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Non-perturbative matching of 3/4-flavor Wilson coefficients with a position-space procedure

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N_f in Weak Hamiltonian

$$H_W = \Sigma_i\, w_i^{4f}(\mu)\, O_i^{4f}(\mu)$$

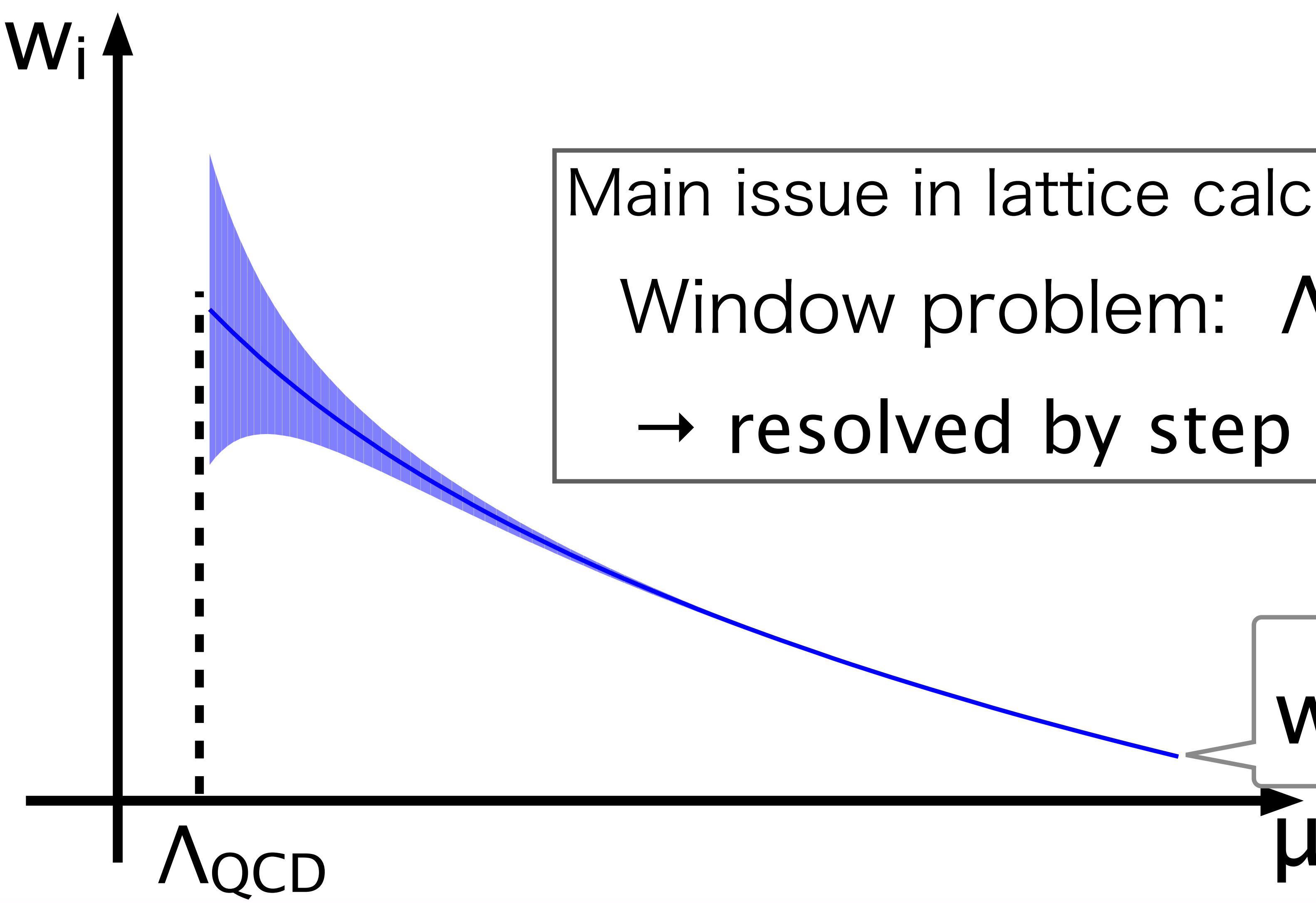
$$= \Sigma_i\, w_i^{3f}(\mu)\, O_i^{3f}(\mu)$$

$$= \ldots$$

We can use either 3f or 4f for WMEs

WMEs w/ 4-flavor operators

$$\langle f|H_W|i\rangle = \Sigma_i\, \underbrace{w_i^{4f}(\mu)}_{\text{pQCD}}\, \underbrace{\langle f|O_i^{4f}(\mu)|i\rangle}_{\text{LQCD}}$$



Main issue in lattice calculation is

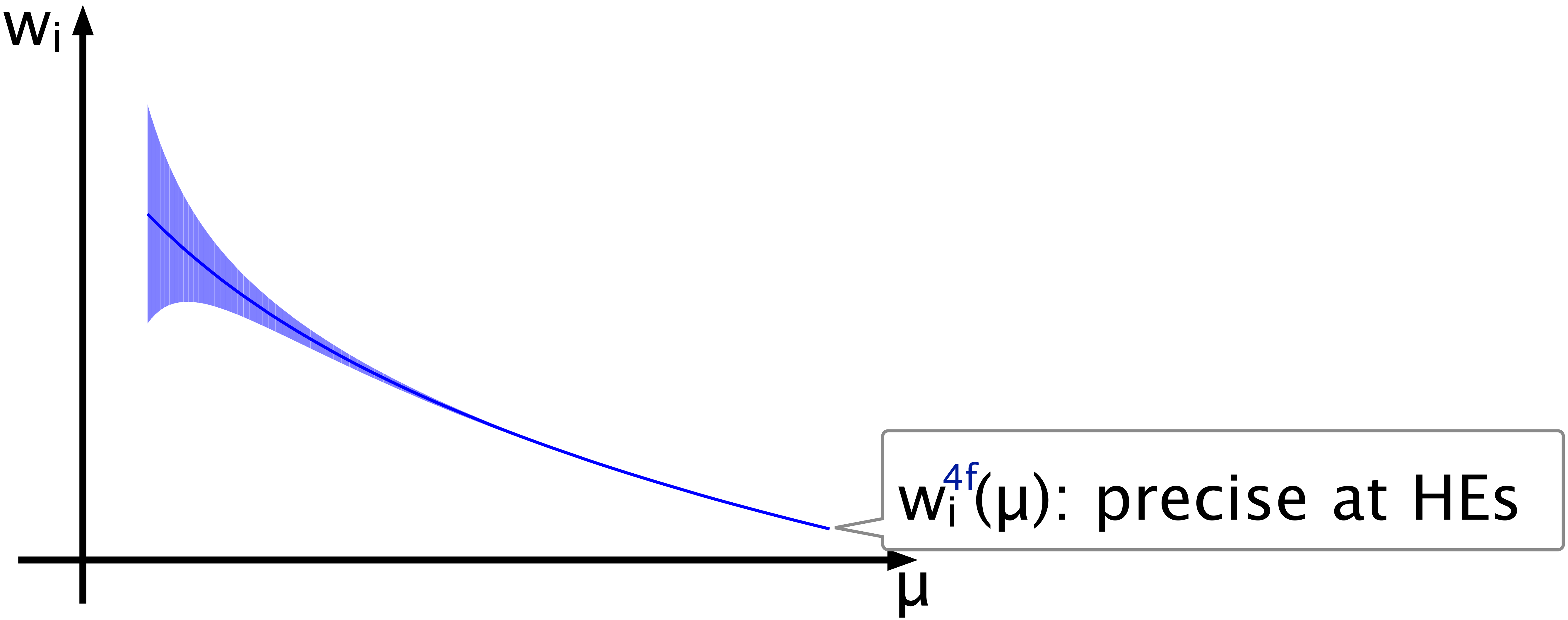
Window problem: $\Lambda_{QCD} \ll \mu \ll a^{-1}$

