quasi-GPDs from Lattice QCD

TMD Collaboration

Martha Constantinou

Temple University

USQCD All-Hands Meeting

Brookhaven National Lab

April 27, 2019

Project Participants

PI:

- **M. Constantinou** — Temple University

Core members:

- **C. Alexandrou** — Univ. of Cyprus & Cyprus Institute
- **K. Cichy** — Adam Mickiewicz University
- **K. Hadjiyiannakou** — Cyprus Institute
- **K. Jansen** — DESY, Zeuthen
- **A. Scapellato** — University of Cyprus
- **F. Steffens** — University of Bonn

Local Students:

- **C. Lauer** — PhD student, Temple University
- **S. Vishnoi** — Undergraduate student, Temple University
- **J. Delmar** — PhD student (Fall 2019), Temple University

Extended Twisted Mass Collaboration

TMD Collaboration

Outline

- Motivation
- Theoretical Background
- Lattice Setup
- Renormalization
- Existing expertise
- Project Readiness - Preparatory work
- Requested Resources
- Summary

Motivation

Distribution Functions:

- Universal tools to study hadron structure
- Global fit analyses of DIS data: main source of information
- Global fits not without ambiguities
- PDFs/GPDs parameterized in terms of off-forward matrix elements of non-local light-cone operators (Not accessible in Euclidean lattice)
- Lattice QCD: long-standing history of moments of PDFs (via OPE), but reconstruction of PDFs not feasible (gauge noise, mixing)
- Alternative approaches proposed, e.g.: quasi-PDFs, good lattice cross sections, high moments (auxiliary heavy quark), hadronic tensor, OPE w/o OPE
- We propose calculation using the quasi-distributions approach

Motivation

- Understanding the internal partonic structure of hadrons from first principles is a long-term goal of Nuclear Physics. The proposed activities are in accordance with the physics goal of the EIC

- Acquiring USQCD resources will allow us to obtain state-of-the-art lattice results, and optimize our algorithms in preparation for the exascale era (among the main pillars of DOE's mission)

- The proposed activities will contribute to the scientific goals and broader impact of the DOE funded **TMD Topical Collaboration**

"To address the challenges of extracting novel quantitative information about the nucleon's internal landscape"

"To provide compelling research, training, and career opportunities for young nuclear theorists"

- 18 Institutions
- Theory, Phenomenology, Lattice QCD
- 2 tenure-track positions: Temple, NMSU
- Support for young researchers

Theoretical Background

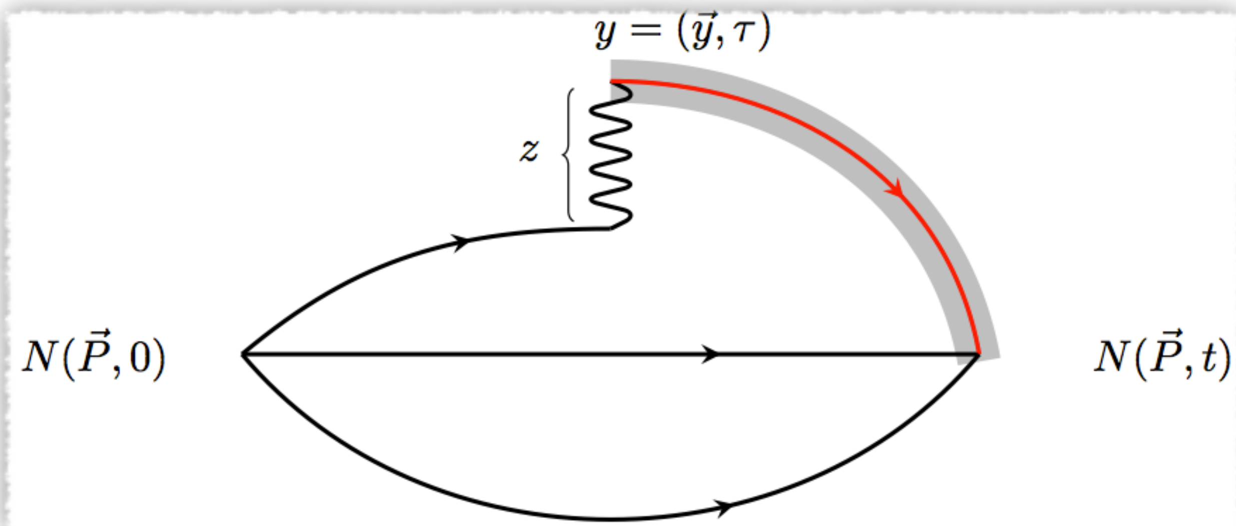
Quark quasi-Distributions

X. Ji, Phys. Rev. Lett. 110 (2013) 262002 [arXiv:1306.1539]

- **Fourier transform of operators for fast moving hadrons coupled with non-local Wilson-line operators**

$$\tilde{q}_\Gamma^{\mathrm{GPD}}(x,\xi,t,\mu^2,P_3) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z}\,\langle N(P_3)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P')\rangle_{\mu^2}$$

$t = Q^2\,,\quad \xi \equiv -\frac{Q_3}{2P_3}\,,\quad \xi$: Quasi-skewness = light-cone skewness $+\ \mathcal{O}\left(\frac{1}{P_3^2}\right)$



Diagrammatic representation of nucleon 3pt-function (connected) of non-local operator containing a Wilson line of length z

- Sink momentum fixed to $(0,\, 0,\, P_3)$
- Source momentum (P'):

