Nucleon Quark-Gluon Structure with Clover-Wilson Fermions

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(LHP collaboration)

Outline

- High momentum nucleon form factors
- TMD amplitudes on a lattice
- (Future) quark orbital angular momentum (qOAM)

Motivation for High-Mom. Form Factor Study

Current experimental status :

- proton: up to $Q^2 \sim 8.5\ \mathrm{GeV}^2$
- neutron: up to $Q^2 \sim 3.4\ \mathrm{GeV}^2$

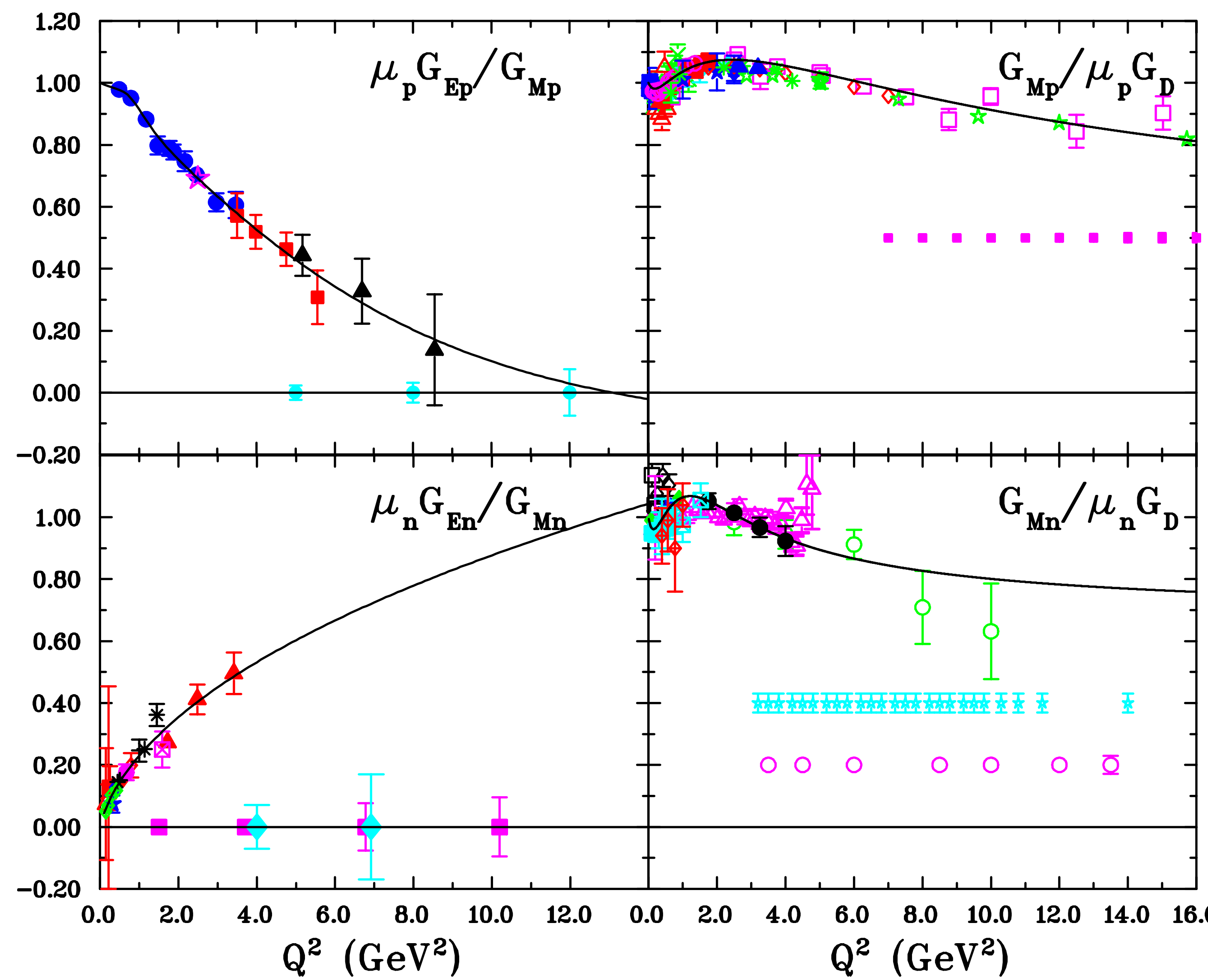
Ongoing experimental activity @CEBAF

- high-resolution spectrometer, Hall A
  - G_M^p up to $Q^2 = 17.5\ \mathrm{GeV}^2$
- Halls B, C
  - G_M^n up to $Q^2 = 14\ \mathrm{GeV}^2$
  - G_E^n/G_M^n up to $Q^2 = 6.9\ \mathrm{GeV}^2$
- new Super-BigBite Spectrometer, Hall A
  - G_E^p/G_M^p up to $Q^2 = 15\ \mathrm{GeV}^2$
  - G_E^n/G_M^n up to $Q^2 = 10.2\ \mathrm{GeV}^2$
  - G_M^n up to $Q^2 = 18\ \mathrm{GeV}^2$

Theory implications :

- Transition to perturbative scaling
- Phenomenology of nucleon constituents
- Input to GPD analysis

V. Punjabi et al.
[arXiv: 1503.01452]



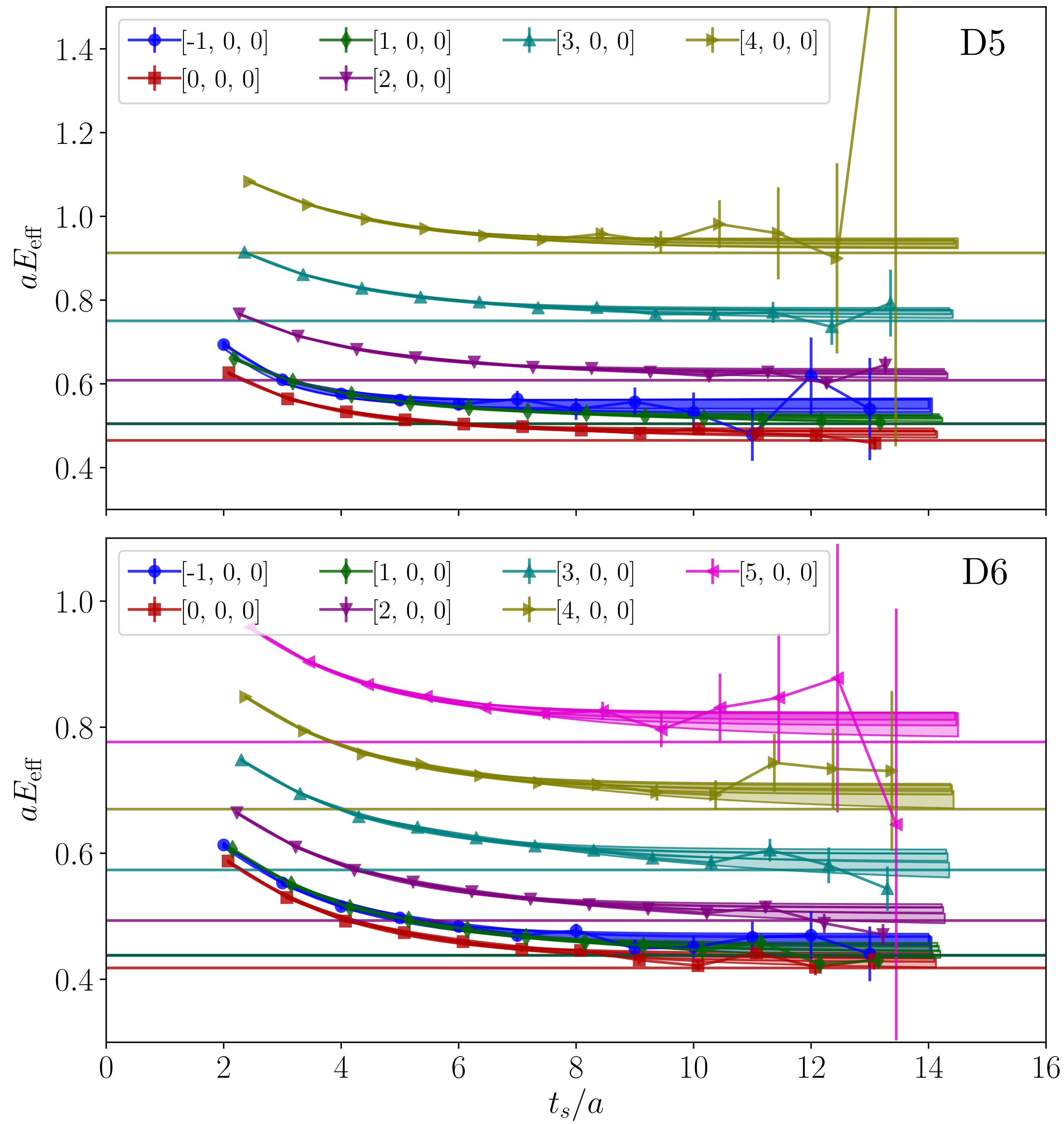
Simulation details

- Current: two Nf=2+1 Wilson-clover ensembles [JLab/W&M]
- different mπ and lattice volumes, ≈ same lattice spacing 0.09fm