→ resolved by step scaling, etc…

$w_i^{4f}(\mu)$: precise at HEs

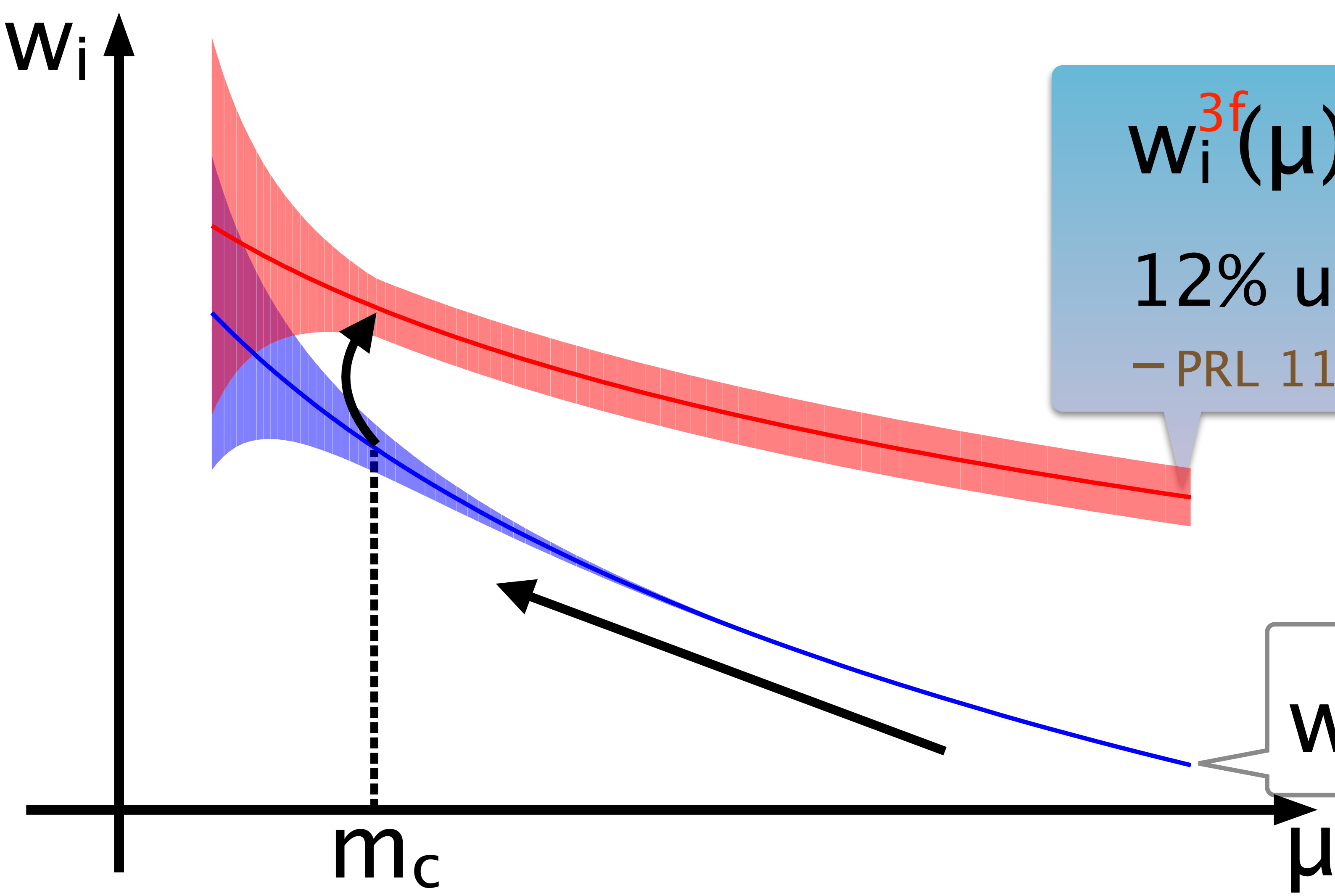
WMEs w/ 3-flavor operators

$$\langle f|H_W|i\rangle = \Sigma_i \; \underbrace{w_i^{3f}(\mu)}_{\text{pQCD}} \; \underbrace{\langle f|O_i^{3f}(\mu)|i\rangle}_{\text{LQCD}}$$



WMEs w/ 3-flavor operators

$$\langle f|H_W|i\rangle = \Sigma_i\, \underbrace{w_i^{3f}(\mu)}_{\text{pQCD}}\ \underbrace{\langle f|O_i^{3f}(\mu)|i\rangle}_{\text{LQCD}}$$



$w_i^{3f}(\mu)$: large uncertainty

12% uncertainty for $K \to \pi\pi$

– PRL 115,212001(2015)

$w_i^{4f}(\mu)$: precise at HEs

WMEs w/ 3-flavor operators

$$\langle f|H_W|i\rangle = \Sigma_i\, w_i^{3f}(\mu)\, \langle f|O_i^{3f}(\mu)|i\rangle$$

~~pQCD~~ LQCD

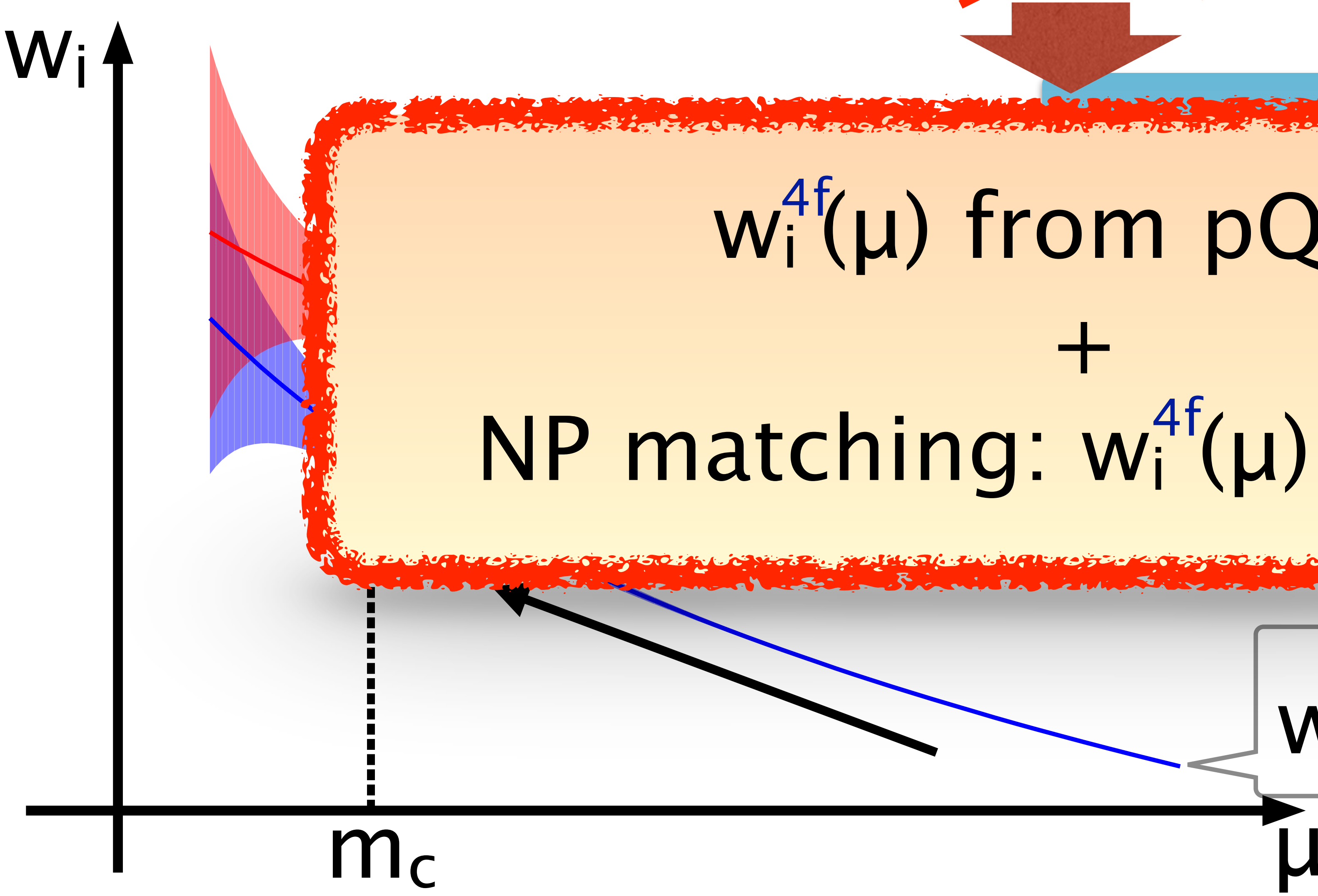
rtainty

→ ππ

$w_i^{4f}(\mu)$ from pQCD

+

NP matching: $w_i^{4f}(\mu) \rightarrow w_i^{3f}(\mu)$

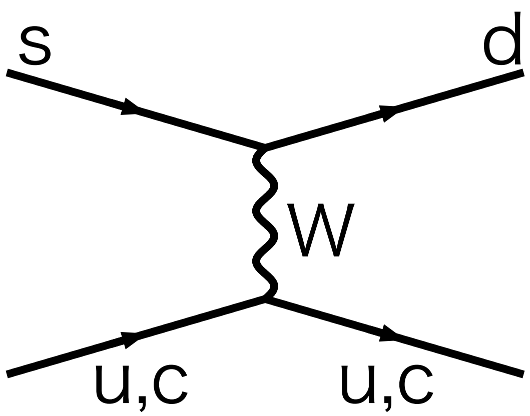


$w_i^{4f}(\mu)$: precise at HEs

$w_i^{3f}(\mu) \neq w_i^{4f}(\mu)$?