  $(0,\, 0,\, P_3)$ for quasi-PDFs

  $(P'_1,\, P'_2,\, P'_3)$ for quasi-GPDs

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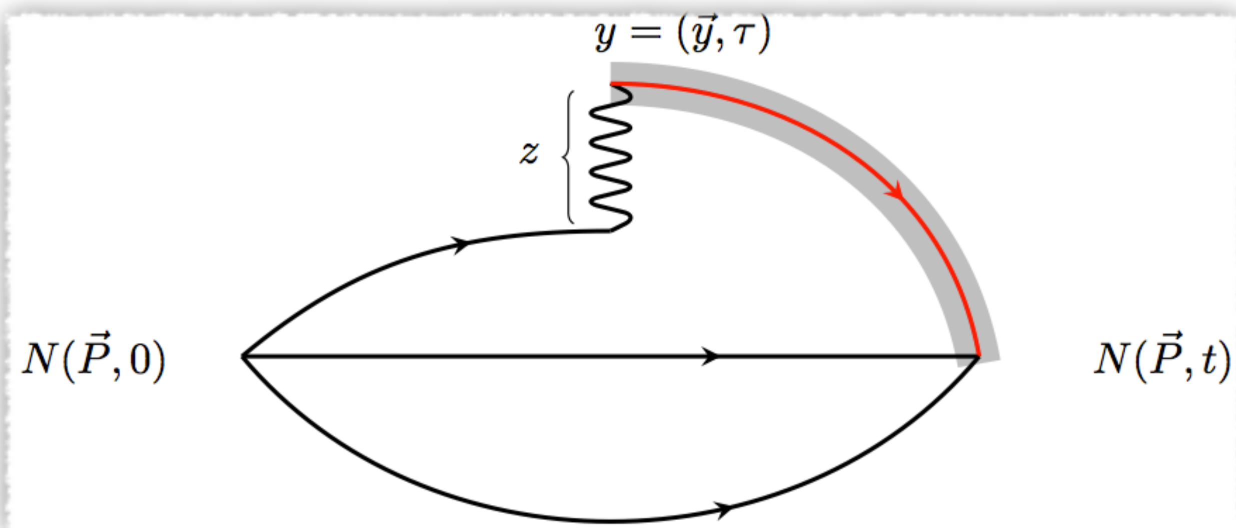

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  (P'_1, P'_2, P'_3) for quasi-GPDs ⟵ **Proposed Activities**

Quark quasi-Distributions

- **Renormalization (complex functions, presence of mixing)**

- **Fourier transform to momentum space (x)**

$$\tilde{q}(x,\mu^2,P_3) = \int \frac{dz}{4\pi}\, e^{ixP_3 z}\langle N\,|\,\overline{\psi}(z)\Gamma\mathscr{A}(z,0)\psi(0)\,|\,N\rangle$$

- **Decomposition for unpolarized & helicity quasi-GPDs** $\tilde{H}\,,\ \tilde{E}\,:$

$$\tilde{q}_\Gamma^{\rm GPD}(x,\xi,t,\mu^2,P_3) = \bar{u}(P_3)\left\{\tilde{H}^\Gamma(x,\xi,t,P_3,\mu)\,\Gamma + \tilde{E}^\Gamma(x,\xi,t,P_3,\mu)\frac{\Delta_\rho[\gamma_\rho,\Gamma]}{4M}\right\}u(P')$$

- **Large momentum effective theory (LaMET) necessary to match quasi-PDFs and standard PDFs**

$$\tilde{G}^\Gamma(x,\xi,t,P_3,\mu) = \int_{-1}^{1}\frac{dy}{|y|}\,C_G^\Gamma\left(\frac{x}{y},\frac{\xi}{y},\frac{\mu}{yP_3}\right)G^\Gamma(y,\xi,t,\mu) + \mathcal{O}\left(\frac{M^2}{P_3^2},\frac{\Lambda_{\rm QCD}^2}{P_3^2}\right)$$

Y.S. Liu, W. Wang, J. Xu, Q.-A. Zhang, J.-H. Zhang, S. Zhao, Y. Zhao, arXiv:1902.00307

- **Target mass corrections (elimination of residual m_N/P_3)**

Lattice Setup
&
Computational Plan

Lattice Setup

- **Nf=2+1+1 twisted mass fermions & clover term**

Ensemble	lattice spacing a	Volume $(L^3 \times T)$	m_π (MeV)	$m_\pi L$
cA211.32	0.093 fm	$32^3 \times 64$	270	4

- **Nucleon momentum, excited states and statistics**

$P_3 = 1.7$ GeV			$P_3 = 2.1$ GeV			$P_3 = 2.5$ GeV		
$T_{\rm sink}$	$N_{\rm conf}$	$N_{\rm meas}$	$T_{\rm sink}$	$N_{\rm conf}$	$N_{\rm meas}$	$T_{\rm sink}$	$N_{\rm conf}$	$N_{\rm meas}$
$8\,a$	100	1600	$8\,a$	200	3200	$8\,a$	300	4800
$10\,a$	200	3200	$10\,a$	350	5600	$10\,a$	500	8000
$12\,a$	300	4800	$12\,a$	500	8000	$12\,a$	700	11200

- **Momentum transfer up to ~2 GeV^2**
  (131 values up to (±3,0,0) & permutations)

- **Increase of statistics and Signal Improvement**
  - **6 directions of Wilson line: ±x, ±y, ±z with P_3 in same direction**
  - **Stout Smearing: 0, 5, 10 steps**
  - **Momentum smearing: tuned for each nucleon momentum**

[G. Bali et al., PRD93, 094515 (2016)]

Renormalization

Renormalization of quasi-PDFs

- Indispensable component of lattice calculation

- Calculation in lattice QCD pert. theory revealed for the *first time* crucial aspects of nonlocal Wilson line operators renormalization:

  M. Constantinou, H. Panagopoulos, Phys. Rev. D96 (2017) 054506 [arXiv:1705.11193]

  - certain Dirac structures have mixing in lattice regularization including the one employed for the unpolarized PDF: γ_z
  - renormalization prescription was proposed for all operators
  - proposed new operator for unpolarized PDF: γ_t
  - led to development of non-perturbative renorm. prescription (RI-type)

  C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, H. Panagopoulos, F. Steffens, Nucl. Phys. B 923 (2017) 394 (Frontiers Article) [arXiv:1706.00265]

- Comparison with global fits becomes real possibility with a concrete renormalization prescription