D5-ensemble: $\beta = 6.3$, $a = 0.094$ fm, $a^{-1} = 2.10$ GeV		
$32^3 \times 64$, $L = 3.01$ fm	$a\mu_l$	-0.2390
	$a\mu_s$	-0.2050
	κ	0.132943
	$C_{\rm sw}$	1.205366
	m_π (MeV)	280
	$m_\pi L$	4.26
	Statistics	86144
D6-ensemble: $\beta = 6.3$, $a = 0.091$ fm, $a^{-1} = 2.17$ GeV		
$48^3 \times 96$, $L = 4.37$ fm	$a\mu_l$	-0.2416
	$a\mu_s$	-0.2050
	κ	0.133035
	$C_{\rm sw}$	1.205366
	m_π (MeV)	170
	$m_\pi L$	3.76
	Statistics	50176

- Computational resources: BNL Institutional Cluster
- Software: Qlua + QUDA-MG for propagators + QUDA-based contractions on GPU (including straight and staple gauge link paths for qPDF and TMD operators)

Effective Energy

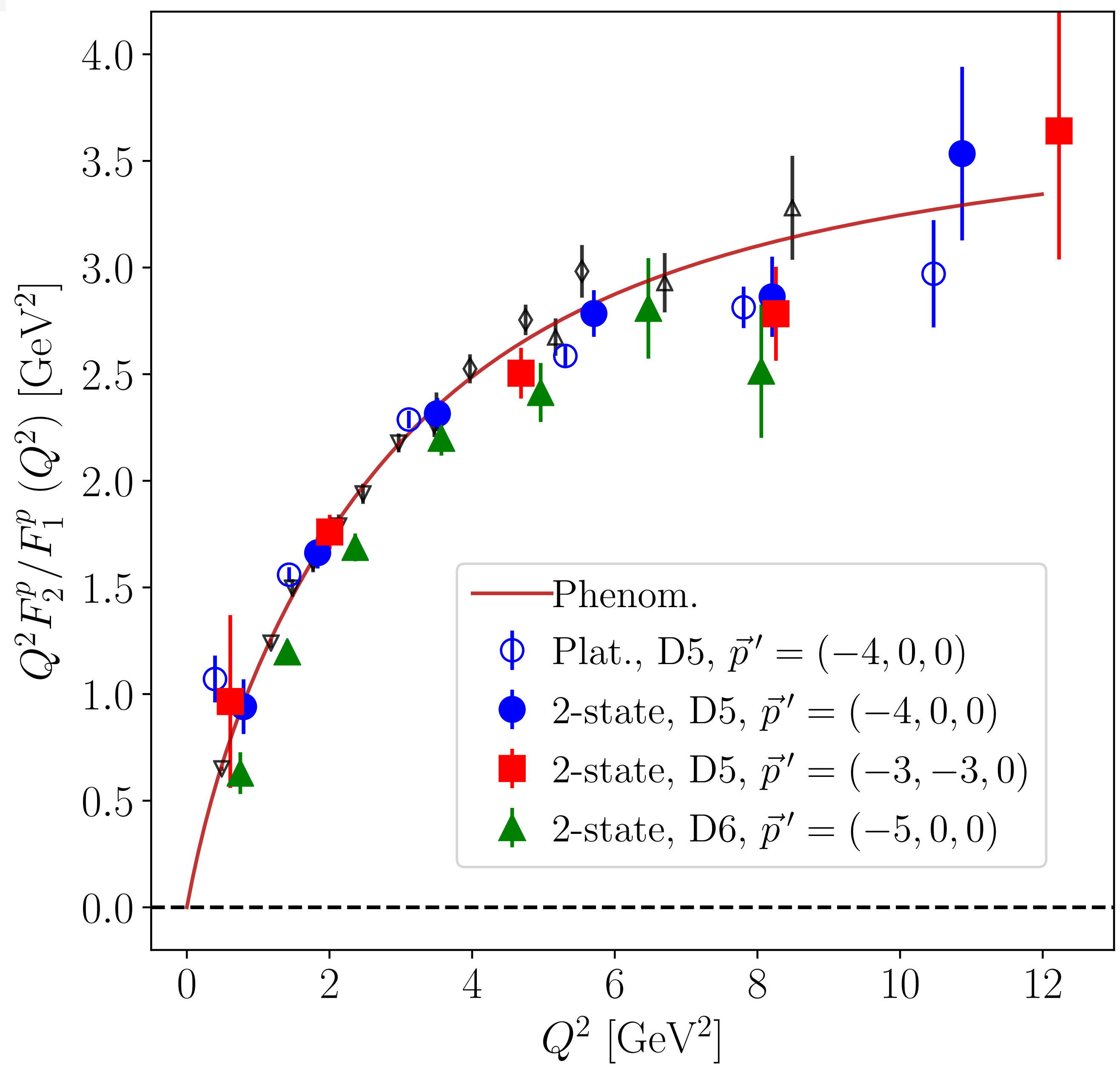


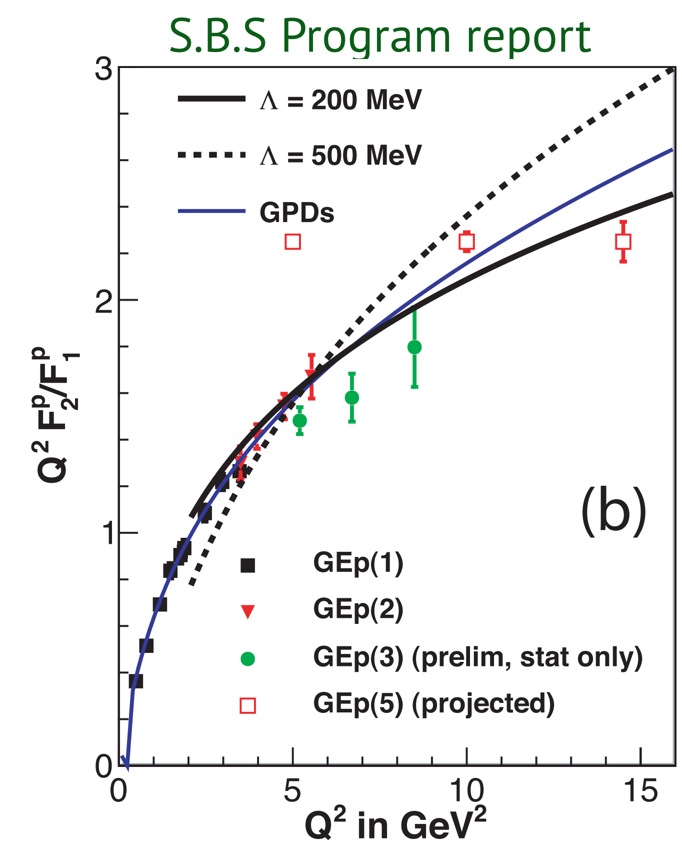
Potential excited state contamination is a major concern

$$E^N_{\rm eff}(t_s) = \log\left(\frac{C(\vec{p},t_s)}{C(\vec{p},t_s+1)}\right) \xrightarrow{t_s \gg 1} E_N$$

- two-state fits to our lattice data are of good quality
- horizontal line from $E = m_N^2 + p^2$ using lattice value of m_N
- ground state energy slightly overestimates cont. dispersion relation
- excited states faint after ~ $t_s/a = 9$