- Of coarse sea charm effects ⇒ $w_i^{3f}(\mu) \neq w_i^{4f}(\mu)$
  - Maybe small difference → neglect in this project
- If O_i^{4f} involves charm…

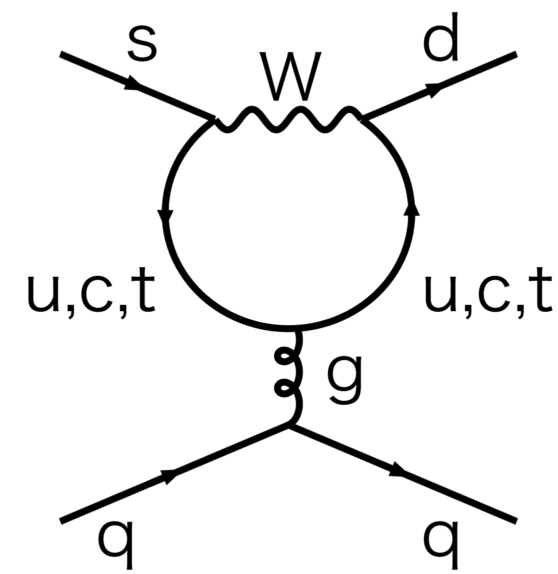
  Ex)



current–current

$O_i^u = (\bar{s}d)_{V-A}\,(\bar{u}u)_{V-A}$

$O_i^c = (\bar{s}d)_{V-A}\,(\bar{c}c)_{V-A}$



QCD penguin

$O_i = (\bar{s}d)_{V-A}\,\Sigma_q\,(\bar{q}q)_{V\pm A}$

  - Corresponding w_i's in 3f & 4f different
  - w_i^{3f} necessary if MEs calculated with O_i^{3f}

$K \to \pi\pi$ by RBC/UKQCD (2015)

- 2+1 DWF / Iwasaki + DSDR gauge action
- a^{-1} = 1.38 GeV

  ⇒ too coarse to introduce charm

  ⇒ 3-flavor operators for MEs
  & perturbative 4/3-flavor matching

  ⇒ 12% systematic uncertainty

- NP matching (obtained from finer lattices) is desired

Contents

- Introduction
- New strategy in a gauge invariant procedure
  - Two-point functions in position space
- Measurement Strategy
  - 107 contractions needed
  - $16^3 \times 32$ test calculation done
  - Main calculation expected to be ready in mid May
- Technique for reducing discretization errors
  - Average over spheres

NP 4f–3f matching in position Sp.

$$H_W = \Sigma_i\, w_i^{4f}(\mu)\, O_i^{4f}(\mu) = \Sigma_i\, w_i^{3f}(\mu)\, O_i^{3f}(\mu)$$

- This means:

  $$\Sigma_i <\bar{O}(x)\, O_i^{4f}(\mu;y)^\dagger> w_i^{4f}(\mu) = \Sigma_i <\bar{O}(x)\, O_i^{3f}(\mu;y)^\dagger> w_i^{3f}(\mu)$$

  for any operator $\bar{O}(x)$

  at LDs: $1/|x-y| \ll m_c$

- Relation b/w w_i^{4f} & w_i^{3f} can be obtained by choosing appropriate number of $\bar{O}(x)$'s

  ⇒ We choose

  $$\bar{O}(x) = O_i^{3f}(\mu;x)$$

NP 4f–3f matching in position Sp.

$$H_W = \Sigma_i\, w_i^{4f}(\mu)\, O_i^{4f}(\mu) = \Sigma_i\, w_i^{3f}(\mu)\, O_i^{3f}(\mu)$$

$$\langle O_j^{3f}(\mu;x)\, H_W(y)^\dagger \rangle$$

$$\Sigma_j\, G_{ij}^{3f-4f}(\mu;x-y)\, w_j^{4f}(\mu) = \Sigma_j\, G_{ij}^{3f-3f}(\mu;x-y)\, w_j^{3f}(\mu)$$

$$G_{ij}^{nf-n'f}(\mu;x-y) = \langle O_i^{nf}(\mu;x)\, O_j^{n'f}(\mu;y)^\dagger \rangle$$

$$w_i^{3f}(\mu) = \Sigma_{jk}\, (G^{3f-3f}(\mu;x-y))_{ij}^{-1}\, G_{jk}^{3f-4f}(\mu;x-y)\, w_k^{4f}(\mu)$$

☆ Gauge invariant & free from contact terms

⇒ can prevent mixing with irrelevant operators

$\Delta S = 1$ 4-quark operators

3-flavor

Type	Q_i
current-current	$Q_1 = (\bar{s}_\alpha d_\alpha)_L(\bar{u}_\beta u_\beta)_L$ $Q_2 = (\bar{s}_\alpha d_\beta)_L(\bar{u}_\beta u_\alpha)_L$
QCD penguin	$Q_3 = (\bar{s}_\alpha d_\alpha)_L \sum_q^{3f} (\bar{q}_\beta q_\beta)_L$ $Q_4 = (\bar{s}_\alpha d_\beta)_L \sum_q^{3f} (\bar{q}_\beta q_\alpha)_L$ $Q_5 = (\bar{s}_\alpha d_\alpha)_L \sum_q^{3f} (\bar{q}_\beta q_\beta)_R$ $Q_6 = (\bar{s}_\alpha d_\beta)_L \sum_q^{3f} (\bar{q}_\beta q_\alpha)_R$
EW penguin	$Q_7 = \frac{3}{2}(\bar{s}_\alpha d_\alpha)_L \sum_q^{3f} e_q(\bar{q}_\beta q_\beta)_R$ $Q_8 = \frac{3}{2}(\bar{s}_\alpha d_\beta)_L \sum_q^{3f} e_q(\bar{q}_\beta q_\alpha)_R$ $Q_9 = \frac{3}{2}(\bar{s}_\alpha d_\alpha)_L \sum_q^{3f} e_q(\bar{q}_\beta q_\beta)_L$ $Q_{10} = \frac{3}{2}(\bar{s}_\alpha d_\beta)_L \sum_q^{3f} e_q(\bar{q}_\beta q_\alpha)_L$

7 independent operators

4-flavor

Type	P_i
current-current	$P_1 = (\bar{s}_\alpha d_\alpha)_L(\bar{u}_\beta u_\beta)_L$ $P_1^c = (\bar{s}_\alpha d_\alpha)_L(\bar{c}_\beta c_\beta)_L$ $P_2 = (\bar{s}_\alpha d_\beta)_L(\bar{u}_\beta u_\alpha)_L$ $P_2^c = (\bar{s}_\alpha d_\beta)_L(\bar{c}_\beta c_\alpha)_L$
QCD penguin	$P_3 = (\bar{s}_\alpha d_\alpha)_L \sum_q^{4f} (\bar{q}_\beta q_\beta)_L$ $P_4 = (\bar{s}_\alpha d_\beta)_L \sum_q^{4f} (\bar{q}_\beta q_\alpha)_L$ $P_5 = (\bar{s}_\alpha d_\alpha)_L \sum_q^{4f} (\bar{q}_\beta q_\beta)_R$ $P_6 = (\bar{s}_\alpha d_\beta)_L \sum_q^{4f} (\bar{q}_\beta q_\alpha)_R$
EW penguin	$P_7 = \frac{3}{2}(\bar{s}_\alpha d_\alpha)_L \sum_q^{4f} e_q(\bar{q}_\beta q_\beta)_R$ $P_8 = \frac{3}{2}(\bar{s}_\alpha d_\beta)_L \sum_q^{4f} e_q(\bar{q}_\beta q_\alpha)_R$ $P_9 = \frac{3}{2}(\bar{s}_\alpha d_\alpha)_L \sum_q^{4f} e_q(\bar{q}_\beta q_\beta)_L$ $P_{10} = \frac{3}{2}(\bar{s}_\alpha d_\beta)_L \sum_q^{4f} e_q(\bar{q}_\beta q_\alpha)_L$

9 independent operators

Color trivialization by Fierz trf.