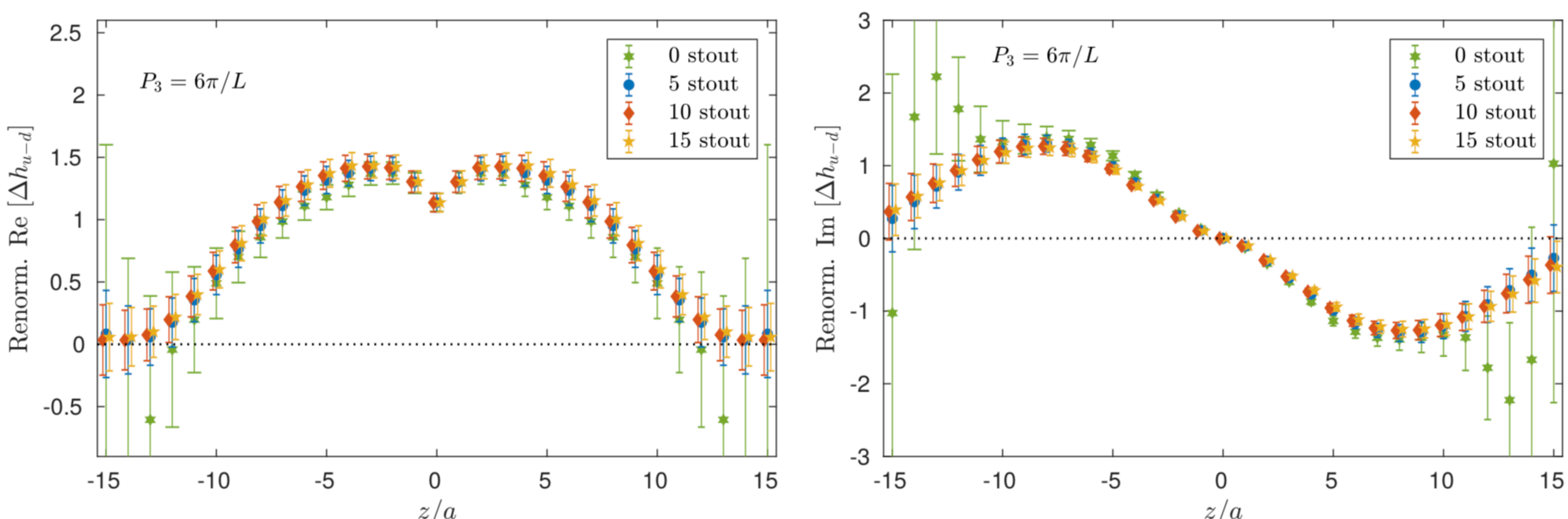
Renormalization of quasi-GPDs

- **Non-perturbative RI' scheme**

$$\frac{Z_{\mathcal{O}}^{\mathrm{RI}'}(z,\mu_0,m_\pi)}{Z_q^{\mathrm{RI}'}(\mu_0,m_\pi)}\frac{1}{12}\mathrm{Tr}\left[\mathcal{V}(z,p,m_\pi)\left(\mathcal{V}^{\mathrm{Born}}(z,p)\right)^{-1}\right]\Big|_{p^2=\mu_0^2}=1$$

$$Z_q^{\mathrm{RI}'}(\mu_0,m_\pi)\frac{1}{12}\mathrm{Tr}\left[\left(S(p,m_\pi)\right)^{-1}S^{\mathrm{Born}}(p)\right]\Big|_{p^2=\mu_0^2}=1$$

- **Also applicable for cases of mixing**
- **Conversion to the MSbar scheme using perturbation theory**
- **Renormalized matrix elements independent of stout smearing**



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Renormalization of quasi-GPDs

We will address systematic uncertainties similar as in

C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, A. Scapellato, F. Steffens, [arXiv:1902.00587]

- **Five ensembles for chiral extrapolation: am_{PS}=0.168, 0.192, 0.213, 0.230, 0.243 (350-550 MeV)**

$$Z_{\mathcal{O}}^{\mathrm{RI}'}(z,\mu_0,m_\pi) = Z_{\mathcal{O},0}^{\mathrm{RI}'}(z,\mu_0) + m_\pi^2\, Z_{\mathcal{O},1}^{\mathrm{RI}'}(z,\mu_0)$$

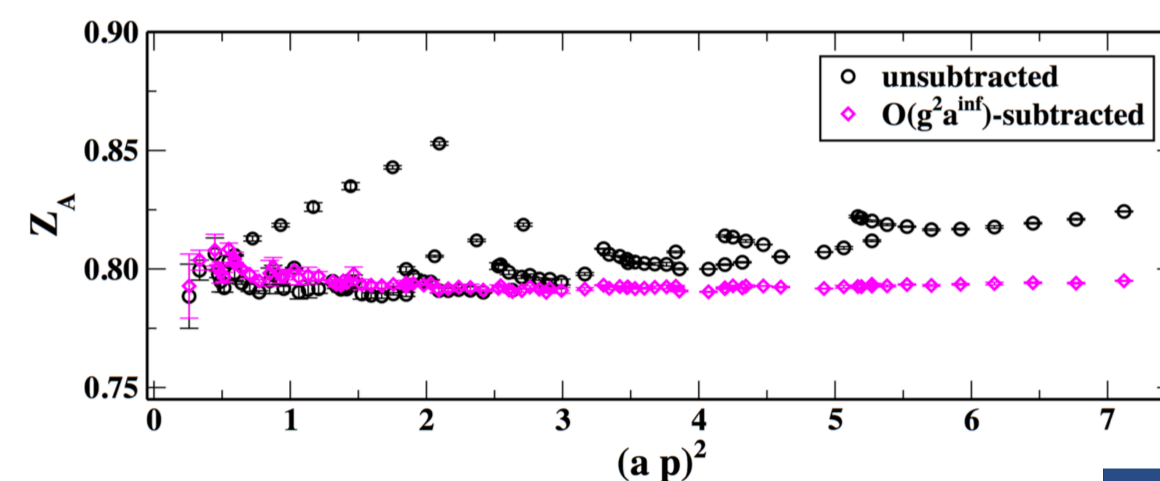
- **Several RI renormalization scales to convert to MSbar and remove residual dependence on initial scale**

$$Z_{\mathcal{O}}^{\overline{\mathrm{MS}}}(z,\bar\mu,\mu_0) = \mathcal{Z}_{\mathcal{O},0}^{\overline{\mathrm{MS}}}(z,\bar\mu) + (a\,\mu_0)^2\, \mathcal{Z}_{\mathcal{O},1}^{\overline{\mathrm{MS}}}(z,\bar\mu)$$

- **Conversion to Modified MS (MMS) scheme**

$$\mathcal{Z}_{\mathcal{O},0}^{\mathrm{M}\overline{\mathrm{MS}}}(z,\bar\mu) = \mathcal{Z}_{\mathcal{O},0}^{\overline{\mathrm{MS}}}(z,\bar\mu)\, \mathcal{C}^{\overline{\mathrm{MS}},\mathrm{M}\overline{\mathrm{MS}}}$$

- **Subtraction of $\mathcal{O}(g^2 a^\infty)$ artifacts similarly to local currents**