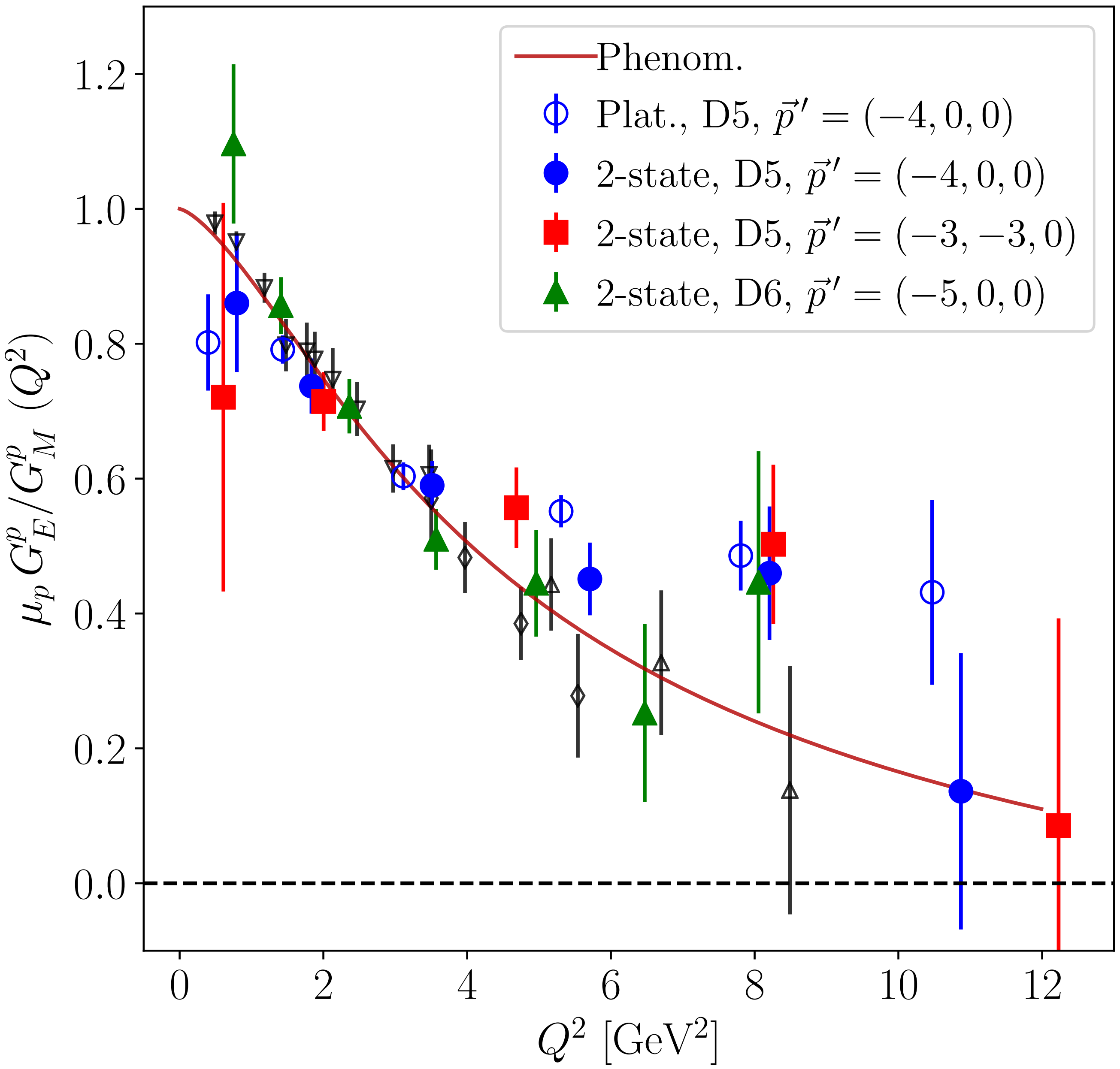
Form Factor Results I: F_2/F_1 for the Proton

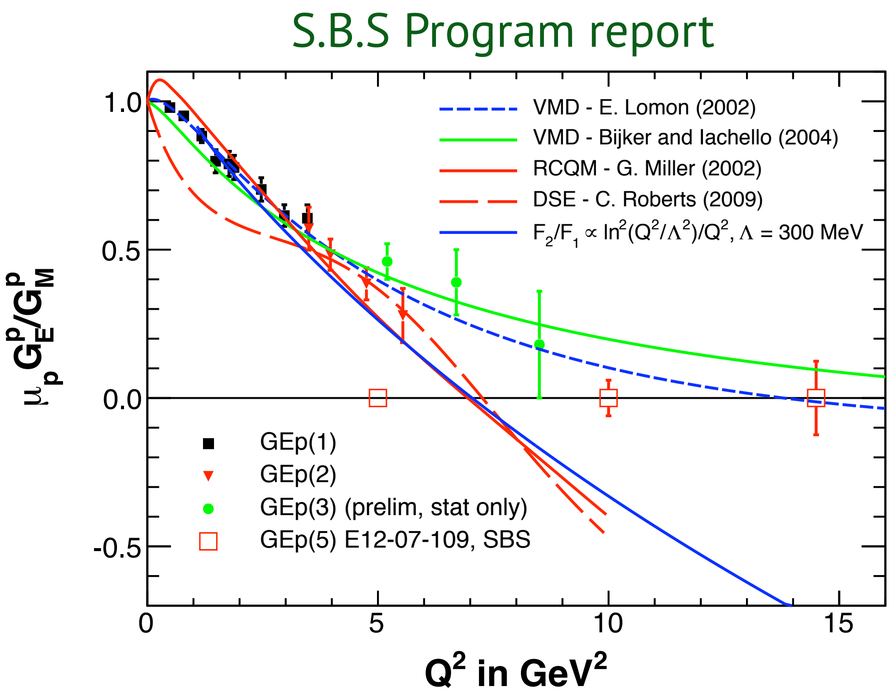




- experimental data: up to Q^2 ~8.5 GeV^2 W. M. Alberico et al. [arXiv: 0812.3539]
- Q^2 - dependence compares well with exp. data and phenom. parametrization
- $Q^2 F_2^p / F_1^p (Q^2) \sim \log^2[Q^2/\Lambda]$ scaling reproduced A.V. Belitsky et al. [arXiv: hep-ph/0212351]
- consistency between on-axis / x-y diagonal boost momentum for D5

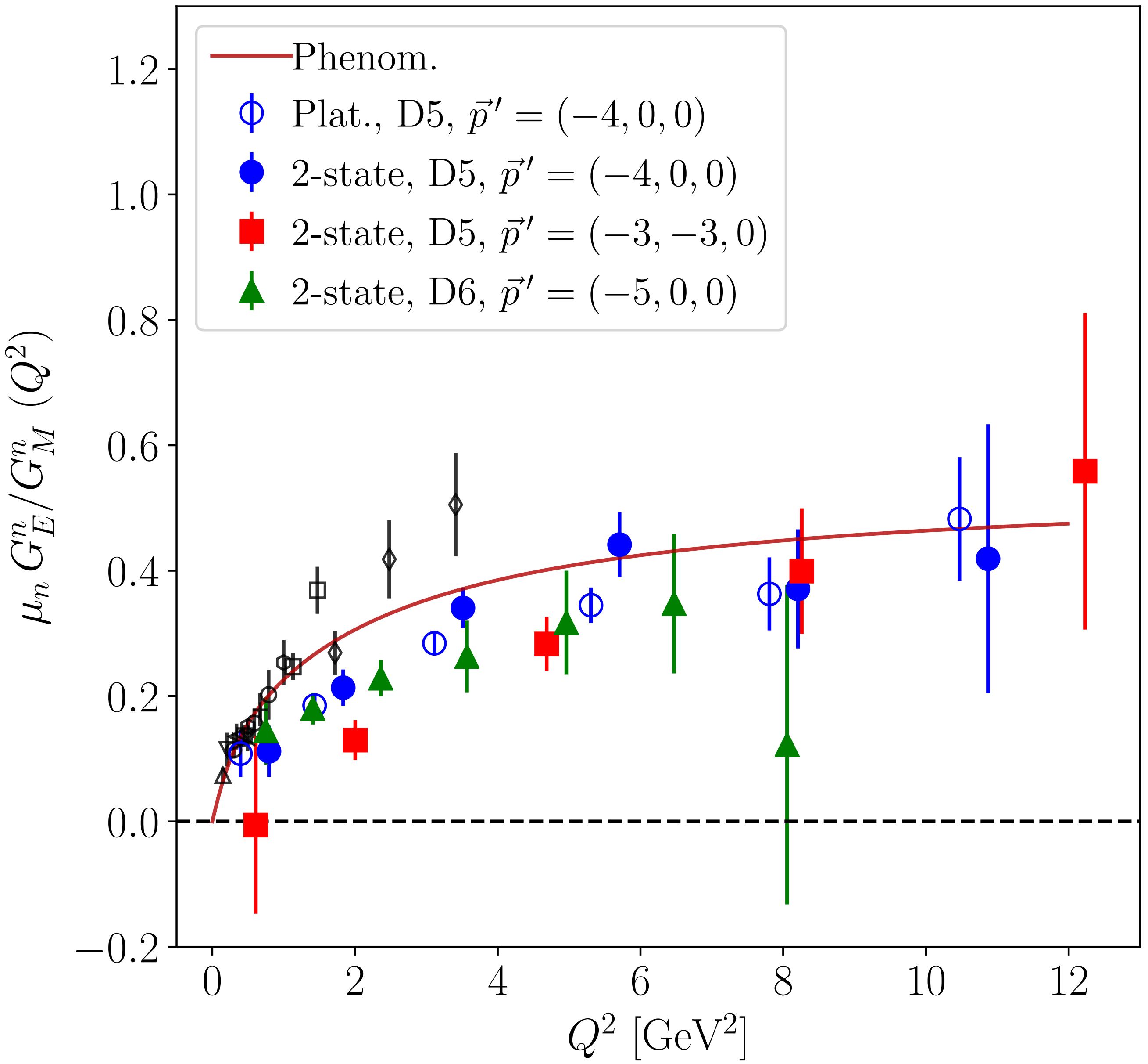
Form Factor Results II: G_E/G_M for the Proton

