Nuclear Matrix Elements for Neutrinoless Double Beta Decay from Lattice QCD

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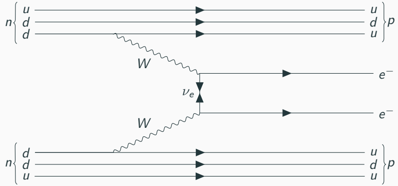
Neutrinoless Double Beta Decay ($0\nu\beta\beta$)

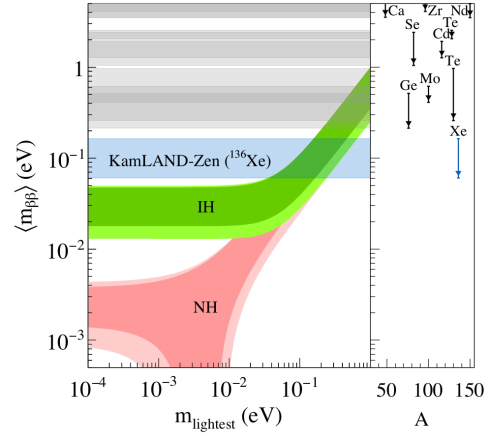
- Hypothetical nuclear decay mode: ${}^{A}_{Z}N \rightarrow {}^{A}_{Z+2} N' + e^- + e^-$
- Many interesting implications:
  - Neutrinos are Majorana particles
  - Constraints on absolute neutrino masses
  - Lepton number violation (baryogenesis)
- Focus on long-distance, light Majorana exchange mechanism
  - Motivated by SM EFT: Majorana neutrino mass appears at dim. 5
  - Higher order short-distance contributions at dims. 7, 9, ...



Experiment	Isotope	$T_{1/2}^{0\nu}$ Bound
Gerda	$^{76}\mathrm{Ge}$	$> 8 \times 10^{25}$ yrs
Cuore	$^{130}\mathrm{Te}$	$> 1.5 \times 10^{25}$ yrs
KamLAND-Zen	$^{136}\mathrm{Xe}$	$> 1.1 \times 10^{26}$ yrs

$0\nu\beta\beta$ Searches

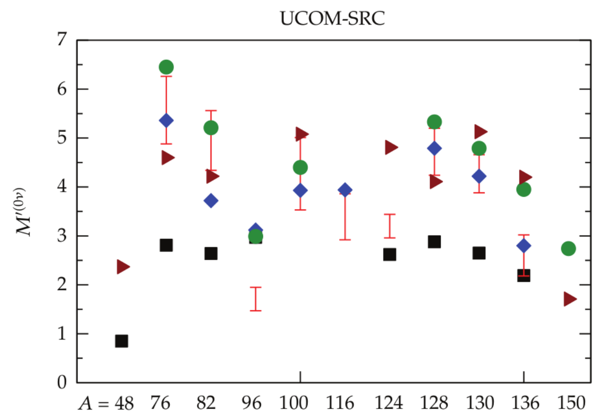
- NME relates half-life to neutrino mass: $\left(T_{1/2}^{0\nu}\right)^{-1} \propto \left|m_{\beta\beta}\right|^2 G^{0\nu} \left|M^{0\nu}\right|^2$
- $m_{\beta\beta} = \sum_i U_{ei}^2 m_i$



- Next-generation experiments will probe IH region in the coming decade

$0\nu\beta\beta$ Searches

- ...but computing relevant NMEs is hard!



- **Goal:** address $M^{0\nu}$ in LQCD for light Majorana exchange mechanism
    1. Use $M^{0\nu}_{nn\rightarrow ppee}$ as input to many-body EFT to predict NMEs of nuclei
    2. Compute $M^{0\nu}$ for light nuclei directly and probe systematics of models

Proposed Calculation

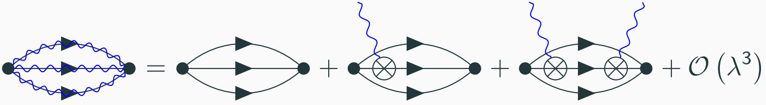
- **We will compute $M^{0\nu}$ for the process $nn \to ppee$**
  - Fundamental, nucleon-level process inducing $0\nu\beta\beta$ of nuclei
  - Need LQCD input for unknown SD LEC g_{ν}^{NN} [PRL 120, 202001 (2018)]
- Start with $m_\pi \approx 800$ MeV, $32^3 \times 48$ Wilson-clover gauge field ensemble
  - Experience from previous NPLQCD calculations
  - Control over signal-to-noise problem for multi-nucleon states
  - $|pp\rangle$ and $|nn\rangle$ are bound states $\Rightarrow$ exponentially suppressed FV effects
- **Requesting 2.1M Sky-core-hours on USQCD KNL resources for 1000 measurements, targeting $\sim 5\%$ precision**
  - 48 wall-source quark propagators per measurement
  - Full time-translation averaging
- Preliminary work:
  1. $2\nu\beta\beta$ amplitude for the process $nn \to ppee\overline{\nu}_e\overline{\nu}_e$ [PRD 96, 054505 (2017)]
  2. $0\nu\beta\beta$ amplitude for the process $\pi^- \to \pi^+ ee$ [arXiv:1811.05554 (2018)]

