Neutrino Nucleon Scattering and Hadronic Tensor

Keh-Fei Liu, Jian Liang, Yi-Bo Yang and Terry Draper

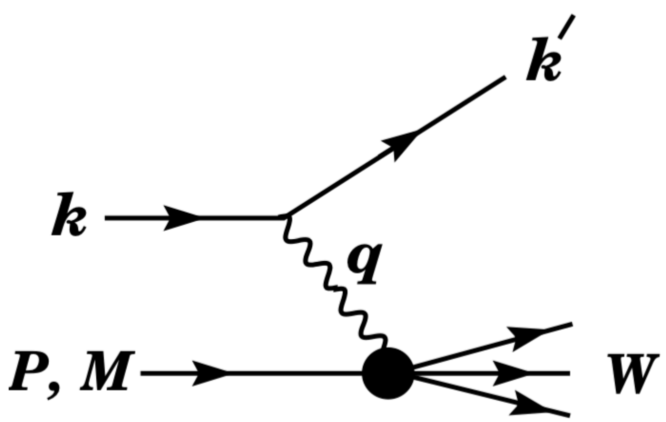
χQCD collaboration

04/27/2019 USQCD all-hands meeting@BNL

Neutrino-nucleus scattering

- To understand more about neutrinos, new long-baseline neutrino experiments are in preparation.

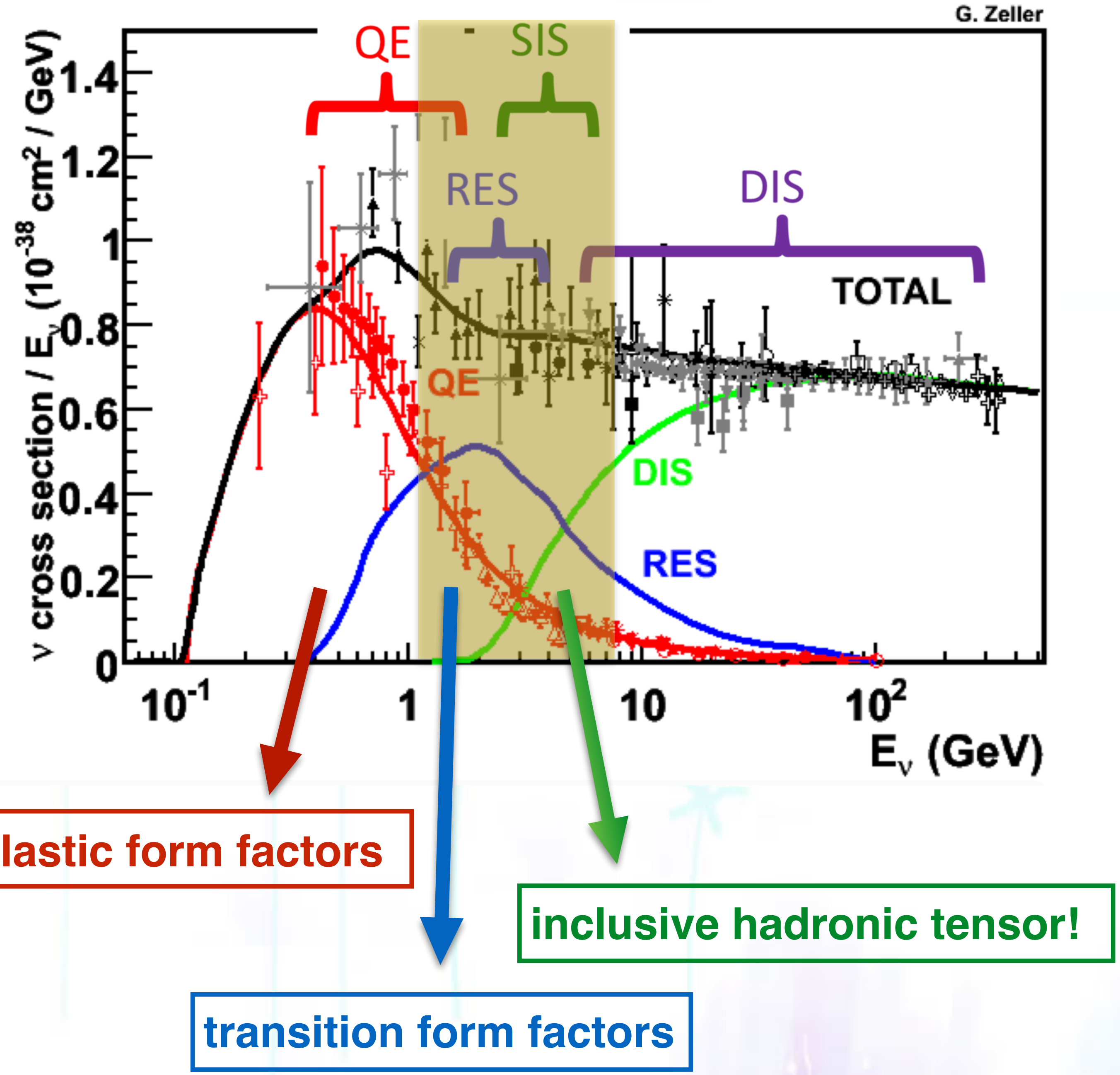
- $\nu A \to \nu N$



- theoretical input about nucleon structure is needed.

- DUNE@LBNF FERMILAB with neutrino energy ~1-~7 GeV, including all QE, RES, SIS and DIS

J.A. Formaggio and G.P. Zeller, RMP84, 1307 (2012); Teppei Katori's talk



No direct calculation on the lattice

$$W_{\mu\nu} = \frac{1}{4\pi}\int d^4z e^{iq\cdot z}\left\langle p,s\left|\left[J_\mu^\dagger(z)J_\nu(0)\right]\right|p,s\right\rangle$$, Im part of the forward Compton amplitude

Lattice QCD: Euclidean field theory using the path-integral formalism

time dependent matrix element can be problematic

Minkowski
$$W_{\mu\nu} = \frac{1}{4\pi}\int d^4z e^{iq\cdot z}\left\langle p,s\left|\left[J_\mu^\dagger(z)J_\nu(0)\right]\right|s,s\right\rangle$$
$$= \frac{1}{2}\sum_n \int \prod_i^n \left[\frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i}\right]\langle p,s|J_\mu^\dagger(0)|n\rangle\langle n|J_\nu(0)|p,s\rangle(2\pi)^3\delta^4(q-p_n+p)$$