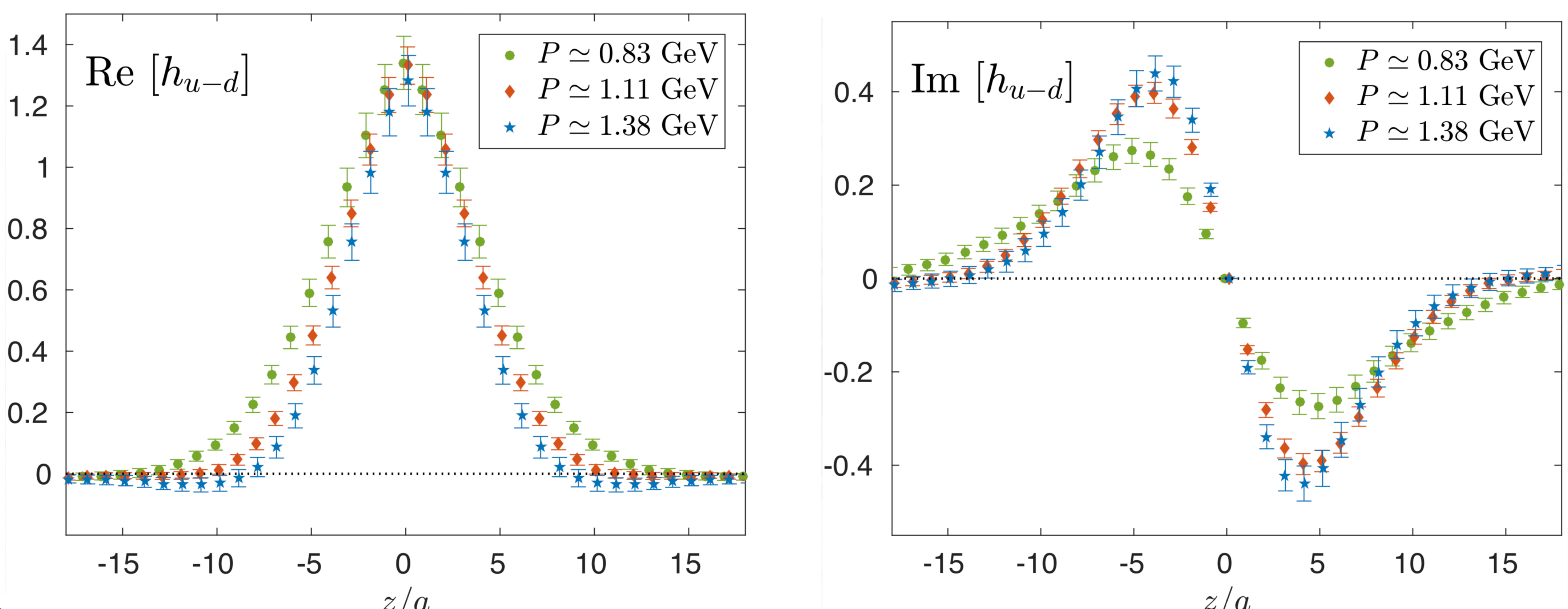
Existing Expertise

Results on quasi-PDFs

Alexandrou, Cichy, Constantinou, Jansen, Scapellato, Steffens, Phys.Rev.Lett. 121 (2018) 112001 [arXiv:1803.02685]
Alexandrou, Cichy, Constantinou, Jansen, Scapellato, Steffens, Phys. Rev. D 98 (2018) 091503 [arXiv:1807.00232]
Alexandrou, Cichy, Constantinou, Hadjiyiannakou, Jansen, Scapellato, Steffens, [arXiv:1902.00587]

- First calculation at the physical point including all components
- Use of $\gamma 0$ (perpendicular to Wilson line) for unpolarized (no mixing)

[M. Constantinou, H. Panagopoulos, Phys. Rev. D 96 (2017) 054506, [arXiv:1705.11193]



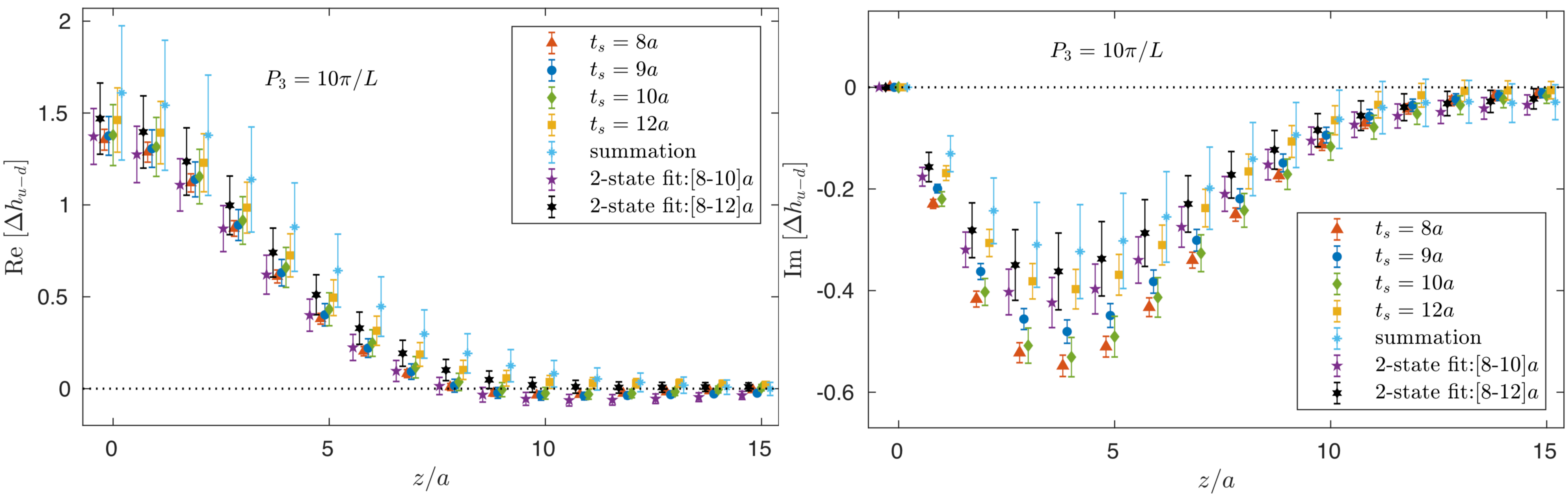
Results on quasi-PDFs

Analyses techniques:

