Excited State Spectroscopy & QCD

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Pushing spectroscopy in multiple directions

- Physical limit
  - Gauge generation on 48^3x512 [(6 fm)^3 x 18fm]
    - Multigrid & setup within HMC integrator → **big speedups on GPUs**
  - Propagators for "annihilation" quark lines (JLab+ALCC)
  - ➡ Not the first priority for highly excited state analysis
    - ➡ but can tackle low energy resonances

- 2-body coupled channel scattering
  - More to sort in formalism
    - spinning particles
  - Quark mass dependence
    - understanding couplings and how resonances "turn on"

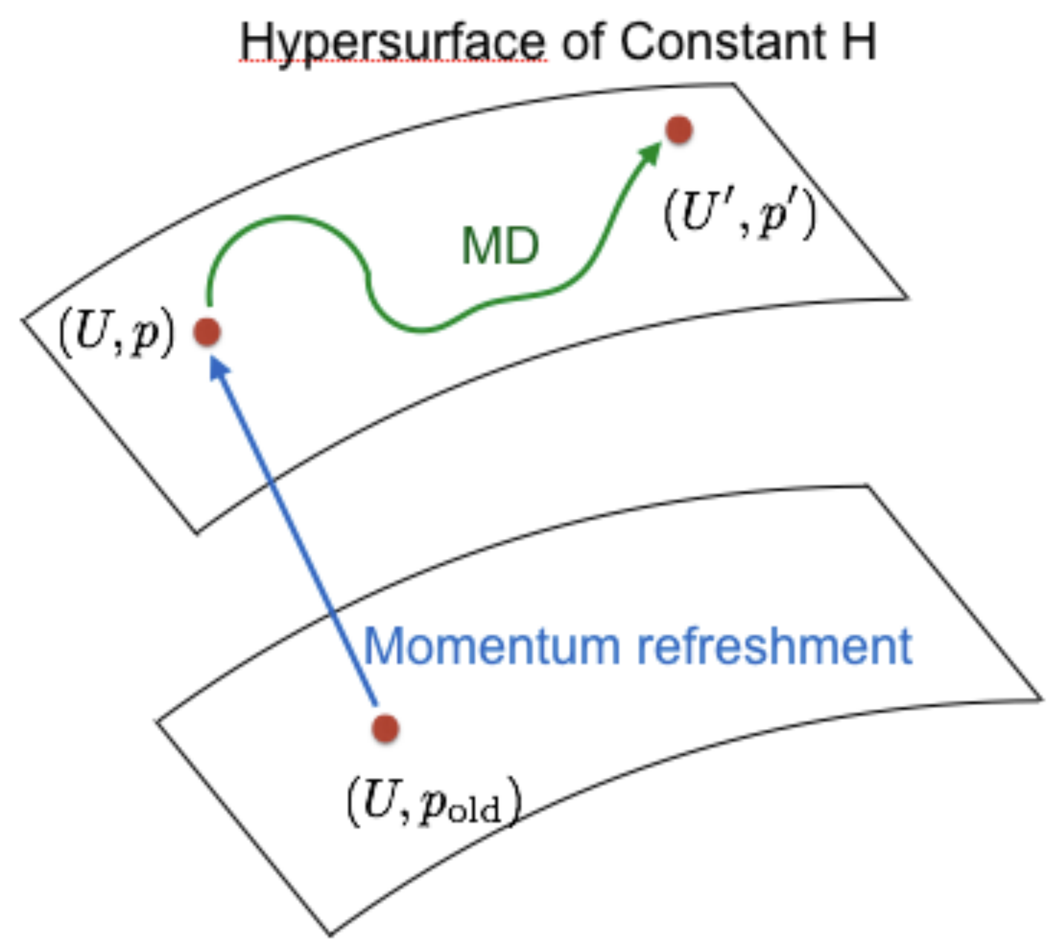
- First forays into 3-body scattering

Accelerating gauge field generation

- Hamilton's equations - 1st order coupled differential eqns.
  - improve with force gradient integrator + mass precond.+…

- Each integration step: sparse matrix solution
  ➡ Use Adaptive Multigrid

- More opportunities
  - Multi-mass twisted Clover: $M^{\dagger}M$
  - Multi-right-hand sides



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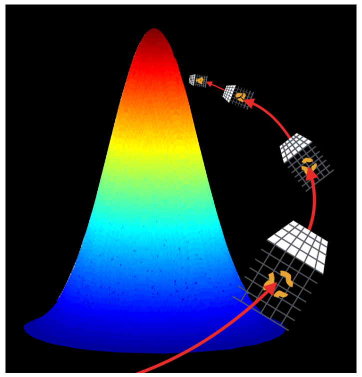
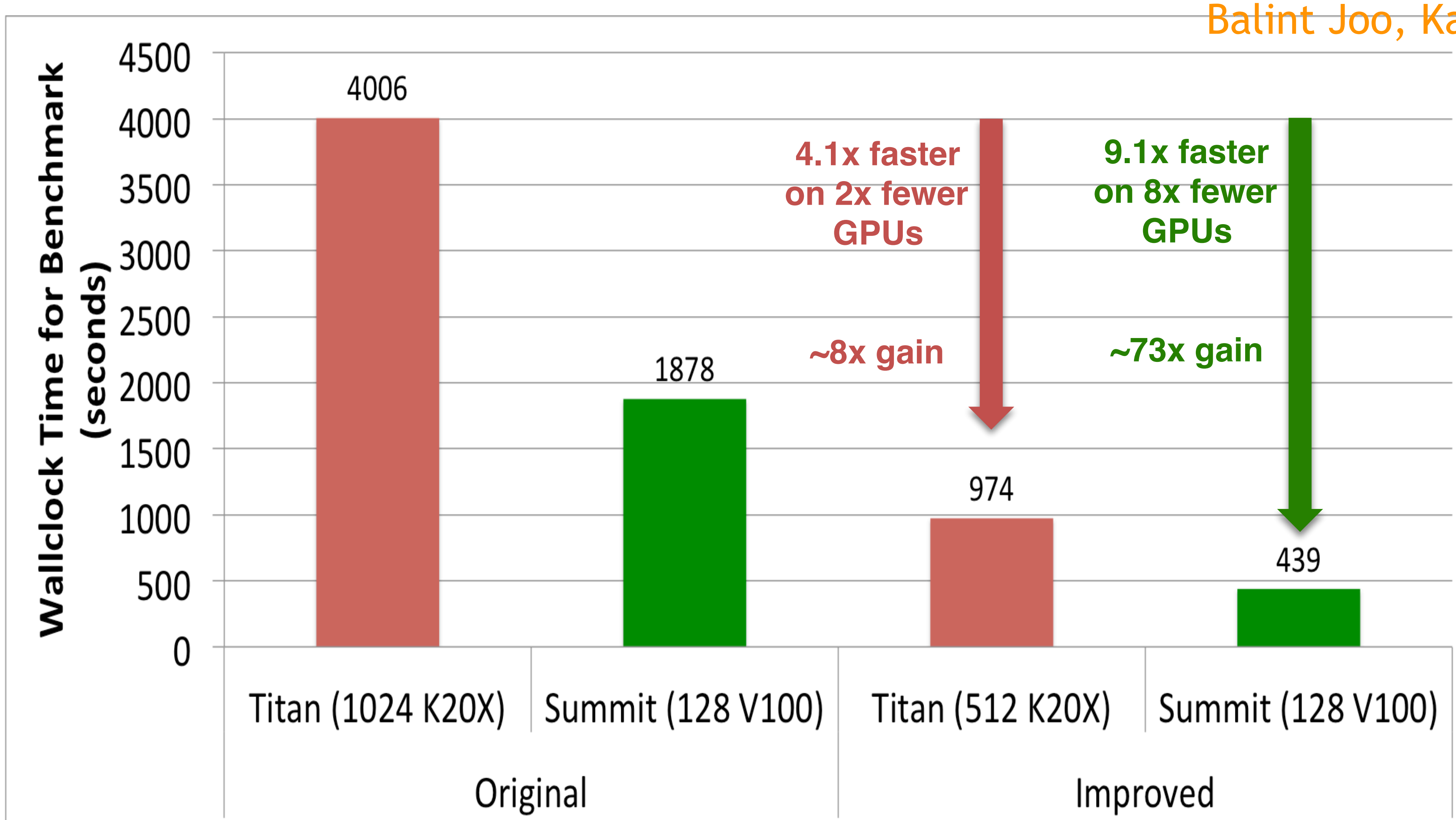
Accelerating gauge field generation

Balint Joo, Kate Clark





- ~9x wallclock speed-up on Summit using 8x fewer GPUs than Titan: ~73x improvement in computational efficiency. Recent tweaks, now 83x.
- **Physical limit generation (more) affordable. More gains expected.**
- Looking for next gains in propagators (multiple right-hand sides…)

Two-body coupled-channel scattering

He, JHEP 0507 011
Hansen, PRD86 016007
Briceno, PRD88 094507
Guo, PRD88 014051

- Extension of Luscher formalism generalized to multi-channels

$$\det\left[\left(\left[t^{(\ell)}(E)\right]^{-1}_{ij} + i\rho_i(E)\,\delta_{ij}\right) - \delta_{ij}\,\mathcal{M}_\ell\big(p_i(E)L\big)\right] = 0$$

scattering matrix — phase space — known functions

matrices in partial-wave space ..

- However, this is **one equation** for **multiple unknowns** (per energy level) $\frac{1}{2}N(N+1)$ for *N* channels
  - parameterize the energy dependence of *t*
  - try to describe a spectrum globally

"Energy-dependent" analysis

Case study: rho resonance

variational analysis of a matrix of correlation functions

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j(0) | 0 \rangle$$
$$= \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0 | \mathcal{O}_i | \mathfrak{n} \rangle \langle \mathfrak{n} | \mathcal{O}_j | 0 \rangle$$

operator basis: ‘single-meson’ + ‘meson-meson’

$$\overline{\psi} \boldsymbol{\Gamma} \psi$$

(& tetraquark & …)

$$\sum_{\hat{\mathbf{p}}_1, \hat{\mathbf{p}}_2} C(\mathbf{p}_1, \mathbf{p}_2; \mathbf{p}) \, M_1(\mathbf{p}_1) \, M_2(\mathbf{p}_2)$$

$$\mathbf{p} = \tfrac{2\pi}{L} [n_x, n_y, n_z]$$

maximum momentum guided by non-interacting energies

$$\sqrt{m_1^2 + \mathbf{p}_1^2} + \sqrt{m_2^2 + \mathbf{p}_2^2}$$

Case study: rho resonance

variational analysis of a matrix of correlation functions

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\mathcal{O}_j(0) | 0 \rangle = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0 | \mathcal{O}_i | \mathfrak{n} \rangle \langle \mathfrak{n} | \mathcal{O}_j | 0 \rangle$$

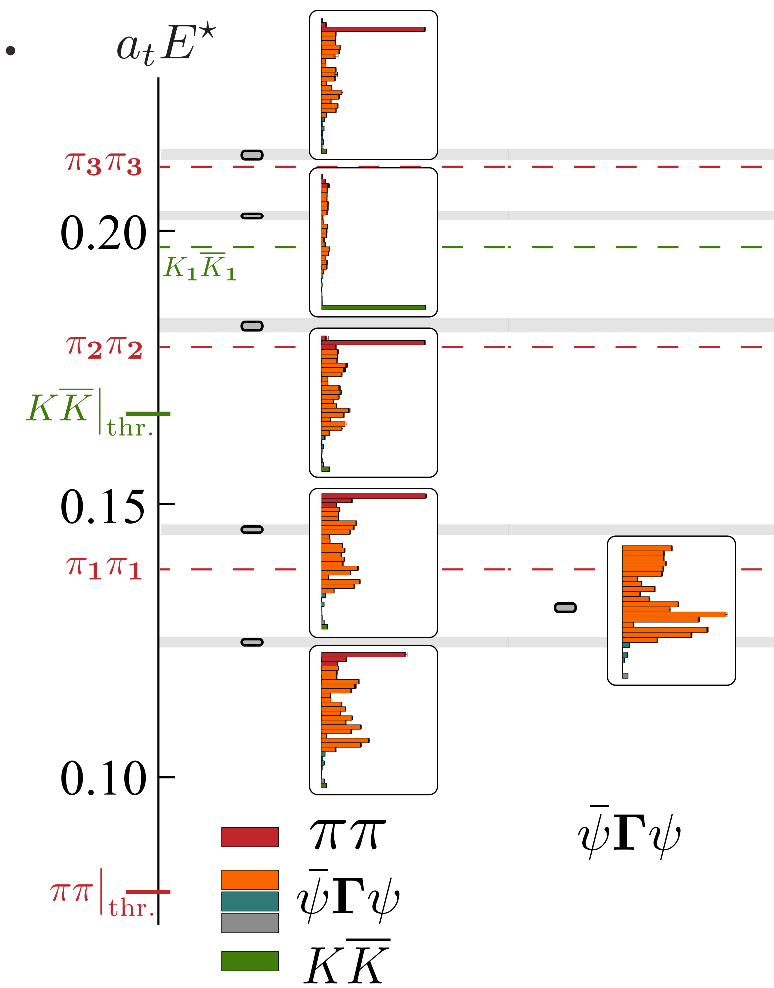
operator basis: ‘single-meson’ + ‘meson-meson’

$$\bar{\psi}\mathbf{\Gamma}\psi \qquad \sum_{\hat{\mathbf{p}}_1, \hat{\mathbf{p}}_2} C(\mathbf{p}_1, \mathbf{p}_2; \mathbf{p})\, M_1(\mathbf{p}_1)\, M_2(\mathbf{p}_2)$$

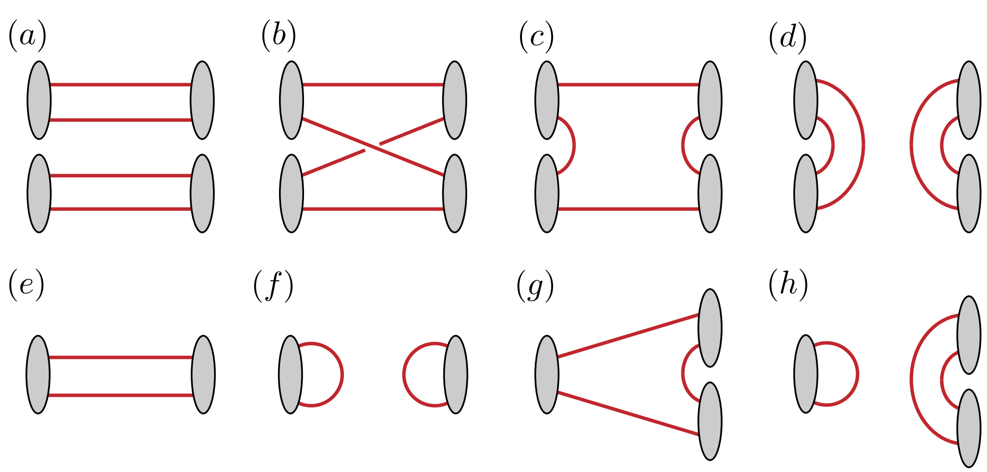
$$\bar{\psi}\mathbf{\Gamma}\psi$$

$$\sum_{\hat{\mathbf{p}}_1, \hat{\mathbf{p}}_2} C(\mathbf{p}_1, \mathbf{p}_2; \mathbf{p})\, M_1(\mathbf{p}_1)\, M_2(\mathbf{p}_2)$$

