Ioffe time Distributions

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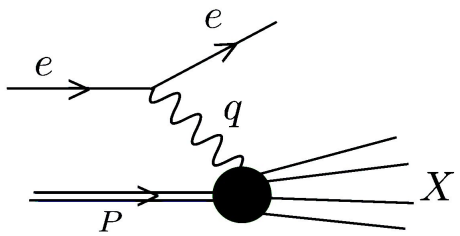
Outline

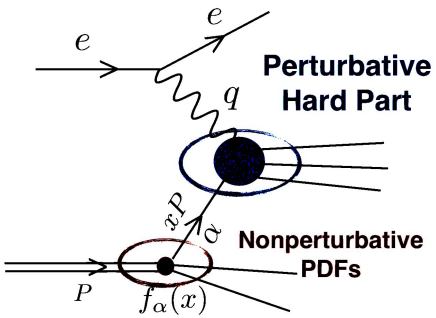
- What are parton distributions?
- Lattice Cross Sections
  - Matrix elements with space like separations which can be factorized to PDFs
  - pseudo-PDF
  - Vector-Axial current correlations
- Plans for the future
  - Combined lattice data fits
  - Distillation

Parton Distribution Functions

- Cross section factorization
  - DIS example

$$d\sigma_h = f_{h/q} \otimes d\sigma_q$$





- Light cone matrix element definition

$$f_{h/q}(x,\mu^2) = \int d(\xi^- p^+) e^{\pm i x(\xi^- p^+)} \qquad p = (p^+, \frac{m^2}{2p^+}, 0_T)$$

$$\langle h(p)|\bar{\psi}_q(0,\xi^-,0_T)\gamma^+ W((0,\xi^-,0_T);0)\psi_q(0)|h(p)\rangle_{\mu^2}$$

- Contains Non-perturbative structure of hadrons
  - TMDs, GPDs, and Wigner distributions
  - One of leading theory errors in many hadronic cross sections

Incomplete list of PDF ideas for the lattice

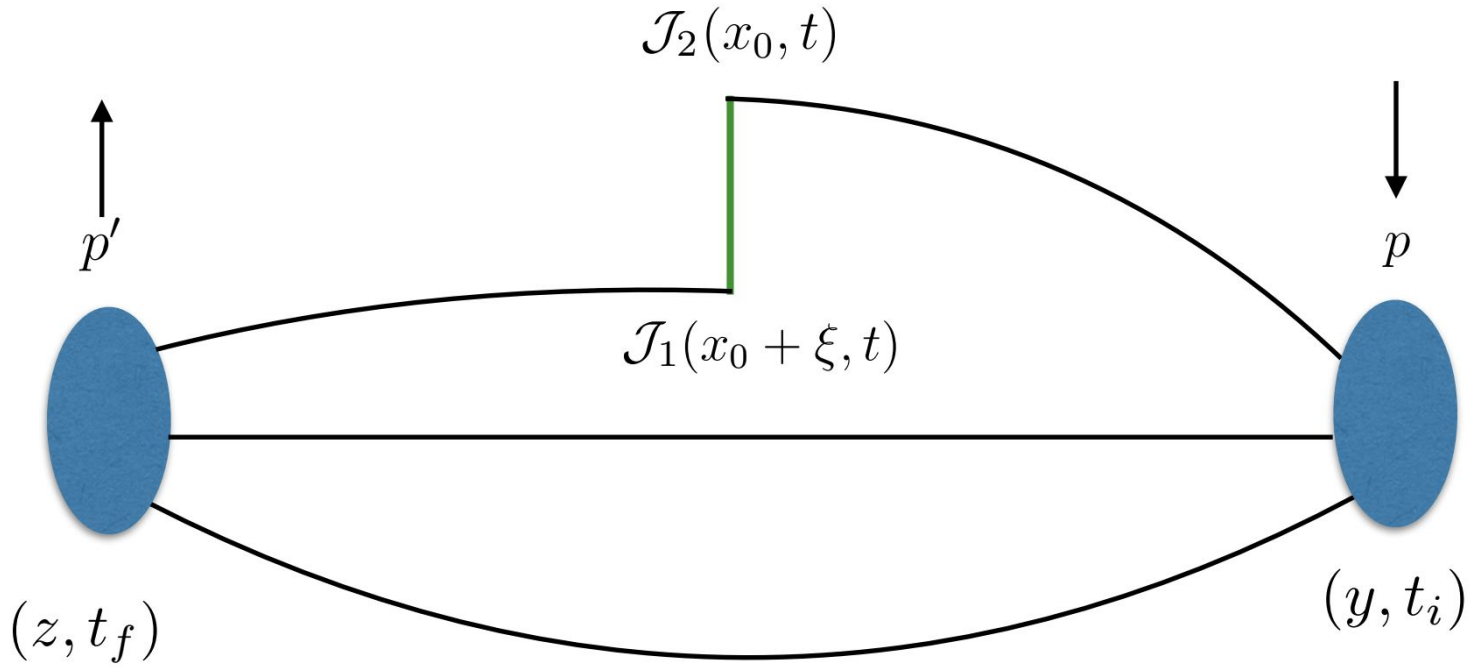
- PDFs cannot be directly calculated on the lattice
  - Light cone and Wick Rotations
  - Finite momentum
- What to do instead
  - “Twist-2” Local Operators
    - Restricted to low moments by reduced rotational symmetry
  - Two current Correlations
    - “Light-like” separated Hadronic Tensor K-F Liu et al Phys. Rev. Lett. 72 1790 (1994) , Phys. Rev. D62 (2000) 074501
    - Good lattice cross sections Y.-Q. Ma J.-W. Qiu (2014) 1404.6860, Y.-Q. Ma, J.-W. Qiu (2017) 1709.03018, Sufian et.al. (2019) 1901.03921
  - Wilson Line Quark Bilinears
    - Quasi PDF X. Ji, Phys.Rev.Lett. 110, (2013), C Alexandrou et.al. (2018) 1803.02685, J.-W. Chen et.al. (2018) 1803.04393
    - Pseudo PDF