- Def: $(\bar{s}d)_L(\bar{q}q)_{R/L} = \bar{s}\gamma_\mu(1-\gamma_5)d \cdot \bar{q}\gamma_\mu(1\pm\gamma_5)q$

- Left–Left operators

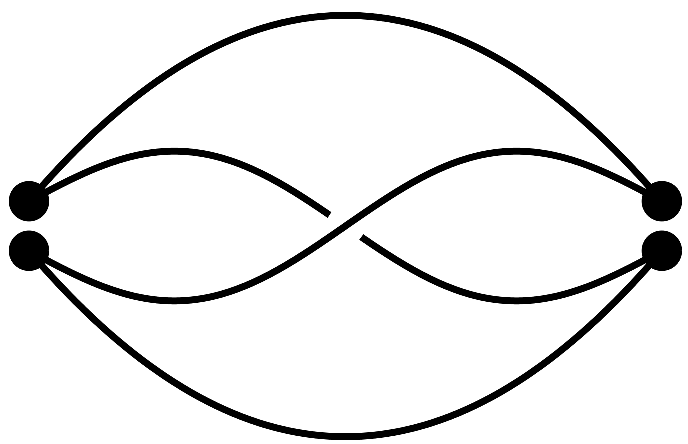
$$(\bar{s}_\alpha d_\beta)_L(\bar{q}_\beta q_\alpha)_L = (\bar{s}_\alpha q_\alpha)_L(\bar{q}_\beta d_\beta)_L$$

- Left–Right operators

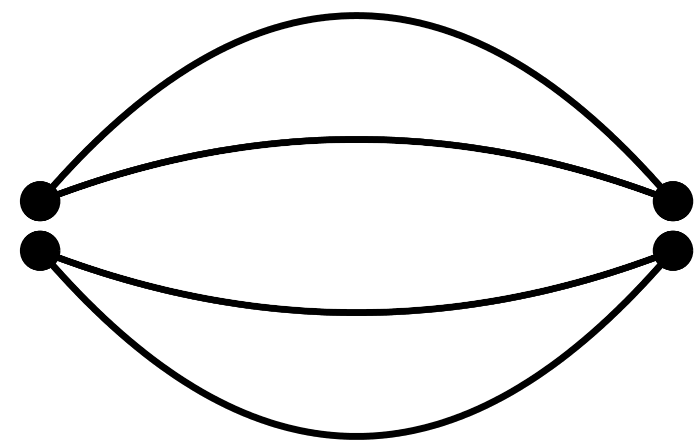
$$(\bar{s}_\alpha d_\beta)_L(\bar{q}_\beta q_\alpha)_R = -2\bar{s}_\alpha(1+\gamma_5)q_\alpha \cdot \bar{q}_\beta(1-\gamma_5)d_\beta$$

Contractions

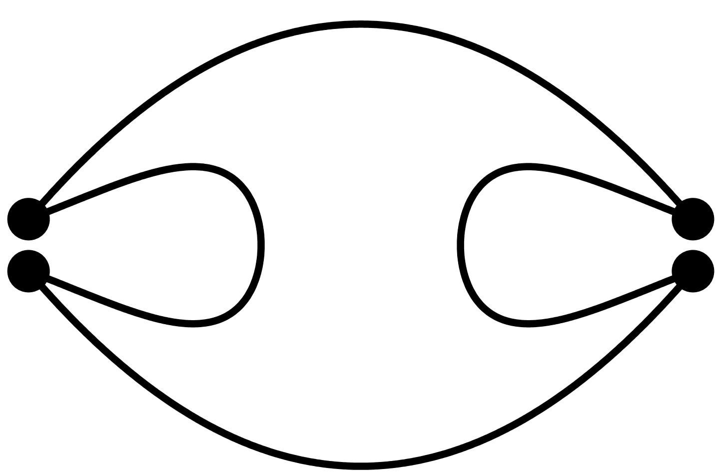
- 4/3-flavor matching should be independent of m_{ud} & m_s
  ⇒ Calculate w/ SU(3) valence quarks + 1 heavy quark



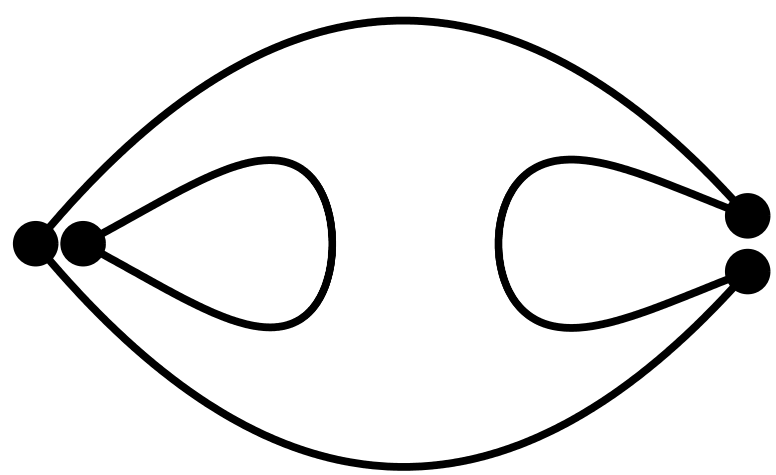
6 contractions



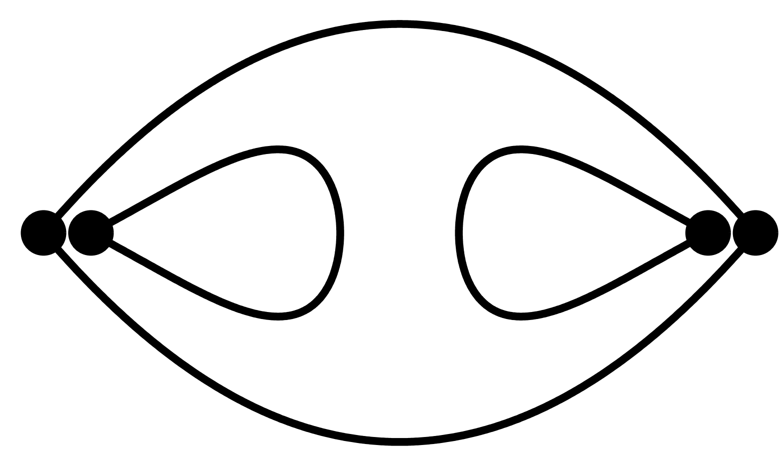
6 contractions



18 contractions



32 contractions



18 contractions

Subtraction of power divergence

- Loop diagram can contain power divergence
  - from power divergence of operators

