^{\mu\nu}\left(1+w\right)h_{A_1}(w) - v^\nu_B\left(v^\mu_B h_{A_2}(w) + v^\mu_{D^*} h_{A_3}(w)\right)\right]$$

- $\mathcal{V}$ and $\mathcal{A}$ are the vector/axial currents in the continuum
- The h_X enter in the definition of $\mathcal{F}$
- We can calculate $h_{A_{1,2,3},V}$ directly from the lattice

Calculating V_{cb} on the lattice: Formalism

- Helicity amplitudes

$$H_{\pm} = \sqrt{m_B\, m_{D^*}}(w+1)\left(h_{A_1}(w) \mp \sqrt{\frac{w-1}{w+1}} h_V(w)\right)$$

$$H_0 = \sqrt{m_B\, m_{D^*}}(w{+}1)m_B\left[(w-r)h_{A_1}(w) - (w-1)\left(r\, h_{A_2}(w) + h_{A_3}(w)\right)\right]/\sqrt{q^2}$$

$$H_S = \sqrt{\frac{w^2-1}{r(1+r^2-2wr)}}\left[(1+w)h_{A_1}(w) + (wr-1)h_{A_2}(w) + (r-w)h_{A_3}(w)\right]$$

- Form factor in terms of the helicity amplitudes

$$\chi(w)\,|\mathcal{F}|^2 = \frac{1-2wr+r^2}{12 m_B\, m_{D^*}\,(1-r)^2}\left(H_0^2(w) + H_+^2(w) + H_-^2(w)\right)$$

Calculating V_{cb} on the lattice: The Roadmap

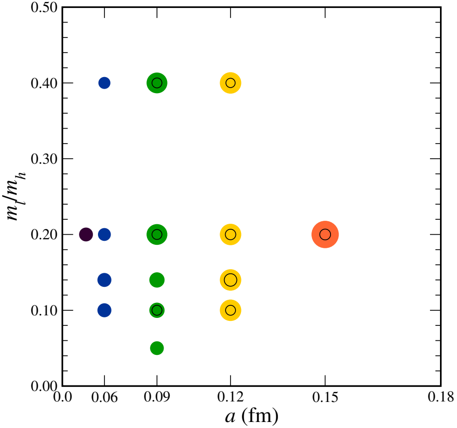
Well established roadmap to improve our calculation:

The all HISQ project

- ASQTAD light + Fermilab heavy
- HISQ light + Fermilab heavy
- HISQ light + HISQ heavy

- **This work**
- Reduces chiral-continuum extrapolation errors, light quark discretization errors
- Removes matching errors, reduces heavy quark discretization errors

Calculating V_{cb} on the lattice: Available ensembles



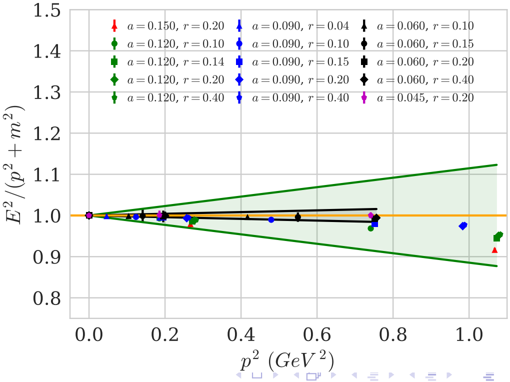
- $N_f = 2+1$ staggered asqtad sea quarks
- Heavy quarks use the fermilab action
- Size of the point proportional to the statistics (min 2372, max 15072)

Calculating V_{cb} on the lattice: Dispersion relation

- Discretization effects coming from the heavy quark break the dispersion relation
- The Fermilab action uses tree-level matching, discretization errors $O(\alpha\, m)$

$$a^2E^2(p_\mu) = (am_1)^2 + \frac{m_1}{m_2}(\mathbf{p}a)^2 + \frac{1}{4}\left[\frac{1}{(am_2)^2} - \frac{am_1}{(am_4)^3}\right](a^2\mathbf{p}^2)^2 - \frac{am_1 w_4}{3}\sum_{i=1}^{3}(ap_i)^4 + O(p_i^6)$$

- As long as the discretization errors are under control, this is all right
- In the Fermilab action we interpret the kinetic mass am_2 as the particle mass



Calculating V_{cb} on the lattice: Current renormalization

- In the coefficients of the terms of our effective theory a dependence arises with the scale (i.e. a)
- The renormalization tries to account for the right dependence
- The scheme we employ is called *Mostly non-perturbative renormalization* of results

$$Z_{V^{1,4},A^{1,4}} = \underbrace{\rho_{V^{1,4},A^{1,4}}}_{\text{Perturbative factor}} \times \underbrace{\sqrt{Z_{V_{bb}} Z_{V_{cc}}}}_{\text{Non-perturbative piece}}$$

- The (relatively large) non-perturbative piece cancels in our ratios
- The (close to one) perturbative piece (matching factor ρ) is calculated at one-loop level for $w = 1$
- The error for $w \neq 1$ is estimated and added to the factor