$M_{GT}^{2\nu}$ for $nn \to ppee\overline{\nu}_e\overline{\nu}_e$ from Lattice QCD

- $SU(3)$-symmetric Wilson-clover lattice with $m_\pi \approx 806$ MeV
- Key technique: **compound propagators** with background axial field $\propto \lambda$

$$S_\lambda(x,y) = S(x,y) + \lambda \int d^4z\, S(x,z) A_3(z) S(z,y) + \mathcal{O}(\lambda^2)$$

- $\mathcal{O}(\lambda^n)$ compound correlation function has n axial current insertions:

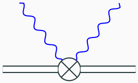


Results for NME:

$$\frac{\Delta}{g_A^2} \frac{M_{GT}^{2\nu}}{6} = -1.04(4)(4)$$

$$\frac{\Delta}{g_A^2} \frac{|\langle pp|A_3^+|d\rangle|^2}{\Delta} = 1.00(3)(1)$$

Matching to pionless EFT:



$$\mathbb{H}_{2,S} = 4.7(1.3)(1.8)\ \mathrm{fm}$$

- $\sim 5\%$ precision from measurements on 1000 trajectories

Neutrinoless Double Beta Decay in the Standard Model

- For typical lattice scales decay is mediated by effective EW Hamiltonian

$$H_W = \frac{G_F}{\sqrt{2}} \left\{ V_{ud} \overline{u}(x) \gamma_\alpha (1-\gamma_5) d(x) \otimes \overline{e}(x) \gamma^\alpha (1-\gamma_5) \nu_e(x) \right\}$$

- Matrix element decomposes into **leptonic** and **hadronic** pieces

$$\int d^4x\, d^4y \left\langle fee \middle| T\{H_W(x) H_W(y)\} \middle| i \right\rangle \propto \int d^4x\, d^4y \left[\overline{u}_e(p_1) \boldsymbol{L_{\alpha\beta}(x,y)} \overline{u}_e^\top(p_2) \right] \boldsymbol{H^{\alpha\beta}(x,y)}$$

$$\boldsymbol{L_{\alpha\beta}(x,y) \equiv \gamma_\alpha (1-\gamma_5) S_\nu(x,y) C^\top (1-\gamma_5) \gamma_\beta^\top}$$

$$\boldsymbol{H_{\alpha\beta}(x,y) \equiv \langle f | T \{ \overline{u}(x) \gamma_\alpha (1-\gamma_5) d(x) \overline{u}(y) \gamma_\beta (1-\gamma_5) d(y) \} | i \rangle}$$

- Develop lattice methods by first computing $\pi^- \to \pi^+ e^- e^-$ amplitude

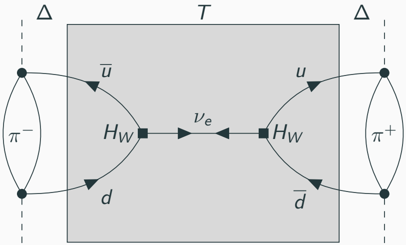
(a) $nn \to ppee$

(b) $\pi^- \to \pi^+ e^- e^-$

Lattice Formalism for $0\nu\beta\beta$

- Adapt Δm_K [arXiv:1406.0916] and rare kaon [arXiv:1701.02858] techniques
- On the lattice, for $E_f = E_i \equiv E$:

$$\mathcal{M}^{0\nu}(T) = |Z|^2 \, e^{-E(T+2\Delta)} \sum_n \frac{\langle fee|H_W|n\rangle\langle n|H_W|i\rangle}{E_n - E} \left[T + \frac{e^{-(E_n-E)T} - 1}{E_n - E} \right]$$



- Choose Δ to suppress excited states
- Extract **ME** from linear fit
- $E_n < E$: exponential contamination
- $E_n \approx E$: quadratic contamination