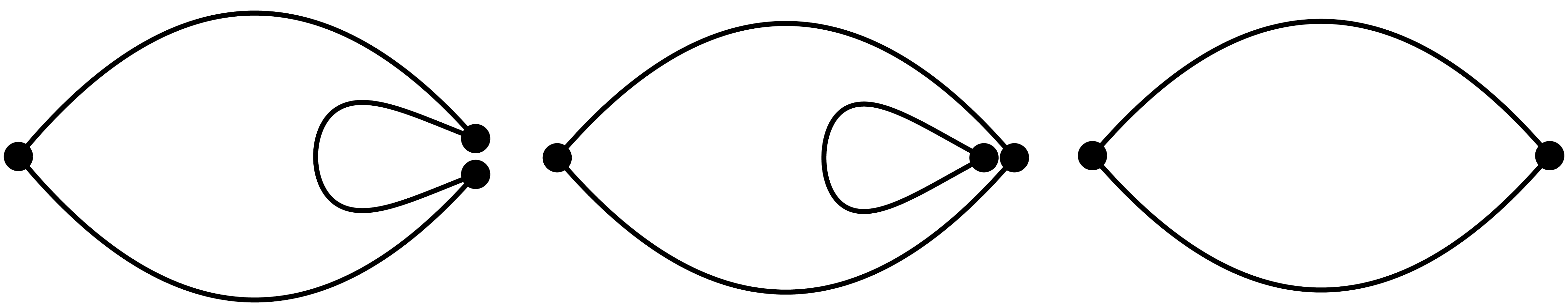
$$O_i \sim \frac{m_q}{a^2}$$

  - Eliminate by redefining

$$O'_i = O_i - C_- \bar{s}(1-\gamma_5)d - C_+ \bar{s}(1+\gamma_5)d$$

with a condition

$$\left\langle \bar{s}(1\pm\gamma_5)d(x) \cdot O'_i(y)^\dagger \right\rangle \Big|_{x-y=x_0} = 0$$



12 contractions 12 contractions 3 contractions

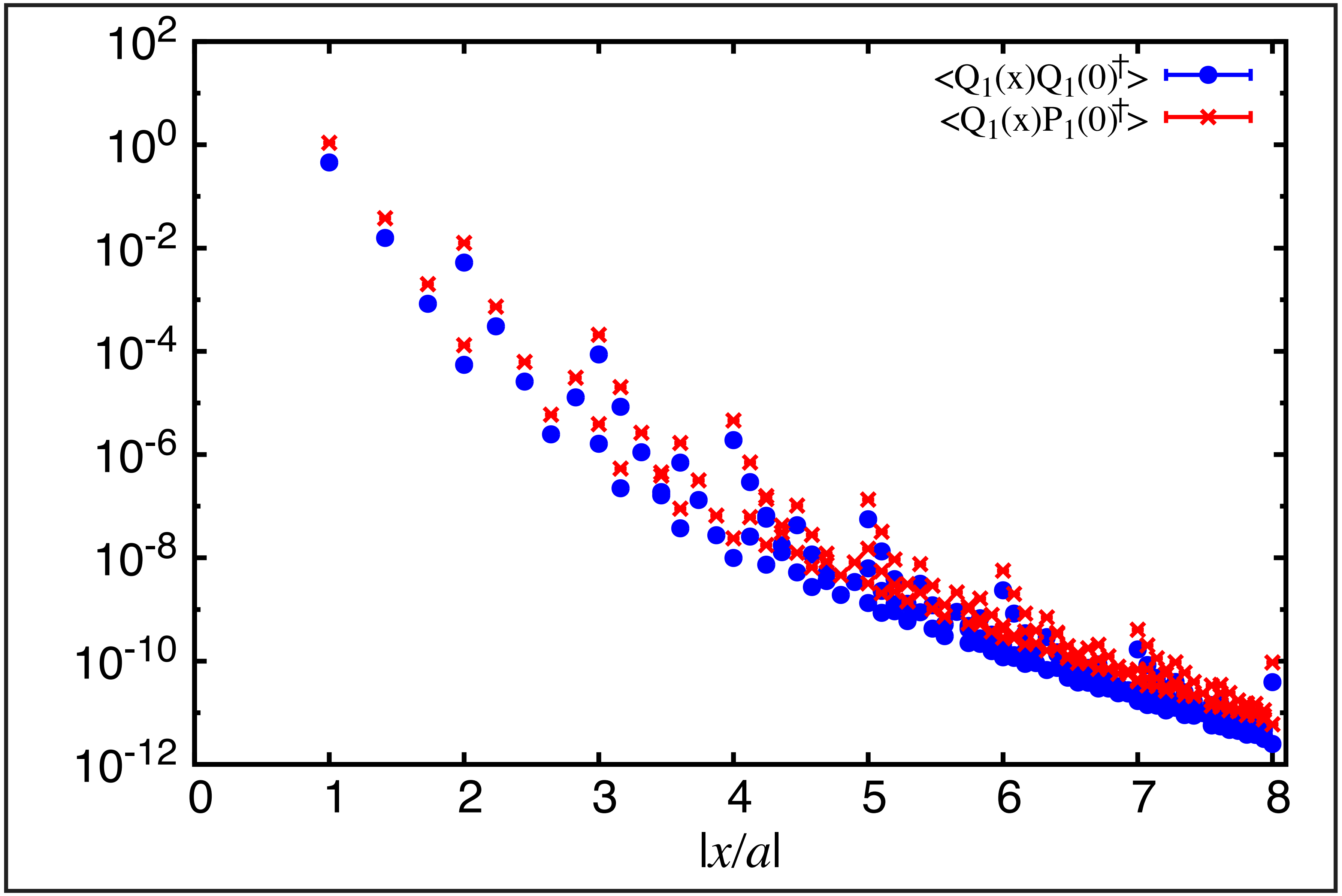
Computational status

- What we have done
  - Test calculation of all 107 contractions on 16^3 x 32
  - using pure random noise A2A propagators
  - single-node calculation
  - Data analysis/checking correctness of code (on going)

- What we are preparing
  - A2A/point-src props using Lanczos algorithm (will be ready by mid May)
  - multi-node calculation on KNL/BNL is currently somehow slow (expected to be figured out in mid May)

- Ensembles in main running
  - 32^3 x 64, 2+1 DWFs, a^{-1} = 2.38, 3.15 GeV

Some results



- Different lattice points distinguished ((1,1,1,1) ≠ (0,0,0,2))

More about source $\bar{O}(x)$

- It can take any form
  - T-slice sum
  - Smearing

$<\bar{O}(\mu;x)\ H_W(y)^\dagger>$

- Our final goal: continuum limit of 4/3-flavor matching
  - to apply it to the result on 1.38 GeV lattice

- We propose an idea to take continuum limit in a more appropriate way
  - Averaging correlators over sphere to get O(4)-symmetric ones
  - Example for Z_m [MT & N. Christ, PRD99,014515]

Average over spheres

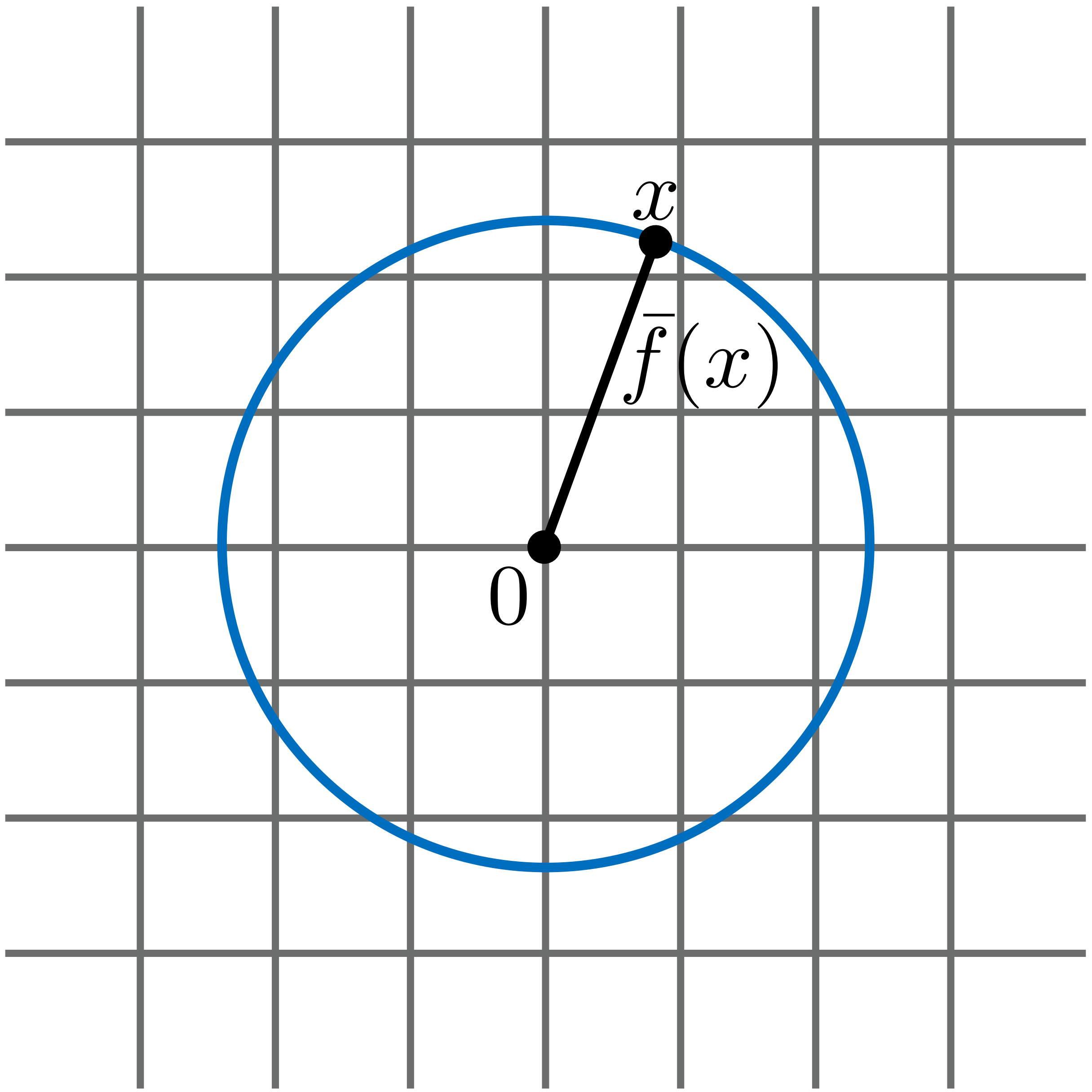
- Evaluate the value of a quantity at each 4d point from values at lattice points, with a guideline