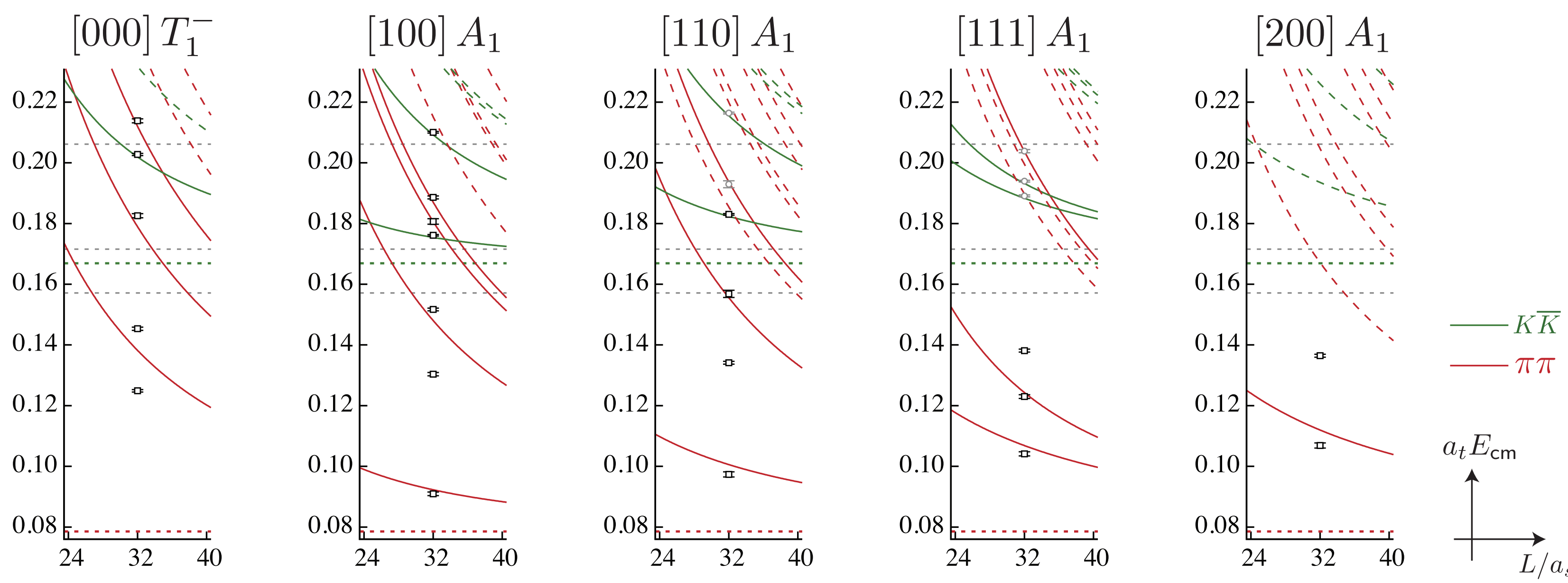
meson-meson operators prove to be vital …



possibly lots of Wick contractions

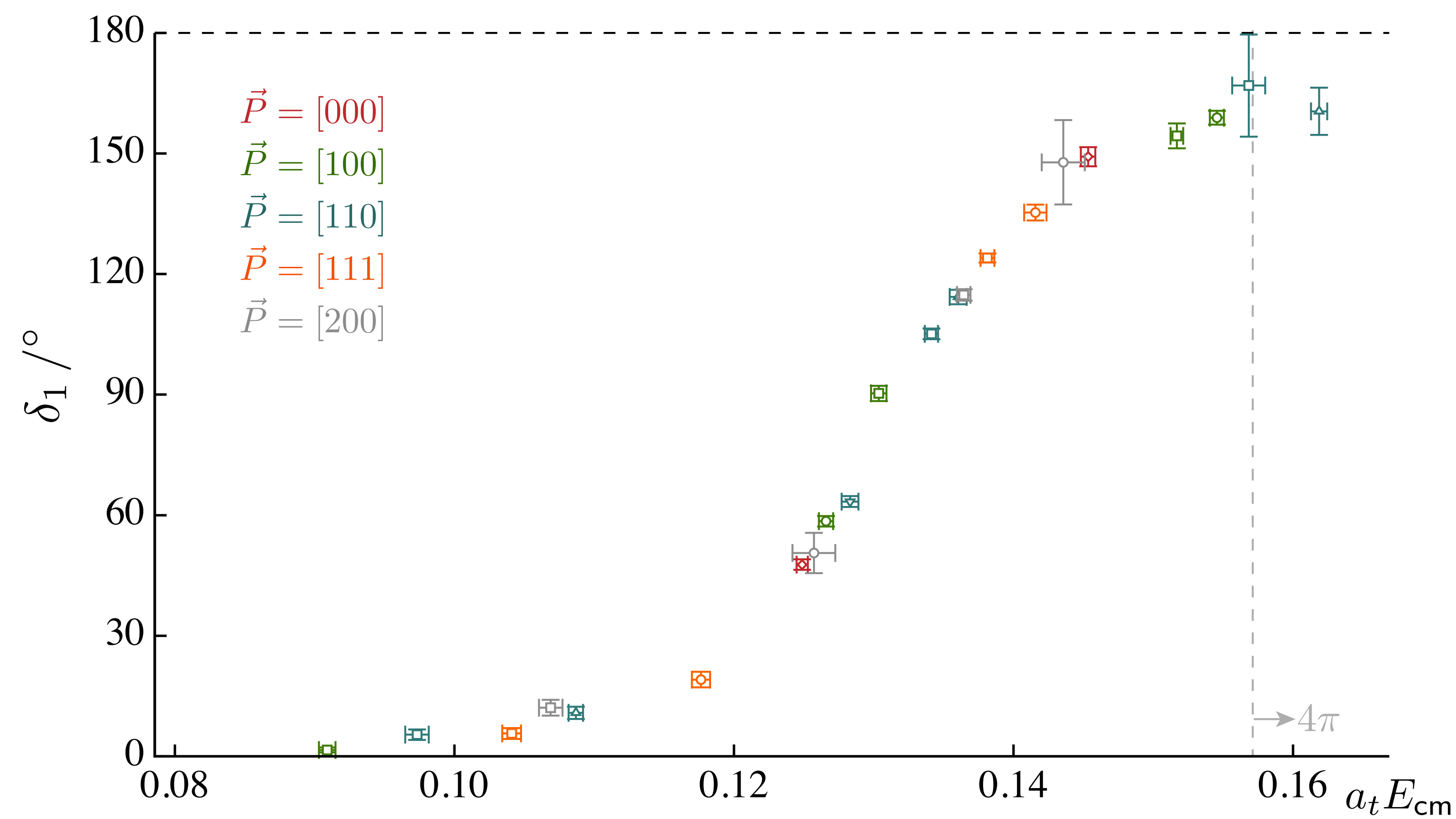


Finite-volume spectrum - moving frames



$m_\pi \sim 236$ MeV

$$\cot \delta(E) = \mathcal{M}(E, L)$$



Non-interacting thresholds and energies as a function of $\vec{k}$

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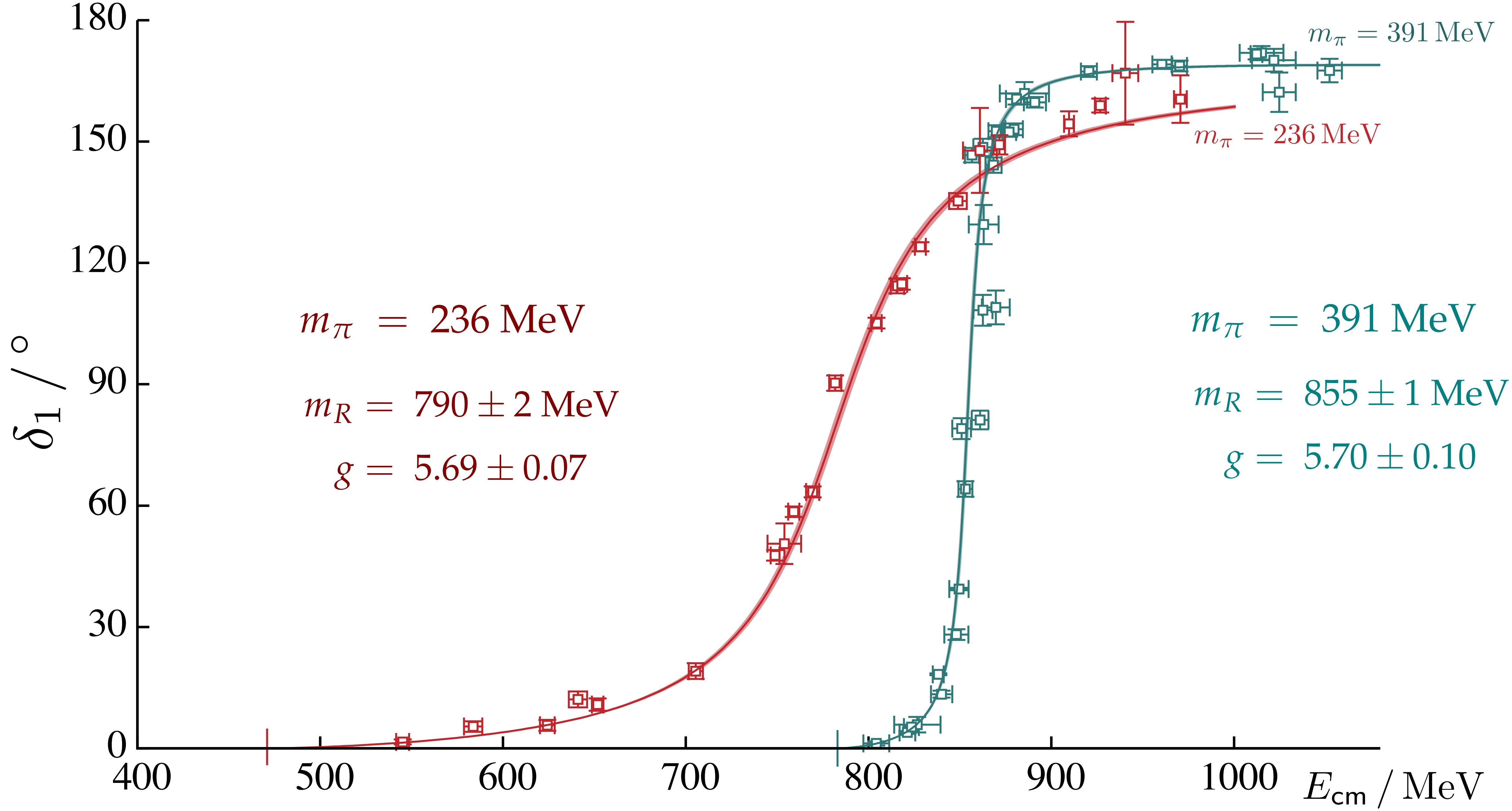
ρ resonance at different pion masses

- BW couplings nearly constant in pion mass (will come back to this later…)



PRD 87 034505

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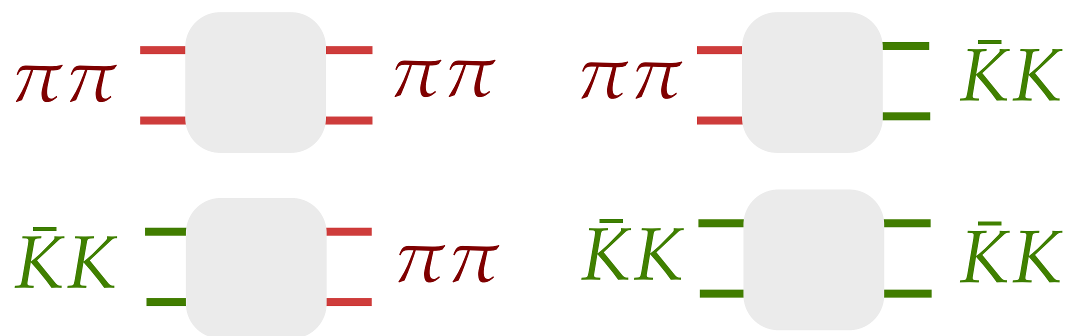
ρ resonance as a coupled channel system

- Parameterize the t-matrix in a unitarity conserving way



$$t_{ij}^{-1}(E) = K_{ij}^{-1}(E) + \delta_{ij}\, I_i(E)$$

$$K_{ij}(E) = \frac{g_i\, g_j}{m^2 - E^2} + \gamma_{ij}$$

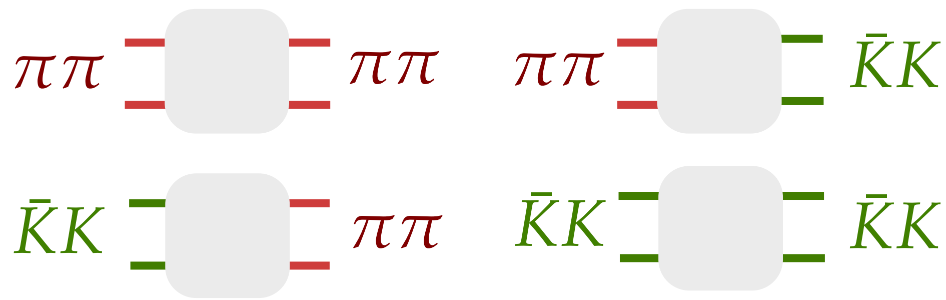
  - Vary the parameters, solving

$$\det\left[\left([t^{(\ell)}(E)]_{ij}^{-1} + i\rho_i(E)\,\delta_{ij}\right) - \delta_{ij}\,\mathcal{M}_\ell(E,L)\right] = 0$$

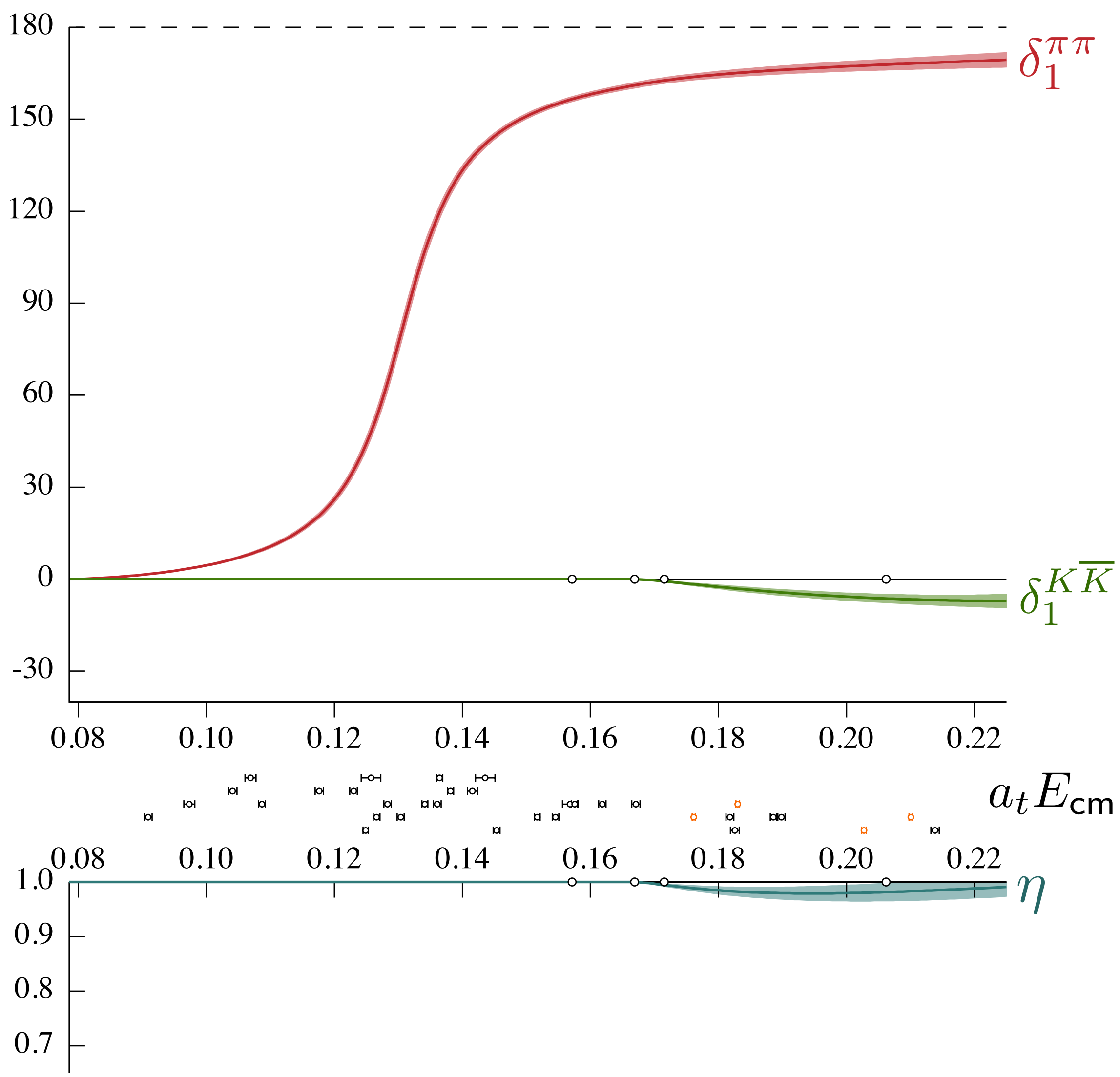
for the spectrum in each irreducible representation & momentum