A. Radyushkin Phys.Lett. B767 (2017), K. Orginos, A Radyushkin, JK, S Zafeiropoulos (2017) 1706.05373

Good Lattice Cross Sections

- Good Experimental Cross Section - An experiment whose results, Form Factors or asymmetries, is sensitive to a particular PDF.
  - DIS, SIDIS, DY, ….
- Good Lattice Cross Section - A lattice QCD calculable matrix element whose result is sensitive to a particular PDF (Matrix element and not actually a cross section)
  - Vector-vector currents, Axial-vector currents, Quark blinears with fields separated by Wilson line, ….

Good Lattice Cross Sections



Factorizable Matrix Elements

- Factorization of a lattice calculable matrix elements
  - Certain Matrix elements can be separated into the same components as experimental cross sections
  - Examples: Two currents with space-like separation, Two quark fields separated by space-like separation

$$\mathcal{M}_i(\nu, z^2) = \int_{-1}^{1} dx \sum_j K_j^i(x\nu, z^2\mu^2) q_j(x, \mu^2)$$

$$\nu = p \cdot z$$

Pseudo Ioffe Time Distributions

- A matrix element of interest

$$M^{\alpha}(z,p) = \langle h(p)|\bar{\psi}(z)\gamma^{\alpha}W(z;0)\psi(0)|\, h(p)\rangle$$

- Lorentz decomposition
  - Physicists love to use of symmetries
  - Choice of p, z, and α can remove higher twist term

$$M^{\alpha}(z,p) = 2\,p^{\alpha}\,M_p(\nu,z^2) + z^{\alpha}M_z(\nu,z^2)$$

- Factorizable Relation to ITDF
  - Perturbatively calculable Wilson coefficients for each parton

A. Radyushkin (2017) 1710.08813
J.-H. Zhang (2018) 1801.03023
T. Izubuchi (2018) 1801.03917

$$I(\nu,\mu^2) = \mathcal{M}(\nu,z^2) + \frac{C_F\alpha_S}{2\pi}\int_0^1 du\left[B(u)\left(\log\left(z^2\mu^2\frac{e^{2\gamma_E}}{4}\right)+1\right) + \left(\frac{4\log(1-u)}{1-u} - 2(1-u)\right)_+\right]\mathcal{M}(u\nu,z^2)$$

Axial-Vector Current Correlations

- A matrix element of interest

$$\mathcal{M}^{\alpha\beta}(\nu, z^2) = \frac{1}{2}\langle p|J_V^\alpha(z)J_A^\beta(0) + J_A^\alpha(z)J_V^\beta(0)|p\rangle$$

