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# Lattice Calculation of ( $x$ -dependent) Parton Distribution Functions + *GPDs*, *TMDs*...

## *SPC Summary 2019*

# Introduction

**Working definition** - calculation of x-dependent PDFs and QDAs (quark distribution amplitudes) from Euclidean-space lattice calculations.

- Quasi-PDF (**qPDF**) interpreted in **LaMET** (Large Momentum Effective Theory) was proposed by X.Ji **X. Ji, Phys. Rev. Lett. 110 (2013) 262002**

$$q(x, \mu^2, P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P | \bar{\psi}(z) \gamma^z e^{-ig \int_0^z dz' A^z(z')} \psi(0) | P \rangle + \mathcal{O}((\Lambda^2/(P^z)^2), M^2/(P^z)^2)$$

*Quasi distributions* approach light-cone distributions in limit of large  $P^z$

$$q(x, \mu^2, P^z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y, \mu^2) + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

- Pseudo-PDF (**pPDF**) recognizing generalization of PDFs in terms of *Ioffe Time*.  $\nu = p \cdot z$  **A. Radyushkin, PLB767 (2017)**

$$\mathcal{M}^\alpha(z, p) = \langle p | \bar{\psi}(z) \gamma^\alpha \exp\left(-ig \int_0^z dz' A^z(z')\right) \psi(0) | p \rangle$$

# Introduction - II

- Good “Lattice Cross Sections” (**LCS**) Ma and Qiu, Phys. Rev. Lett. 120 022003

$$\sigma_n(\omega, \xi^2, P^2) = \langle P | T\{O_n(\xi)\} | P \rangle \quad \text{Expressed in coordinate space}$$

where

$$\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^1 \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\omega, \xi^2, x^2 P^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda_{\text{QCD}}^2)$$

Calculated in LQCD

Structure function

Calculated in perturbation theory (“process dependent”)

Factorize in  $\omega = P \cdot \xi, \xi^2 P^2$  providing  $\xi \ll \frac{1}{\Lambda_{\text{QCD}}}$

Operators

$$P \rightarrow \sqrt{s} \quad \text{Collision energy}$$

$$\xi \rightarrow \frac{1}{Q} \quad \text{Hard Probe}$$

$$O(\xi) = \bar{\psi}(0) \Gamma W(0, 0 + \xi) \psi(\xi) \begin{cases} O_S(\xi) = \xi^4 Z_S^2 [\bar{\psi}_q \psi_q](\xi) [\bar{\psi}_q \psi](0) \\ O_{V'}(\xi) = \xi^2 Z_{V'}^2 [\bar{\psi}_q \xi \cdot \gamma \psi_{q'}](\xi) [\bar{\psi}_{q'} \xi \cdot \gamma \psi](0) \end{cases}$$

# Pseudo-PDF vs Quasi-PDF

Relation between **qPDF** and **pPDF** approaches

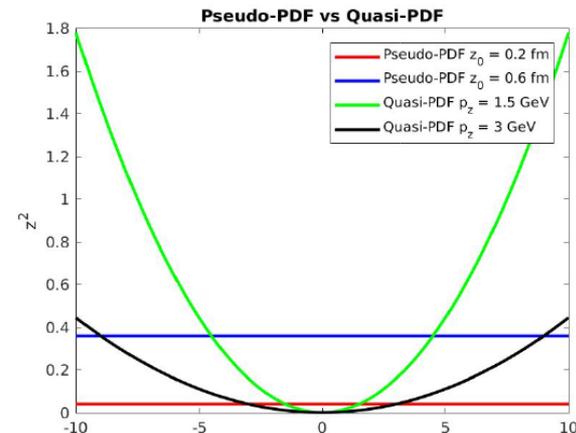
Joe Karpie

- Both integrals of Ioffe-Time Distribution Function
- Computed matrix elements the same
- Should yield same PDF after matching and systematic controls

$$P(x, z_0^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu e^{-i\nu x} M(\nu, z_0^2)$$

$$Q(x, p_z^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu e^{-i\nu x} M(\nu, \frac{\nu^2}{p_z^2})$$

Require sufficiently fine lattice spacings



KF Liu, SJ Dong, PRL72, 1790 (1994)