Want pole mass and couplings of t-matrix

ρ resonance as a coupled channel system

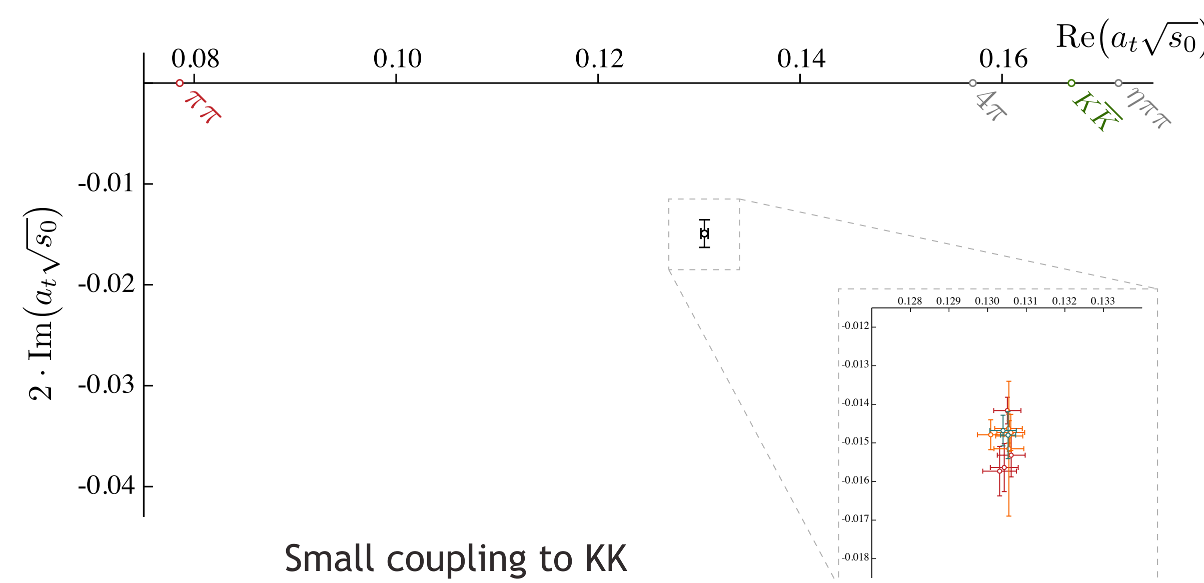


Phase shifts & inelasticity



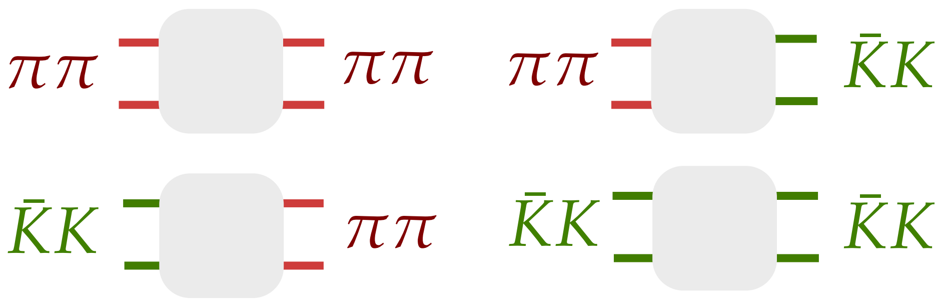
$m_\pi \sim 236$ MeV

t-matrix pole location



Small coupling to KK

ρ resonance as a coupled channel system



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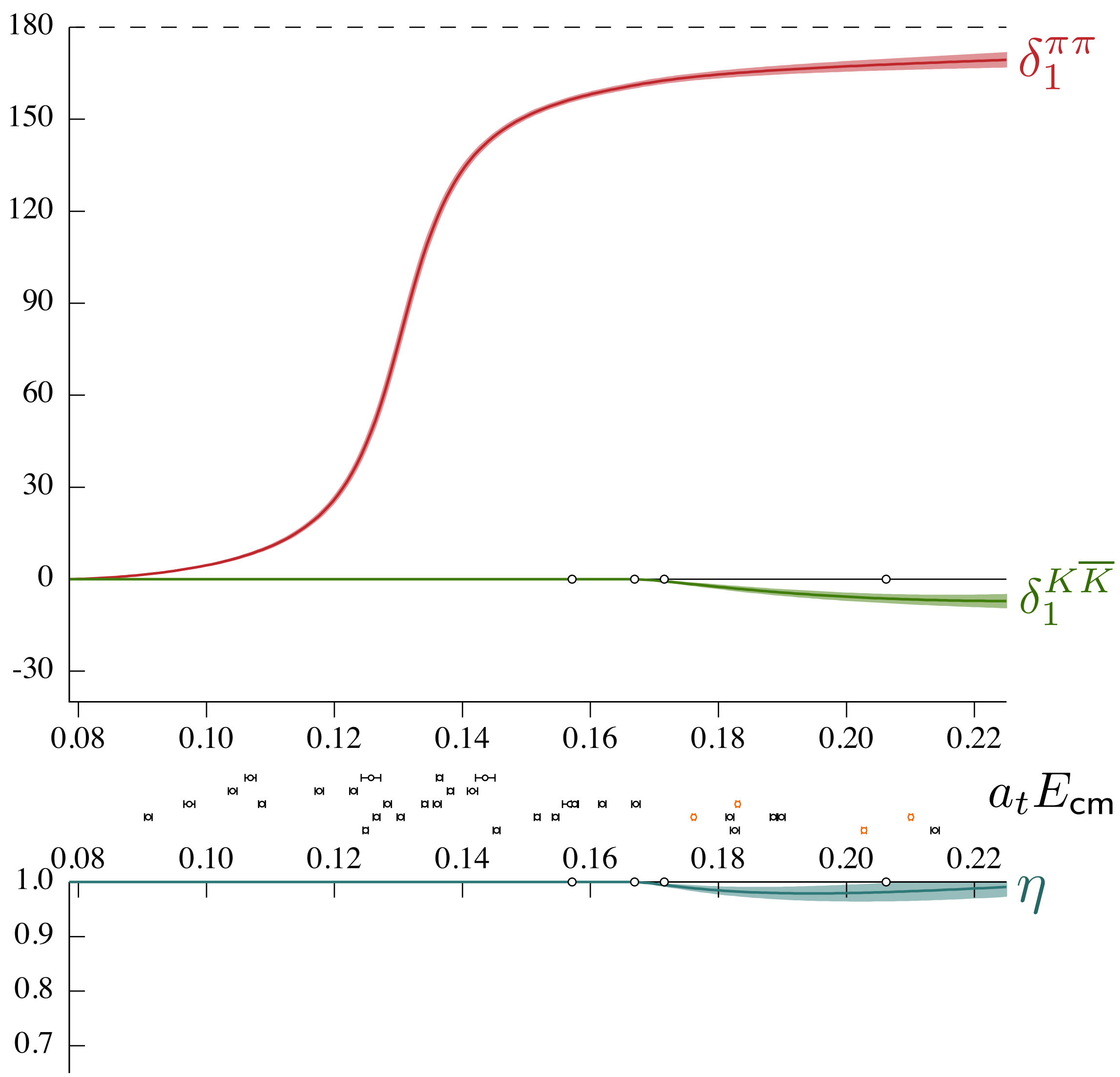

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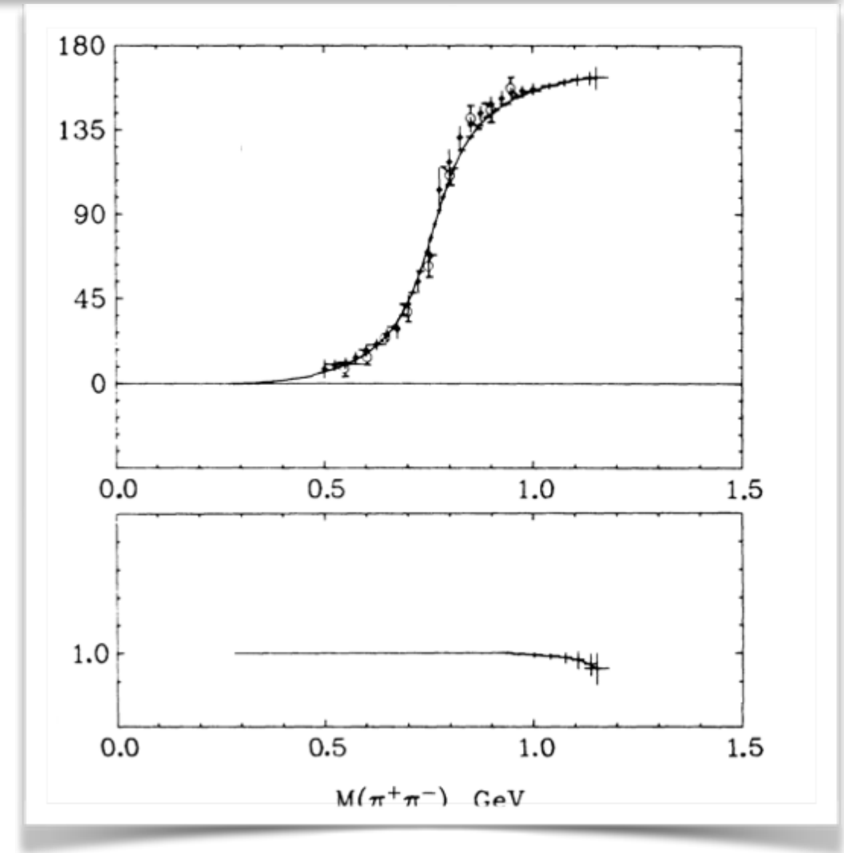

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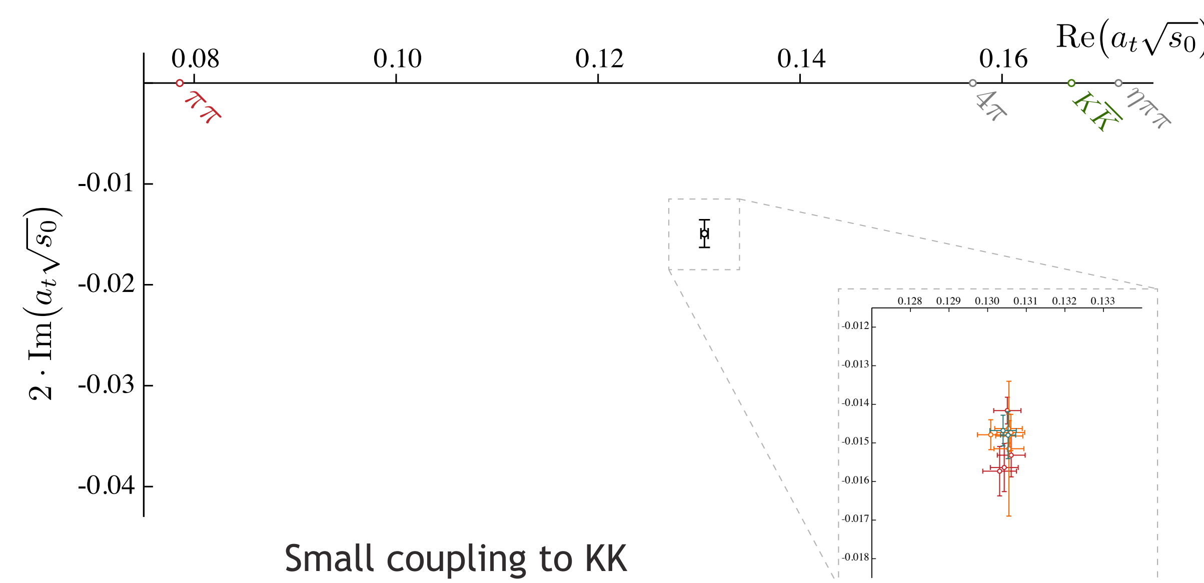
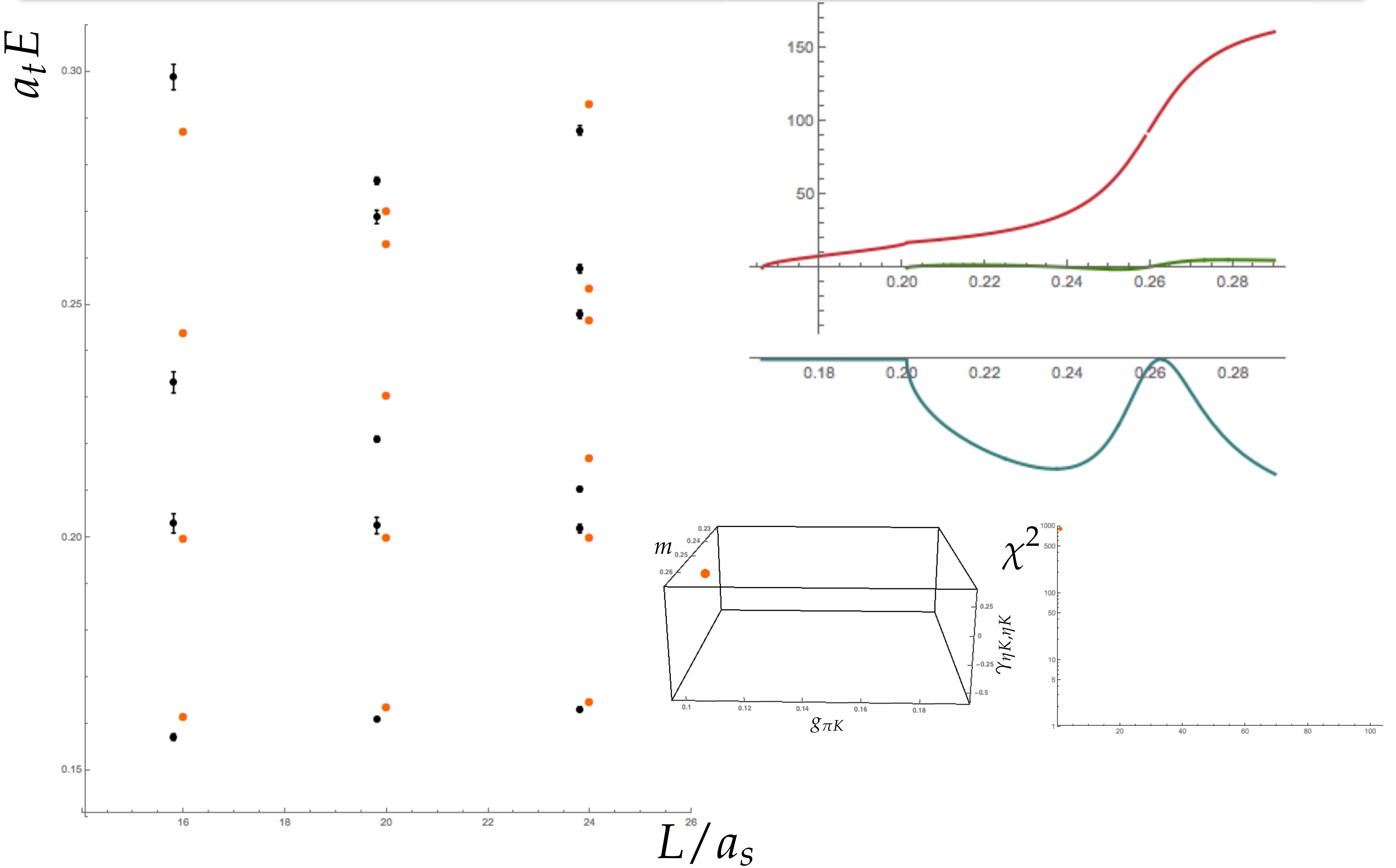


Small coupling to KK

Fun diversion: example of $\pi K/\eta K$



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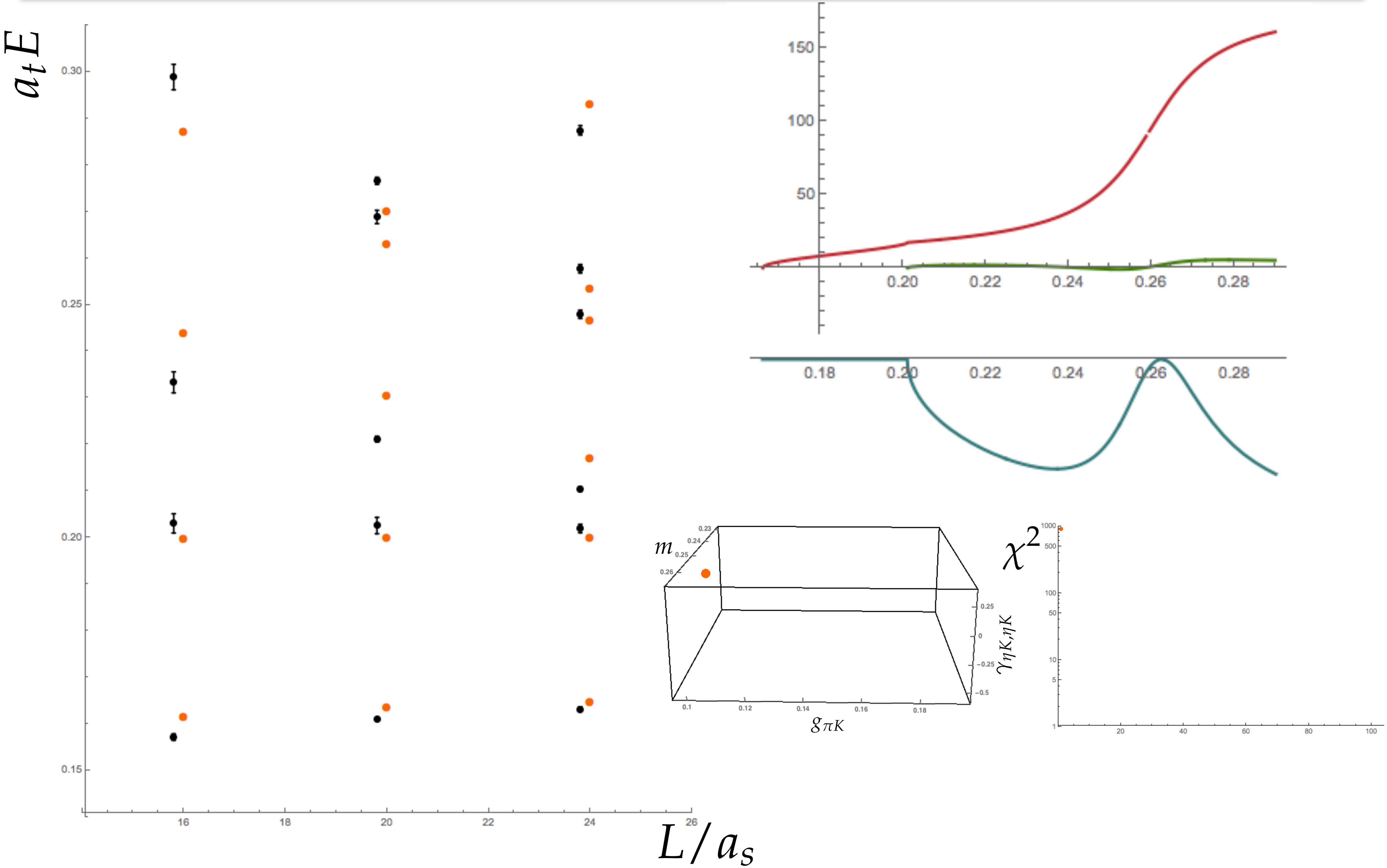