This analysis is **blinded** and the blinding happens at the level of the matching factor

Calculating V_{cb} on the lattice: The chiral-continuum extrapolation

- Our data represents the form factors at non-zero a and unphysical m_π
- Extrapolation to the physical pion mass described by EFTs
  - The EFT describe the a and the m_π dependence
- Functional form explicitly known

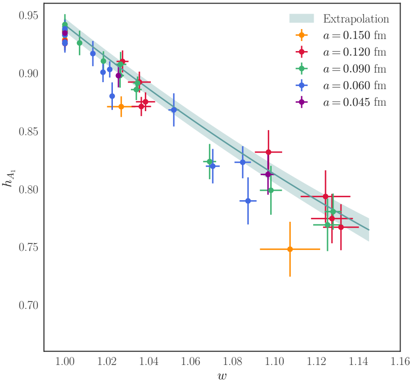
$$h_{A_1}(w) = \underbrace{1 + \frac{X_{A_1}(\Lambda_\chi)}{m_c^2} + \frac{g^2_{D^* D\pi}}{48\pi^2 f_\pi^2 r_1^2} \mathrm{logs}_{\mathrm{SU3}}(a, m_l, m_s, \Lambda_{QCD})}_{\mathrm{NLO}\ \chi\mathrm{PT} + \mathrm{HQET}} -$$

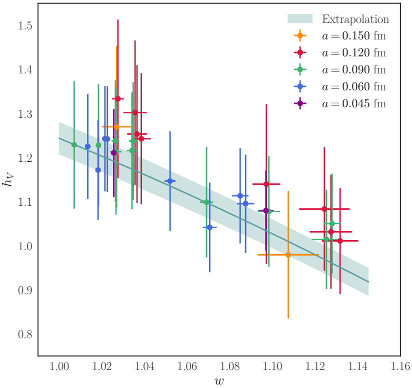
$$\underbrace{\rho^2(w-1) + k(w-1)^2}_{w\ \mathrm{dependence}} \underbrace{+ c_1 x_l + c_2 x_l^2 + c_{a1} x_{a^2} + c_{a2} x_{a^2}^2 + c_{a,m} x_l x_{a^2}}_{\mathrm{NNLO}\ \chi\mathrm{PT}}$$

with

$$x_l = B_0 \frac{m_l}{(2\pi f_\pi)^2}, \qquad x_{a^2} = \left(\frac{a}{4\pi f_\pi r_1^2} \right)^2$$

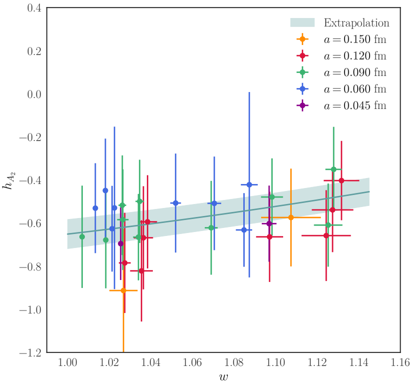
Results: Chiral-continuum fits

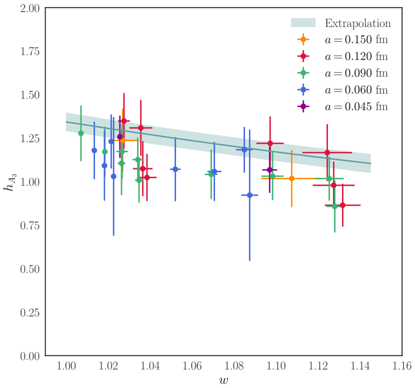




- Preliminary **blinded** results

Results: Chiral-continuum fits





- Preliminary **blinded** results

Analysis: Preliminary error budget

Source	h_V (%)	h_{A_1} (%)	h_{A_2} (%)	h_{A_3} (%)
Statistics	1.1	0.4	4.9	1.9
Isospin effects	0.0	0.0	0.6	0.3
χPT/cont. extrapolation	**1.9**	**0.7**	**6.3**	**2.9**
Matching	*1.5*	*0.4*	*0.1*	*1.5*
Heavy quark discretization	*1.4**	*0.4**	*5.8**	*1.3**

*Estimate, currently subject of intensive study

- **Bold** marks errors to be reduced/removed when using HISQ for light quarks
- *Italic* marks errors to be reduced/removed when using HISQ for heavy quarks
  - Heavy HISQ would introduce new extrapolation errors

Analysis: z-Expansion

- The BGL expansion is performed on different (more convenient) form factors

Phys.Lett. **B**769, 441 (2017), Phys.Lett. **B**771, 359 (2017)

$$g = \frac{h_V(w)}{\sqrt{m_B\, m_{D^*}}} = \frac{1}{\phi_g(z)B_g(z)} \sum_j a_j z^j$$

$$f = \sqrt{m_B\, m_{D^*}}(1+w)h_{A_1}(w) = \frac{1}{\phi_f(z)B_f(z)} \sum_j b_j z^j$$