Single-state fit, Two-state fit, Summation method

Each method can offer important information

Extra care due to nucleon boost, thus convergence of techniques is crucial



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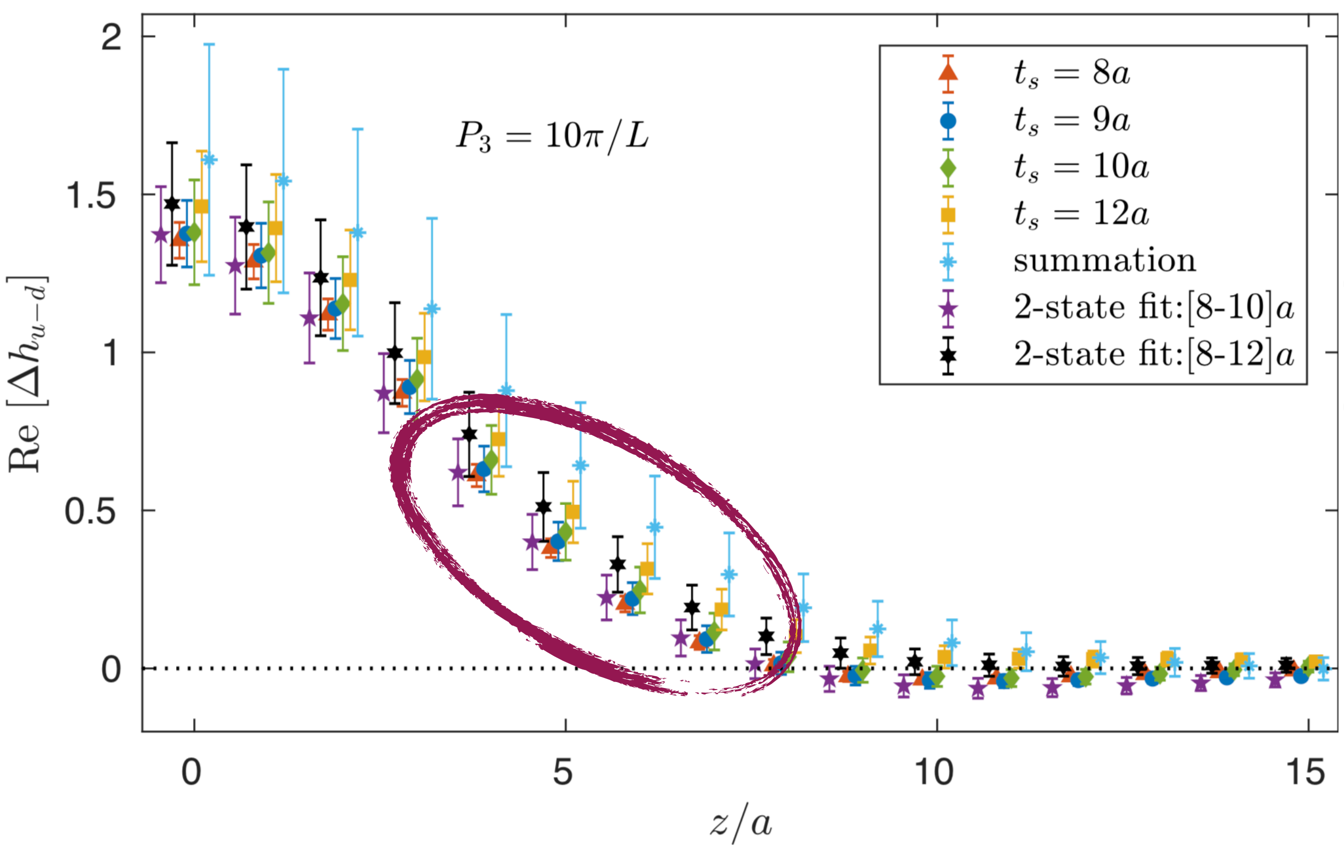
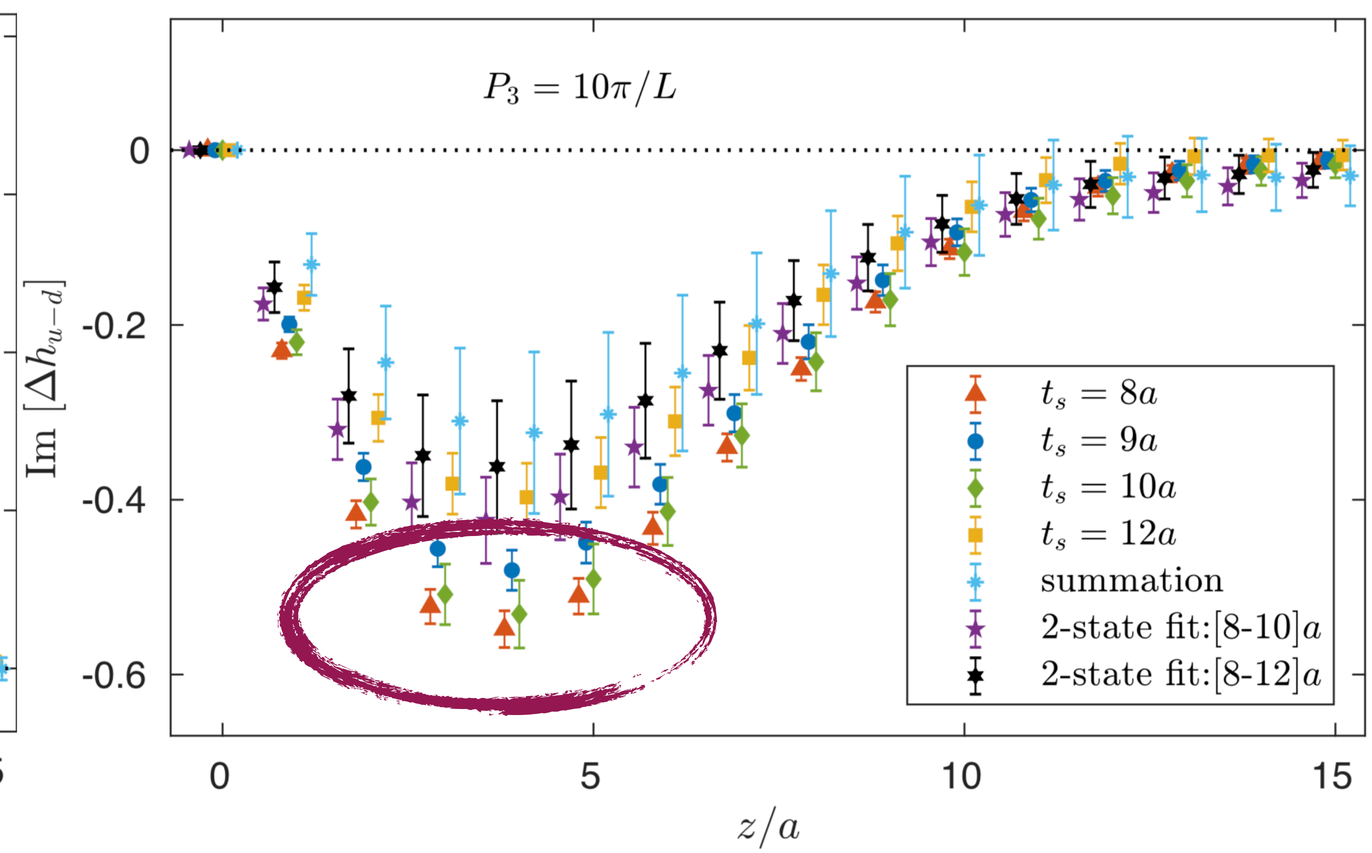
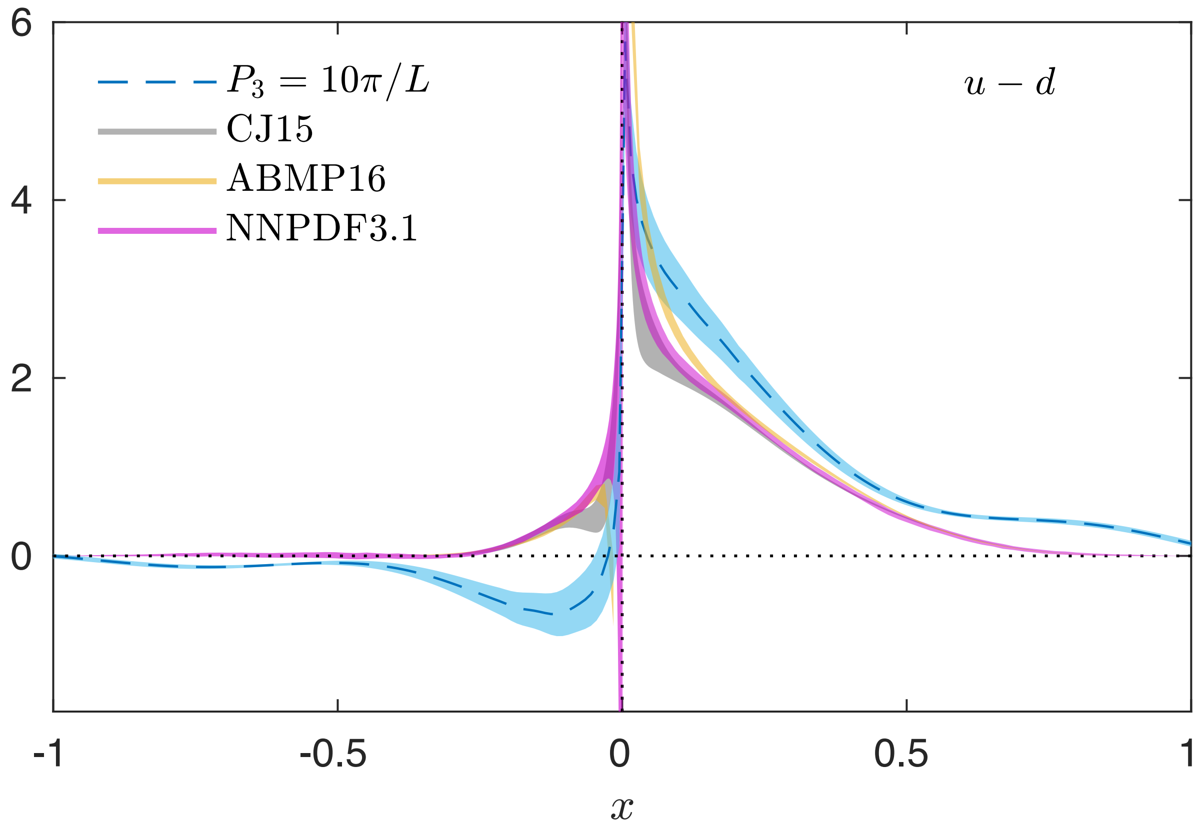
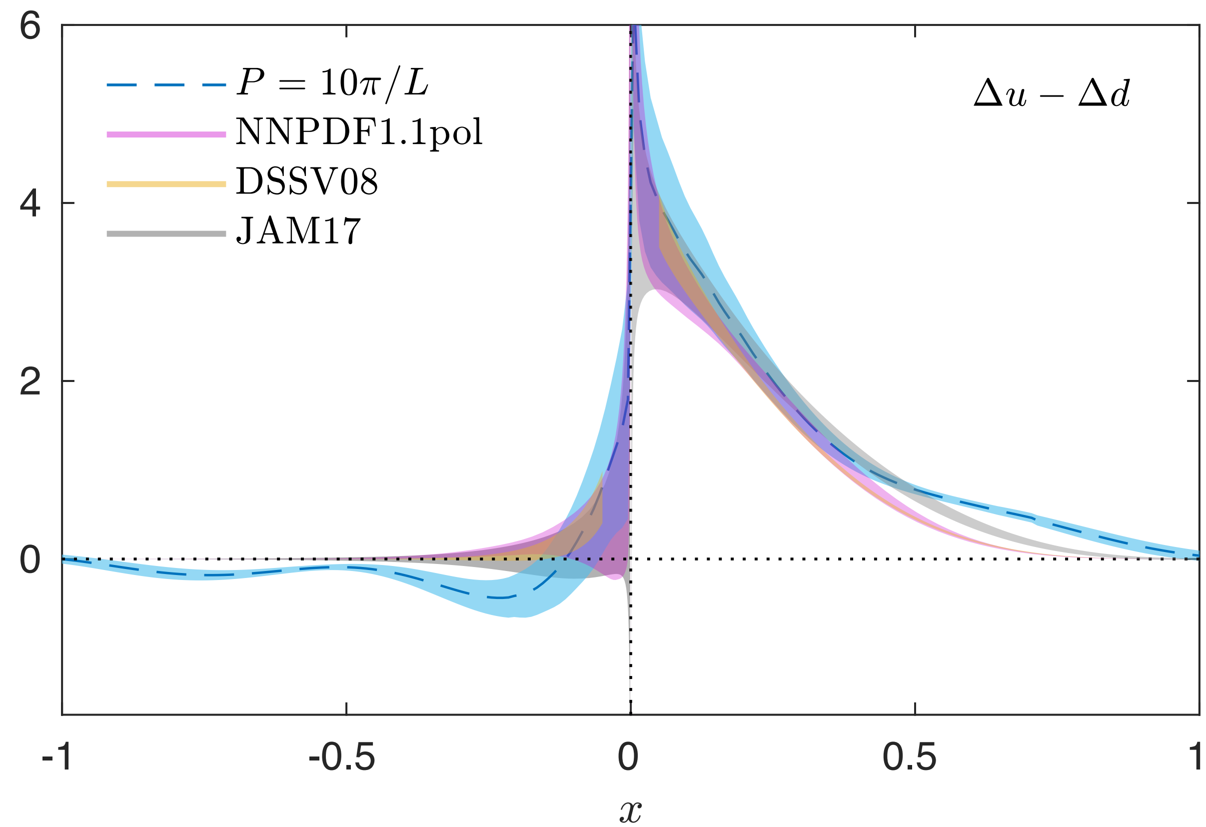