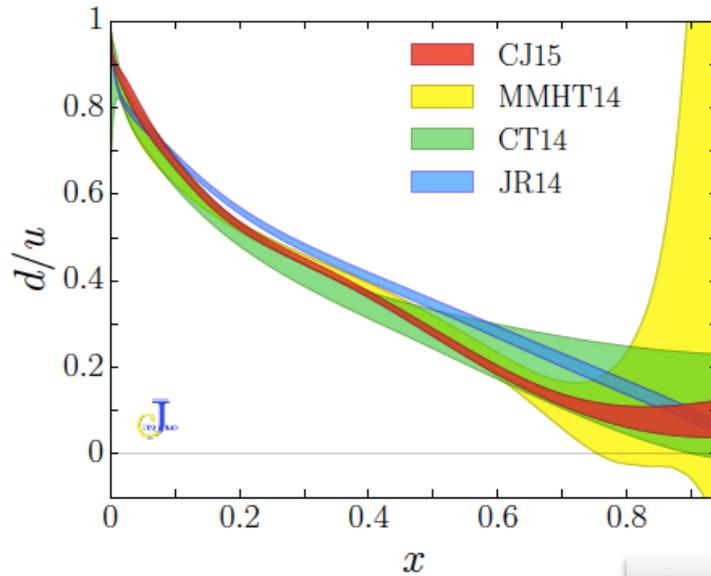
- Hadronic Tensor (**HT**)  $W_{\mu\nu} = \frac{1}{4\pi} \int d^4 z e^{iq \cdot z} \langle p | J_\mu(z)^\dagger J_\nu(0) | p \rangle$

$$C_4(\vec{p}, \vec{q}, \tau) = \sum_{\vec{x}_f} e^{-i\vec{p} \cdot \vec{x}_f} \sum_{\vec{x}_2, \vec{x}_1} e^{-i\vec{q} \cdot (\vec{x}_2 - \vec{x}_1)} \langle N(\vec{x}_f, t_f) J_\mu(\vec{x}_2, t_2) J_\nu(\vec{x}_1, t_1) \bar{N}(\vec{0}, t_0) \rangle$$

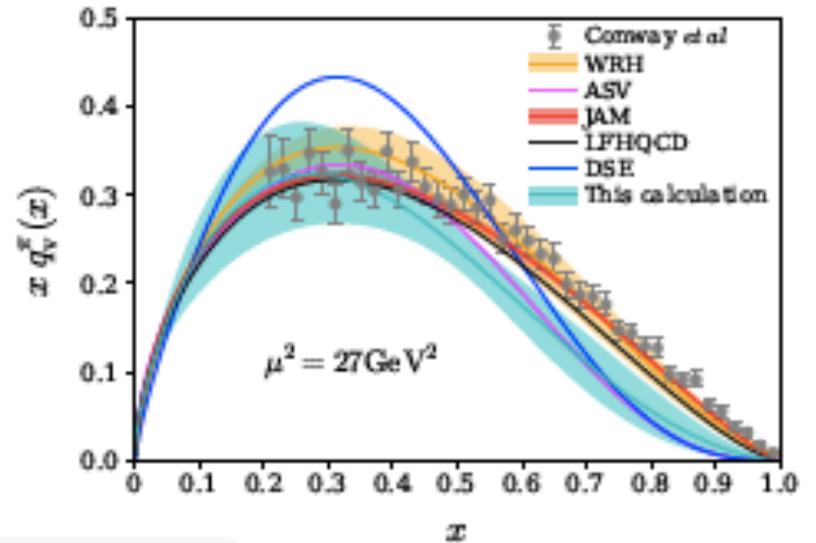
This is a **four-point** function.