Euclidean
$$W'_{\mu\nu} = \frac{1}{4\pi}\sum_n \int dt e^{\left(\nu-(E_n-E_p)\right)t}\int d^3\mathbf{z} e^{i\mathbf{q}\cdot\mathbf{z}}\langle p,s|J_\mu^\dagger(\mathbf{z})|n\rangle\langle n|J_\nu(0)|p,s\rangle$$
$$= \frac{1}{4\pi}\sum_n \boxed{\frac{e^{\left(\nu-(E_n-E_p)\right)T}-1}{\nu-(E_n-E_p)}}\int d^3\mathbf{z} e^{i\mathbf{q}\cdot\mathbf{z}}\langle p,s|J_\mu^\dagger(\mathbf{z})|n\rangle\langle n|J_\nu(0)|p,s\rangle$$

Euclidean hadronic tensor

four-point function with two currents

$$C_4 = \sum_{\boldsymbol{x}_f} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}_f} \sum_{\boldsymbol{x}_2\boldsymbol{x}_1} e^{-i\boldsymbol{q}\cdot(\boldsymbol{x}_2-\boldsymbol{x}_1)} \left\langle \chi_N(\boldsymbol{x}_f, t_f) J_\mu^\dagger(\boldsymbol{x}_2, t_2) J_\nu(\boldsymbol{x}_1, t_1) \bar{\chi}_N(\boldsymbol{0}, t_0) \right\rangle$$

nucleon two-point function

$$C_2 = \sum_{\boldsymbol{x}_f} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}_f} \left\langle \chi_N(\boldsymbol{x}_f, t_f) \bar{\chi}_N(\boldsymbol{0}, t_0) \right\rangle$$

K.F. Liu and S. J. Dong, PRL 72, 1790 (1994)

K.-F. Liu, PRD62, 074501 (2000)

J. Liang et. al., EPJ Web Conf. 175, 14014 (2018)

Euclidean hadronic tensor

$$\tilde{W}_{\mu\nu}(\boldsymbol{p}, \boldsymbol{q}, \tau) = \frac{E_p}{m_N} \frac{\mathrm{Tr}[\Gamma_e C_4]}{\mathrm{Tr}[\Gamma_e C_2]} \to \sum_{\boldsymbol{x}_2\boldsymbol{x}_1} e^{-i\boldsymbol{q}\cdot(\boldsymbol{x}_2-\boldsymbol{x}_1)} \langle p, s \,|\, J_\mu(\boldsymbol{x}_2, t_2) J_\nu(\boldsymbol{x}_1, t_1) \,|\, p, s \rangle$$

$$= \sum_n A_n e^{-\nu_n \tau}, \; \tau \equiv t_2 - t_1, \; \nu_n = E_n - E_p$$

Energy transfer is determined by the energy of the intermediate states.

Back to the Minkowski space

Euclidean hadronic tensor

$$\tilde{W}_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\tau) \sim \sum_n A_n e^{-\nu_n \tau},\ \nu_n \equiv E_n - E_p$$

Formally, an inverse Laplace transform will do

$$W_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\nu) = \frac{1}{i}\int_{c-i\infty}^{c+i\infty} d\tau e^{\nu\tau}\tilde{W}_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\tau)$$

Practically, need to solve the inverse problem of the Laplace transform

$$\tilde{W}_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\tau) = \int d\nu W_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\nu) e^{-\nu\tau}$$

several (O(10)) discrete data points | **continuous function w.r.t. \nu**

lack of information, an ill-posed problem (BG, ME, BR…)

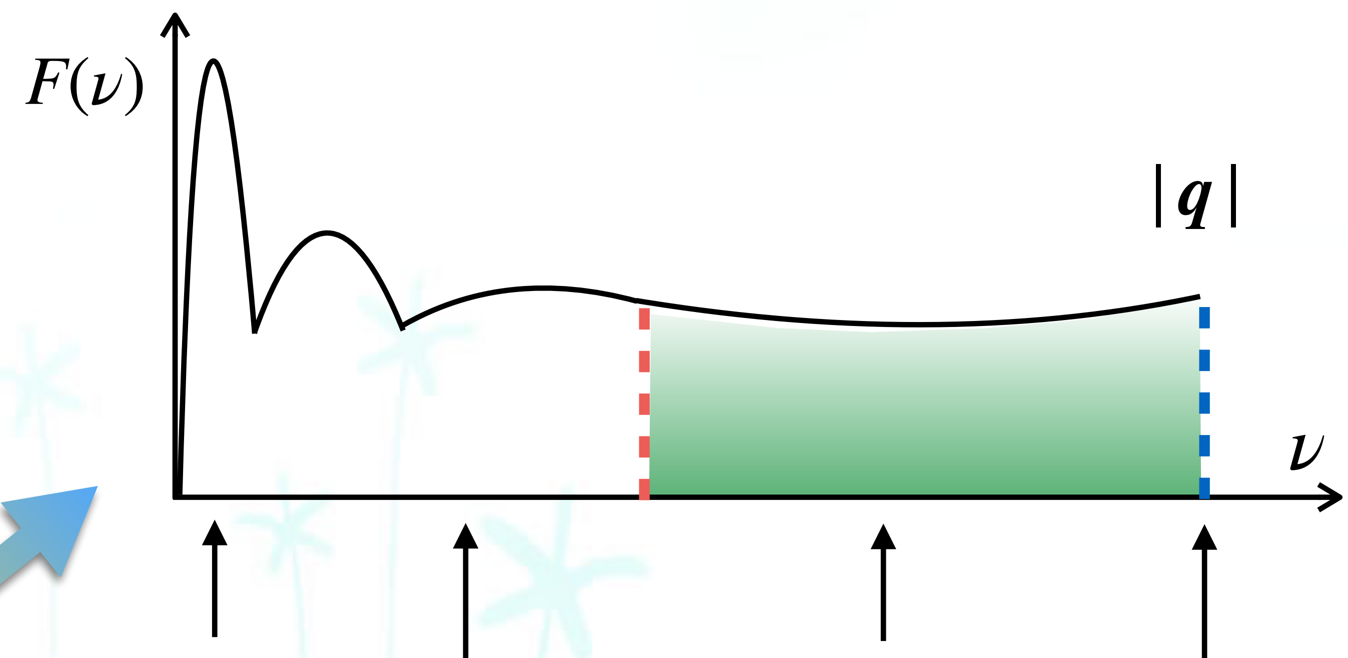
Kinematics

$$C_4 = \sum_{\boldsymbol{x}_f} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}_f} \sum_{\boldsymbol{x}_2 \boldsymbol{x}_1} e^{-i\boldsymbol{q}\cdot(\boldsymbol{x}_2-\boldsymbol{x}_1)} \left\langle \chi_N(\boldsymbol{x}_f, t_f) J^\dagger_\mu(\boldsymbol{x}_2, t_2) J_\nu(\boldsymbol{x}_1, t_1) \bar{\chi}_N(\boldsymbol{0}, t_0) \right\rangle$$