“Spinning” particles - $\pi\omega$ scattering

The ω is a stable state except at very low quark masses

the non-zero spin of the ω introduces new features, e.g. $J^P = 1^+$ in **two** partial-waves $\begin{pmatrix} {}^3S_1 \\ {}^3D_1 \end{pmatrix}$

expect a ***b*₁ resonance**

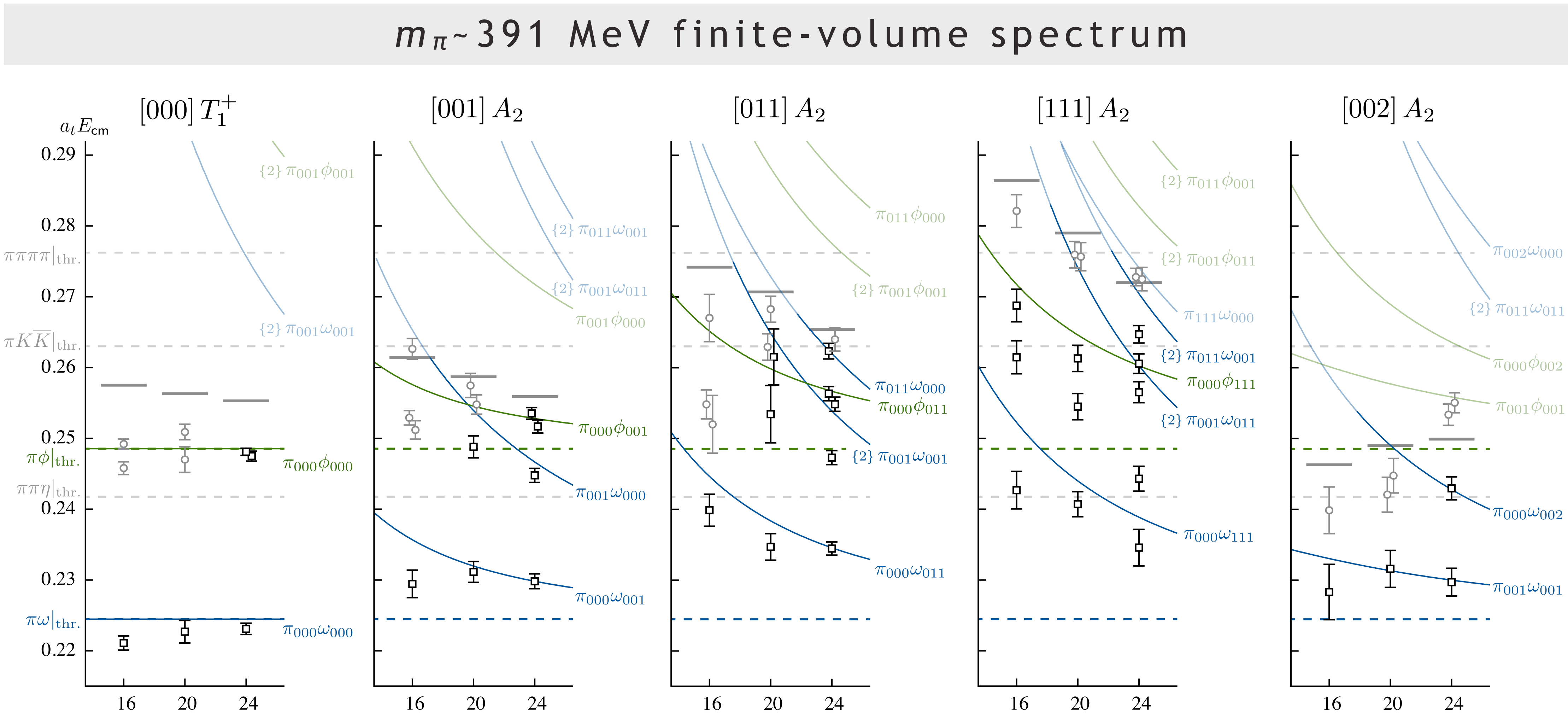
$b_1(1235)$ $\quad I^G(J^{PC}) = 1^+(1^{+-})$

Mass $m = 1229.5 \pm 3.2$ MeV (S = 1.6)
Full width $\Gamma = 142 \pm 9$ MeV (S = 1.2)

$b_1(1235)$ DECAY MODES	Fraction (Γ_i/Γ)	Confidence level	p (MeV/c)
$\omega\pi$	dominant		348
[D/S amplitude ratio $= 0.277 \pm 0.027$]			

$\pi\omega$ scattering

The ω is a stable state at m_π~391 MeV



at low energies, a coupled $\pi\omega$, $\pi\varphi$ scattering system …

FYI: first real foray into 3-body: considered $\rho\eta$ with ρ unstable

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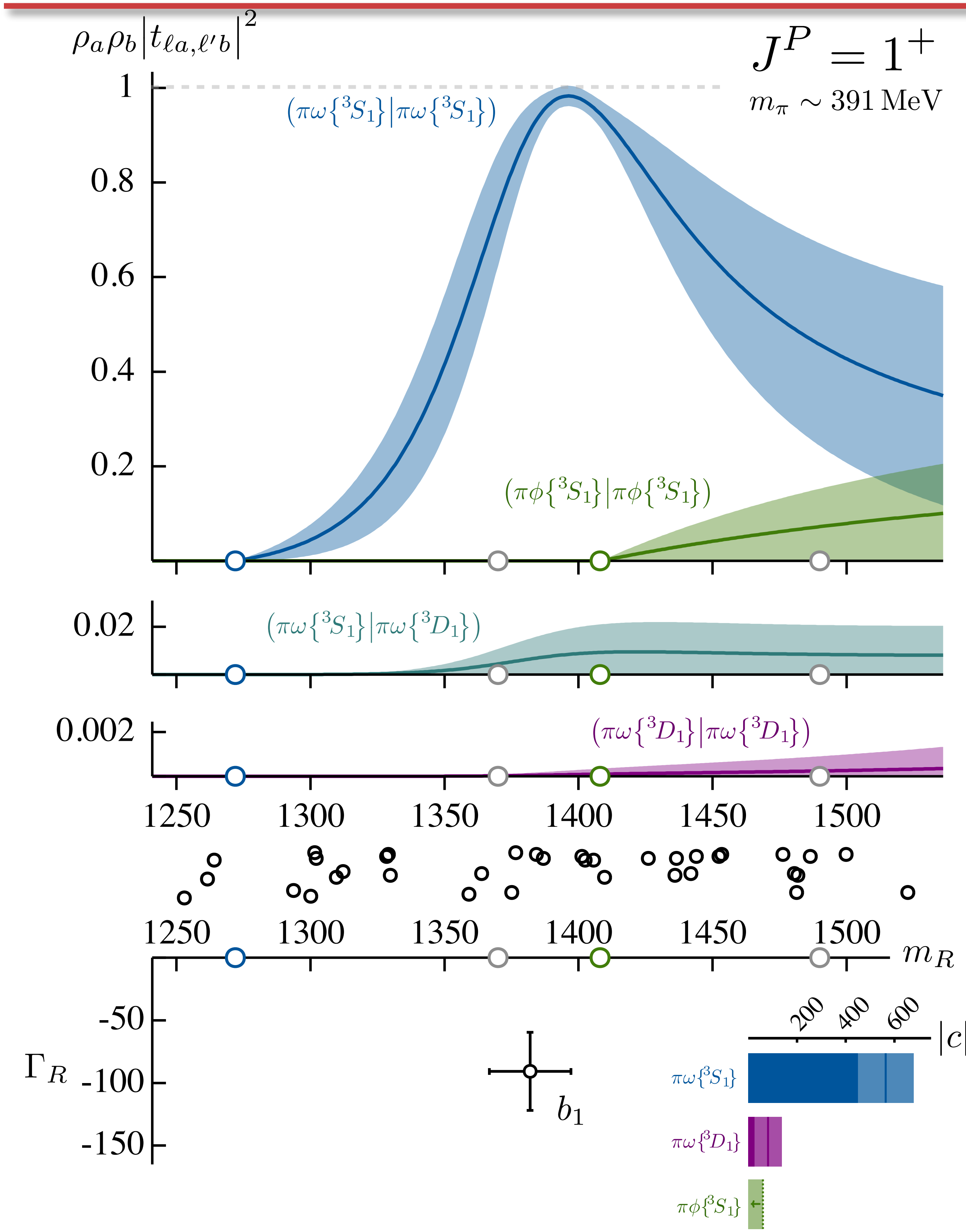
b_1 $\pi\omega/\pi\varphi$ scattering



clear b_1 resonance

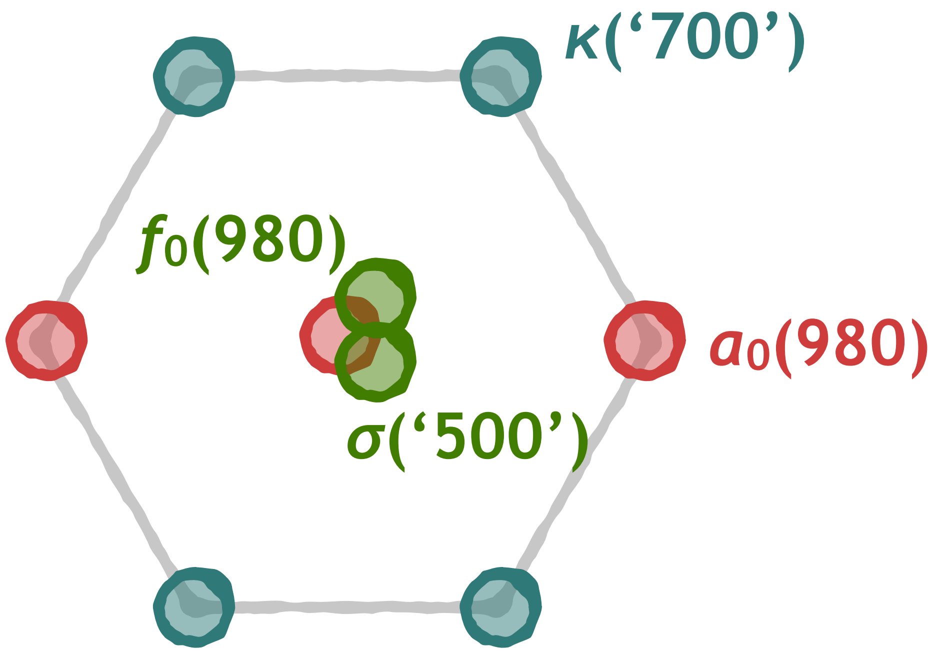
— strong $\pi\omega$ *S*-wave

— weak $\pi\omega$ *D*-wave

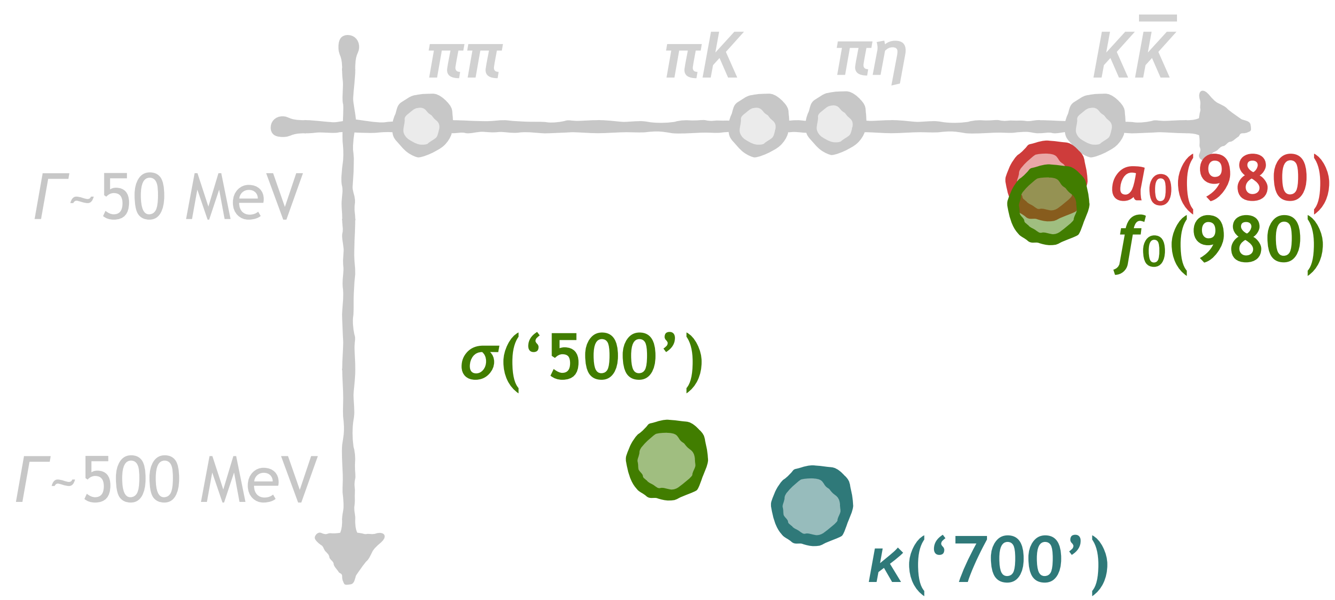
— negligible $\pi\varphi$

Light scalar mesons - empirically

Conventional wisdom: an ‘inverted’ mass nonet

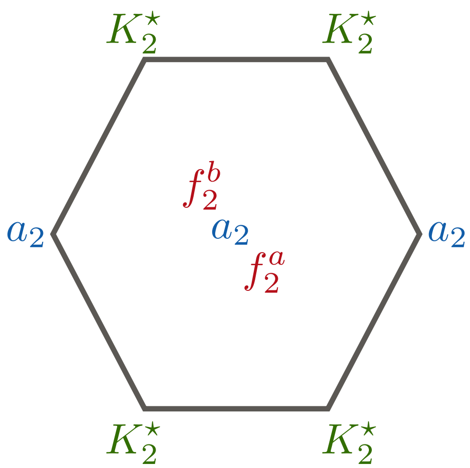


Similar? Vastly different imaginary parts…



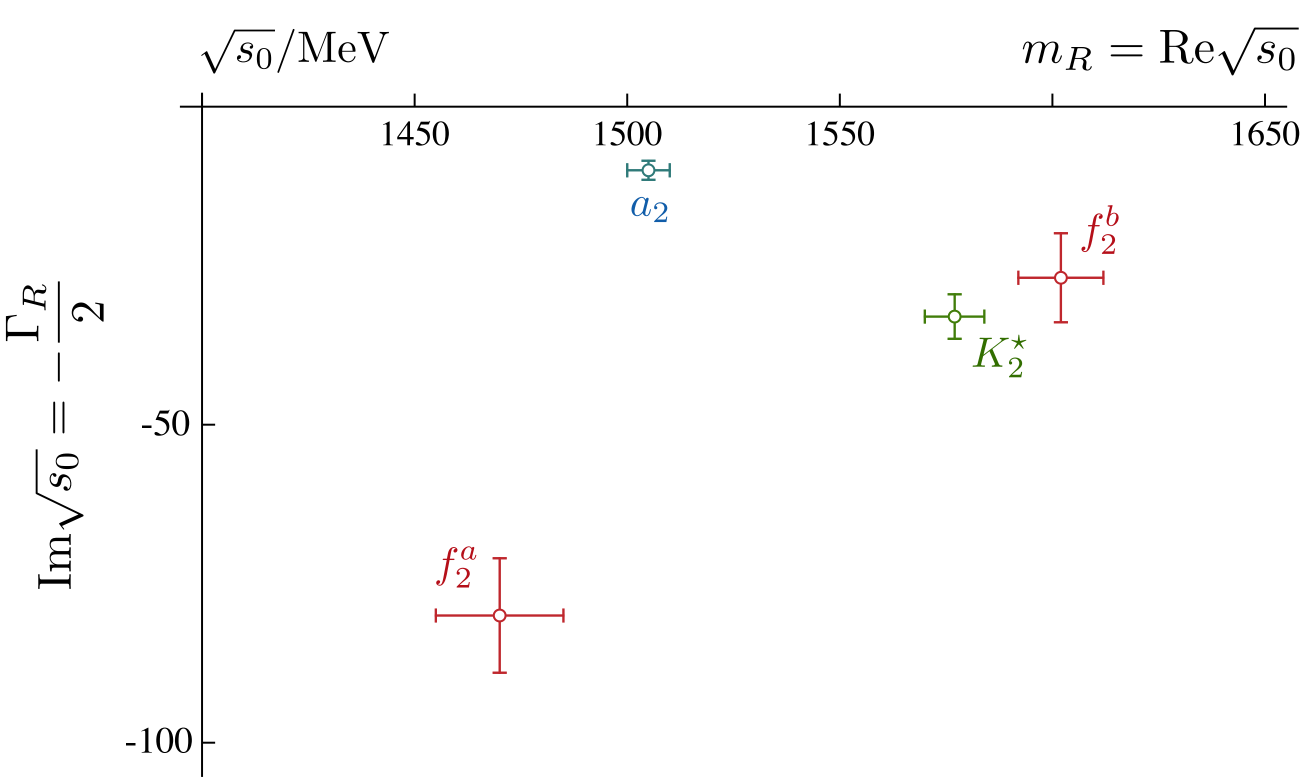
What does QCD have to say?