- **Refinement 1:** explicit double current integration in $\mathcal{O}(V \log V)$ via FFTs
- **Refinement 2:** UV-regulated continuum neutrino propagator
- SD amplitude from MEs of local four-quark operators [PRD 68 (2003) 034016]

Exact Double Current Integration

- Fully utilizing gauge configurations important for suppressing noise
- Build Wick contractions using 1D FFTs [Microw. Opt. Tech. Lett. 31, 28 (2001)]:
  1. Construct partially contracted "neutrino blocks" in $\mathcal{O}(V \log V)$:

$$B_\mu (x; t_1, t_2) = \int d^3 y \, L_{\mu\nu}(x-y) \left[S_u^\dagger (t_1 \to y) \gamma_\nu (1-\gamma_5) S_d (t_2 \to y) \right]$$
$$= \mathscr{F}^{-1} \left[\mathscr{F} (L_{\mu\nu}) \cdot \mathscr{F} \left(S_u^\dagger \gamma_\nu (1-\gamma_5) S_d \right) \right]$$

  2. Contract remaining indices in $\mathcal{O}(V)$

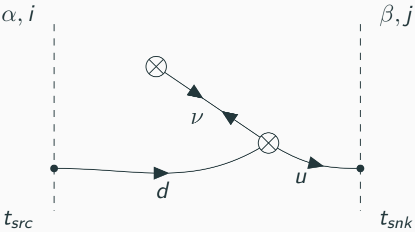


Figure 1: Neutrino block

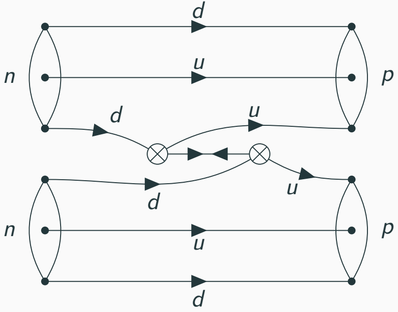
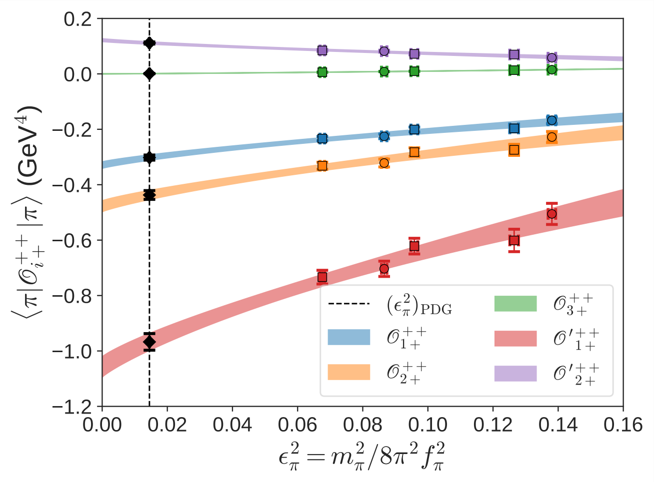


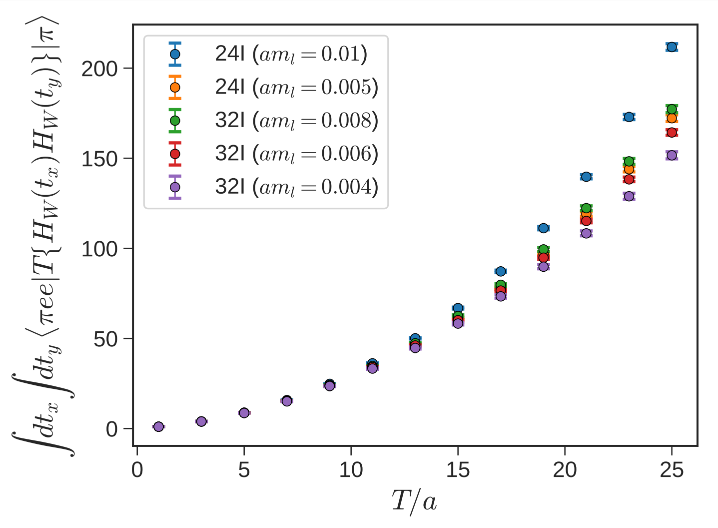
Figure 2: $nn \to ppee$

SD Preliminary Physical Point Extrapolation



- Series of $\{24, 32\}^3 \times 64$ DWF ensembles: 300 MeV $\lesssim m_\pi \lesssim$ 430 MeV
- Fit ansatz: (continuum NLO χPT) + (NLO χPTFV) + ($c_a a^2$)
- Preliminary results compatible with Nicholson et al. [PRL 121, 172501 (2018)]