$$\tilde{W}_{\mu\nu}(\boldsymbol{p}, \boldsymbol{q}, \tau) = \sum_n A_n e^{-(E_n - E_p)\tau},\ \nu_n = E_n - E_p$$

$$\nu_{\min} = E_{n=0} - E_p,\ E^2_{n=0} = m_p^2 + (\boldsymbol{p} + \boldsymbol{q})^2$$

$$\nu_{\max} = |\boldsymbol{q}|$$



Elastic

$x = 1$, **form factors**

RES and SIS

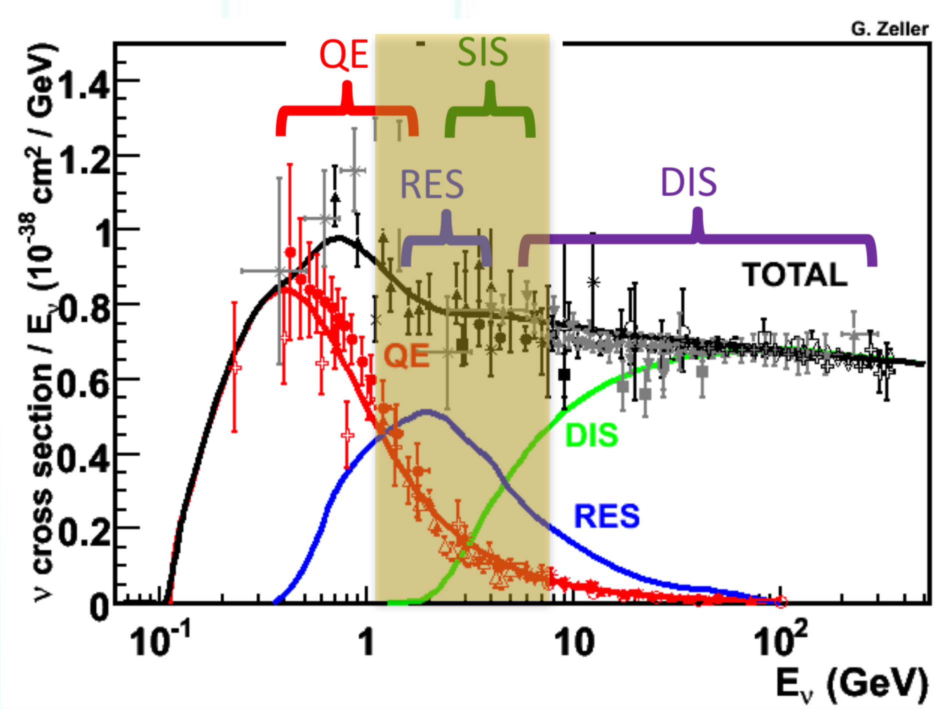
N\pi, \Delta, …, continuous spectrum

DIS region

$$x = \frac{Q^2}{2m\nu}, Q^2 \to \infty, \nu \to \infty$$

$Q^2 = 0$

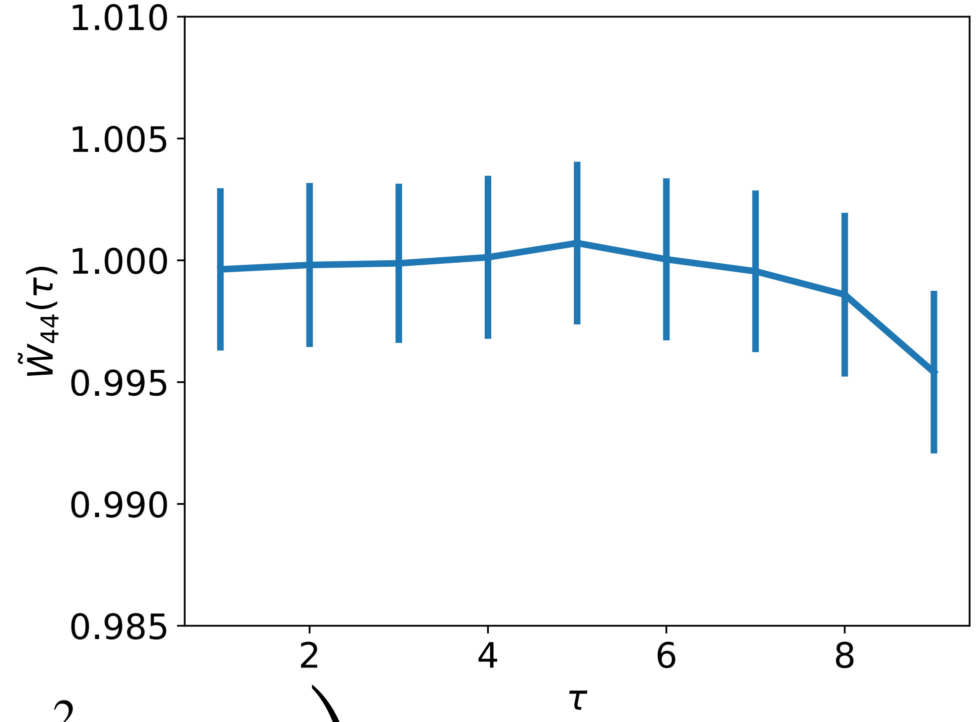
photoproduction



The elastic case

normalized vector current $J_4 = \bar{\psi}\gamma_4\psi$

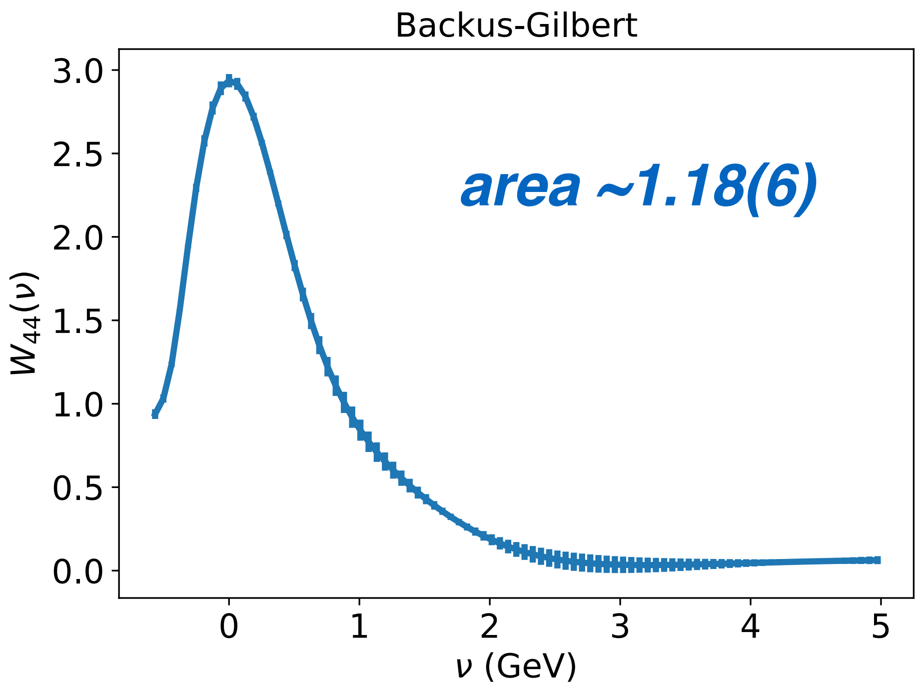