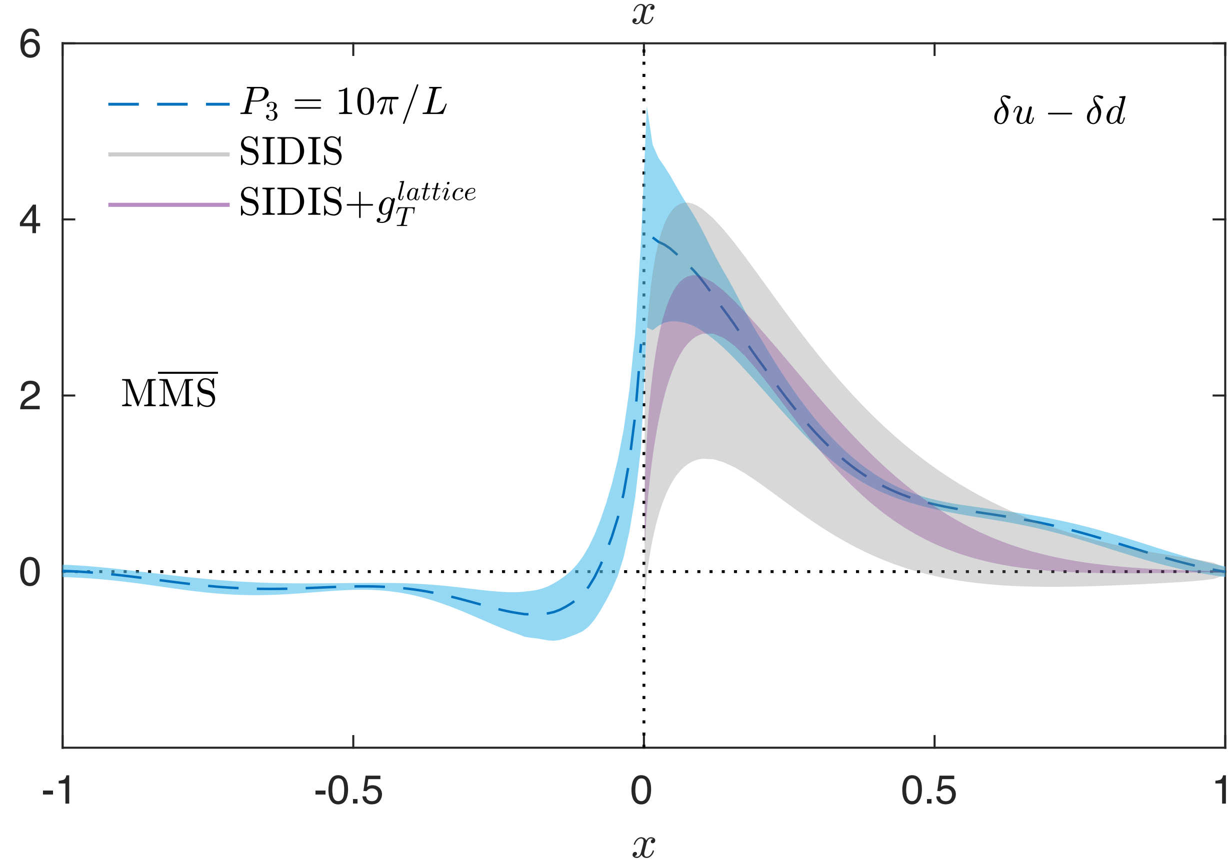


- T_{sink}=8a heavily contaminated by excited states
- T_{sink}=9a-10a not consistent with T_{sink}=12 within uncertainties

Results on quasi-PDFs







- **investigation of excited states with various techniques**
- **matching from MMS to MS scheme (particle number conservation)**
- **transversity PDFs particularly interesting due to poor estimates from global fits**

Preparatory Work

Preliminary results for quasi-GPDs

- **Results for sink momentum P”=(0,0,P_3) (1.25 GeV)**
- **Various Q^2 and ξ. Representative examples:**