# Impact

d/u at large x

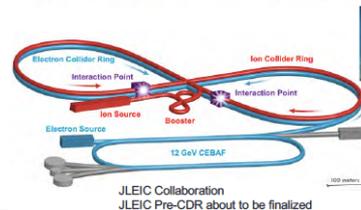
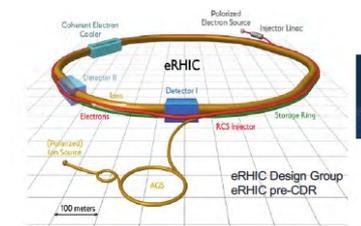


Large-x behavior of pion PDF



Imaging the Quarks

Imaging the Gluons



# Progress and Challenges

G.C. Rossi, M. Testa, arXiv:1706.04428

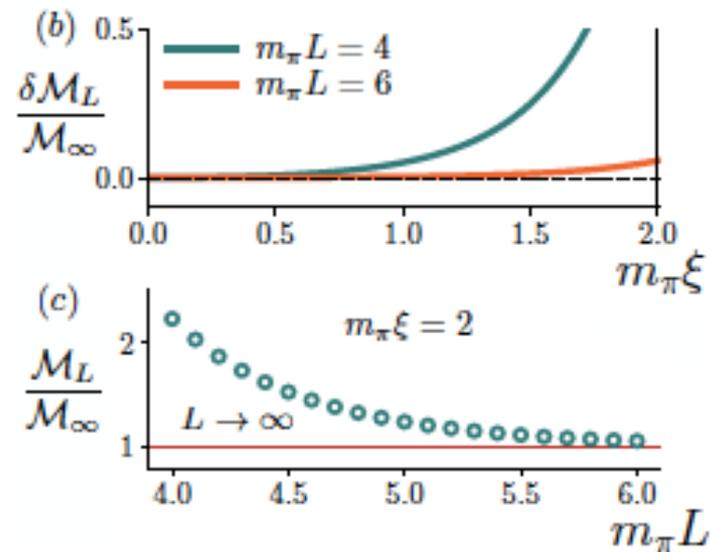
Do moments computed from non-local operators yield finite results in continuum limit? - *Yes*

J Karpie, K Orginos, S Zafeiropoulos, arXiv:1807.10933

Finite volume effects in non-local operators? - *they introduce another scale*

R. Briceno, J. Guerrero, M. Hansen, C. Monahan, arXiv:1805.01034

Depends on details of matrix element



# Progress and Challenges - II

“Inverse Problem” - ill-posed inverse Fourier transform.

$$\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^1 \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\omega, \xi^2, x^2 P^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda_{\text{QCD}}^2)$$

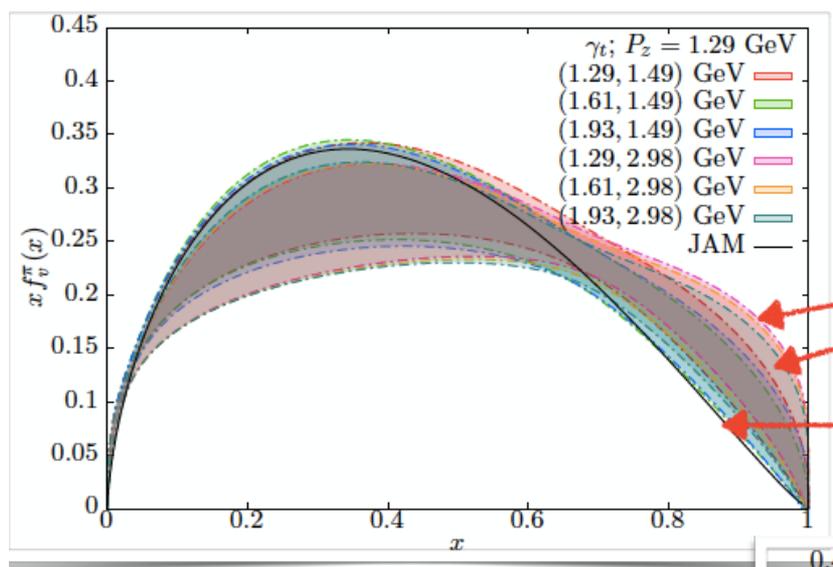
Extract PDF?

Calculate on Lattice

Calculate in PQCD

Exactly same challenge as global fitting community!