$$\tilde{W}_{44}(\boldsymbol{p}=0,\, \boldsymbol{q}=0,\, \tau) \overset{\tau\to\infty}{=} |\langle N|J_4|N\rangle|^2 e^{-(M_p - M_p)\tau}$$
$$= F_1^2(q^2=0) = g_V^2 = 1$$

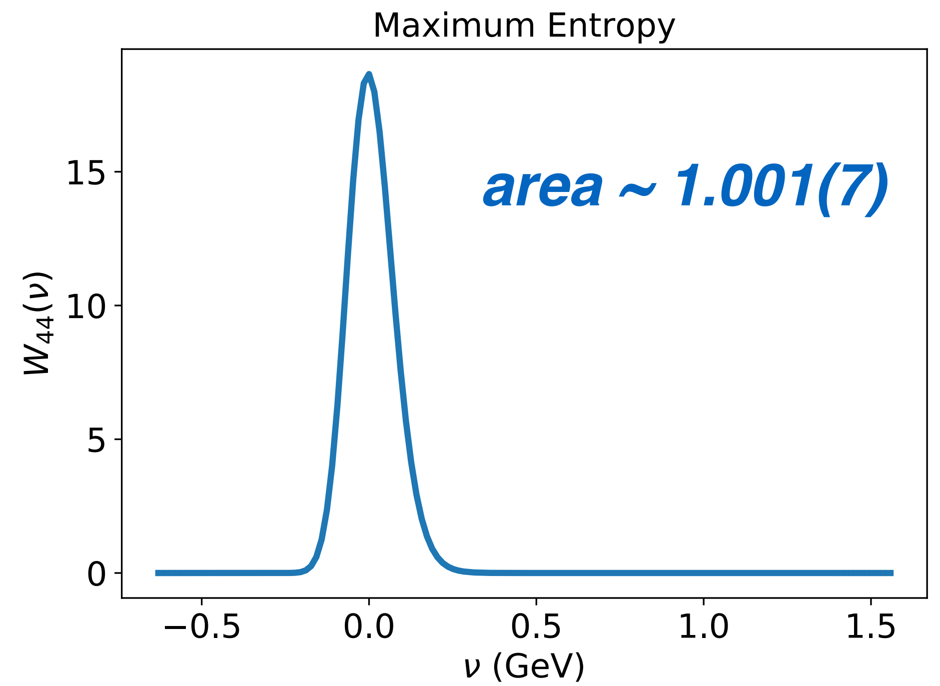


inverse $\tilde{W}_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\tau) = \int d\nu W_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\nu) e^{-\nu\tau}$

$$W_{44}(q^2,\, \nu) = \delta(q^2 + 2m_N\nu)\frac{2m_N}{1 - q^2/4{m_N}^2}\left(G_E^2(q^2) - \frac{q^2}{4M_N^2}G_M^2(q^2)\right)$$

$$\overset{q^2=0}{=} \delta\nu G_E^2(q^2=0) = \delta\nu g_V^2 = \delta\nu$$ **delta function at zero**