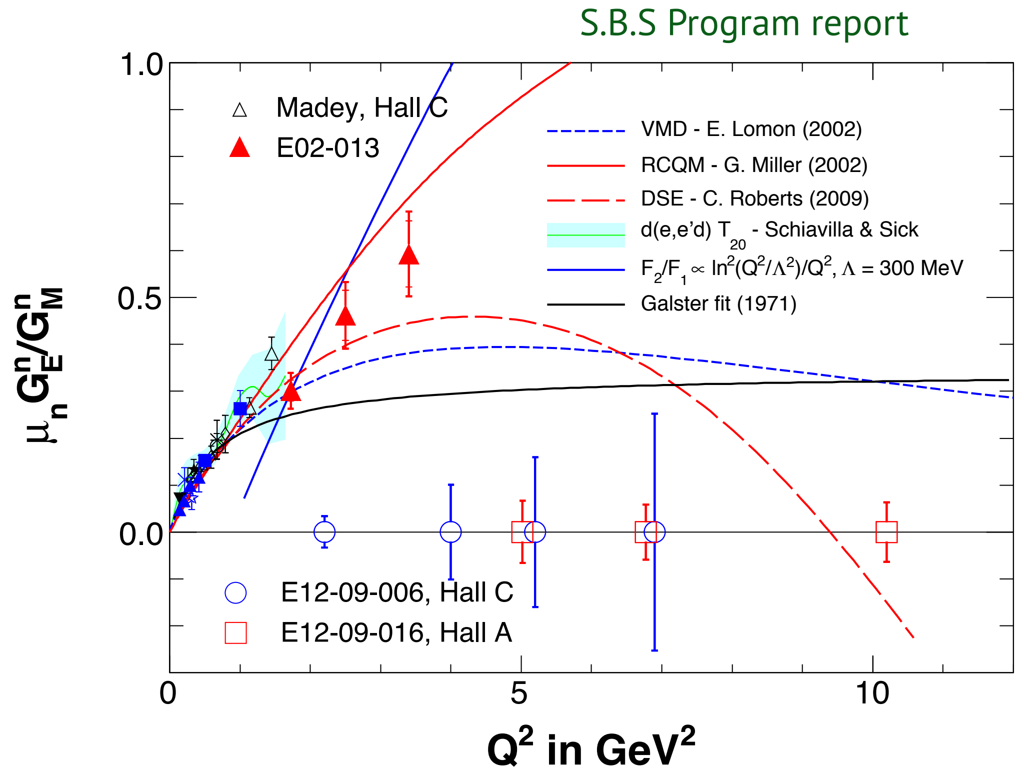




- experimental data: up to $Q^2 \sim 8.5$ GeV^2
- consistency between our lattice data
- good agreement with experiment / phenomenology for proton up to $Q^2 \sim 6$ GeV^2
- variety in theoretical predictions: lattice data support smoother approach towards

Form Factor Results III: G_E/G_M for the Neutron

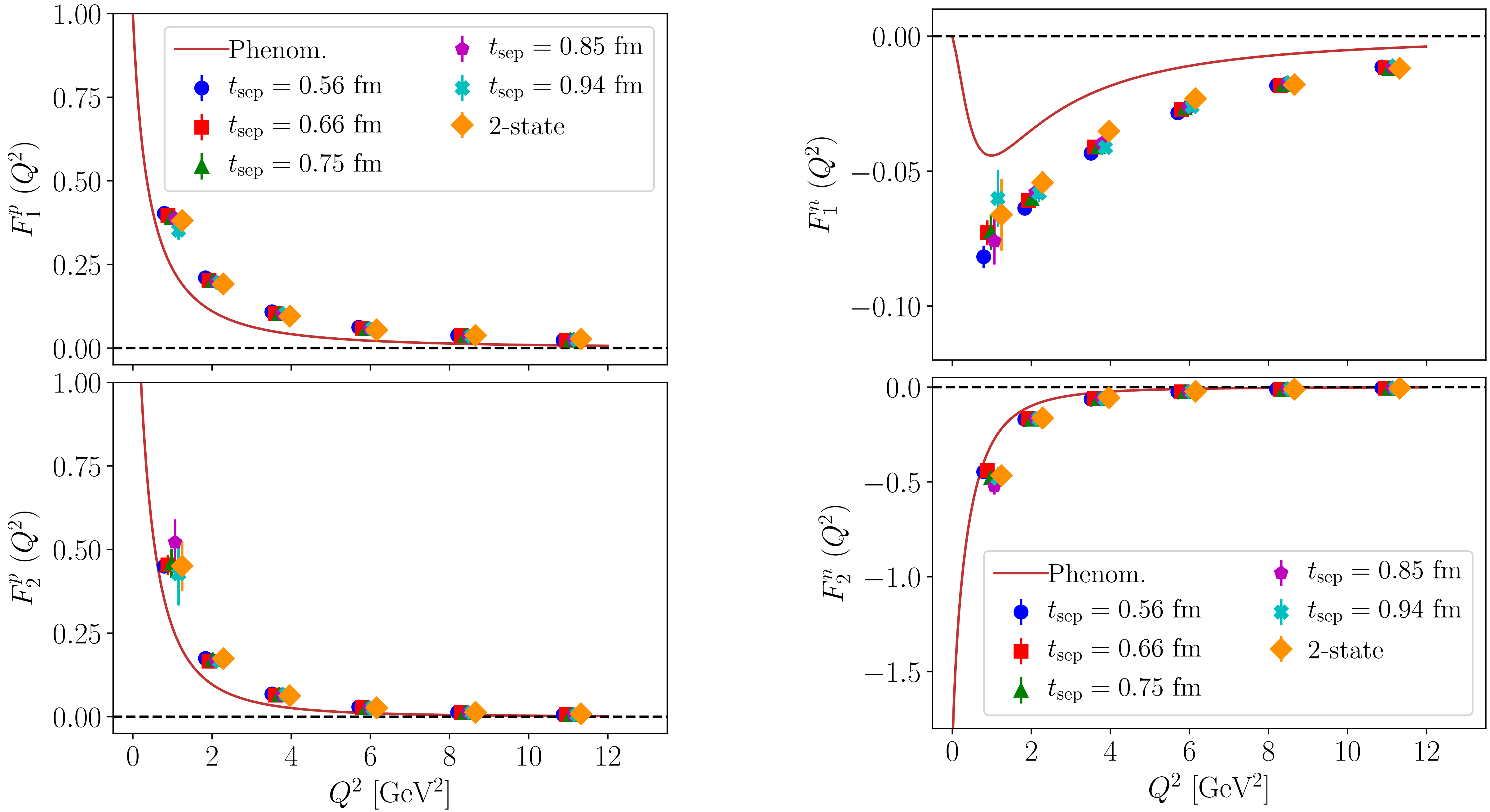




Experiment & Phenomenology

- experimental data: up to $Q^2 \sim 3.4$ GeV^2
- neutron: out lattice data underestimate experiment / phenomenology
  - disconnected diagrams?
- same qualitative behavior

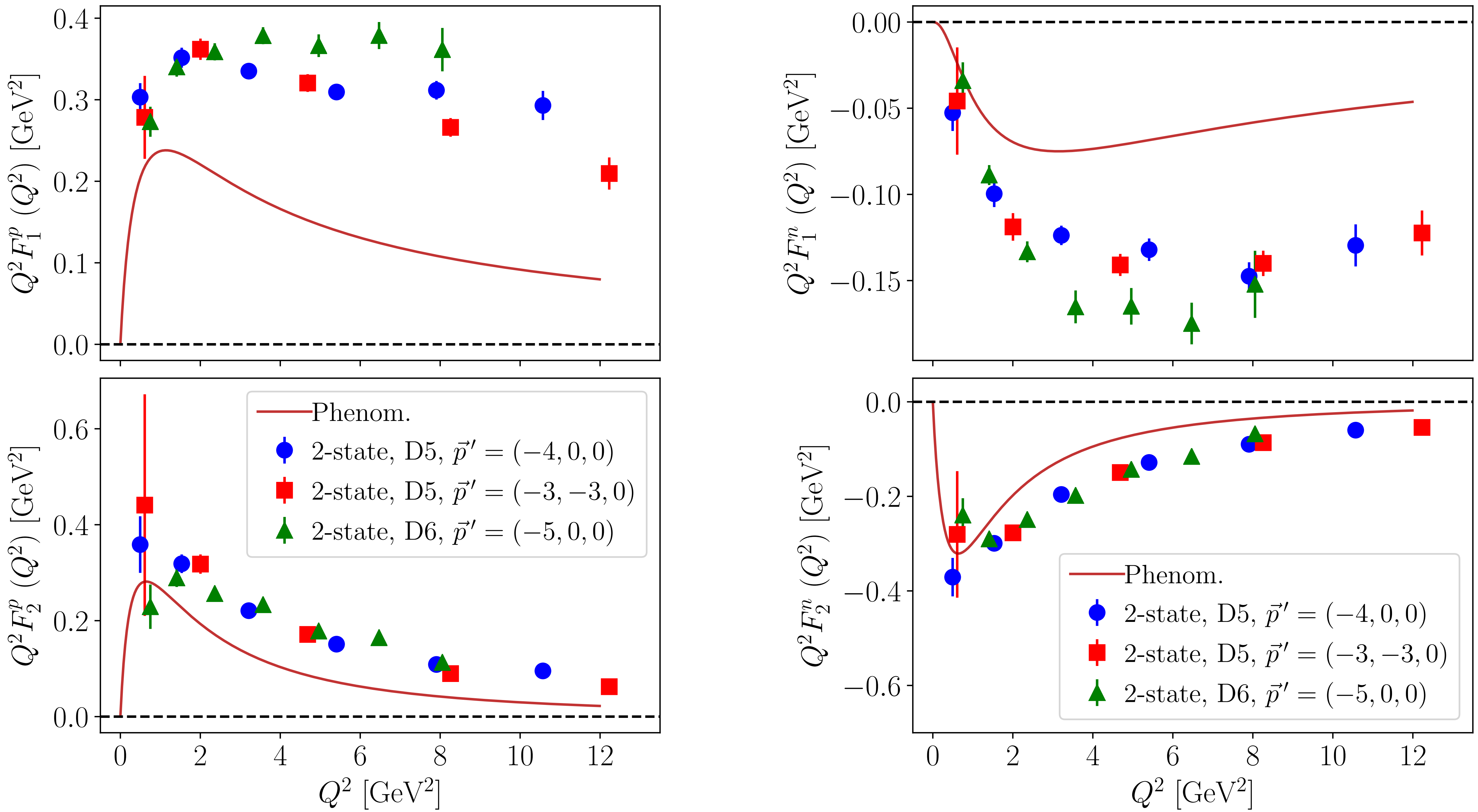
Form Factor Results IV: Individual Form Factors