N. Karthik, APS 2019

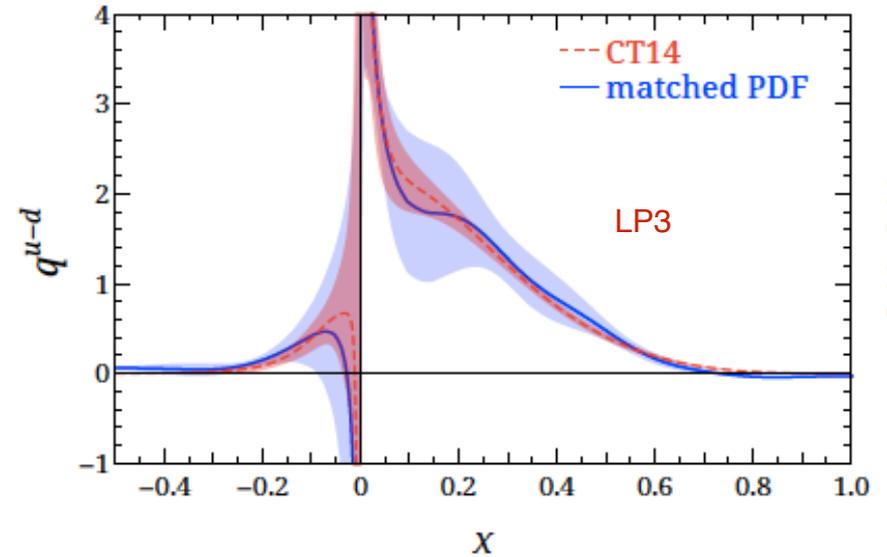
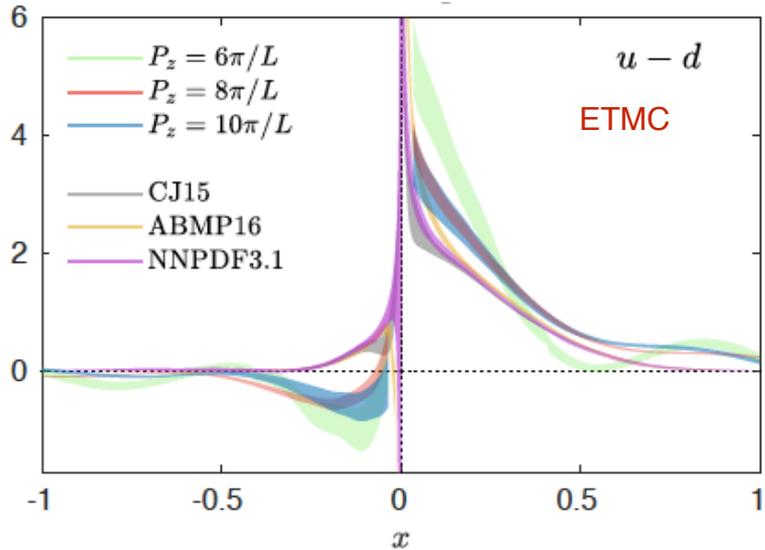


Pion PDF - use  
*phenomenological  
parametrization*

$$f(x) = x^\alpha (1 - x)^\beta$$

# Some Highlights...

From C. Monahan, Lattice 18



*Isovector Distributions at physical pion mass*



- Gluon PDFs and isoscalar structure
- 3D Imaging of Hadrons

# Proposals

**Team:** Constantia Alexandrou, Krzysztof Cichyd, Kyriakos Hadjiyiannakou, Karl Jansen, Colin Lauer, Aurora Scapellato, Fernanda Steffens

**Project:** Quasi-GPDs from Lattice QCD

**Request:** 12M SKX core-hours, 15TB Disk, 10TB Tape

Nf=2+1+1 TM fermions, 270 MeV Pion

**Team:** T. Izubuchi, L. Jin, C. Kallidonis, N. Karthik, S. Mukherjee, P. Petreczky, C. Shugert, and S. Syritsyn

**Project:** Computing Pion Parton Distribution Function on Fine Lattice - Continuation

**Request:** 140K K80 GPU, 80TB Disk, 100TB Tape

**Approach:** qPDF + pPDF

HISQ sea and Clover valence at very fine lattice spacings of 0.04 fm to get to high momenta.