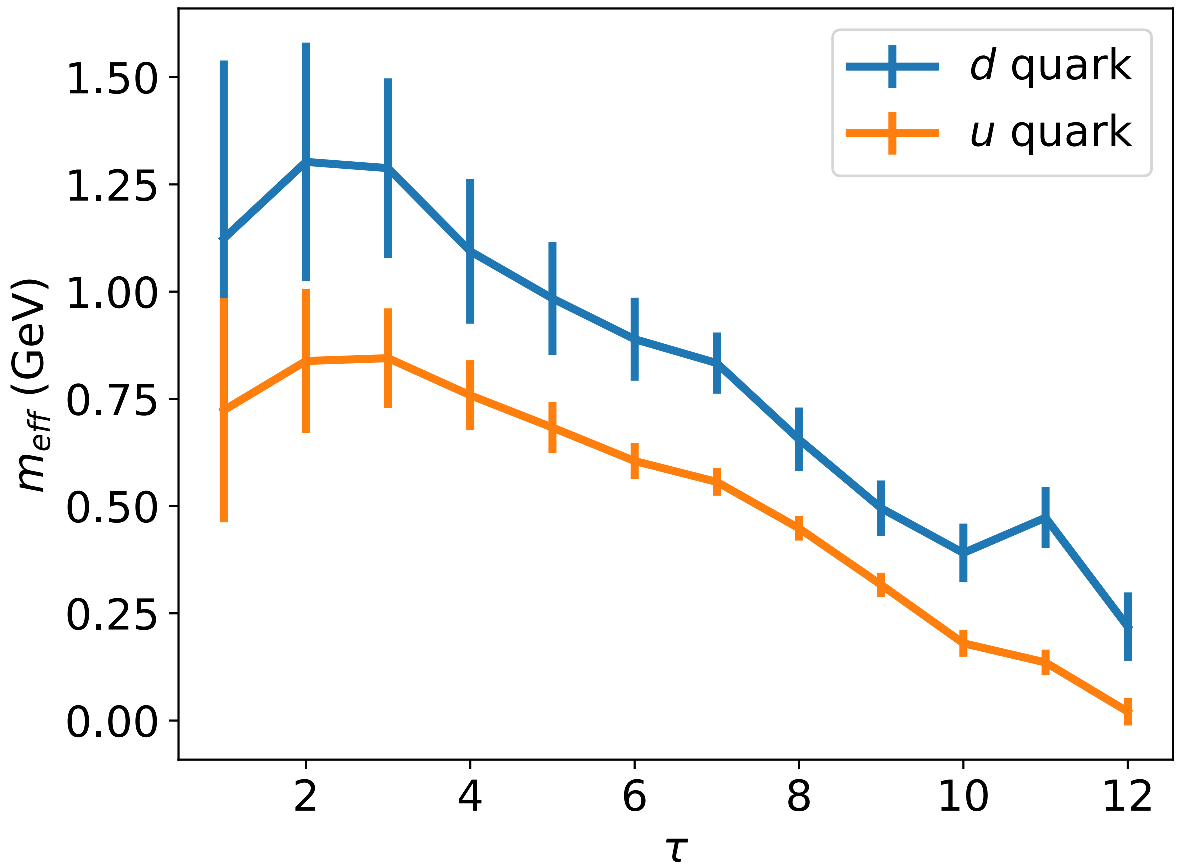


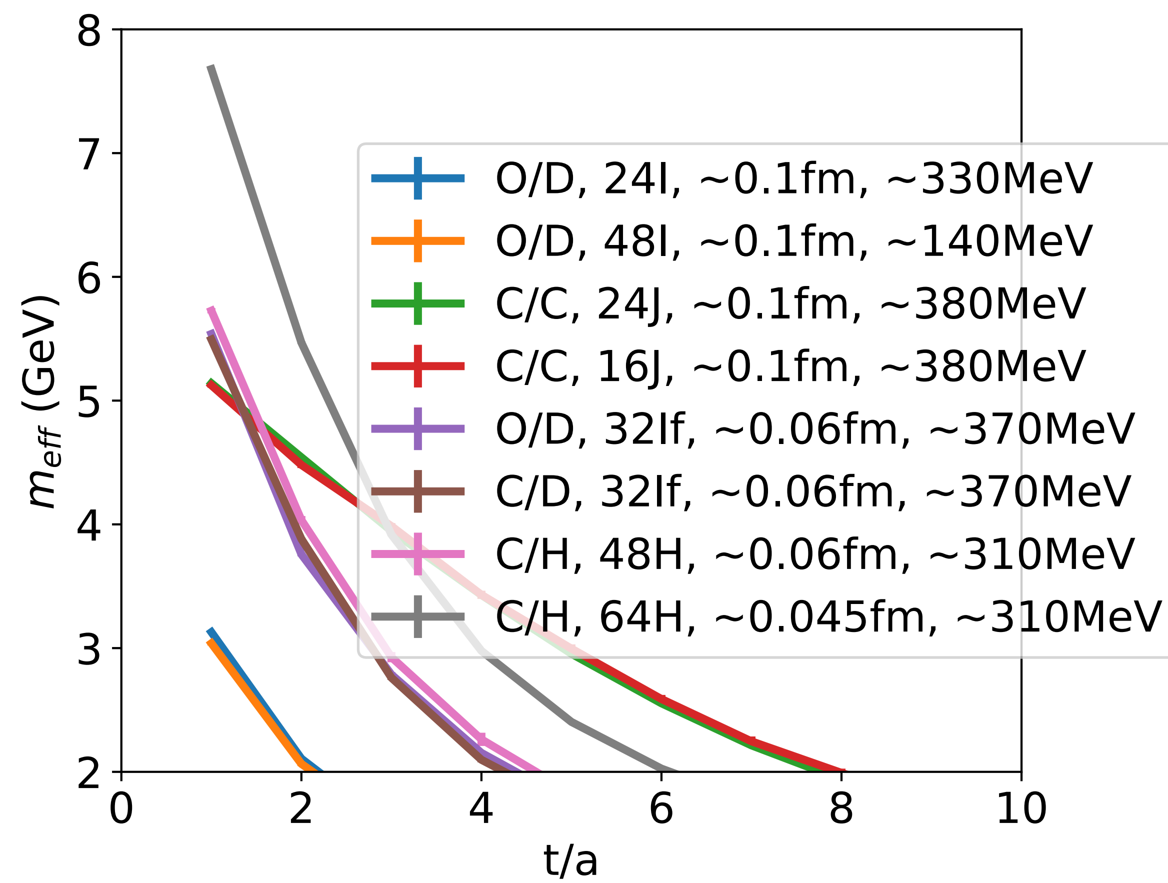
note, different x scale

Case with large momentum transfers

$$\tilde{W}_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\tau) = \sum_n A_n e^{-\nu_n \tau} \quad E_p \sim 2.15 \text{ GeV} \quad |\boldsymbol{q}| \sim 3.57 \text{ GeV} \quad a_s \sim 0.12 \text{ fm}, \quad \xi = 3.5$$

check the effective energy of the Euclidean hadronic tensor $\nu_{\text{eff}} = \log[\tilde{W}(\tau)/\tilde{W}(\tau+1)]$





$\nu_{\max} \sim E_n - E_p \sim 1 \text{ GeV} \quad E_n \sim 3.2 \text{ GeV}$ NOT large enough energy transfer!

lattice artifacts: finite volume? finite lattice spacing?! unphysical pion mass?

Summary and outlook

- Calculating the hadronic tensor on the lattice would be helpful to the neutrino experiments.

- This might be the only lattice approach that can have inclusive results in all the QE, RES and SIS regions.

- We can have reasonable results for the elastic contributions.

- We find that the lattice spacing plays an important role to reach higher excited states (larger energy transfers).

- We are working on lattices with smaller lattice spacings (~0.045) to have better results.

- More applications. E.g., parton physics if we can have large enough momentum and energy transfers.