$$\bar{f}(x) = \eta(f^{\mathrm{lat}}; x)$$

※ details in following slides

- Take the average over the sphere for each distance |x|

$$\hat{f}(|x|) = \frac{1}{2\pi^2} \oint_{S^3(|x|)} \mathrm{d}\Omega \, \bar{f}(x)$$



Potential $O(a^1)$ error (1-dim)

- Defs:
  - f_n: lattice value at site n
  - $f(x)$: "continuum limit" : $f_n = f(an) + O(a^2)$

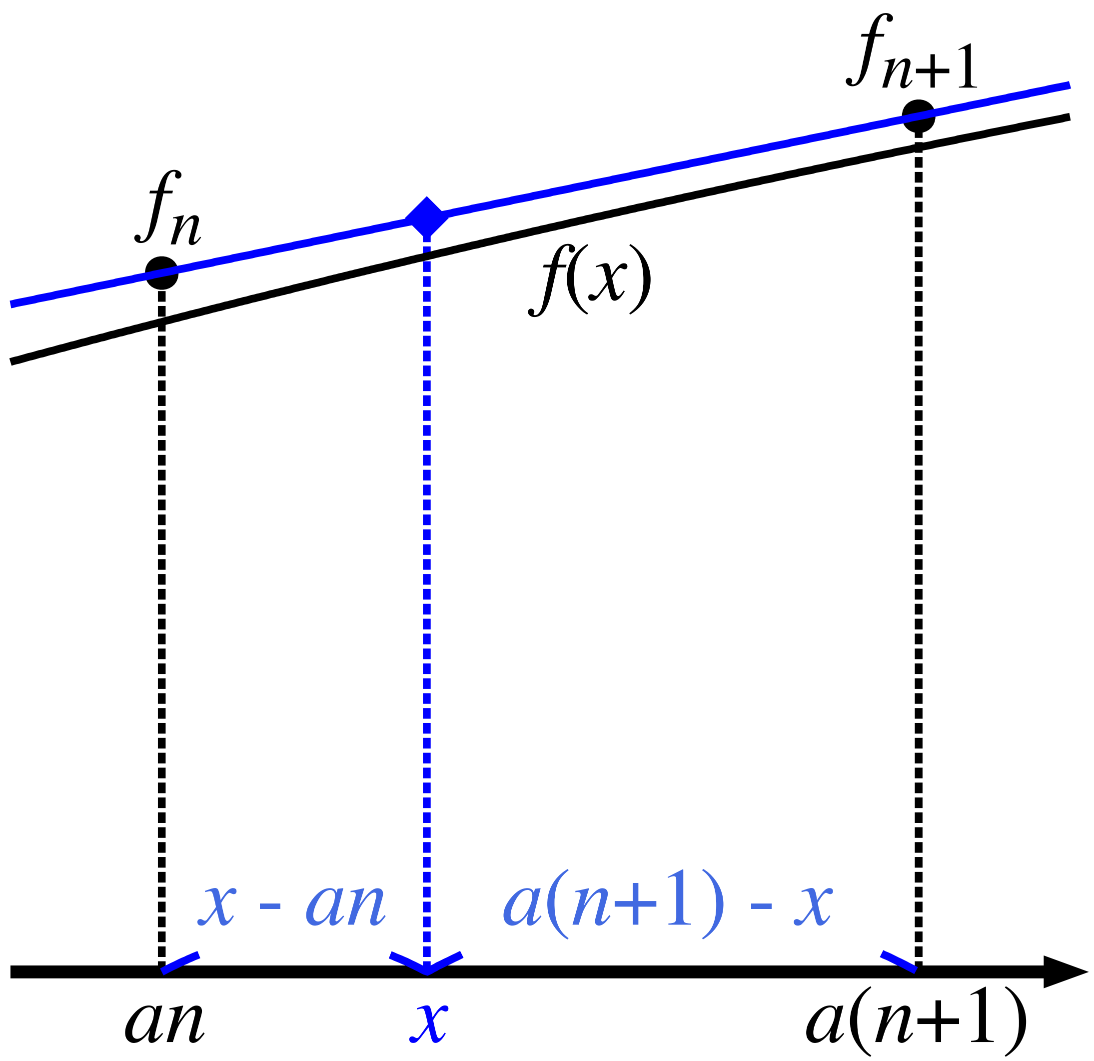
- Estimation $\bar{f}(x)$ should satisfy
  - $\bar{f}(x) = f(x) + O(a^2)$

- Potential $O(a^1)$ error in $\bar{f}(x)$
  - $f_n = f(an) + O(a^2)$

    $= f(x) + \underbrace{f'(x)\cdot(an-x)}_{O(a^1)} + O(a^2)$
  - $\bar{f}(x)$ is calculated using f_n's $\Rightarrow O(a^1)$ can appear
  - Balanced combination needed

Evaluation of $\bar{f}(x)$ (1-dim)

- Linear interpolation

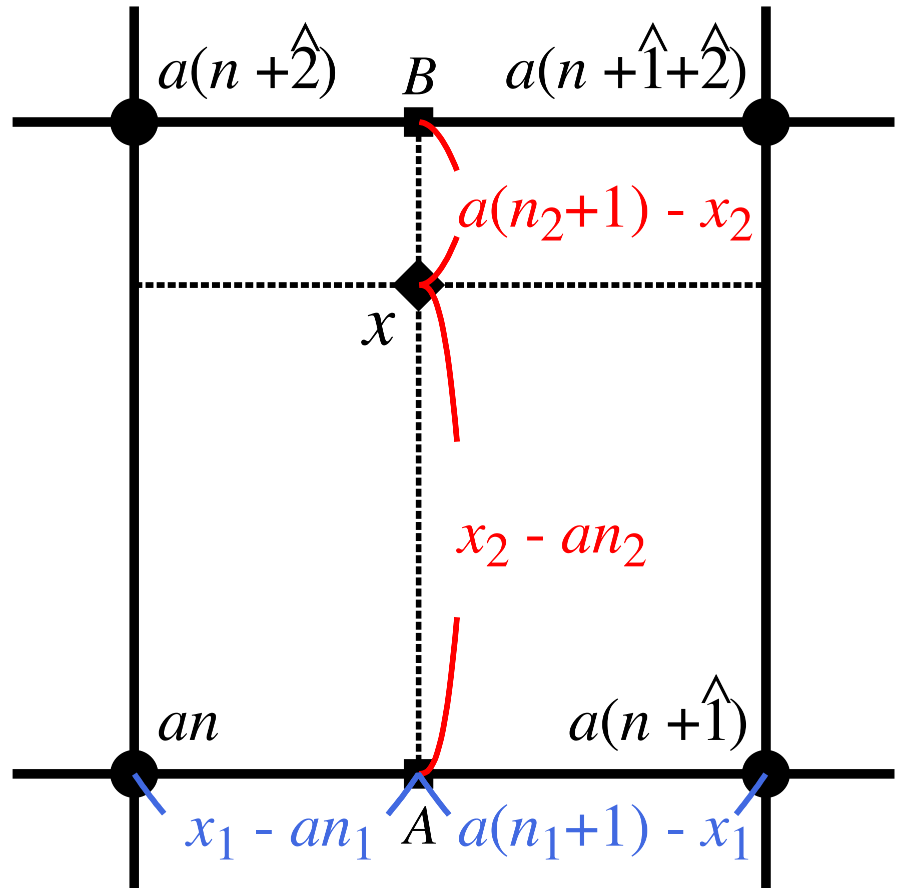


$$\bar{f}(x) = \frac{(a(n+1) - x)f_n + (x - an)f_{n+1}}{a} = f(x) + O(a^2)$$

Accurate up to $O(a^2)$

Evaluation of $\bar{f}(x)$ (2-dim)