**Team:** Huey-Wen Lin, Zhouyou Fan, Ruizi Li, Rui Zhang, Jiunn-Wei Chen, Jian-Hui Zhang

**Project:** Bjorken-x Dependence of Gluon Parton Distribution Functions on the Lattice

**Request:** 15M SKX core hours; 59K K80 GPU hours

**Approach:** qPDF

Clover on HISQ at a  $\sim 0.06$ fm, 310 MeV Pion Mass + physical. Broad program including GPDs.

# Proposals

**Team:** Bipasha Chakraborty, Robert Edwards, Balint Joo, Kostas Orginos, Jianwei Qiu, David Richards, Raza Sufian, Frank Winter, Carl Carlson, Colin Egerer, Joe Karpie, Tanjib Khan, Wayne Morris, Anatoly Radyushkin, Savvas Zafeiropoulos

**Project:** Parton Distribution Functions and Amplitudes of the Pseudoscalar Mesons and Nucleon from Lattice QCD

**Request:** 89M KNL core hours; 900K GeForce GPU hours

**Approach:** pPDF, using Wilson-line and current-current matrix elements  
Clover on clover, using distillation,  $a \sim 0.09$  fm, Pion Mass around 170 MeV.

**Team:** Jian Liang, Keh-Fei Liu, Yi-Bo Yang and Terrence Draper

**Project:** Neutrino-Nucleon Scattering and Hadronic Tensor

**Request:** 12M SKX core hours, 80TB Disk, 300 TB Tap

**Approach:** HT

Clover on HISQ at valence at  $a \sim 0.045$ fm, 310 MeV pion. Extend calculation down to resonance and shallow-elastic regions

# CFNS Workshop on Lattice Parton Distribution Functions

17-19 April 2019  
Physics department (Building 510)  
US/Eastern timezone

## Overview

Timetable

Registration

BNL Guest Registration (GIS)

Participant List

## CFNS Administration

✉ [irachel@bnl.gov](mailto:irachel@bnl.gov)

☎ 631-344-3500

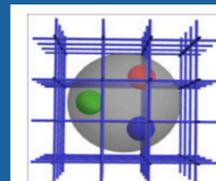
## Updates:

The workshop will be held in the CFNS seminar room located in the second floor of the Physics Department (Building 510). Here is the route from the main entrance to Building 510: <https://www.bnl.gov/maps/510map.php>.

For lunch, an option is the cafeteria located in the Berkner hall which is an 8 minute walk from the Physics building. For a complete set of on-site and off-site options for food, see here: <https://www.bnl.gov/staffservices/foodservices.php>.

On 18th Thursday, we will have the workshop dinner at Desmond's (https://eastwindlongisland.com/desmonds-restaurant/) from 5:30pm to 8:00pm.

The CFNS Workshop on Lattice Parton Distribution Functions will be held at the Jefferson Laboratory from Wednesday, April 17th to Friday, April 19th.



W. K. Kellogg  
Biological Station  
MICHIGAN STATE UNIVERSITY

## Parton Distributions and Lattice Calculations (PDFLattice 2019)

25-27 September 2019  
Other Institutes  
America/Detroit timezone

Search... 🔍

## Overview

Timetable

Registration

Participant List

Travel Information

Venue Photos

First Bulletin

Contact: Huey-Wen Lin

✉ [hwlin@pa.msu.edu](mailto:hwlin@pa.msu.edu)

The second workshop on Parton Distributions and Lattice Calculations (PDFLattice2019) will be hosted at the [MSU Kellogg Biological Station](#) (near Kalamazoo MI). Sessions will start at 9am on Wednesday 25 September and will end at 1pm on Friday 27 September 2019.

The workshop continues the series started at the University of Oxford in 2017 and aims at bringing together the global PDF analysis and lattice-QCD communities to explore ways to improve the current understanding of the distributions of partons in the nucleon.

The format of the workshop will be similar to the one adopted in the previous edition, that is few short talks and ample time for discussions will be scheduled. In addition to a follow-up on the topics discussed in Oxford, one full day will be dedicated to talk about issues and opportunities in the determination of transversity. The rest of the workshop will include a brainstorming session on what lattice QCD could do to help determine Generalized Parton Distributions (GPDs), and a session to plan how the output of the workshop will be documented.