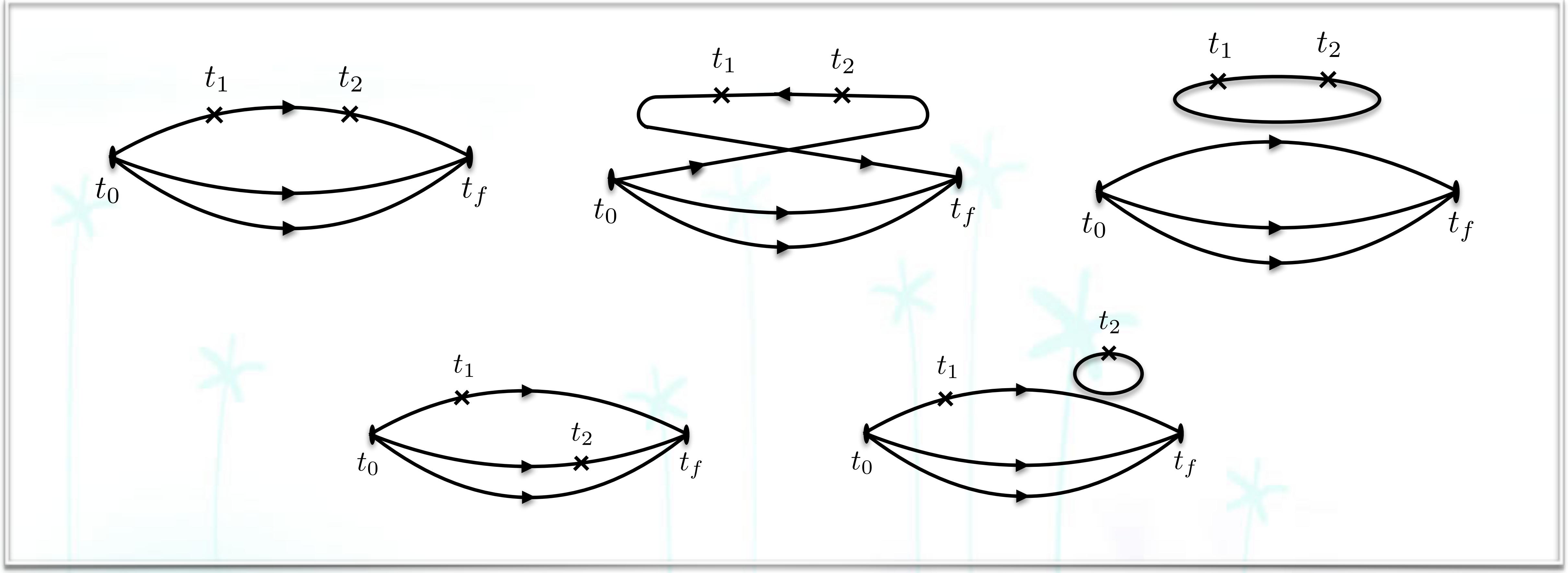
Thank you for your attention!

backup slides

Contractions

$$C_4 = \sum_{\boldsymbol{x_f}} e^{-i\boldsymbol{p}\cdot\boldsymbol{x_f}} \sum_{\boldsymbol{x_2}\boldsymbol{x_1}} e^{-i\boldsymbol{q}\cdot(\boldsymbol{x_2}-\boldsymbol{x_1})} \left\langle \chi_N(\boldsymbol{x_f}, t_f) J_\mu(\boldsymbol{x_2}, t_2) J_\nu(\boldsymbol{x_1}, t_1) \bar{\chi}_N(\boldsymbol{0}, t_0) \right\rangle$$

$$\chi_N = \left[u_1^T C\gamma_5 d\right] u_2$$



More contractions if we consider different types of the two currents: vector, axial vector, neutral or charged, various quark flavors …

No disconnected insertions are considered in the current plan.

The latter two are suppressed when the momentum and energy transfers are large.

Tests on two-point functions

$$C_2(\tau) = e^{-m_1\tau} + e^{-m_2\tau} + e^{-m_3\tau}$$

$$C_2(\tau) = \int d\omega \rho(\omega) e^{-\omega\tau}$$

- **Backus-Gilbert (BG)**

  G. Backus and F. Gilbert, Geophysical Journal International 16, 169 (1968)

- **Maximum Entropy (ME)**

  E Rietsch et. al., JOURNAL OF GEOPHYSICS, 42:489 (1977)

  M. Asakawa et. al., Prog. Part. Nucl. Phys. 46, 459 (2001)

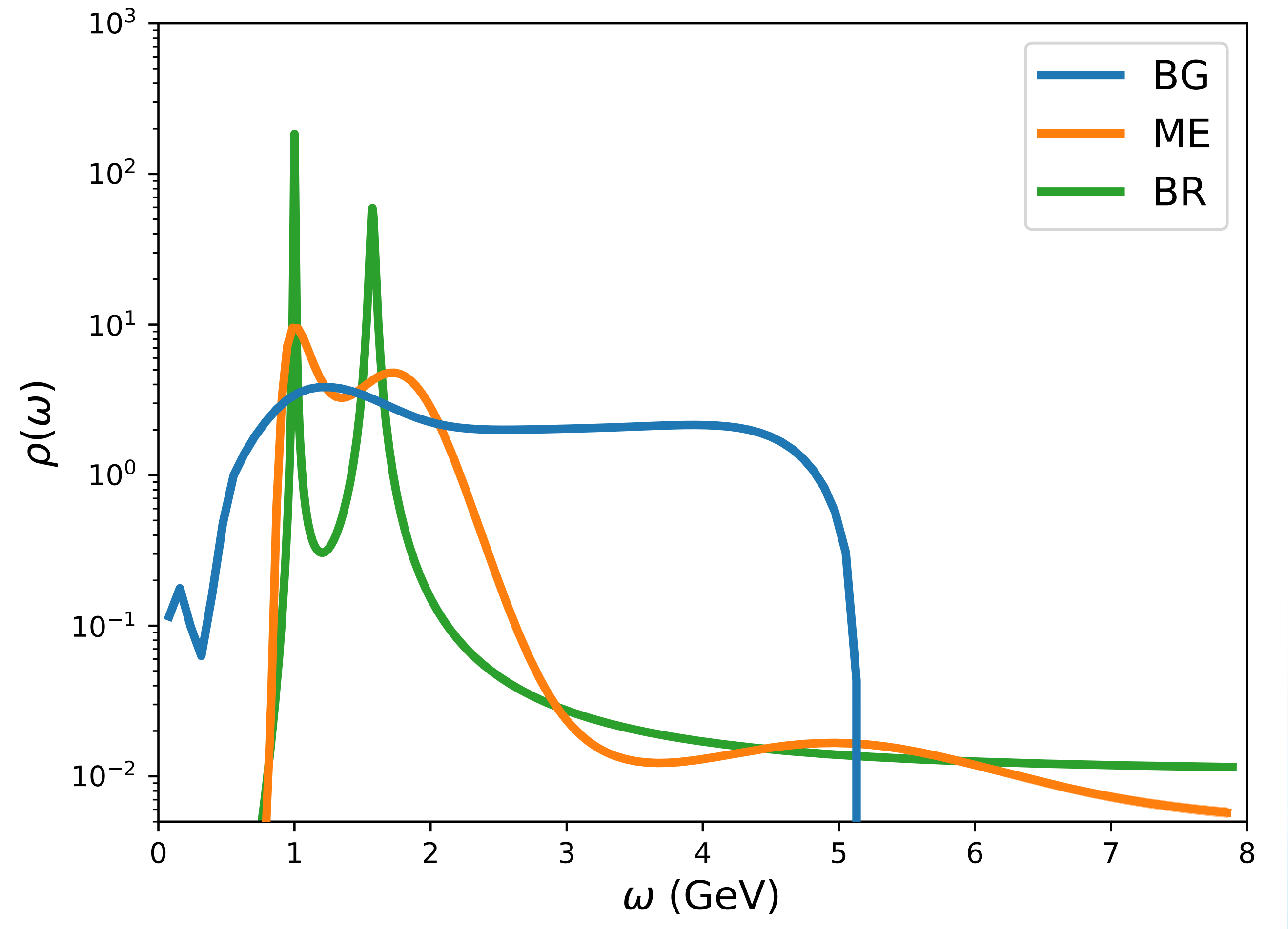
- **Bayesian Reconstruction (BR)**

  Y. Burnier and A. Rothkopf, PRL 111, 182003 (2013)

mock two-point function data: 3 single exponentials with mass 1.0, 1.5 and 1.8 GeV respectively, a~0.1 fm, Nt=20, S/N=100