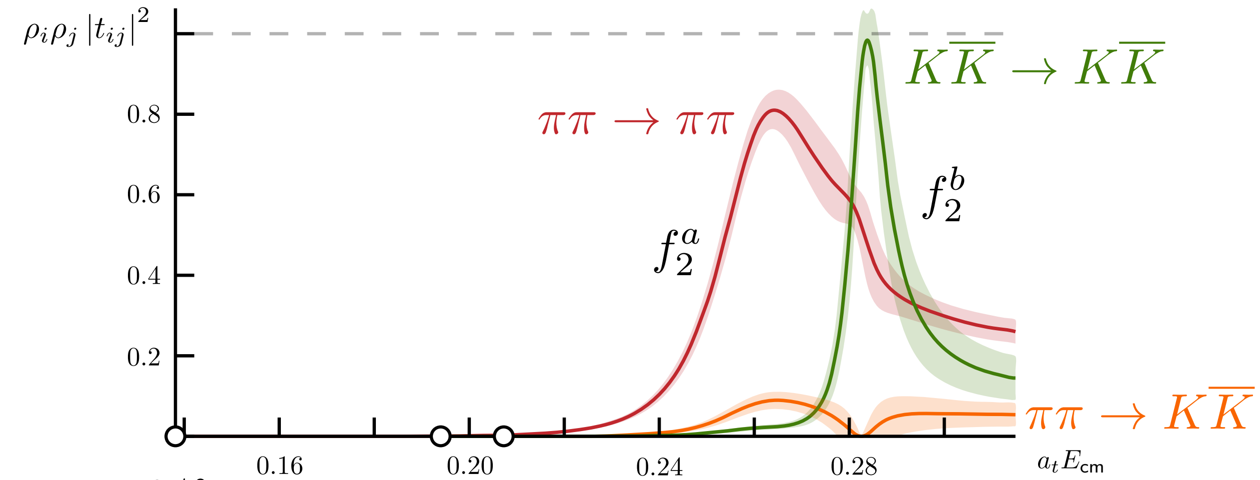
Lightest tensors at m_π=391 MeV

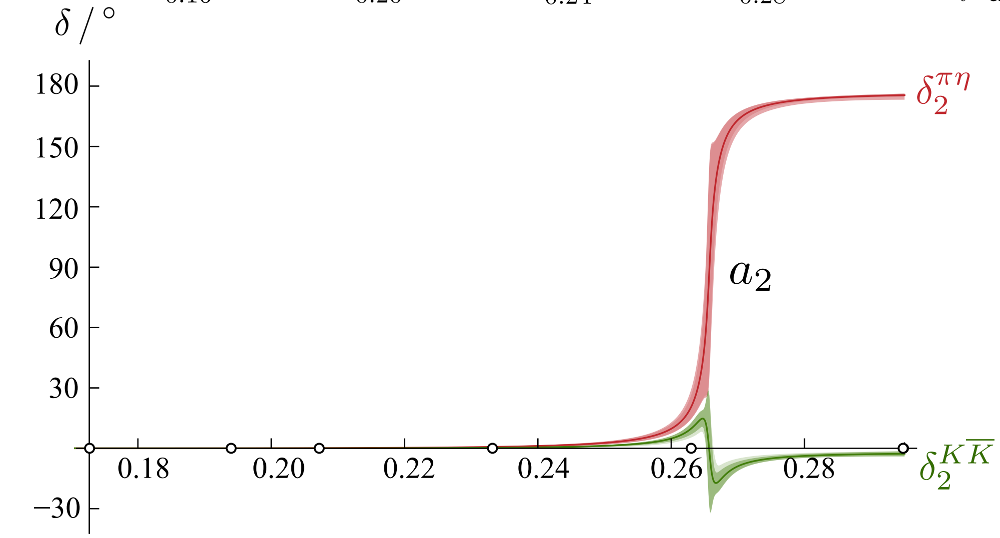


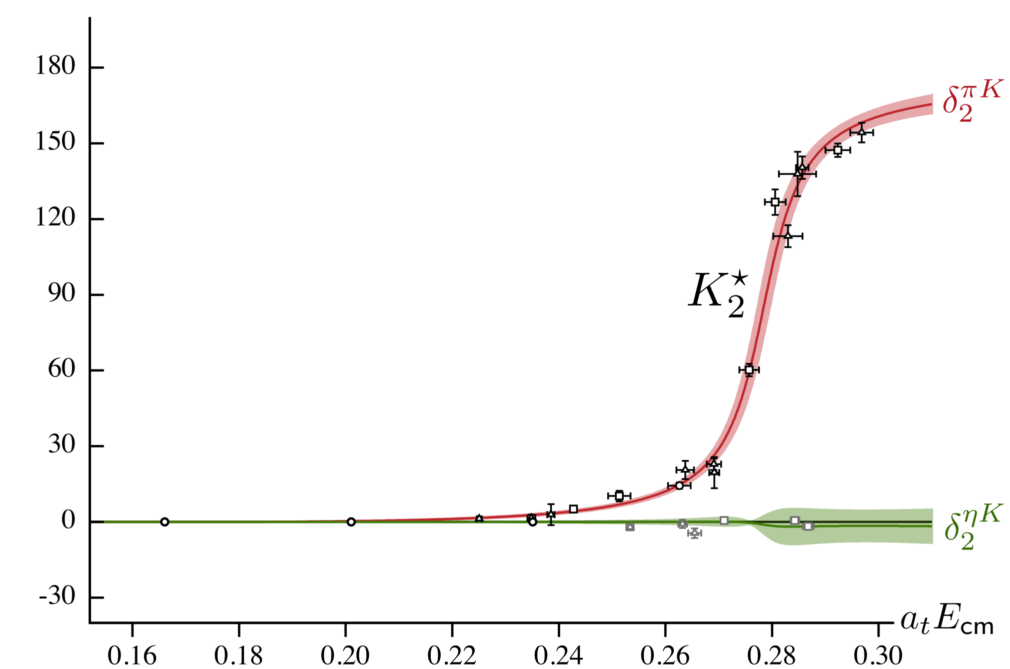
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

$$\sqrt{s_0} = m_R - i\frac{\Gamma_R}{2}$$









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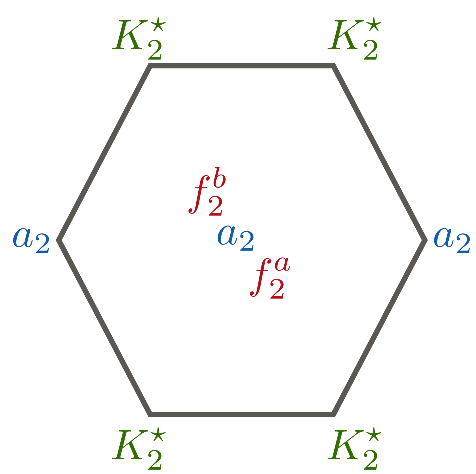
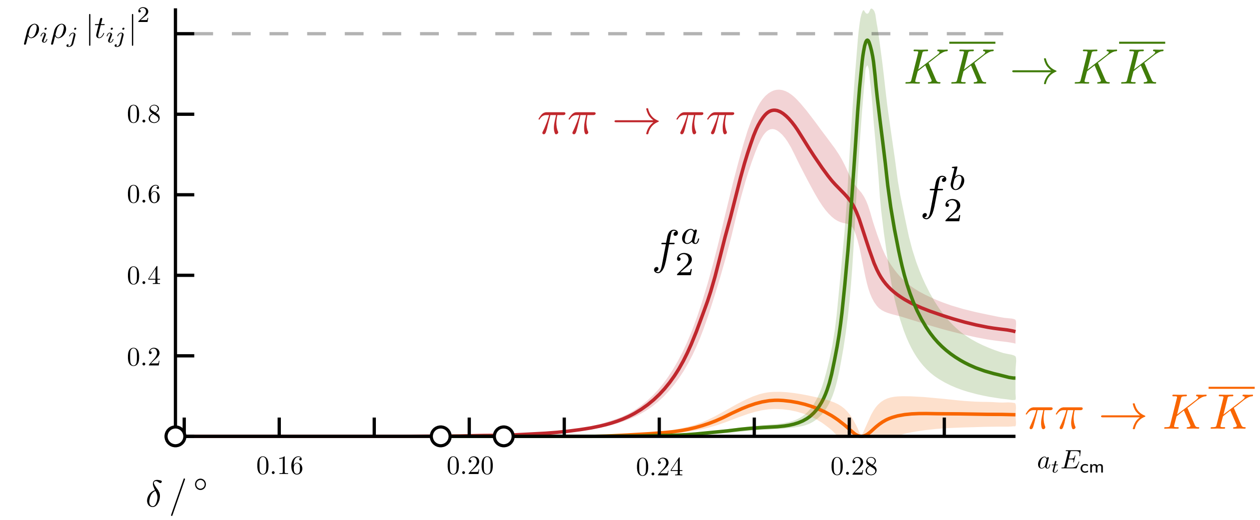
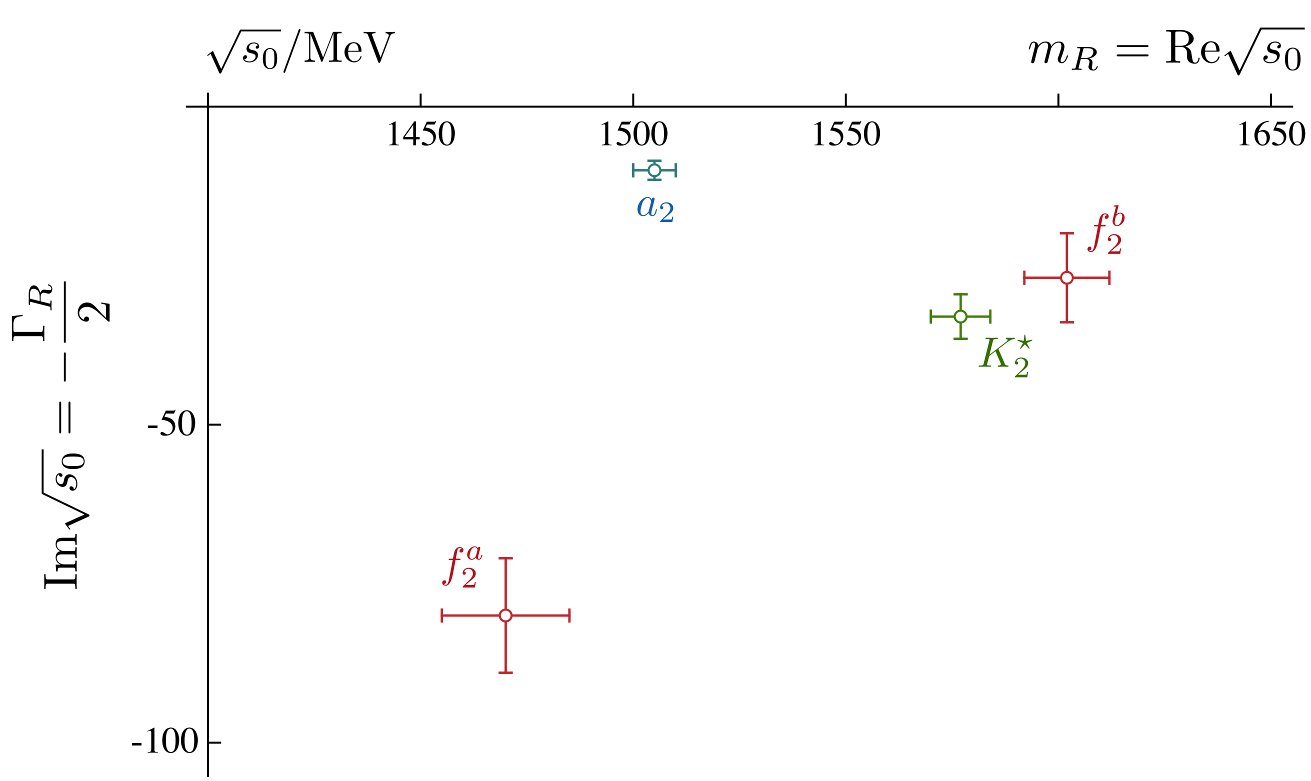
Lightest tensors at m_π=391 MeV





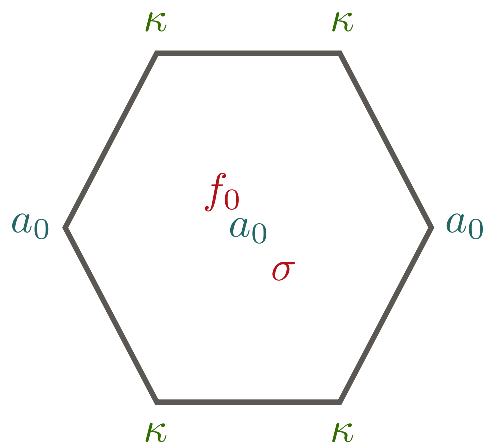
PDG $f_2(1270)$ $\pi\pi$ 84% $f_2(1525)$ $K\bar{K}$ 89%