- Bilinear interpolation



$$\bar{f}(x) = \frac{(a(n_2+1)-x_2)\bar{f}(A) + (x_2 - an_2)\bar{f}(B)}{a}$$

$$= a^{-2}\left(\ a(n_1+1)-x_1 \quad x_1 - an_1\ \right)\begin{pmatrix} f_n & f_{n+\hat{2}} \\ f_{n+\hat{1}} & f_{n+\hat{1}+\hat{2}} \end{pmatrix}\begin{pmatrix} a(n_2+1)-x_2 \\ x_2 - an_2 \end{pmatrix}$$

$$= f(x) \underline{+ O(a^2)}$$

Evaluation of $\bar{f}(x)$ (4-dim)

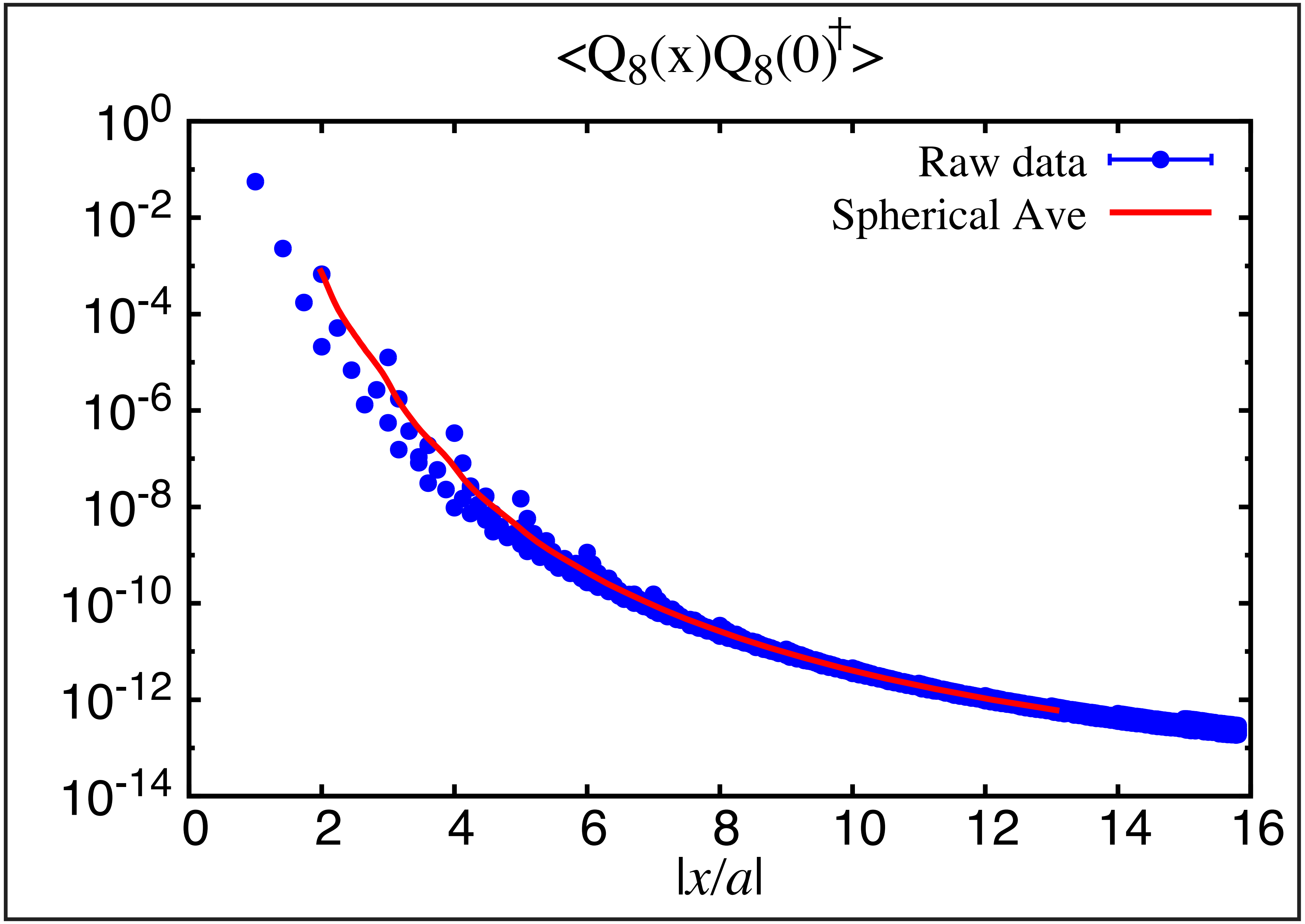
- Quadrilinear interpolation

$$\bar{f}(x) = a^{-4} \sum_{i,j,k,l=0}^{1} \Delta_{1,i}\Delta_{2,j}\Delta_{3,k}\Delta_{4,l} f_{n+i\hat{1}+j\hat{2}+k\hat{3}+l\hat{4}}$$

$$\Delta_{\mu,i} = |a(n_\mu + 1 - i) - x_\mu|$$

  – Accurate up to $O(a^2)$

Some results



Summary

- NP 4f/3f matching desired for $K \to \pi\pi$ calculation
- Position-space procedure
  - Gauge invariant
  - Free from mixing w/ irrelevant operators
- Measurement almost ready for main running
- New treatment for discretization errors is proposed
  - Interpolation + Spherical average
  - Good for continuum limit

Previous effort in mom Sp.

- Condition

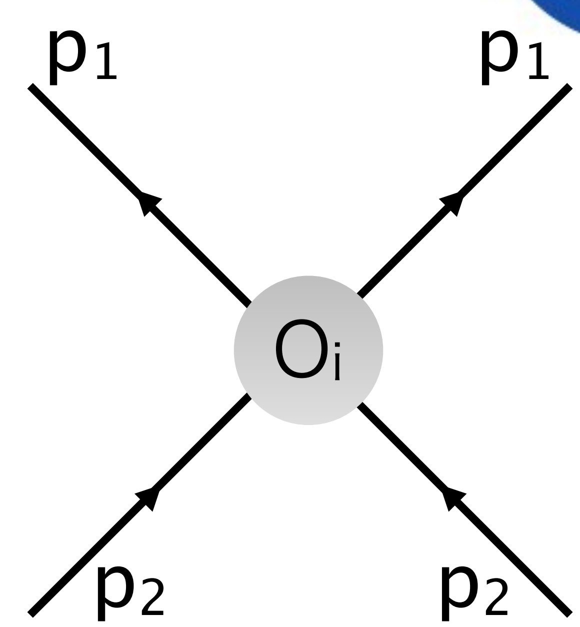
$$P^{abcd}_{\alpha\beta\gamma\delta}\ \Lambda^{abcd}_{\alpha\beta\gamma\delta}\,(O^{3f}_i(\mu);p_1,p_2)\ w^{3f}_i(\mu)$$