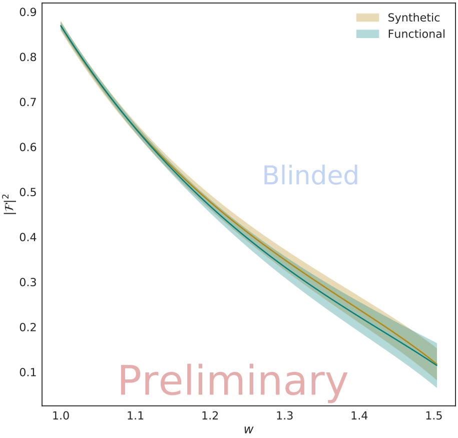
$$\mathcal{F}_1 = \sqrt{q^2} H_0 = \frac{1}{\phi_{\mathcal{F}_1}(z)B_{\mathcal{F}_1}(z)} \sum_j c_j z^j$$

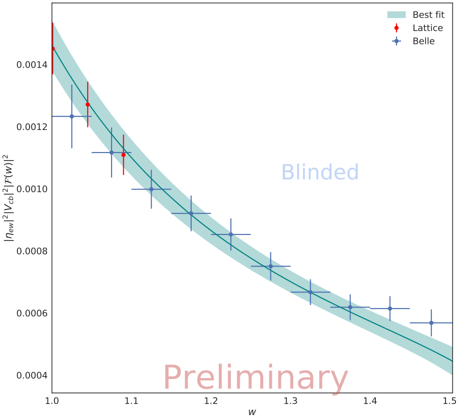
$$\mathcal{F}_2 = \frac{\sqrt{q^2}}{m_{D^*}\sqrt{w^2-1}} H_S = \frac{1}{\phi_{\mathcal{F}_2}(z)B_{\mathcal{F}_2}(z)} \sum_j d_j z^j$$

- Constraint $\mathcal{F}_1(z=0) = (m_B - m_{D^*})f(z=0)$
- Constraint $(1+w)m_B^2(1-r)\mathcal{F}_1(z=z_{\text{Max}}) = (1+r)\mathcal{F}_2(z=z_{\text{Max}})$
- BGL (weak) unitarity constraints (all HISQ will use strong constraints)

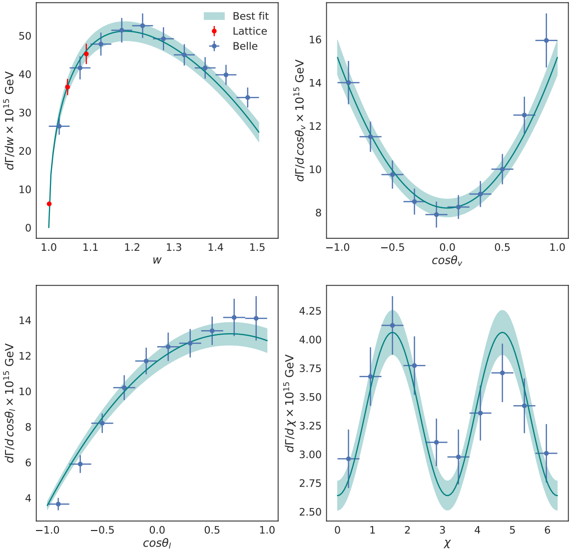
$$\sum_j a_j^2 \le 1, \qquad \sum_j b_j^2 + c_j^2 \le 1, \qquad \sum_j d_j^2 \le 1$$

Results: Pure-lattice prediction and joint fit



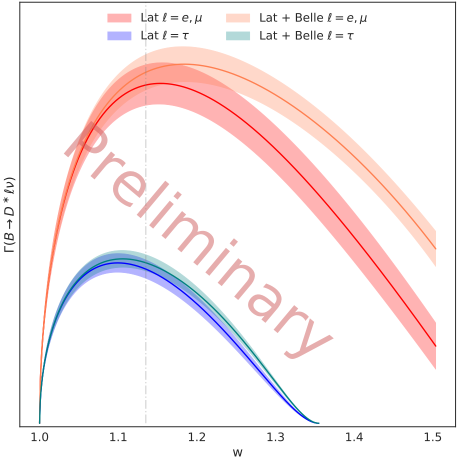


Results: Angular bins



Results: $R(D^*)$

- Pure lattice QCD prediction of $R(D^*)$
- Includes constraint $\mathcal{F}_1(w_{\rm Max}) = \frac{1+r}{(1+w)m_B^2(1-r)}\mathcal{F}_2(w_{\rm Max})$



Conclusions

What to expect

- Errors might not be improved compared to previous lattice estimations
- The main new information of this analysis won't come from the zero-recoil value, but from the slope
- Main sources of errors of our form factor seem to be
  - χPT-continuum extrapolation
  - HQ discretization
  - Matching

The future

- Well established roadmap to reduce errors in our calculation with the all-HISQ ensembles
  - Light HISQ quarks + heavy Fermilab quarks aim to reduce χPT-cont. extrapolation errors
  - Light HISQ quarks + heavy HISQ quarks aim to reduce discretization and matching errors
  - Joint $B \to D$ and $B \to D^*$ analysis to benefit from strong unitarity constraints
- This roadmap is to be followed in other processes involving other CKM matrix elements