LD Preliminary Lattice Results



- Expect divergence $\propto e^{m_\pi T}$ from $|e\overline{\nu}_e\rangle$ intermediate state
- Preliminary 16I analysis [arXiv:1811.05554]; analysis in progress for this data

Estimated Cost for $nn \to ppee$

- NPLQCD $M^{2\nu}_{nn\to ppee\nu\nu}$: $\mathcal{O}(1000)$ measurements $\Rightarrow \sim 5\%$ precision
- Benchmarks for $M^{0\nu}_{\pi\to\pi ee}$ code on KNLs (CPS + FFTW + Grid):

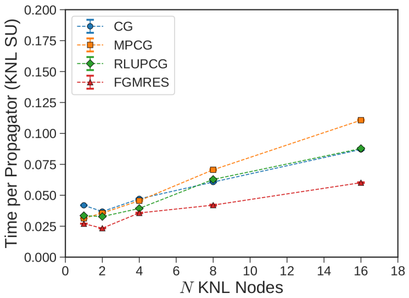


Figure 3: Propagators

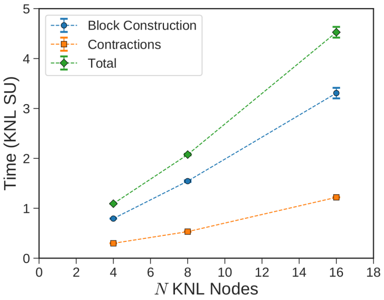


Figure 4: Contractions

- Extrapolate to cost of $M^{0\nu}_{nn\to ppee}$ by scaling: $(4!)^2/4 = 144$

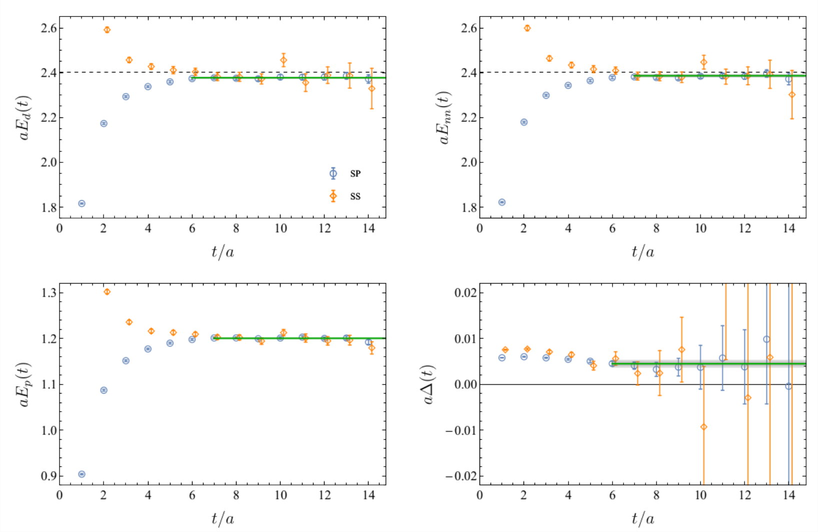
	Propagators	2-Point and 3-Point	4-Point	**Total**
Per Meas.	$48 \times 1.38 = 66.2$	302.4	1676.8	**2044.4**
Requested (1000 Meas.)	6.6×10^4	3.0×10^5	1.7×10^6	$\mathbf{2.1 \times 10^6}$

Questions?

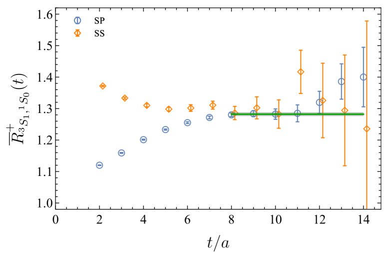
Extra Slides

Deuteron and Dinucleon Masses at $m_\pi \approx 800$ MeV

$|pp\rangle$, $|d\rangle = |pn\rangle$, and $|nn\rangle$ are bound states $\Rightarrow$ FV corrections $\sim e^{-ML}$



$\langle pp|A_3|d\rangle$, $\beta_A^{(2)}$, and $M_{GT}^{2\nu}$



- Top: $|\langle pp|A_3|d\rangle|^2$
- Bottom left: $\beta_A^{(2)}$
  ($^1S_0$ isotensor axial polarizability)
- Bottom right: $M_{GT}^{2\nu}$