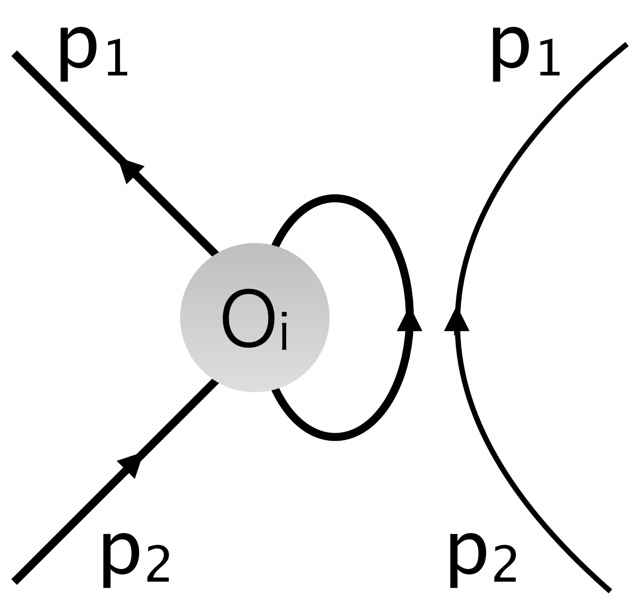
$$\|$$

$$P^{abcd}_{\alpha\beta\gamma\delta}\ \Lambda^{abcd}_{\alpha\beta\gamma\delta}\,(O^{4f}_i(\mu);p_1,p_2)\ w^{4f}_i(\mu)$$

G-fixed amputated Green's function

flavor, color and spin projector





- Condition valid in $|p_{1,2}| \ll m_c$
- Statistical error
  - $|p_{1,2}|$ = 1.2 GeV → 10%
  - $|p_{1,2}|$ = 0.6 GeV → 50%
-

Why mom procedure so bad?

- Gauge fixing
  - Large Gribov noise
    - Gauge condition does not have a unique solution on the gauge orbit
    - Gauge-dependent quantities have some ambiguity
  - Mixing with gauge-noninvariant operators
- Off-shell condition
  - Mixing with operators that vanish by EoM

☆ All significant at small $p_{1,2}$

☆ Position-space procedure is free from all of these

Quark mass renormalization

- $Z_m = Z_S^{-1}$
- Position-space renormalization of scalar current

$$Z_S^{\overline{MS}/lat}(\mu, 1/a)^2 \, \Pi_S^{lat}(1/a; x) = \Pi_S^{\overline{MS}}(\mu; x)$$

$$Z_S^{\overline{MS}/lat}(\mu, 1/a) = \sqrt{\frac{\Pi_S^{\overline{MS}}(\mu; x)}{\Pi_S^{lat}(1/a; x)}}$$

  - ✦ Π_S: two-point function of scalar currents
  - ✦ $Z_S = Z_P$ if chirally symmetric lattice action (DWF in this work)
- We analyze

$$\widetilde{m}_q^{\overline{MS}}(\mu; x; a) = m_q^{\text{bare,phys}}(a) \sqrt{\frac{\frac{1}{2}(\Pi_S^{\text{lat}}(1/a; x) + \Pi_P^{\text{lat}}(1/a; x))}{\Pi_S^{\overline{MS}}(\mu; x)}}$$

Lattice calculation

- Ensembles
  - 2+1 Domain-wall fermions
  - 3 lattice spacings: 1.7−3.1 GeV
  - Pion masses: 300−420 MeV

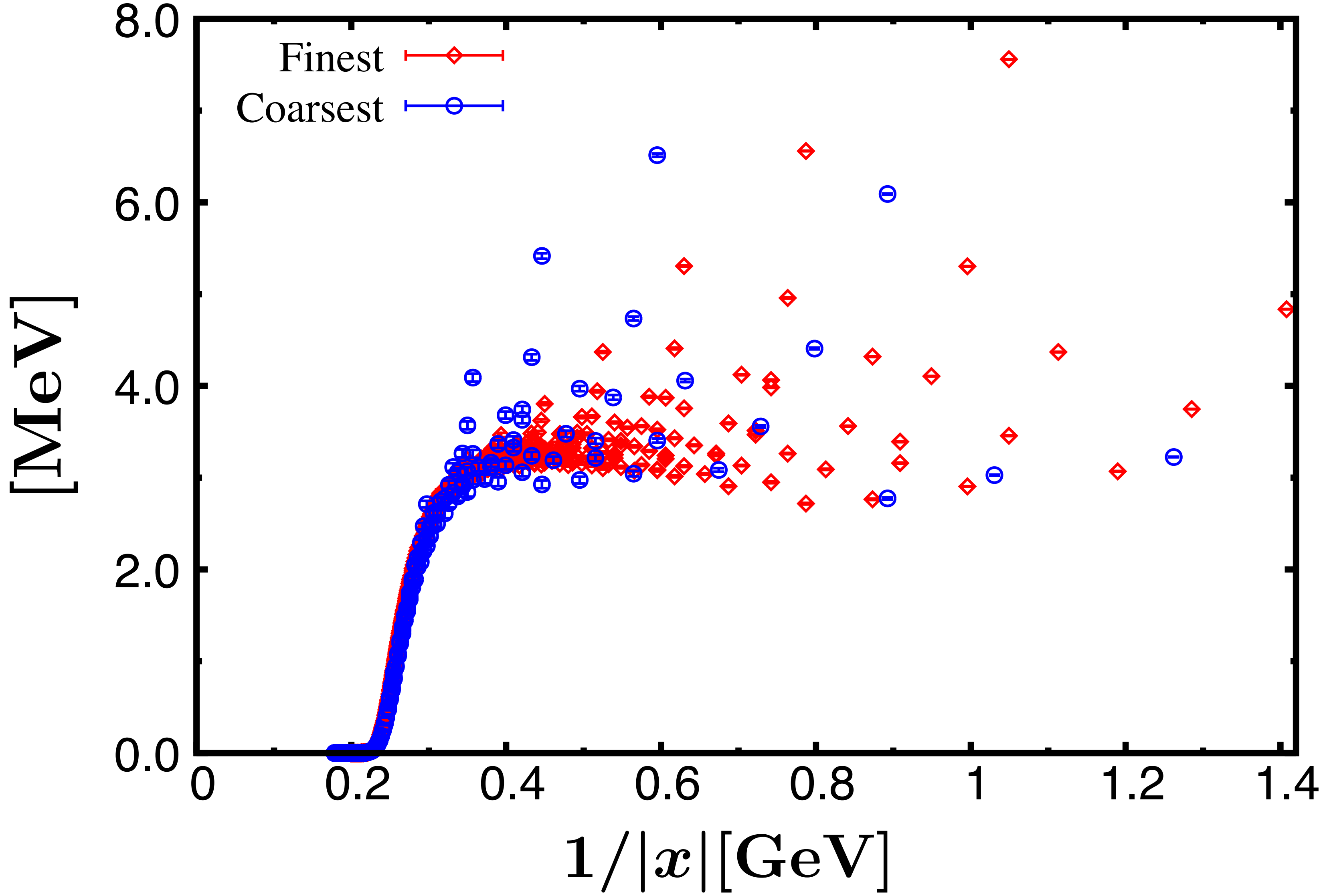
- For each ensemble, we analyze

$$\widetilde{m}_q^{\overline{\mathrm{MS}}}(\mu; x; a) = \underline{m_q^{\mathrm{bare,phys}}(a)} \sqrt{\frac{\frac{1}{2}(\Pi_S^{\mathrm{lat}}(1/a; x) + \Pi_P^{\mathrm{lat}}(1/a; x))}{\Pi_S^{\overline{\mathrm{MS}}}(\mu; x)}}$$

[RBC/UKQCD (2016)]

Supposed to be a constant Z_m if in renormalization window

$\widetilde{m}_q^{\overline{MS}}(3\,\text{GeV};x)$



- Different lattice points distinguished ((1,1,1,1) vs (0,0,0,2))
- Large discretization errors

Result for spherical average

- Sphere average of

$$\widetilde{m}_q^{\overline{\rm MS}}(\mu;x;a) = m_q^{\rm bare,phys}(a)\sqrt{\frac{\frac{1}{2}(\Pi_S^{\rm lat}(1/a;x)+\Pi_P^{\rm lat}(1/a;x))}{\Pi_S^{\overline{\rm MS}}(\mu;x)}}$$

- Able to calculate at any distance
- Plateau seen better for finer lattices