bad resolution of BG

BR is shaper and more stable than ME



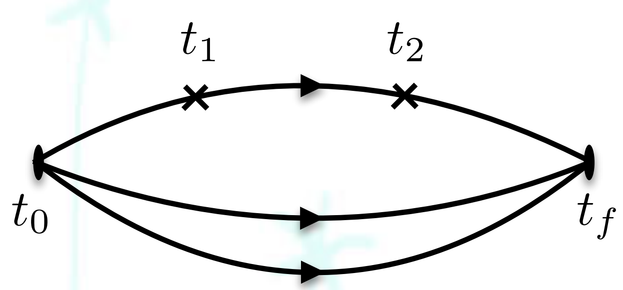
Lattice setups

clover anisotropic lattice, $24^3 \times 128,\ a_t \sim 0.035\ \text{fm},\ m_\pi \sim 380\ \text{MeV},\ \frac{2\pi}{L} \sim 0.42\ \text{GeV}$

H.-W. Lin et al., PRD 79, 034502 (2009)

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x, Q^2) + \frac{\hat{P}_\mu \hat{P}_\nu}{P \cdot q} F_2(x, Q^2)$$

$\mu = \nu = 1$ and $p_1 = q_1 = 0$ $\quad W_{11}(\nu) = F_1(x, Q^2)$



two sequential-sources for each 4-point function
554 configurations, 16 source positions

The *x*-range can be reached on this lattice is roughy [0.05, 0.3] by combining different kinematic setups.

This calculation:

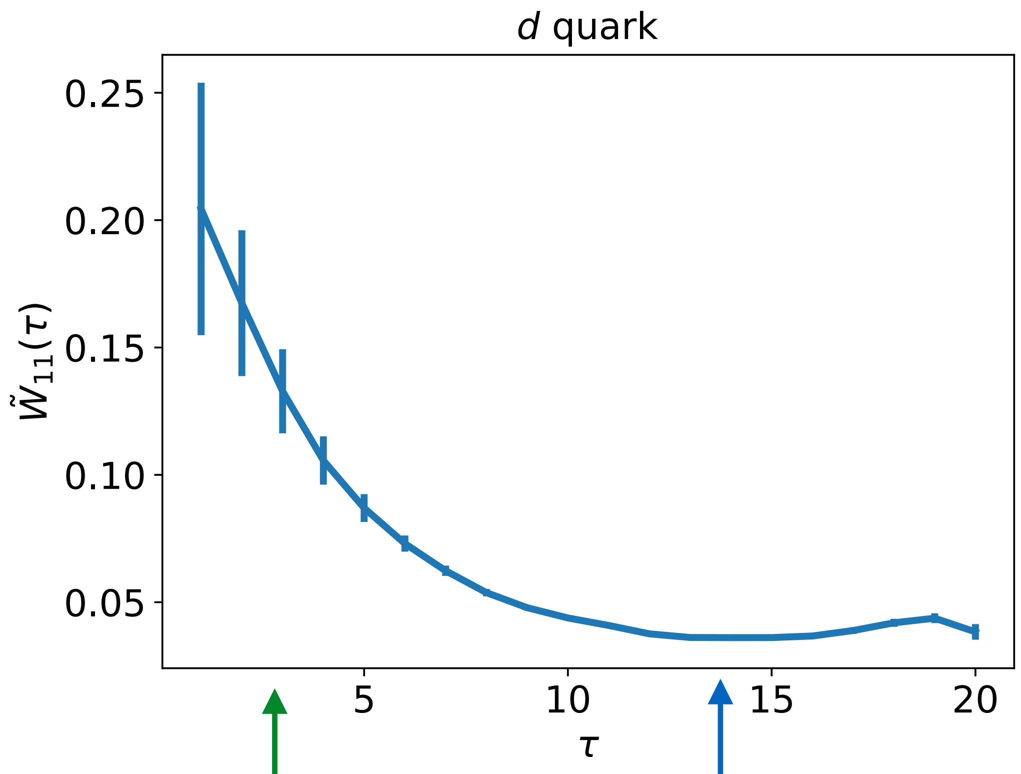
$\boldsymbol{p}$	$\boldsymbol{q}$	E_p	$E_{n=0}$	$\lvert\boldsymbol{q}\rvert$	ν	Q^2	x
(0,3,3)	(0,-6,-6)	2.15	2.15	3.57	[2.96, 3.28]	[4, 2]	[0.16, 0.07]