- shallow trend towards phenom. with increasing source-sink separation
- similar qualitative behavior, overestimation of phenom. prediction

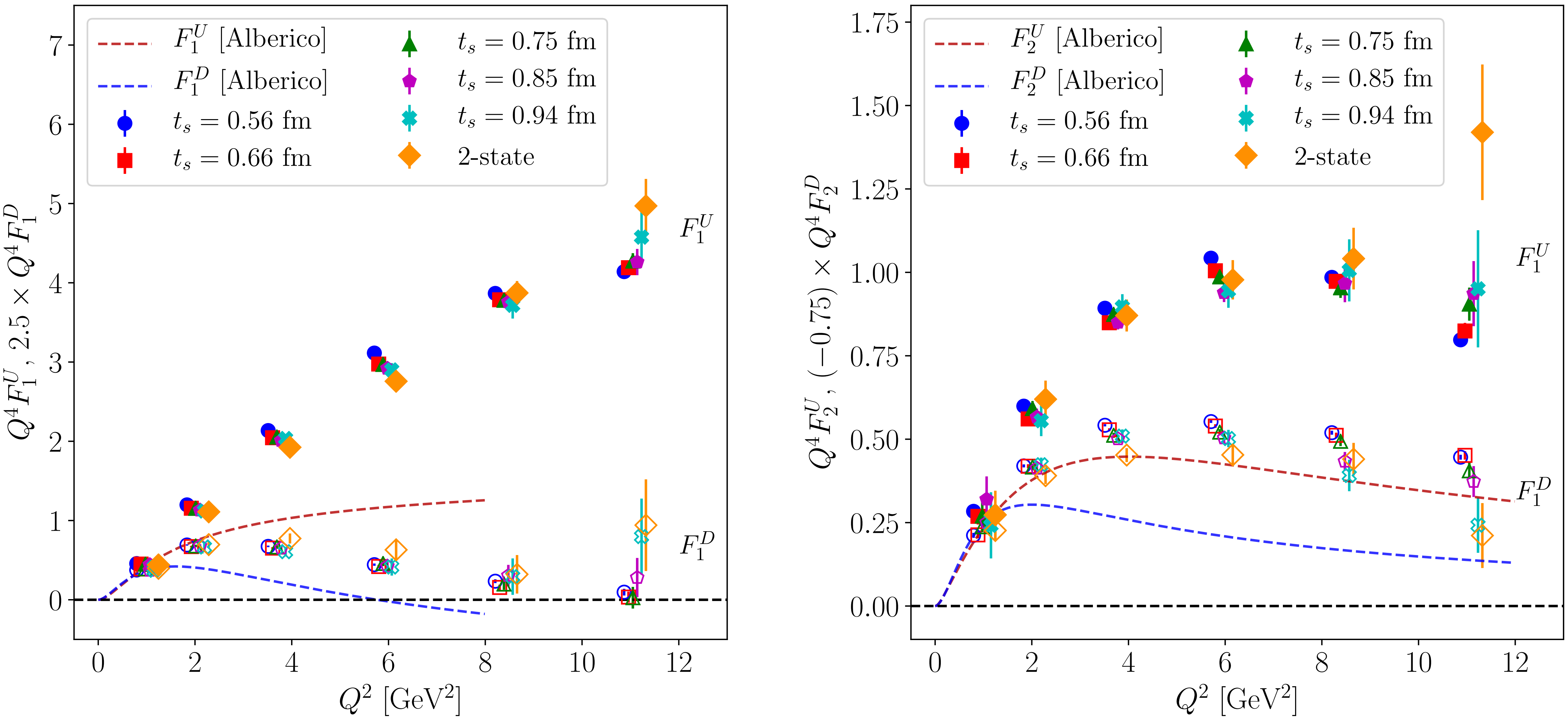
(comparison to [W. M. Alberico et al.])[arXiv: 0812.3539]

Form Factor Results IV: Individual Form Factors



- discrepancies for individual form factors
- *a thorough investigation is needed*

Form Factor Contributions from u,d Quarks



- discrepancies observed for form factors of up- and down- quarks
- qualitative agreement of characteristic features

High-Momentum TODO List

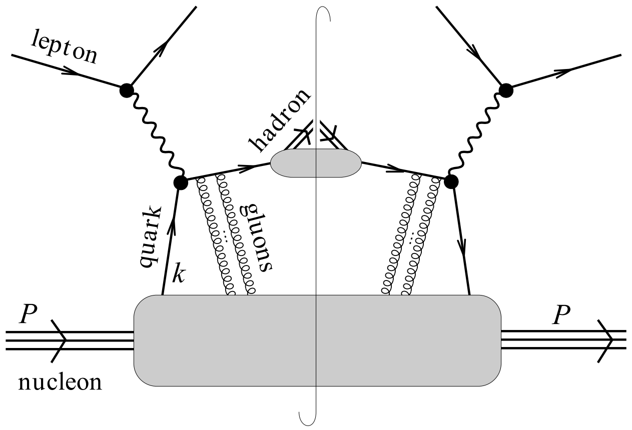
- understand/resolve disagreement for individual form factors F_1, F_2
- complete investigation of excited state effects
- consider other systematic effects
  - $O(a)$ improvement
  - physical pion mass
  - continuum extrapolation
- disconnected diagrams on the way

TMD Calculations on a Lattice

Transerse-mom.dependent quark PDFs probed in SIDIS

$$l + N(P) \longrightarrow l' + N(P_h) + X$$

One of the primary EIC goals to study 3D nucleon image



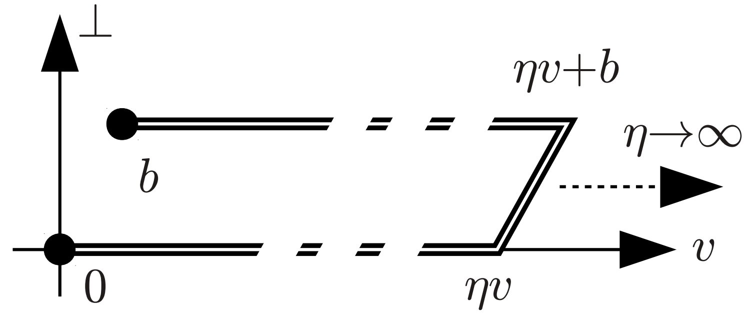
- TMD operator definition involves Wilson "staple" with "legs" along the light cone; rapidity divergences suggest making it spacelike [Aybat, Collins, Qiu, Rogers]

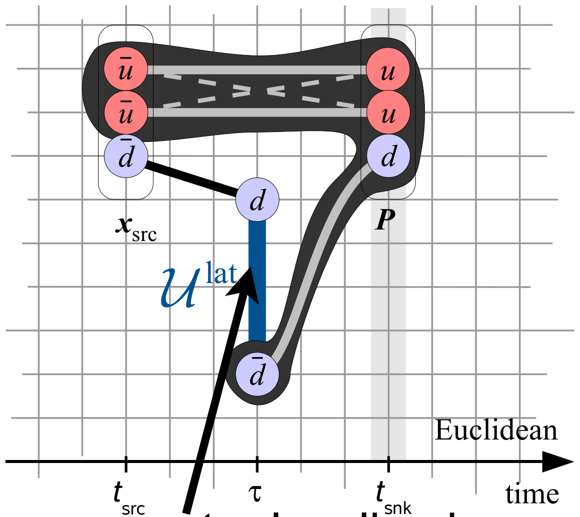
- Non-local lattice operator [B.Musch, Ph.Hagler, et al '10]

$$\Phi(b, P, S, \hat{\zeta}, \mu) = \frac{1}{2}\langle P, S|\, \bar{q}(0)\, \Gamma\, \mathcal{U}(\eta v, b)\, q(b)\, |P, S\rangle$$

spacelike link path to account for final state interactions

$\mathcal{U} =$





operator localized at Euclidean time τ

- LC limit of spacelike staple: Collins-Soper parameter $\hat{\zeta} = \dfrac{P \cdot v}{m_N\,|v|} \to \infty$

Large momentum P >> m_N is required...