- Lorentz decomposition
  - Choice of $\alpha = x$ and $\beta = y$ isolates the first Lorentz structure

$$\mathcal{M}^{\alpha\beta}(\nu, z^2) = \epsilon^{\alpha\beta\gamma\delta}p_\gamma z_\delta T_1(\nu, z^2) + (p_\alpha z_\beta - p_\beta z_\alpha)T_2(\nu, z^2)$$

- Factorizable Relation to PDF
  - Perturbatively calculable Wilson coefficients for each parton
  - Currently only tree-level matching

$$T_1(\nu, z^2) = \int_0^1 dx \frac{cos(\nu x)}{\pi^2} q_v(x, 1/z^2)$$

Extracting the Valence Quark distribution

- First attempt to avoid ill posed inverse Fourier transform

$$\mathcal{M}_i(\nu, z^2) = \int_{-1}^{1} dx \sum_j K_j^i(x, \nu, z^2\mu^2) q_j(x, \mu^2)$$

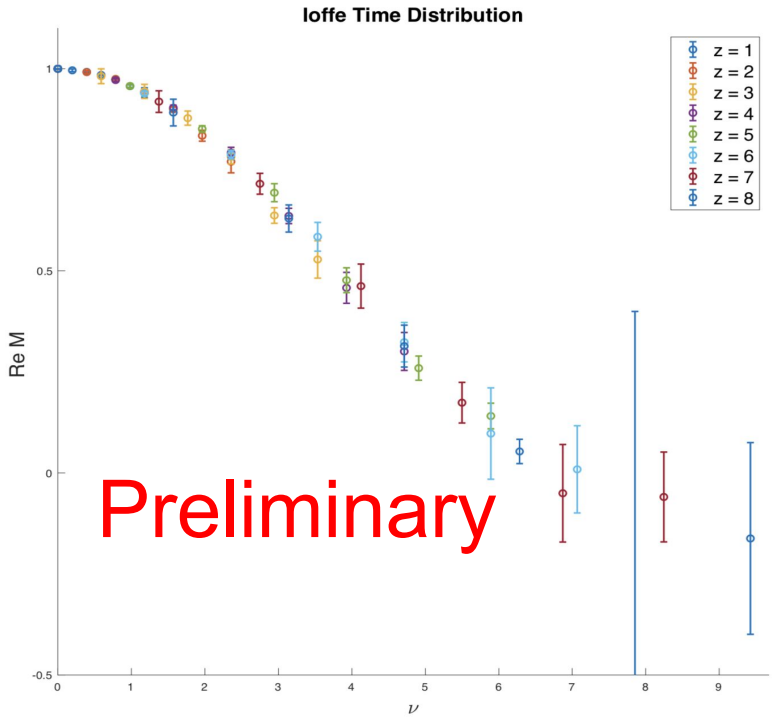
- A general model PDF used by JAM collaboration for fitting

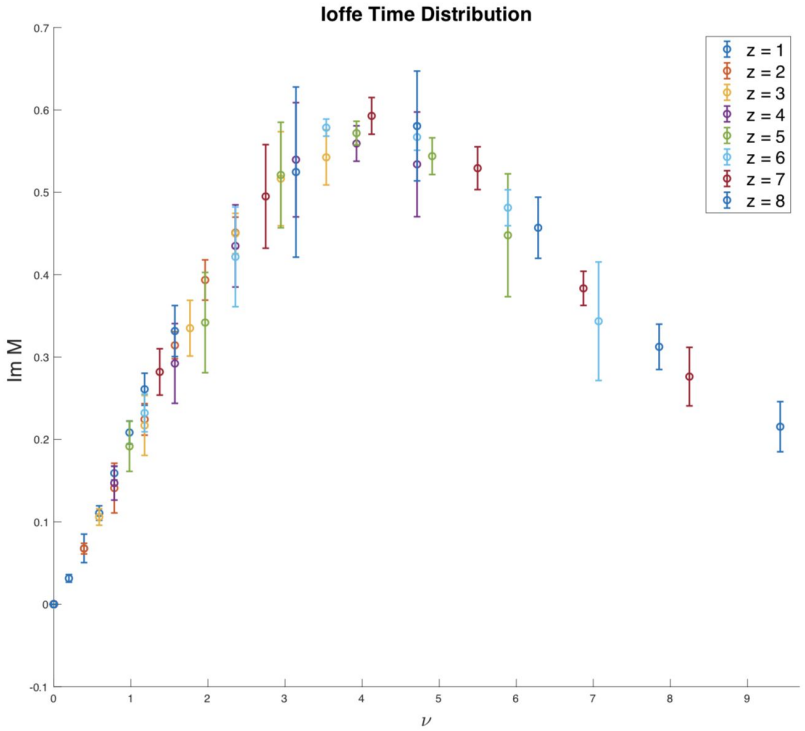
$$f_{abcd}(x) = N_{abcd}\ x^a(1-x)^b(1 + c\sqrt{x} + d\,x\,)$$