flavor tagging by decays



Used as motivation in
GlueX detector upgrade proposal

Lightest scalars at m_π=391 MeV



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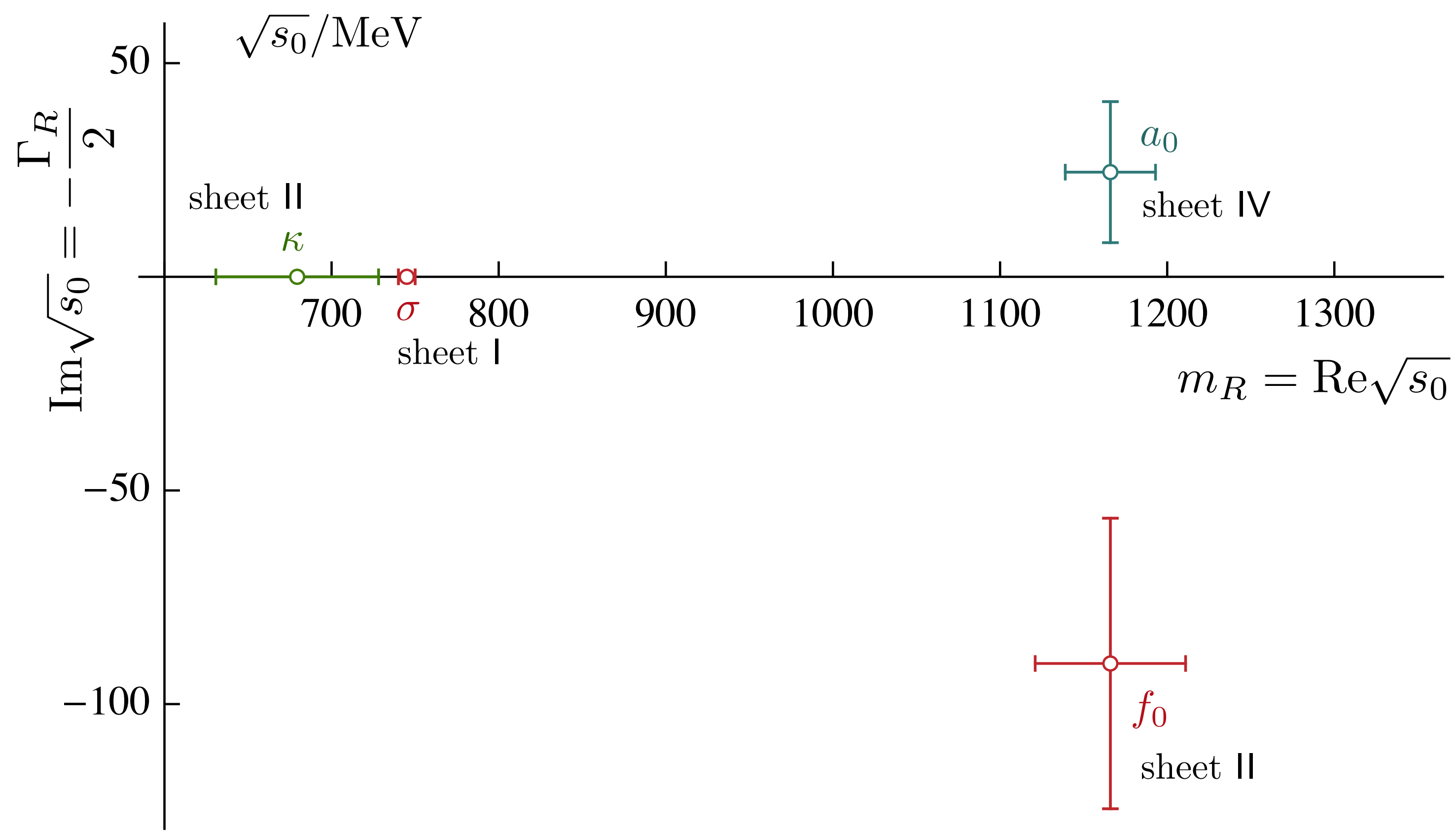
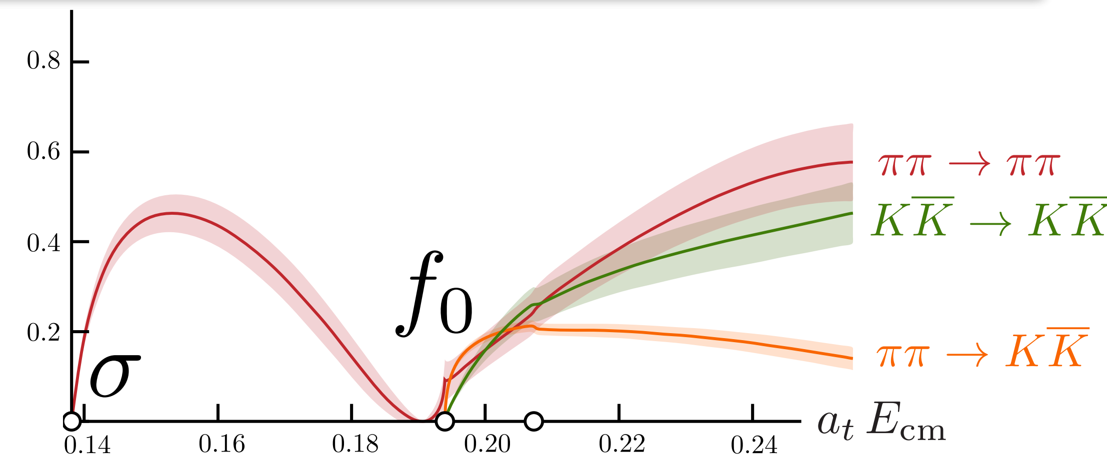
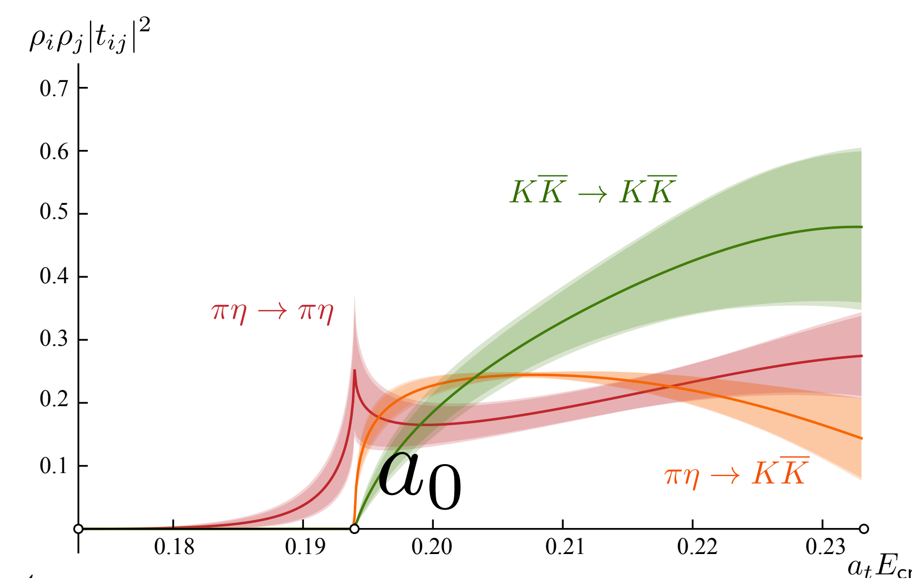
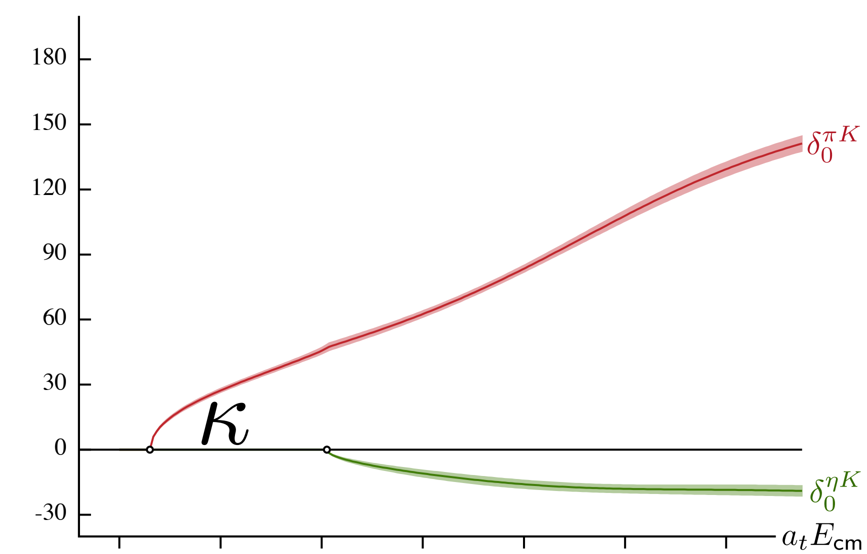
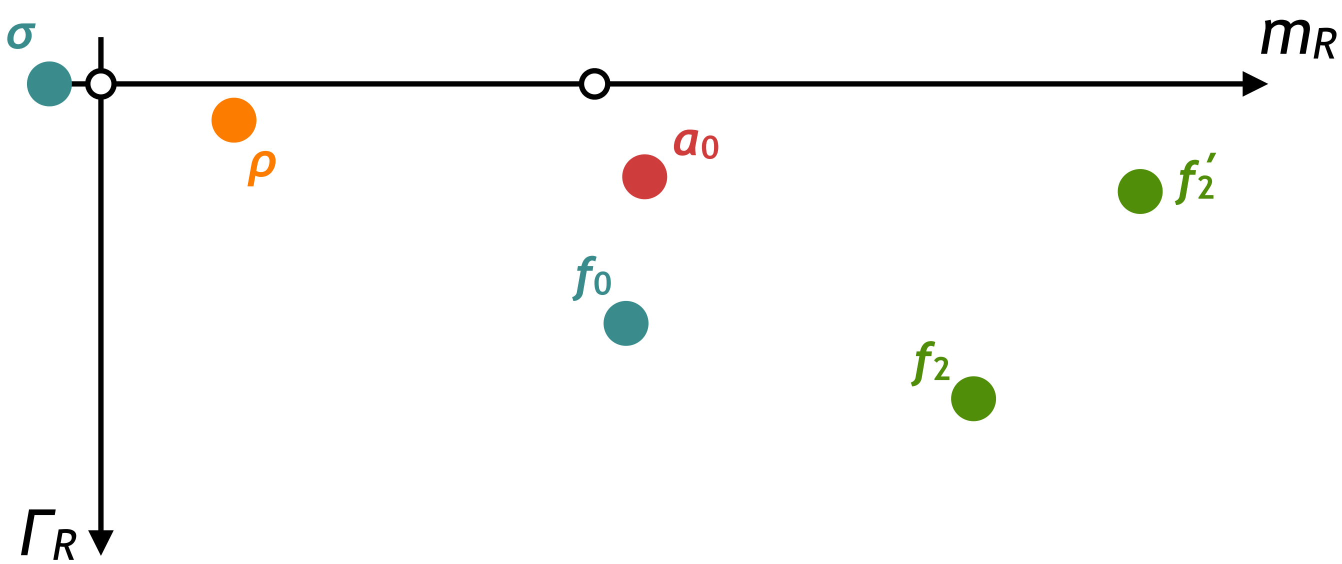