Lattice TMD Amplitudes and TM "Shifts"

- Unpolarized parton density (Γ = γ+) in a polarized nucleon
  [B.Musch, Ph.Hagler, et al, PRD85:094510 (2012)]

$$\frac{1}{2P^+}\widetilde{\Phi}^{[\gamma^+]}_{\text{unsub}} = \widetilde{A}_{2B} + i m_N\, \varepsilon_{ij} b_i S_j\, \widetilde{A}_{12B}$$

expanded in invariant amplitudes ⟺ F.T. of TMD

$$\tilde{f}_1^{[1](0)}(b_T^2, \hat{\zeta}, \ldots, \eta v\cdot P) = 2\widetilde{A}_{2B}(b_T^2, \hat{\zeta}, \eta v\cdot P)/\widetilde{S}(b^2, \ldots) \quad \text{unpol. TMD}$$

$$\tilde{f}_{1T}^{\perp[1](1)}(b_T^2, \hat{\zeta}, \ldots, \eta v\cdot P) = -2\widetilde{A}_{12B}(b_T^2, \hat{\zeta}, \eta v\cdot P)/\widetilde{S}(b^2, \ldots) \quad \text{Sivers TMD}$$

[1] = first Mellin moment $\int dx\, x^0(..)$ if *b* is purely transverse

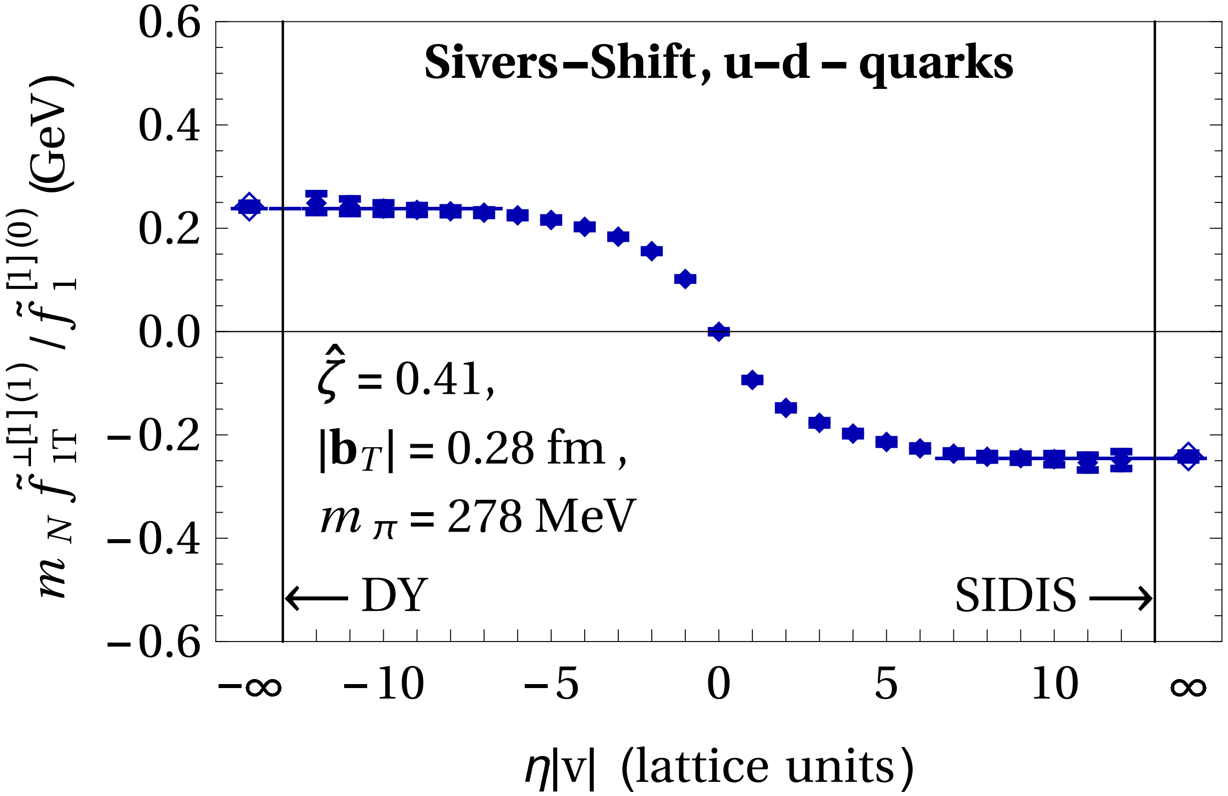
(k) = weighed F.T. $\int dk_T\, (k_T)^{2n}\, (..) \sim \langle (k_T)^{2n}\rangle$ in $\mathbf{b_T}\to 0$ limit

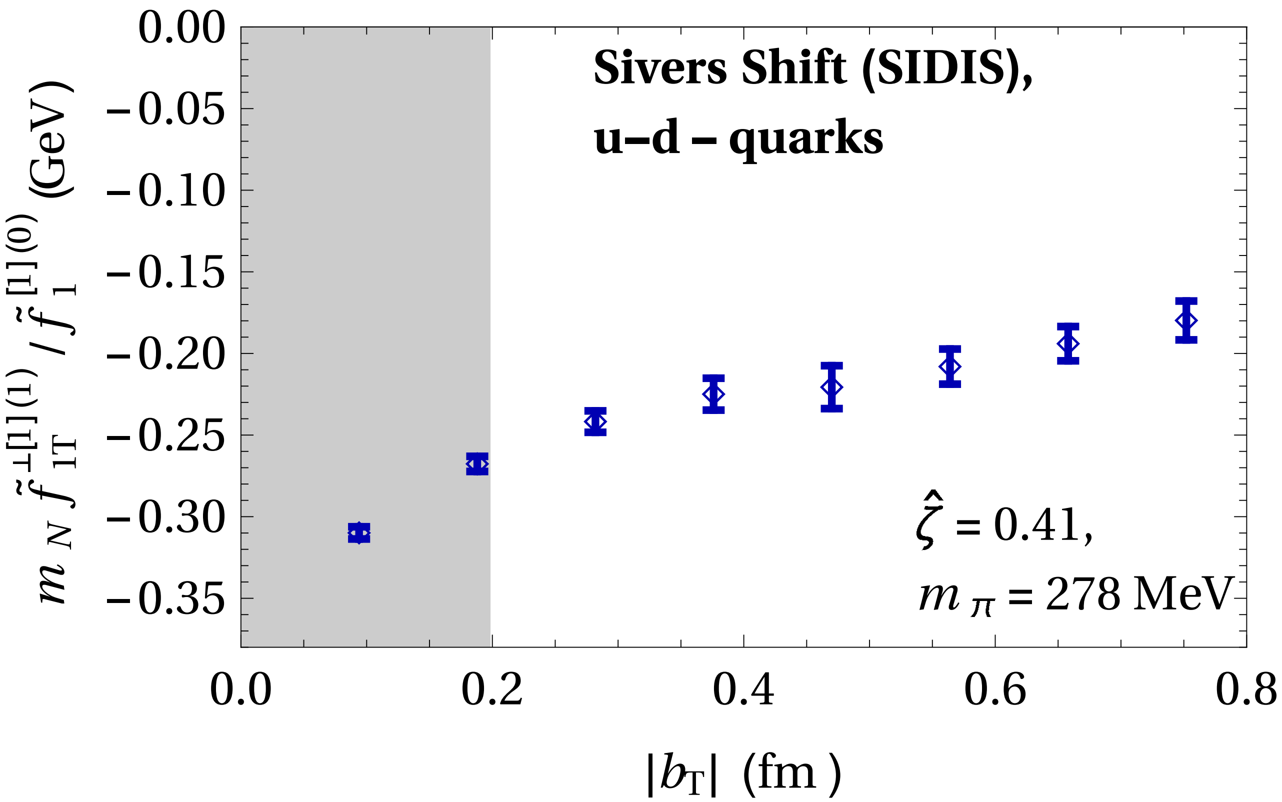
- Soft factor $\tilde{S}(b^2)$ absorbs divergences;
  cancels in the ratio yielding the "Sivers shift"

$$\langle k_y\rangle_{TU} = \left.\frac{\int dx \int d^2k_T\, k_y\, \Phi^{[\gamma^+]}(x, k_T, P, \ldots)}{\int dx \int d^2k_T\, \Phi^{[\gamma^+]}(x, k_T, P, \ldots)}\right|_{S=\hat{x}} = m_N \left[\frac{\tilde{f}_{1T}^{\perp[1](1)}}{\tilde{f}_1^{[1](0)}}\right]_{b_T\to 0}$$