$$N_{abcd} = B(a+1, b+1) + cB(a+\frac{3}{2}, b+1) + dB(a+2, b+1)$$

- Expected lowest order behaviors for nucleon
  - Regge $a = -\frac{1}{2}$
  - Quark counting $b = 3$
  - Corrections

Nucleon pseudo-ITD results
a=0.127fm m_pi=440MeV





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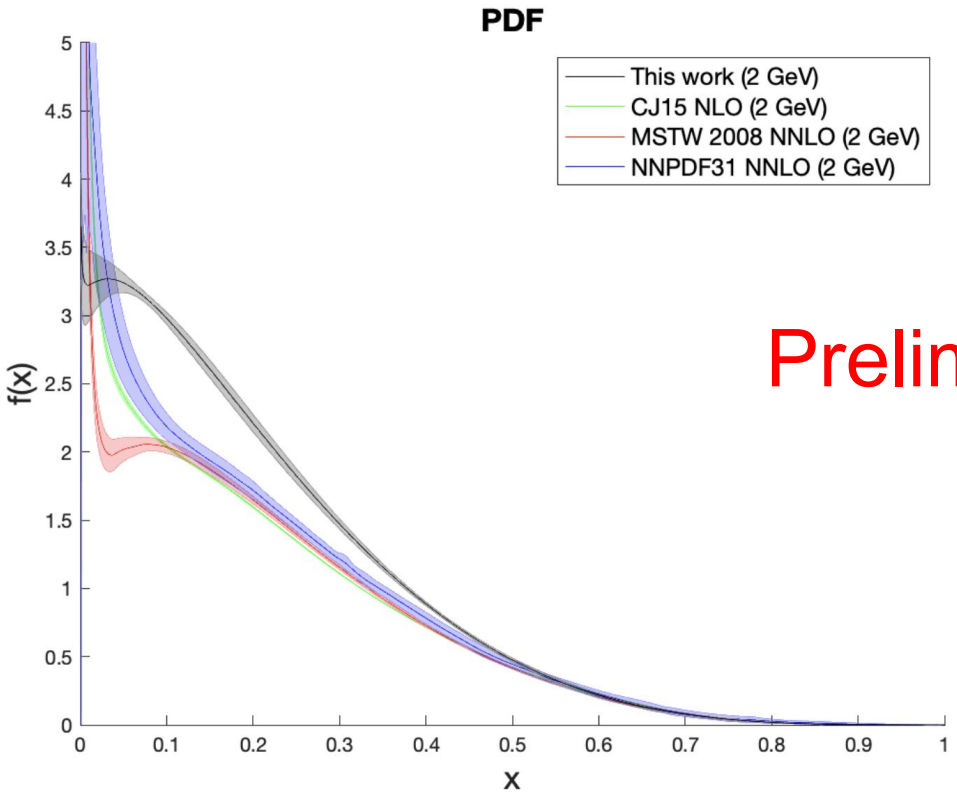
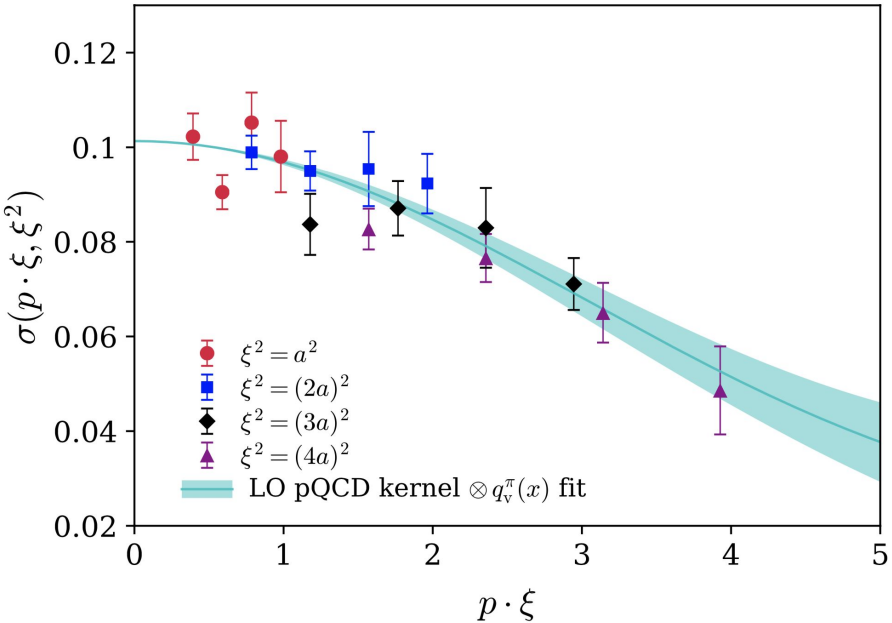
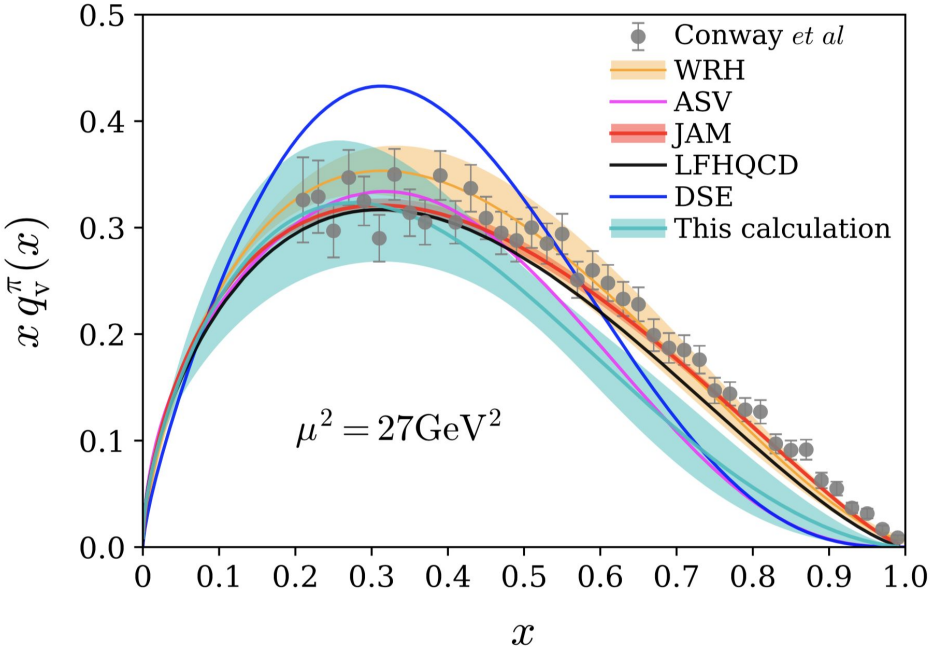