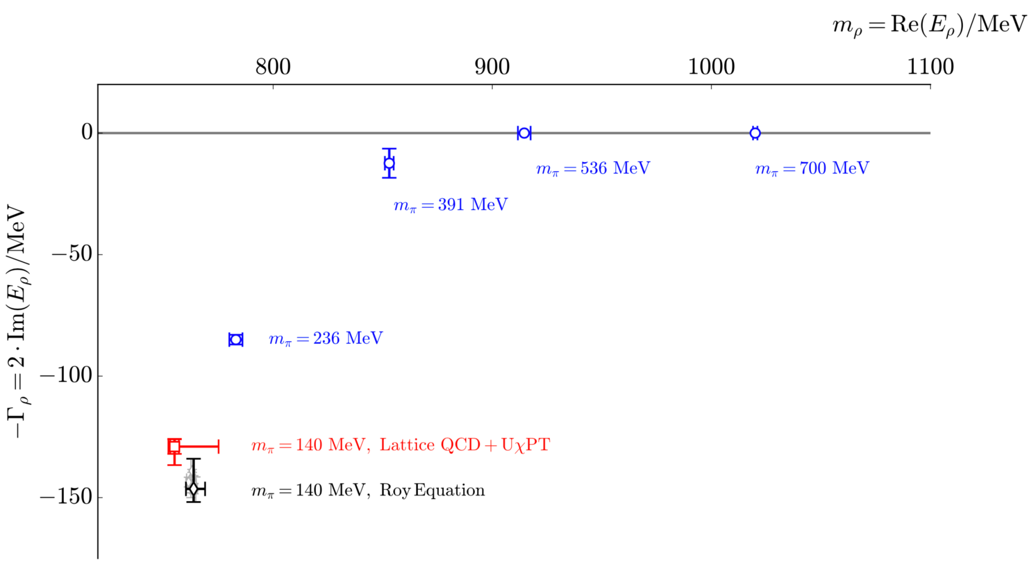


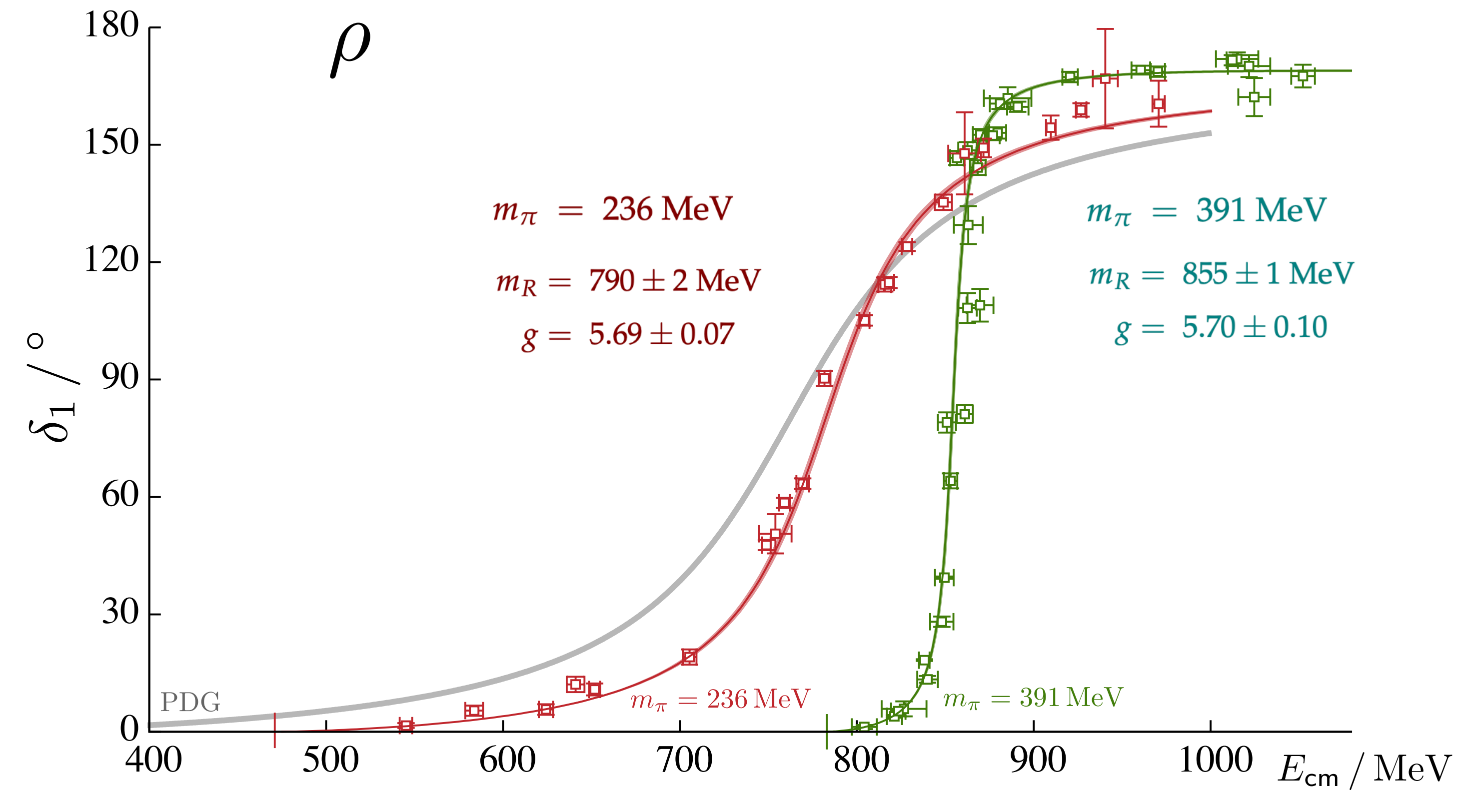


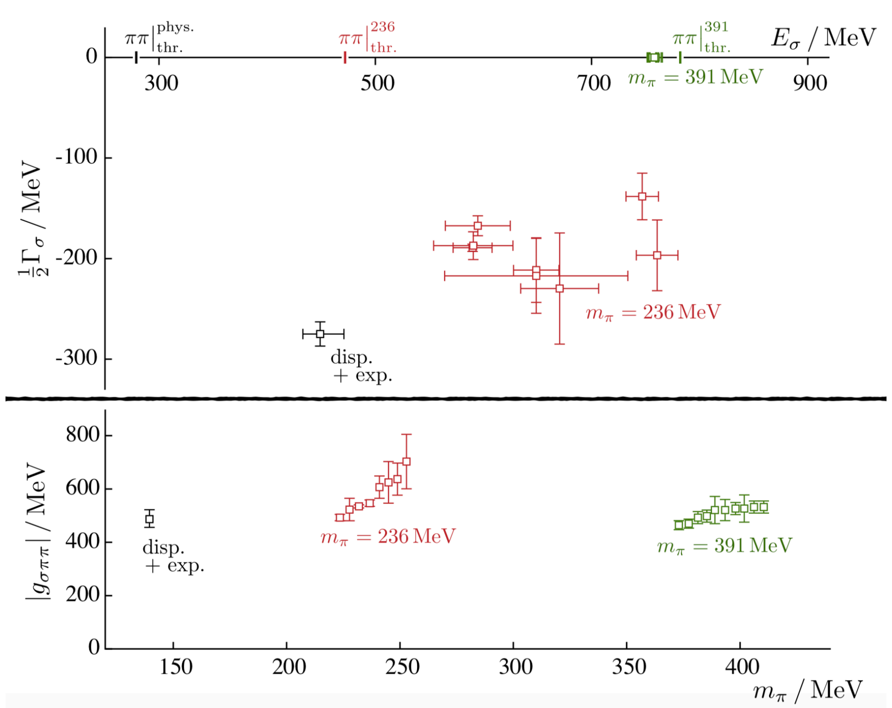
light meson resonances at m_π~391 MeV

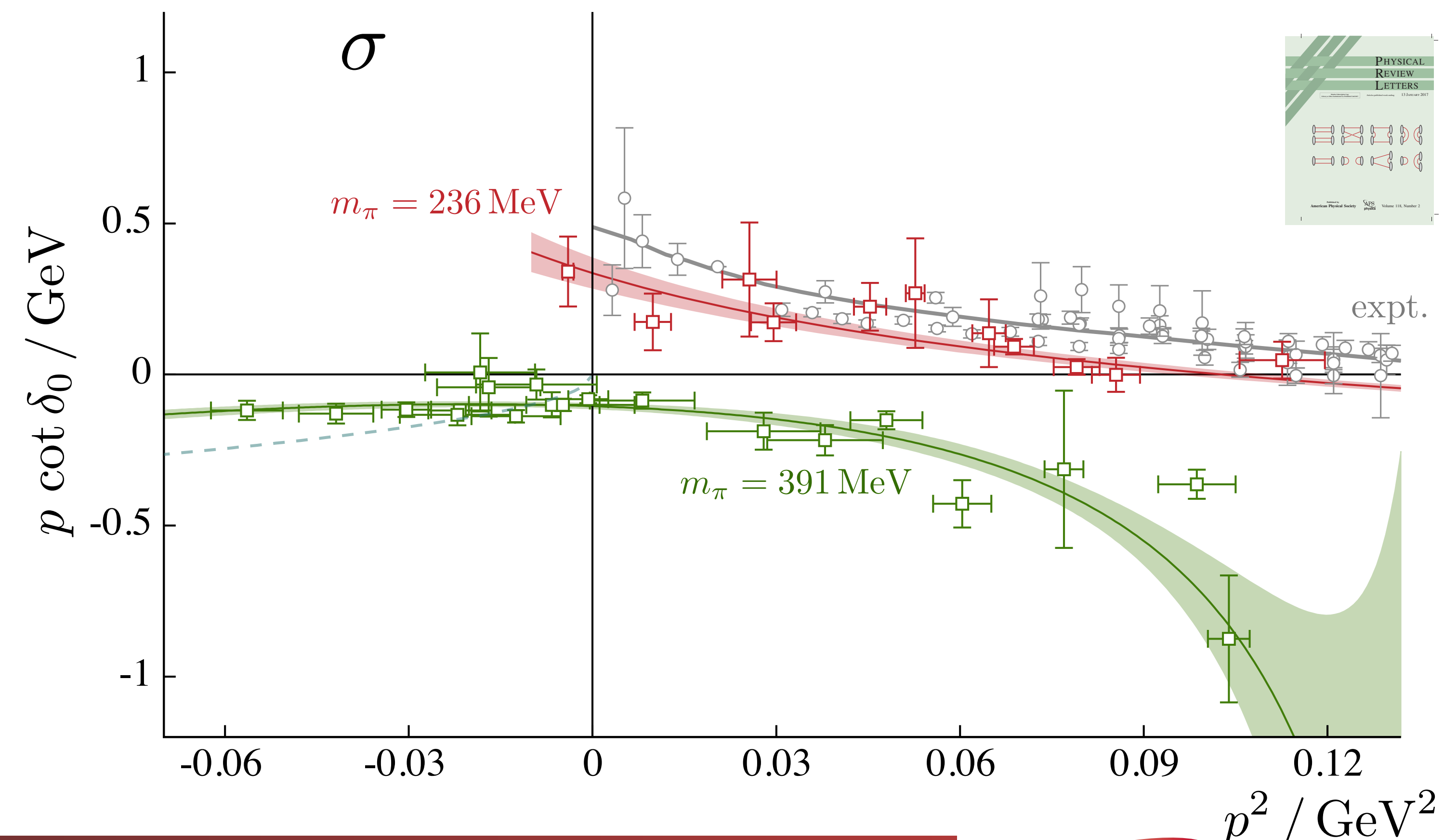


Quark mass dependence: I=0 & 1









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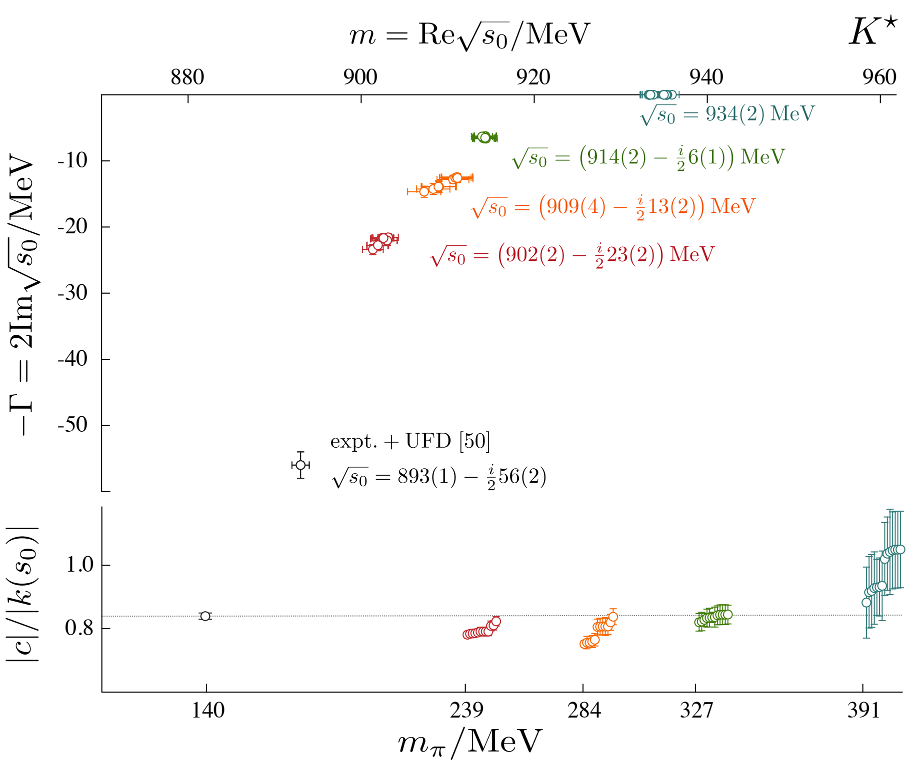
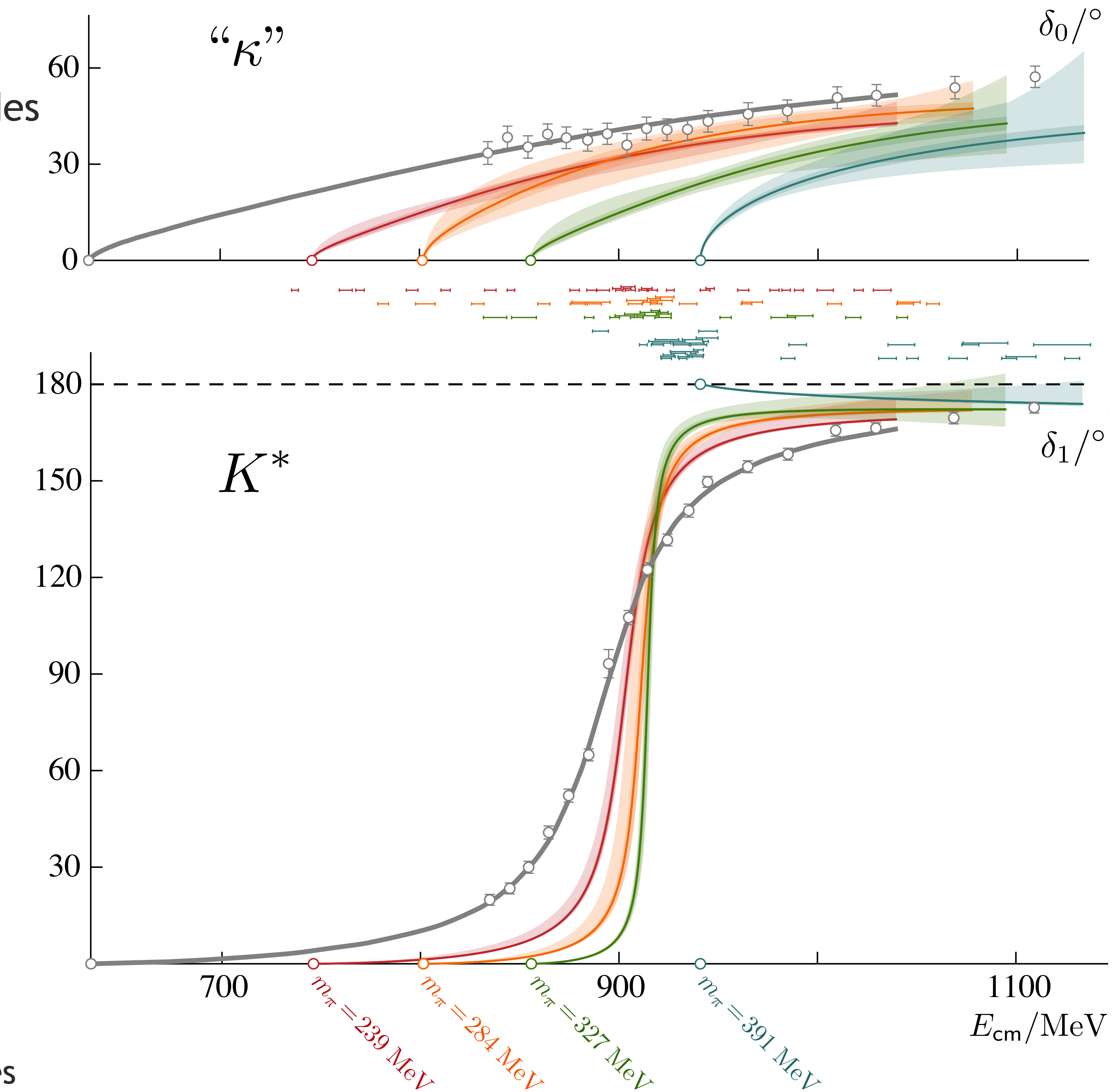
Quark mass dependence: I= 1/2

A more complicated story…
need "t" & "u"-channel amplitudes





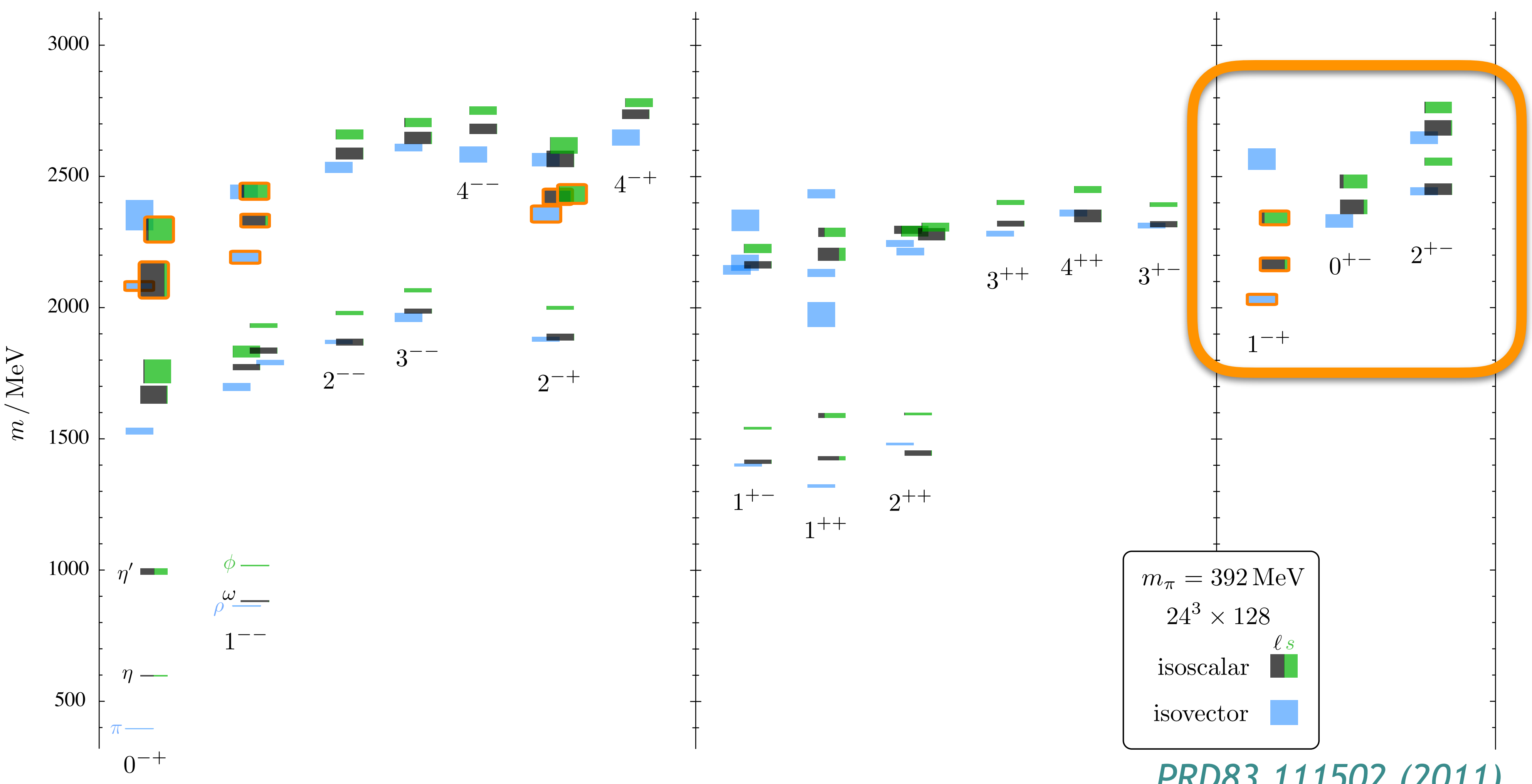
Added new dynamical ensembles

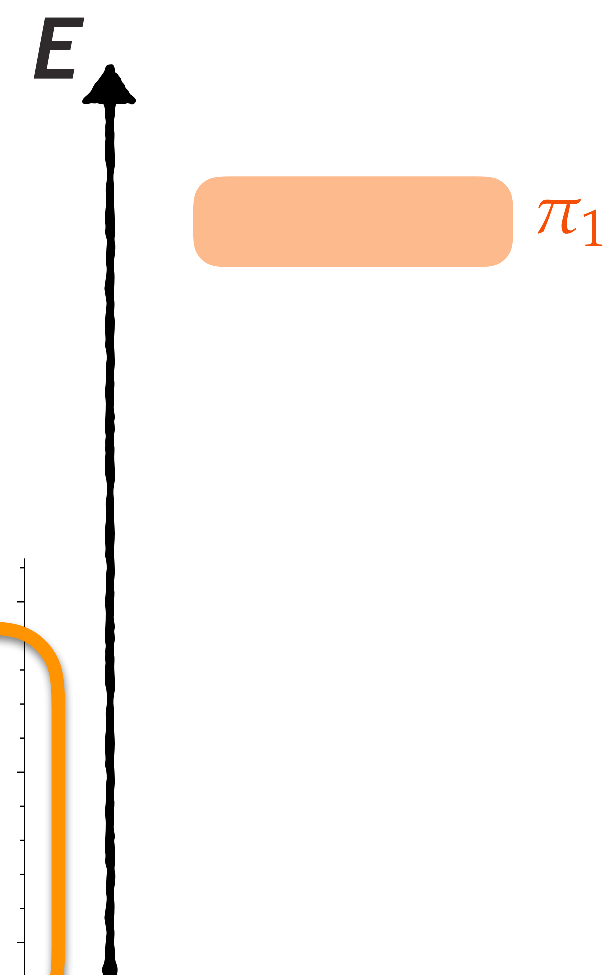
Exotic hybrid mesons in QCD?