- "Generalized shift" for $b \neq 0$

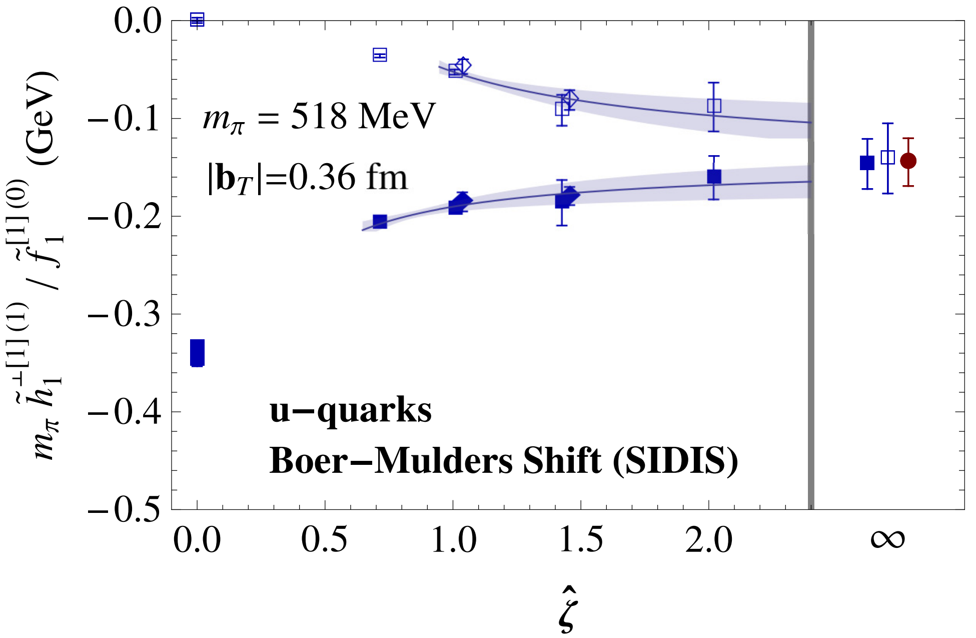
Sivers "Shift" vs. Operator Geometry





Boer-Mulders and Sivers "Shift" vs. CS parameter

Pion Boer-Mulders shift



Collins-Soper parameter limit towards LC

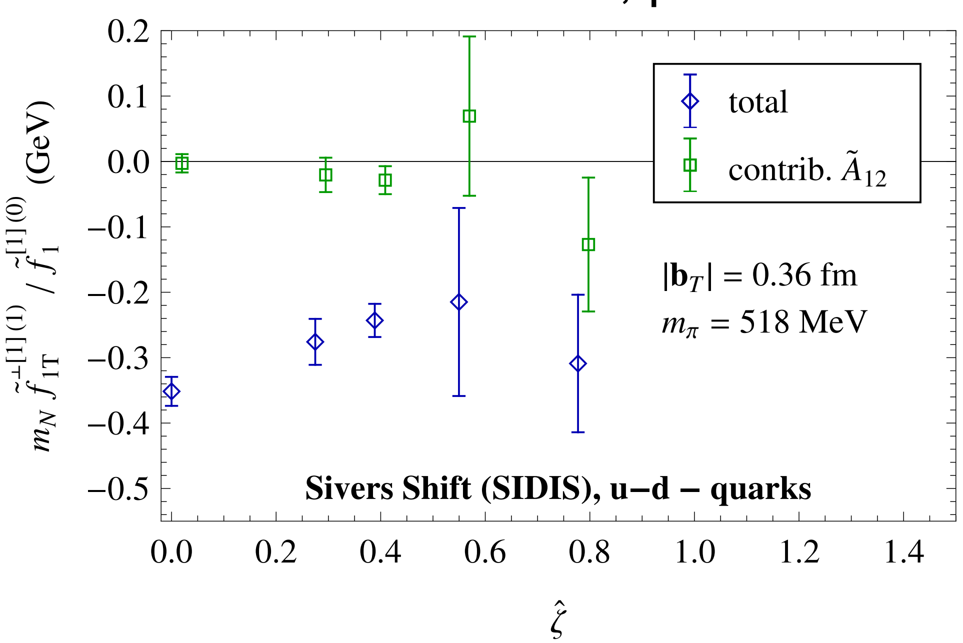
$$\hat{\zeta} = \frac{P \cdot v}{m_N |v|} \to \infty$$

← pion is easier than nucleon

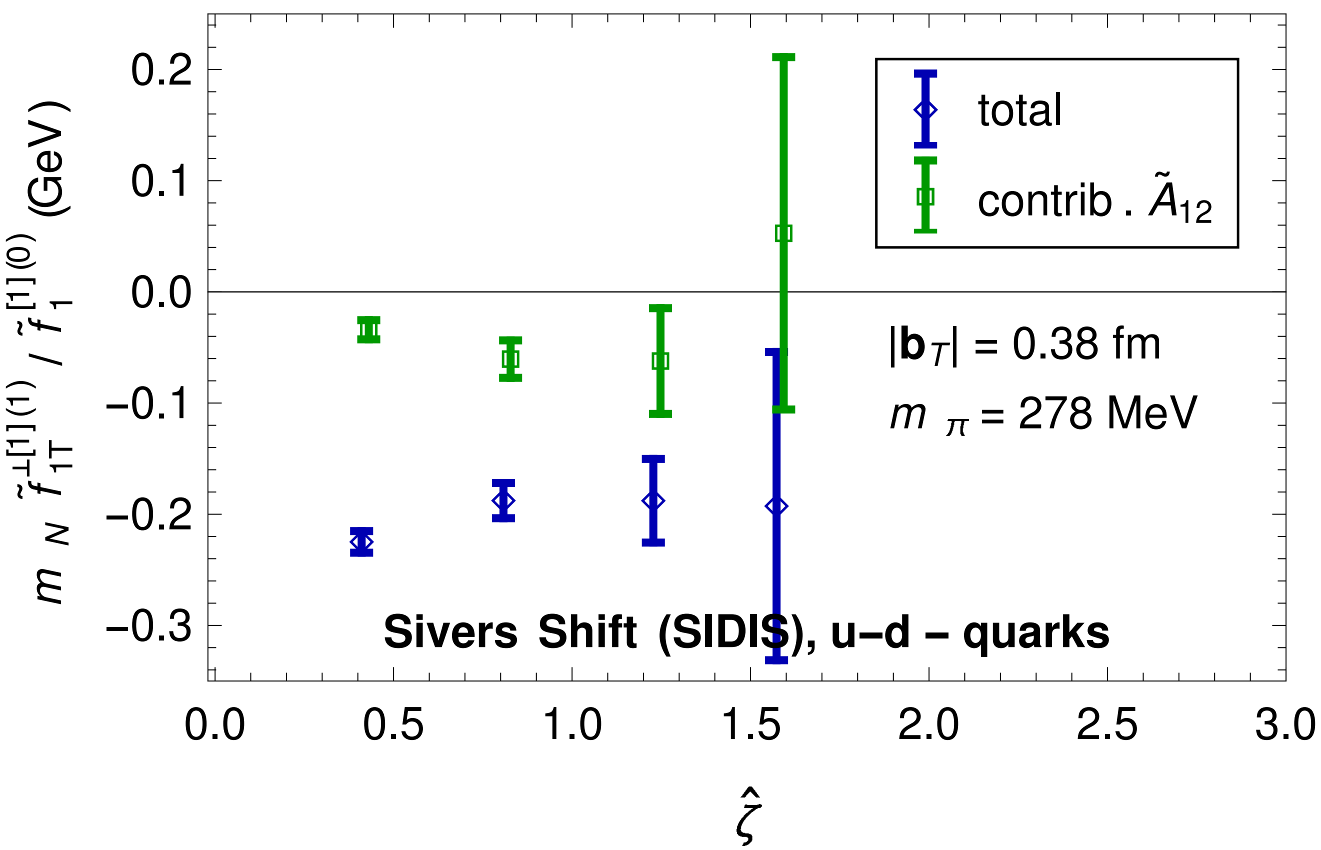
2018-2019 preliminary results (21,500 samples on D5 (reduced m_π)

P_N up to 1.65 GeV (extended ζ range)

Nucleon Sivers shift, previous results



Sivers Shift, nucleon, 2018-2019 alloc.