Preliminary

Pion Axial-Vector Current Correlations



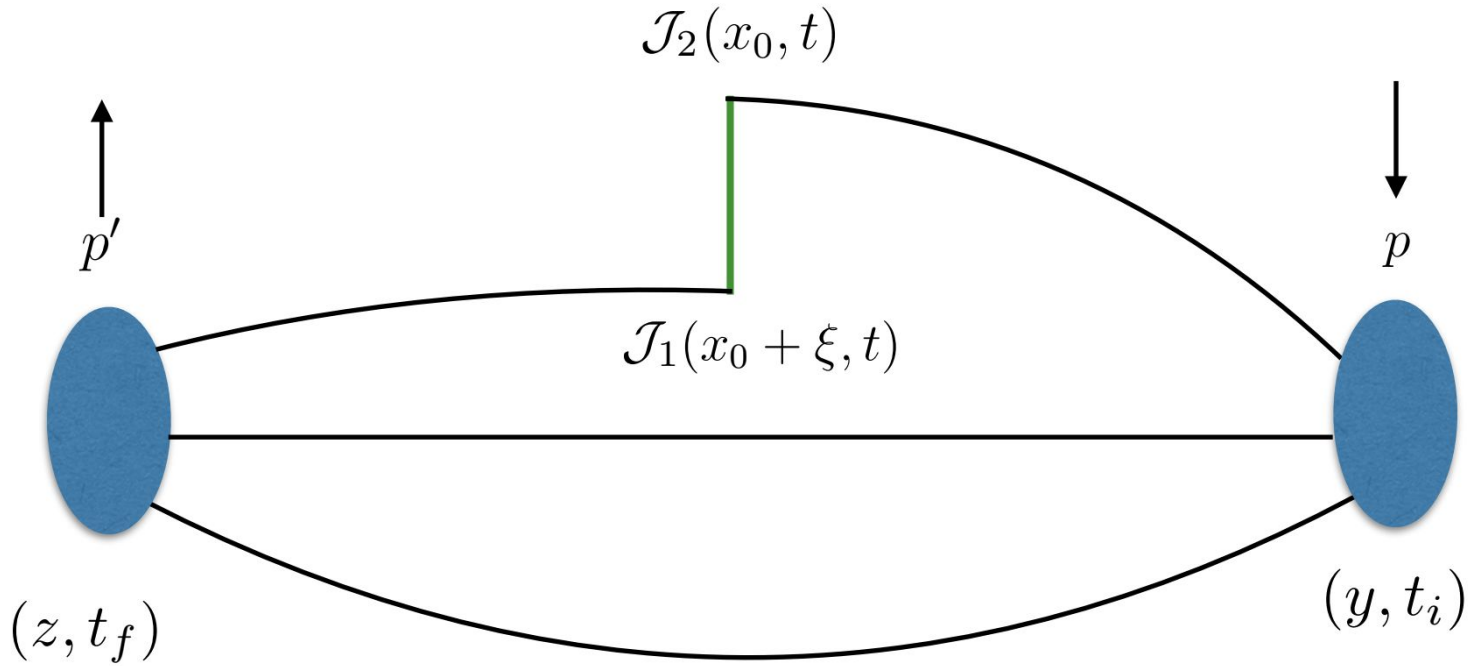


R. Sufian, JK, C. Egerer, K. Orginos, JW. Qiu, and D. Richards (2019) 1901.03921

Proposal for Future Work

- Continue calculation on finer ensemble, a= 0.091fm, with near physical pion mass, 170 MeV
- Calculate more Gamma structures in Wilson line Operator and more current combinations.
- Improve calculation with Distillation and GEVP
- Calculate Distribution Amplitudes
- Perform combined analysis with several LCSs and possibly experimental data