Goal: Predict and understand hybrid mesons from QCD

signals for a 1^{-+} resonance above 1600 MeV

Single particle operator constructions



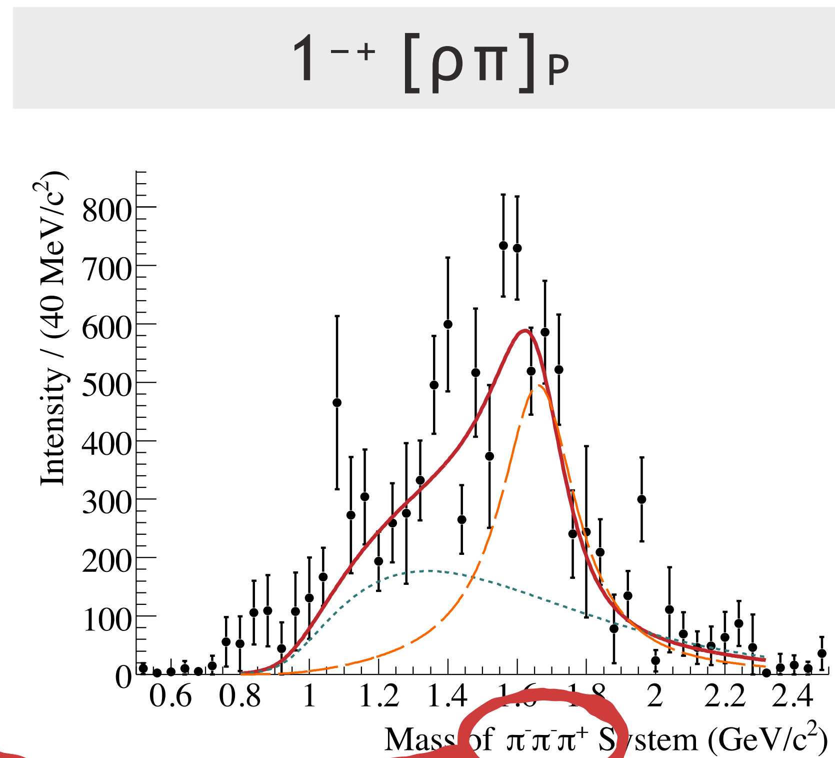


PRD83 111502 (2011)
PRD88 094595 (2013)

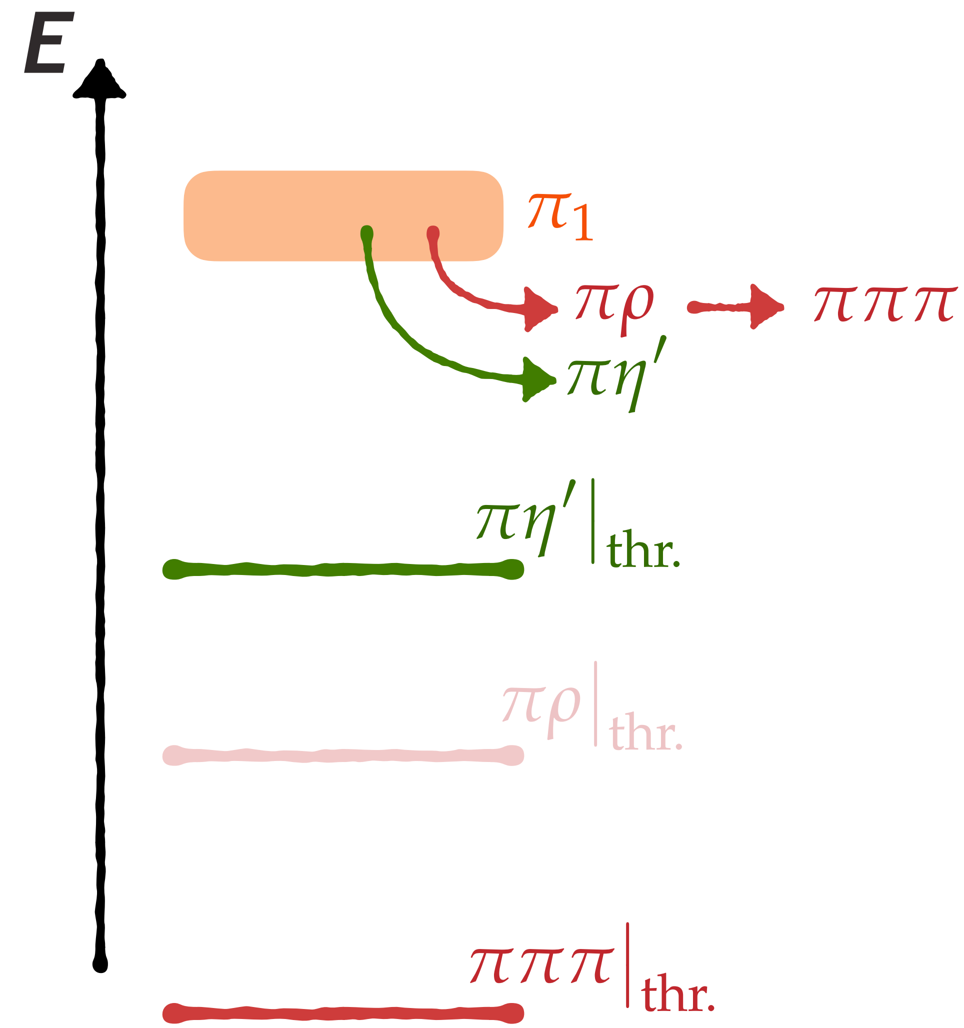
In general, many-body final states

- True final-states can include more than two stable hadrons

$\pi\pi\pi$



COMPASS Pb data
PRL104 241803 (2010)



Three-body formalism

Briceno, Hansen, Sharpe ...

Status of scattering formalisms

Two-body coupled channel

- finite-volume formalism ✔
- amplitude formalism (good) except cross-channels (not-good) 😊 😐
    - opportunities to collaborate with phenom. (e.g., JPAC) ✔

Status of scattering formalisms

Two-body coupled channel

- finite-volume formalism ✔
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Three-body coupled channel

- unitarity in 3-body ✔
- finite-volume formalism for spinless particles
    - coupled 2-body & 3-body ✔
    - determinant condition ✔
    - integral equations 😐
- 3-body amplitude formalism - not well understood! 😕
    - opportunities to collaborate with phenom. (e.g., JPAC)
- need 3-body energies. codes ready and optimized! ✔

Summary

- Rich spectrum of mesons & baryons - exotic & non-exotic hybrids
  - ➡ Direct impact on expt. program -> new expt. proposals
    - ➡ GlueX: kaon PID upgrade, CLAS12: hybrid baryons

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- *Goal is to compute resonance information - decays & branching fractions*
  - S-matrix formalism increasingly important - extended collabs. with JPAC, …

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  - ➡ Direct impact on expt. program -> new expt. proposals
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- *Goal is to compute resonance information - decays & branching fractions*
  - S-matrix formalism increasingly important - extended collabs. with JPAC, …

- Near term:
  - Use multiple volumes over range of pion masses -> poles & couplings
  - Physical limit calculations in progress
  - ➡ Knowledge of even size of branching fractions useful for expt. analysis

- Long term:
  - Exciting developments in near light-cone formalism
  - Envision merging spectroscopy and structure projects
    - ➡ Understanding role of gluonic structures

Progress since 2013

2019

πK elastic scattering (4 light quark masses) **1904.03188**

$\pi\omega$ scattering and the b_1 resonance **1904.04136**

2018

Dynamically-coupled partial-waves in $\rho\pi$ isospin-2 scattering from lattice QCD
A. J. Woss, C. E. Thomas, J. J. Dudek, R. G. Edwards, D. J. Wilson — **JHEP 07 (2018) 043**

Isoscalar $\pi\pi$, $K\overline{K}$, $\eta\eta$ scattering and the σ, f_0, f_2 mesons from QCD
R. A. Briceno, J. J. Dudek, R. G. Edwards, D. J. Wilson — **Phys. Rev. D 97, 054513 (2018)**

2017

Tetraquark operators in lattice QCD and exotic flavour states in the charm sector
G. K. C. Cheung, C. E. Thomas, J. J. Dudek, R. G. Edwards — **JHEP 11 (2017) 033**

Isoscalar $\pi\pi$ scattering and the σ meson resonance from QCD
R. A. Briceño, J. J. Dudek, R. G. Edwards, D. J. Wilson — **Phys. Rev. Lett. 118, 022002 (2017)**

2016

Excited and exotic charmonium, D_s and D meson spectra for two light quark masses from lattice QCD
G. K. C. Cheung, C. O'Hara, G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, D. Tims — **JHEP 12 (2016) 089**

Coupled-Channel $D\pi$, $D\eta$ and $D_s\overline{K}$ Scattering from Lattice QCD
G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, D. J. Wilson — **JHEP 10 (2016) 011**

The $\pi\pi \to \pi\gamma^*$ amplitude and the resonant $\rho \to \pi\gamma^*$ transition from lattice QCD
R. A. Briceño, J. J. Dudek, R. G. Edwards, C. J. Shultz, C. E. Thomas, D. J. Wilson — **Phys. Rev. D 93, 114508 (2016)**

An a_0 resonance in strongly coupled $\pi\eta$, $K\overline{K}$ scattering from lattice QCD
J. J. Dudek, R. G. Edwards, D. J. Wilson — **Phys. Rev. D 93, 094506 (2016)**

2015

The resonant $\pi^+\gamma \to \pi^+\pi^0$ amplitude from Quantum Chromodynamics
R. A. Briceño, J. J. Dudek, R. G. Edwards, C. J. Shultz, C. E. Thomas, D. J. Wilson — **Phys. Rev. Lett. 115, 242001 (2015)**

Coupled $\pi\pi$, $K\overline{K}$ scattering in P-wave and the ρ resonance from lattice QCD
D. J. Wilson, R. A. Briceño, J. J. Dudek, R. G. Edwards, C. E. Thomas — **Phys. Rev. D 92, 094502 (2015)**

Spectroscopy of doubly-charmed baryons from lattice QCD
M. Padmanath, R. G. Edwards, N. Mathur, M. Peardon — **Phys. Rev. D 91, 094502 (2015)**

Excited meson radiative transitions from lattice QCD using variationally optimized operators
C. J. Shultz, J. J. Dudek, R. G. Edwards — **Phys. Rev. D 91, 114501 (2015)**

Resonances in coupled πK, ηK scattering from lattice QCD
D. J. Wilson, J. J. Dudek, R. G. Edwards, C. E. Thomas — **Phys. Rev. D 91, 054008 (2015)**

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Resonances in coupled πK, ηK scattering from quantum chromodynamics
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Decay constants of the pion and its excitations on the lattice
E. V. Mastropas, D. G. Richards — **Phys. Rev. D 90, 014511 (2014)**

Spectroscopy of triply-charmed baryons from lattice QCD
M. Padmanath, R. G. Edwards, N. Mathur, M. Peardon — **Phys. Rev. D 90, 074504 (2014)**

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Toward the excited isoscalar meson spectrum from lattice QCD
J. J. Dudek, R. G. Edwards, P. Guo, C. E. Thomas — **Phys. Rev. D 88, 094505 (2013)**

Excited spectroscopy of charmed mesons from lattice QCD
G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, L. Liu — **JHEP 1305 (2013) 021**

Flavor structure of the excited baryon spectra from lattice QCD
R. G. Edwards, N. Mathur, D. G. Richards, S. J. Wallace — **Phys. Rev. D 87, 054506 (2013)**

Energy dependence of the ρ resonance in $\pi\pi$ elastic scattering from lattice QCD
J. J. Dudek, R. G. Edwards, C. E. Thomas — **Phys. Rev. D 87, 034505 (2013)**

Four-particle effects

We don't know the equation that describes 2π-4π spectrum, but we know it will have the form:

$$\det\left[\begin{pmatrix} F_{2\pi} & \\ & F_{4\pi}\end{pmatrix}^{-1} + \begin{pmatrix} \mathcal{M}_{2\pi,2\pi} & \mathcal{M}_{2\pi,4\pi} \\ \mathcal{M}_{4\pi,2\pi} & \mathcal{M}_{4\pi,4\pi}\end{pmatrix}\right] = 0$$

$F_a(L, E_{L,n})$: finite volume function, (don't know it for 4π)

$\mathcal{M}_{a,b}(E_{L,n})$: scattering amplitude coupling a^{th} and b^{th} channel

$E_{L,n}$: energy level for the n^{th} state, which satisfies the equation above

If $\mathcal{M}_{2\pi,4\pi} \sim \mathcal{O}(\epsilon)$

$$\det\left[F_{2\pi}^{-1} + \mathcal{M}_{2\pi,2\pi}\right] \times \det\left[F_{4\pi}^{-1} + \mathcal{M}_{4\pi,4\pi}\right] + \mathcal{O}(\epsilon) = 0$$

Two spectra that do not talk to each other.

Same argument applies for charmonium systems:

$$(E_{L,n} - E_{J/\Psi}) \times \det\left[F_{Nbody} + \mathcal{M}_{Nbody}\right] + \mathcal{O}(\epsilon) = 0$$

Four-particle effects

Two-point correlation functions:

$$C_{ab}^{2pt.}(t,\mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle = \sum_n Z_{b,n}Z_{a,n}^\dagger e^{-E_n t}$$

Assume only a basis of two-particle operators is used, and $\mathcal{M}_{2\pi,4\pi} \sim \mathcal{O}(\epsilon)$

If the nth state satisfies

$$\det\left[F_{4\pi}^{-1} + \mathcal{M}_{4\pi,4\pi}\right] + \mathcal{O}(\epsilon) = 0$$

The overlap with the nth would be vanishingly small

$$|Z_{2\pi,n}|^2 = |\langle 0|(2\pi)|n, E_{L,n}\rangle|^2 \sim |\mathcal{M}_{2\pi,4\pi}|^2 \sim \mathcal{O}(\epsilon^2)$$

following arguments presented in
Briceño, Hanse & Walker-Loud (2014)

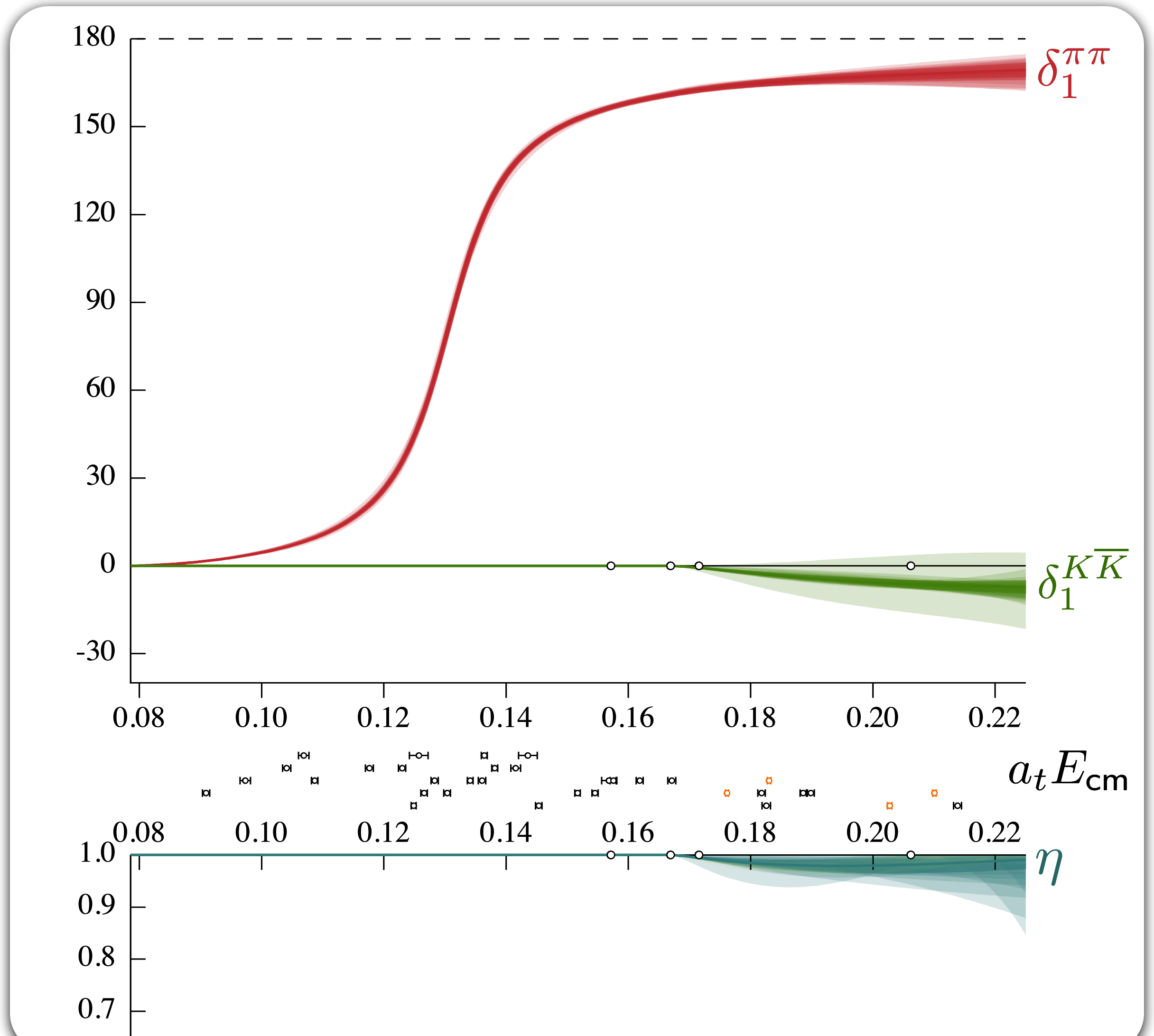
Again, it is this same principle which allows for the study of charmonia on the lattice, despite the fact that any number of light multi-meson states can go on-shell.

Consistency check

If this form were not true:

$$\det\left[\begin{pmatrix} F_{\pi\pi} & \\ & F_{K\overline{K}}\end{pmatrix}^{-1} + \begin{pmatrix}\mathcal{M}_{\pi\pi,\pi\pi} & \mathcal{M}_{\pi\pi,K\overline{K}} \\ \mathcal{M}_{\pi\pi,K\overline{K}} & \mathcal{M}_{K\overline{K},K\overline{K}}\end{pmatrix}\right] \times \det\left[F_{4\pi}^{-1} + \mathcal{M}_{4\pi,4\pi}\right] + \mathcal{O}(\epsilon) = 0$$

and/or the overlap argument were not true, the two-body coupled-channel formalism would not describe the spectrum properly above the 4π threshold



...but it does...

Jefferson Lab