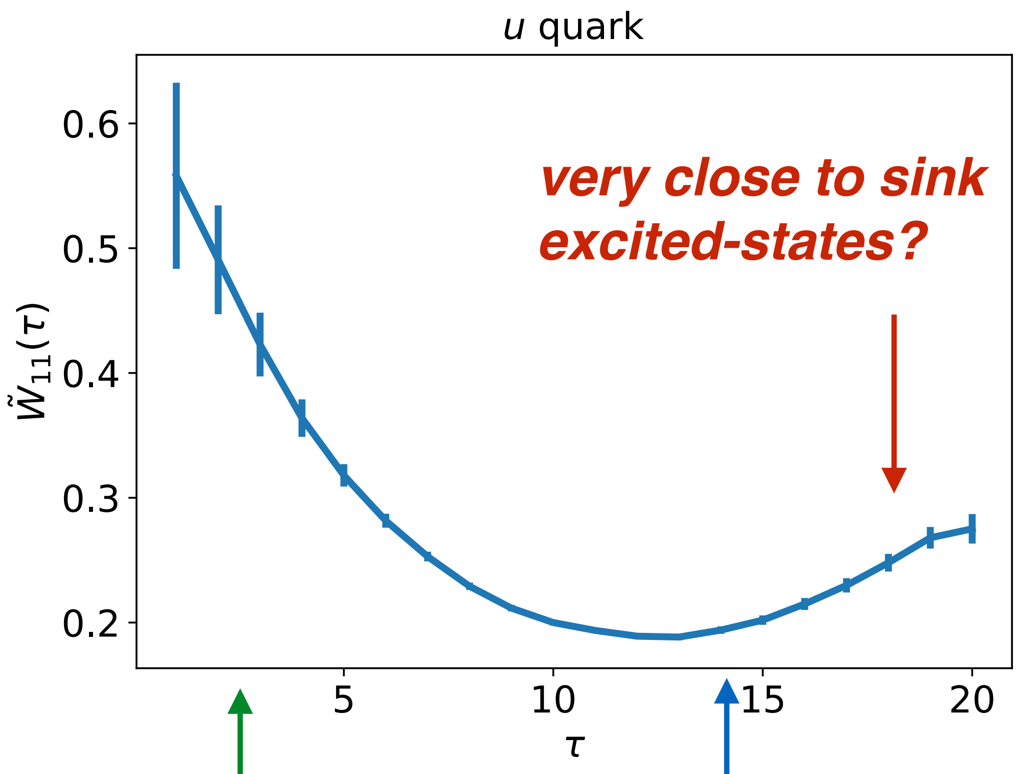
Large momentum transfer

$\boldsymbol{p}$	$\boldsymbol{q}$	E_p	$E_{n=0}$	$\lvert\boldsymbol{q}\rvert$	ν	Q^2	x
(0,3,3)	(0,-6,-6)	2.15	2.15	3.57	[2.96, 3.28]	[4, 2]	[0.16, 0.07]

$\mu = \nu = 1$ and $p_1 = q_1 = 0$ $\quad W_{11}(\nu) = F_1(x, Q^2)$

$$\tilde{W}_{\mu\nu}(\boldsymbol{p}, \boldsymbol{q}, \tau) = \sum_n A_n e^{-(E_n - E_p)\tau} \qquad \boldsymbol{p} + \boldsymbol{q} = -\boldsymbol{p} \qquad E_0 = (m_N^2 + \lvert\boldsymbol{p} + \boldsymbol{q}\rvert^2) = E_p$$





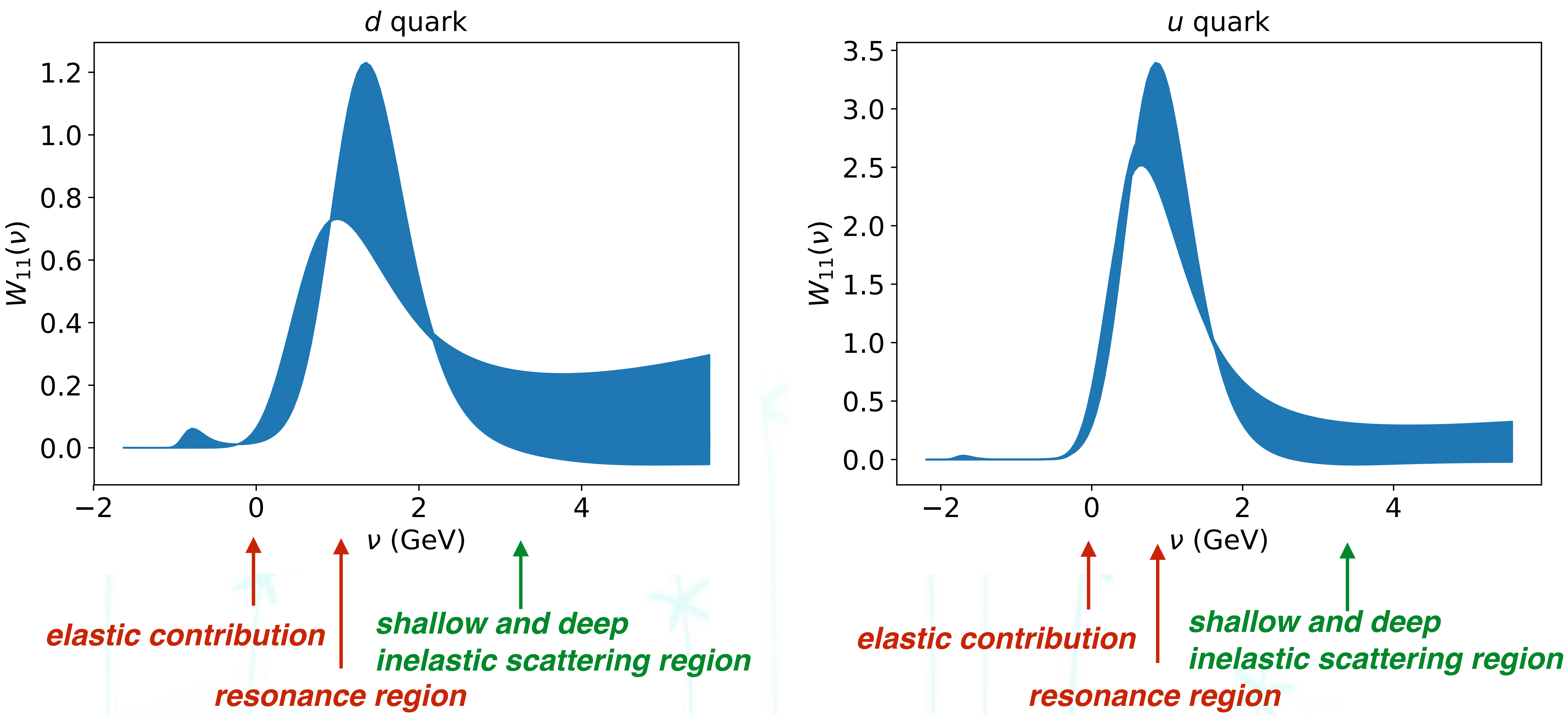
elastic contribution (flat)

higher intermediate-states contribution (exponentially decay)

elastic contribution (flat)

higher intermediate-states contribution (exponentially decay)

Minkowski hadronic tensor (after ME)



- ✦ **Elastic contribution is suppressed by the large momentum transfer.** $G^2(0) \propto \dfrac{1}{\left(1+\frac{Q_{\rm el}^2}{\Lambda^2}\right)^4}$

  $Q^2 \sim 13\ \mathrm{GeV}^2,\ G^2(0) \sim 10^{-5}$
- ✦ **RES contribution is large and relatively stable.**
- ✦ **Large error in the SIS and DIS region, no enough constraint from the data**