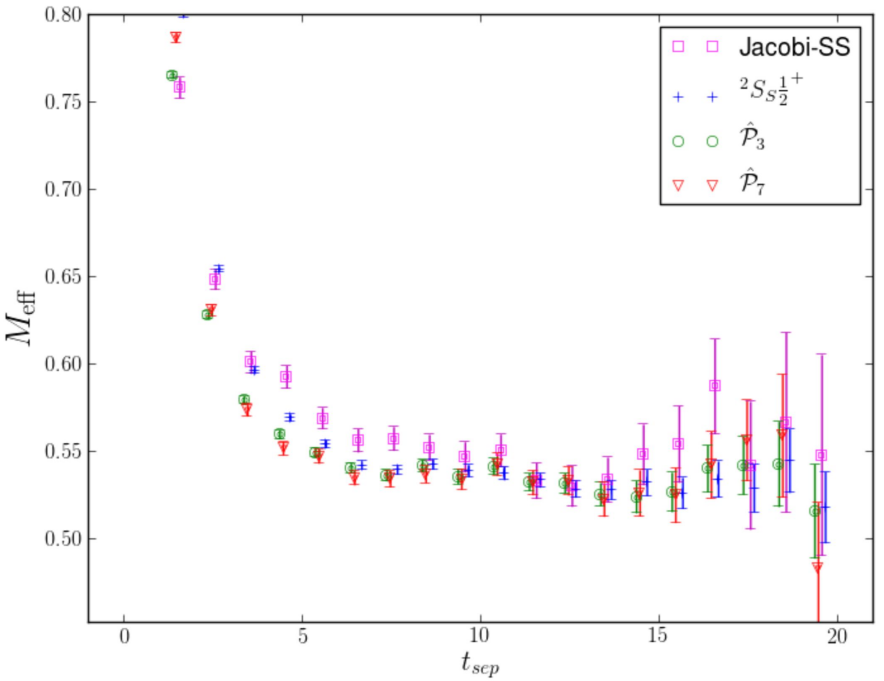
Good Lattice Cross Sections



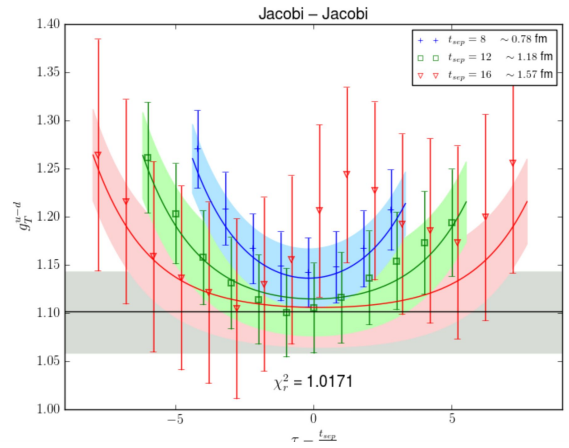
Desire for Distillation

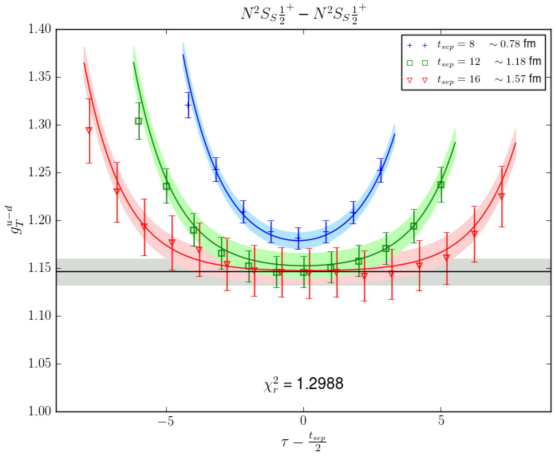
- Improved signal-to-noise and excited state overlap from GEVP
- Generalized perambulators can be calculated for many operators can be constructed without additional propagators
- Reuse generalized perambulators for different hadron-hadron and hadron-vacuum matrix elements
- Only way we came up with for 2 currents in a nucleon

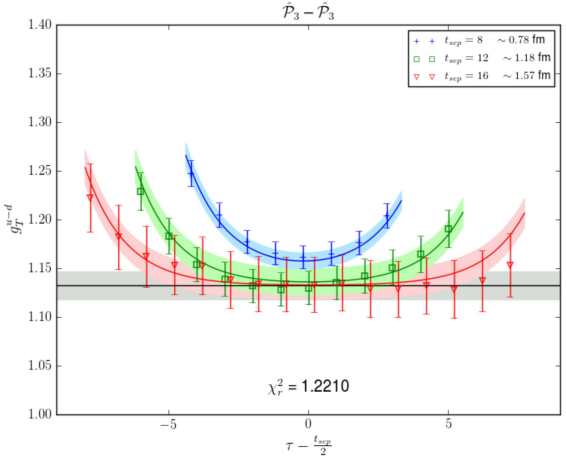
C. Egerer, D. Richards, F. Winter (2018) 1810.09991