Next: Generalize to GTMD ⇒ Quark OAM

$$L_3^{\mathcal{U}} = \int dx \int d^2k_T \int d^2r_T\, (r_T \times k_T)_3\, \mathcal{W}^{\mathcal{U}}(x, k_T, r_T)$$ Wigner distribution

$$= \epsilon_{ij} \frac{\partial}{\partial z_{T,i}} \frac{\partial}{\partial \Delta_{T,j}} \langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle \Big|_{z^+=z^-=0,\ \Delta_T=0,\ z_T\to 0}$$

Perform Δ_T-derivative using Rome method (Phys. Lett. B718 (2012) 589)

$$G(x, y; \vec{p}) = e^{-i\vec{p}(\vec{x}-\vec{y})} G(x, y)$$

$$\frac{\partial}{\partial p_j} G(x, y; \vec{p})\Big|_{\vec{p}=0} = -i \sum_q G(x, z) \Gamma_V^j G(z, y)$$

Clover fermions:
$$\Gamma_V^j G(z, y) = U_j^\dagger(z - \hat{\jmath}) \frac{1 + \gamma^j}{2} G(z - \hat{\jmath}, y) - U_j(z) \frac{1 - \gamma^j}{2} G(z + \hat{\jmath}, y)$$

(further contributions from derivatives of source/sink smearings)

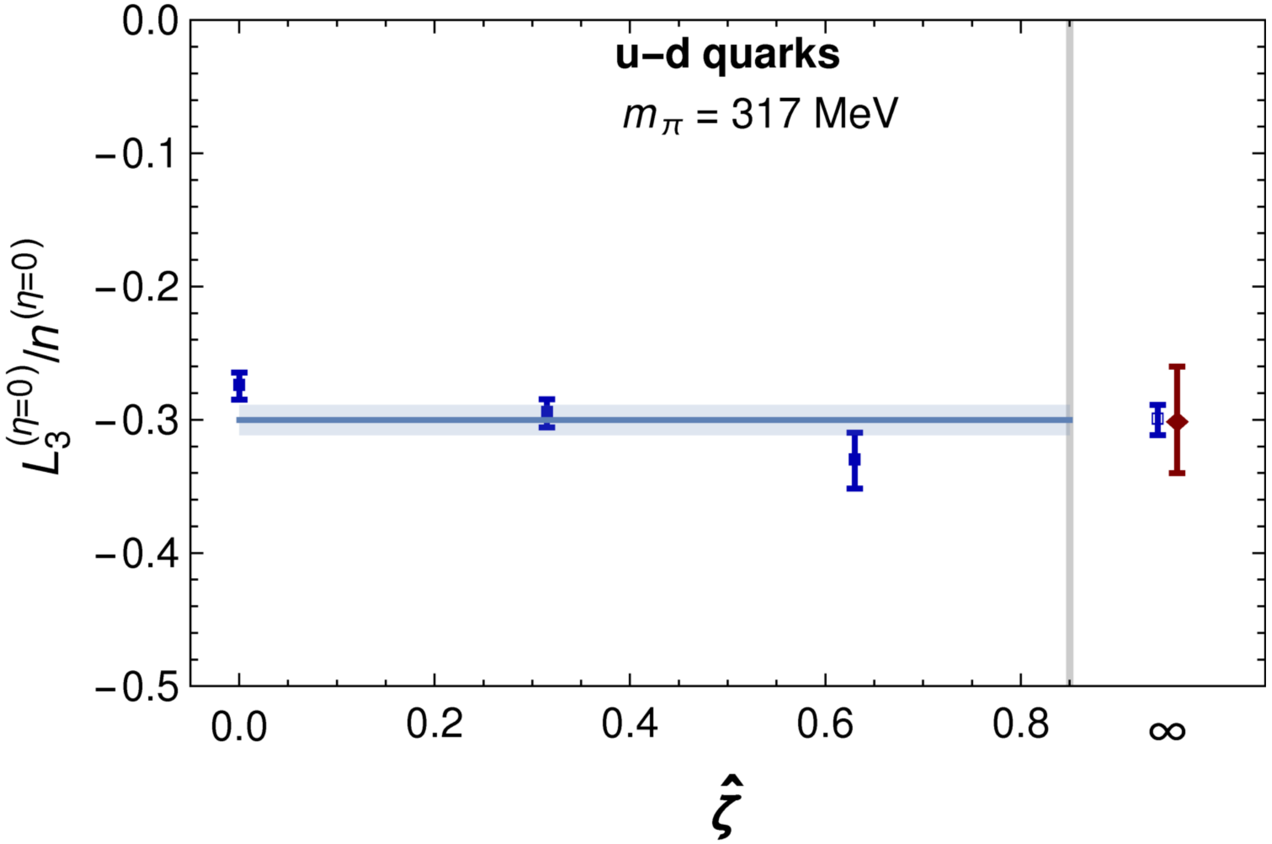
Proton radius calculation with the "Rome method" previously reported in
[N.Hasan, J.Green, et al (LHPC) PRD97:034504(2018)]

Quark OAM: Ji(η = 0) and Jaffe-Manohar (η→∞)

Ji's definition of quark OAM:
straight link ($\eta = 0$)

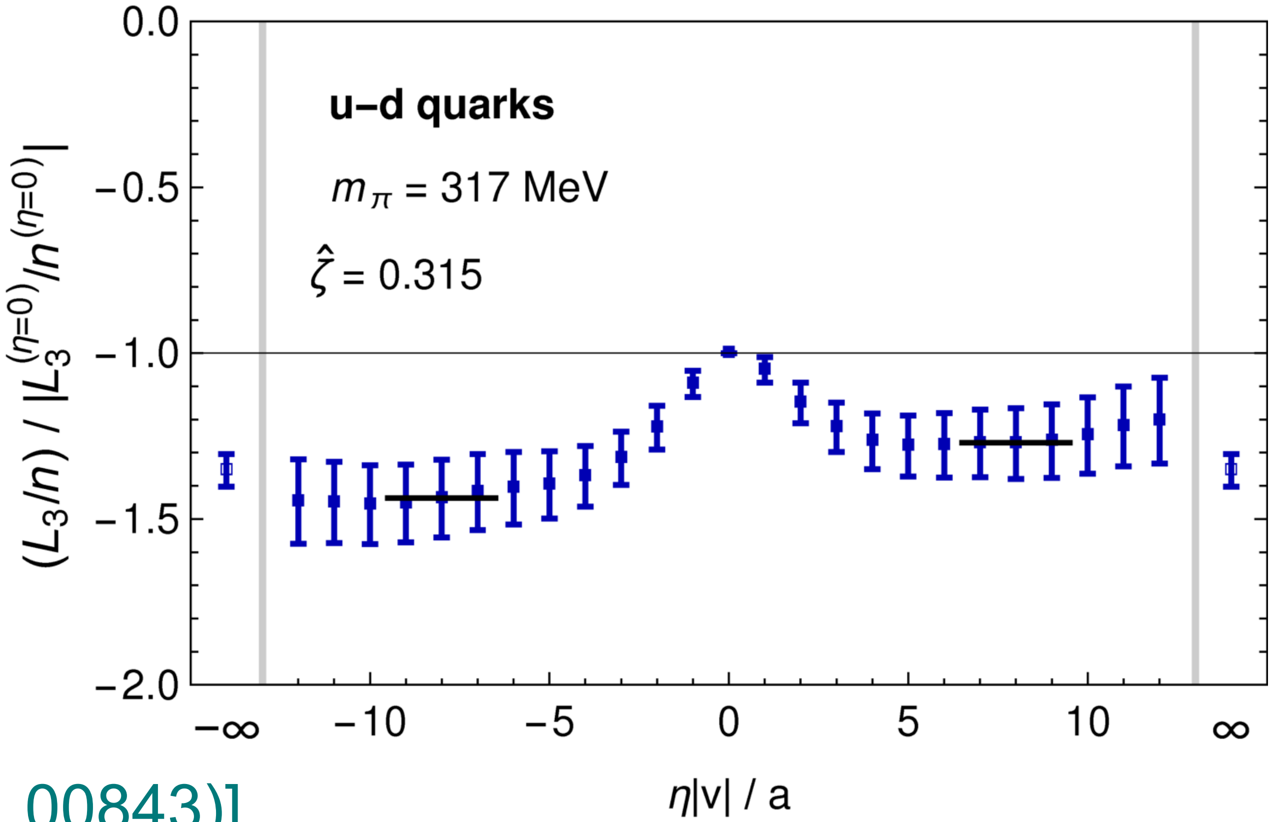
qOAM point from Ji's sum rule
[J. Bratt et al (LHPC) 2010]

better agrement than finite-diff. result
[arxiv:1701.01536]



Jaffe-Manohar's qOAM:
staple $\eta \to \infty$
(Figure: $L_{[JM]}$ / $L_{[Ji]}$ ratio)

struck quark leaving proton in SIDIS accumulates +30-50% orbital torque
[M.Burkardt 2013]



- m_π=317 MeV a=0.114fm ("C13")

[PoS SPIN2018 (2019) 047 (arXiv:1901.00843)]

2019 Request Highlights

- [TMD-T] Repeating TMD calculation on D6 (m_π=170 MeV) with ≈10,000 statistics

  16.2 M Jpsich-equivalent GPU time

- [GTMD-L] Computing quark OAM on D5 (m_π=280 MeV) with ≈21,500 statistics

  13.2 M Jpsich-equivalent GPU time

- [DISCO] Computing disconnected diagrams for high-momentum FF on D6

  12.8 M Jpsich-equivalent GPU time