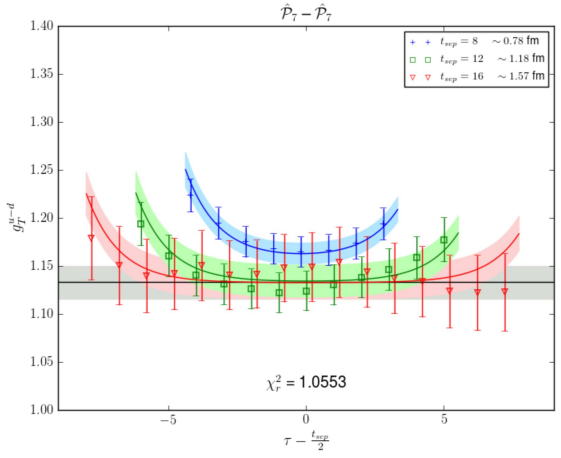


Matrix elements from Distillation and GEVP









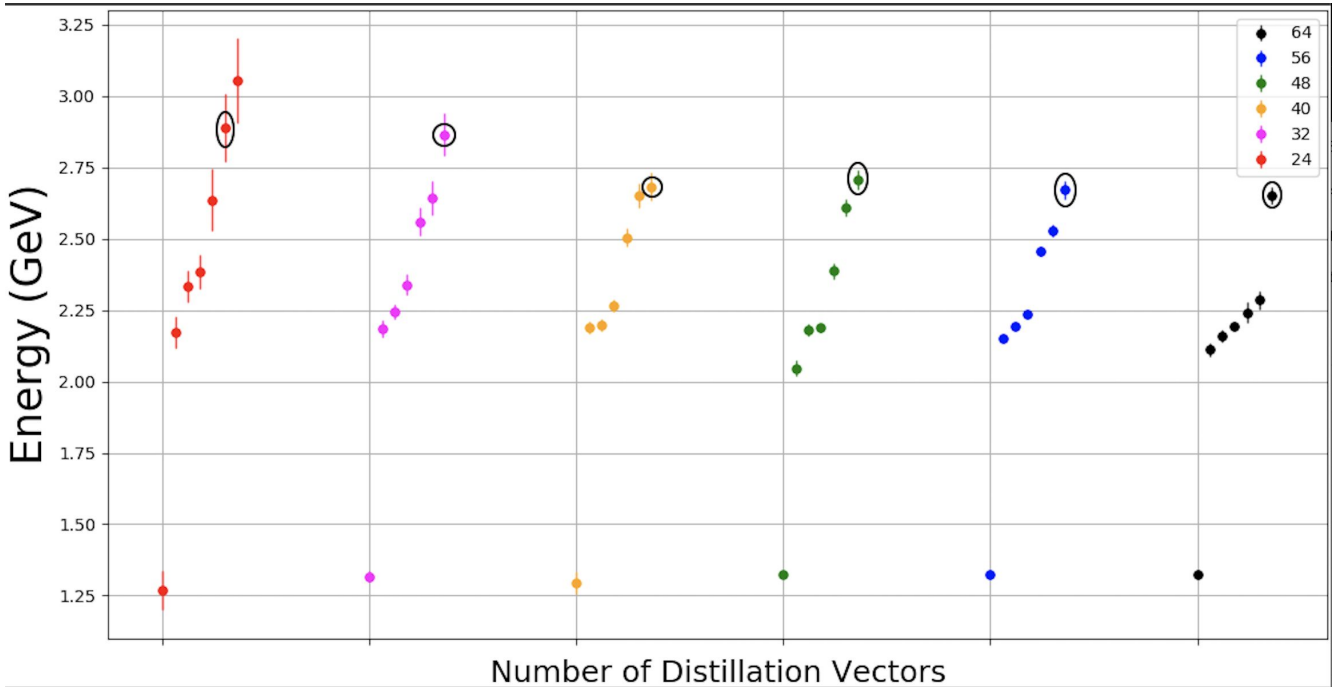
- Tensor charge
- Distillation dramatically improves signal
- GEVP reduces excited states.

C. Egerer, D. Richards, F. Winter (2018) 1810.09991

Distillation in motion

- Dispersion relation of Delta in motion shows that fewer eigen-vectors are needed than previously thought

T. Khan's poster from Lattice 2018



Combined Lattice PDF analysis

- Modern PDFs are extracted from data from many experiments of several types
- There exist many Good Lattice Cross Sections with different systematic errors
- Several Lattice Cross Sections can be fit simultaneously
- Currently planning to calculate following current pairs

$$SS, PP, V_xV_x, V_yV_y, V_xA_y, V_yA_x, A_xA_x, A_yA_y, A_zA_z$$

- Incompatibility between Lattice Data and experimental results can signal if systematic errors are significant

Summary

- PDFs cannot be directly calculated from lattice QCD, but related observables are
- Biggest problem for calculation is limited range of momentum and separations
- Issues with signal and higher state effects
- PDFs can and should be extracted from several observables simultaneously

Thank you for your attention!