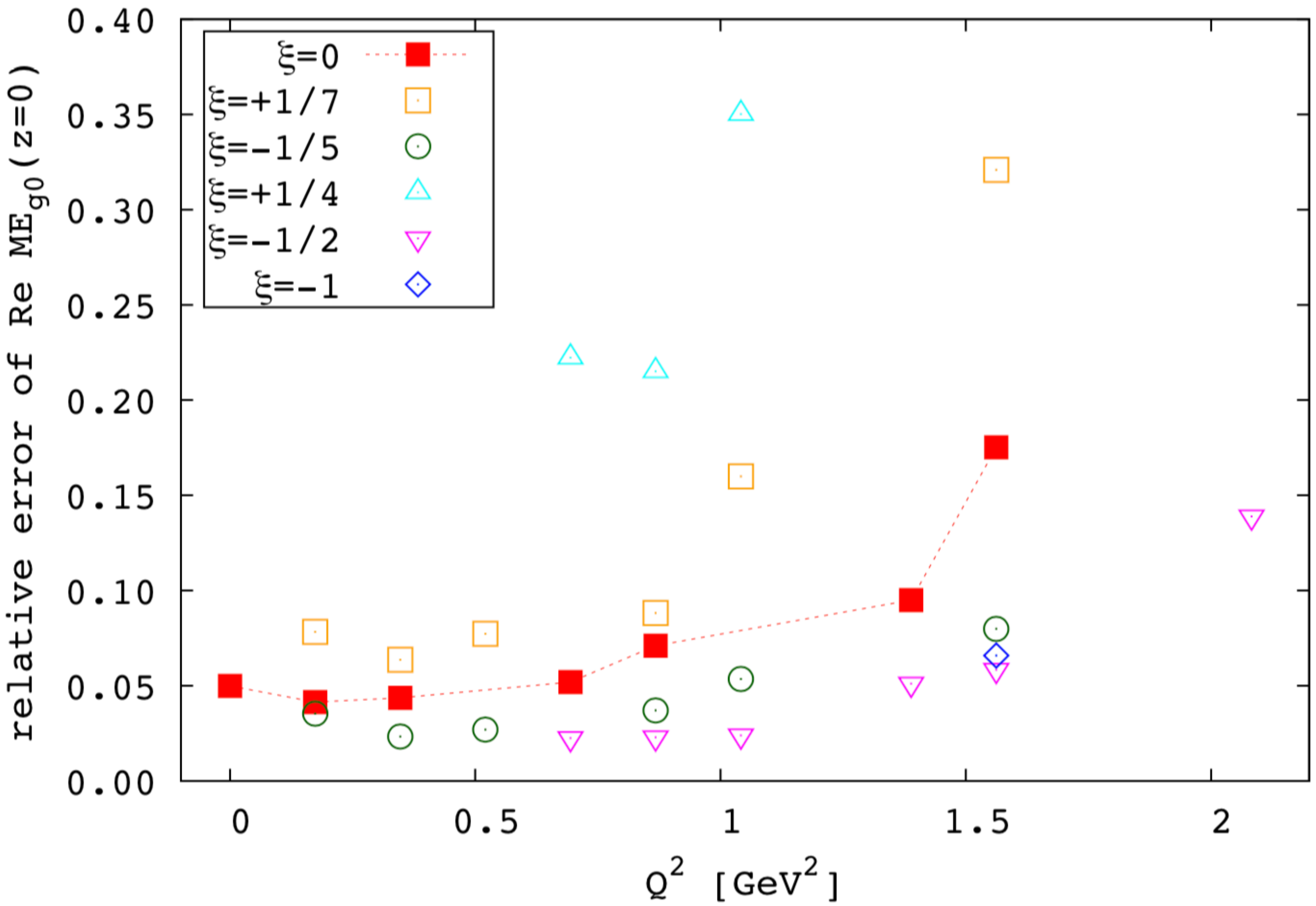
$$\xi = -\frac{P_3'' - P_3'}{P_3'' + P_3'} = -\frac{Q_3}{2P_3}$$

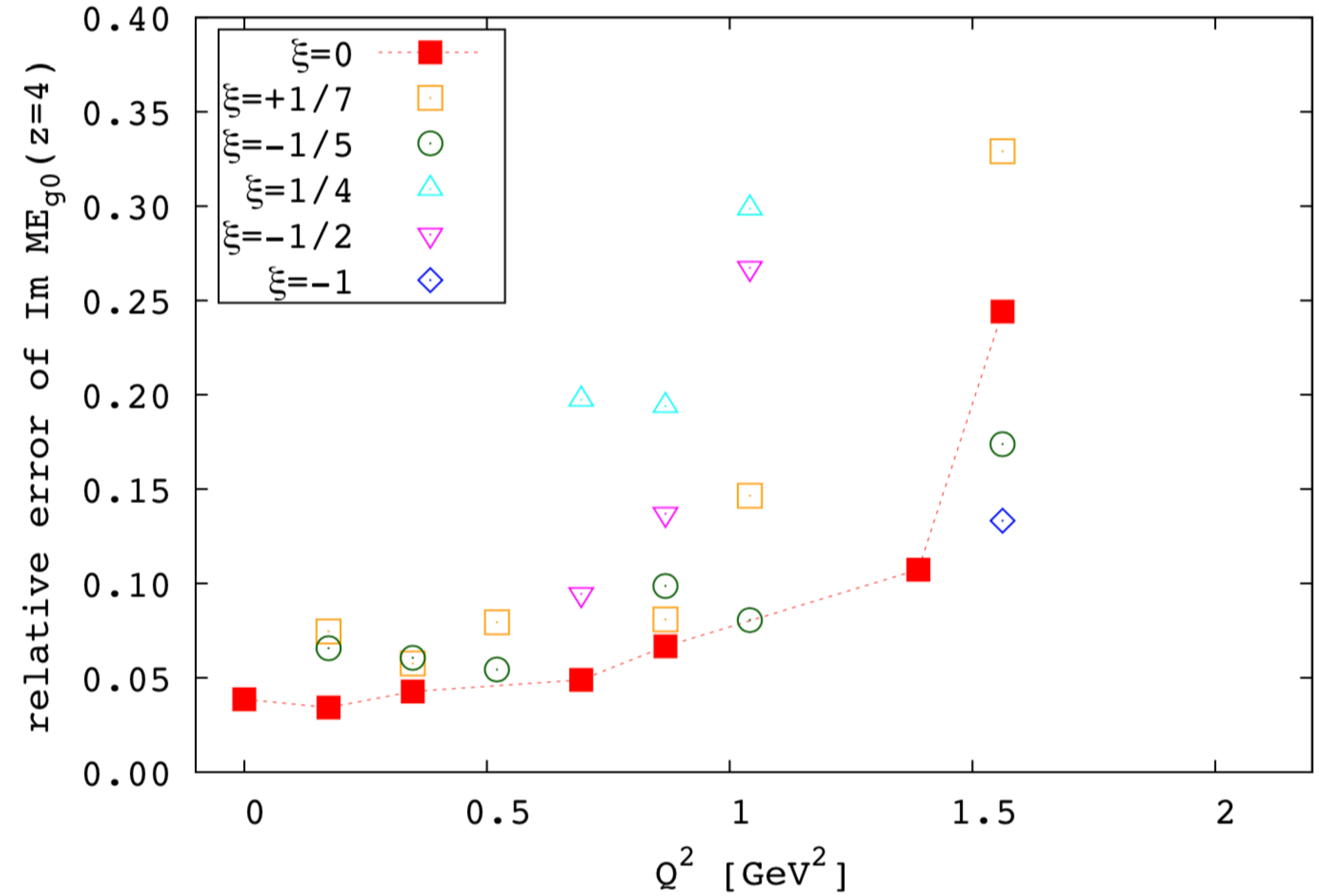
Class	P'	$\|P'\|$ [GeV]	Q	$t = Q^2$	t [GeV^2]	$\frac{P_3'+P_3''}{2}$	Q_3	ξ
0	(0,0,3)	1.25	(0,0,0)	0	0	3	0	0
1	(-1,0,3)	1.32	(1,0,0)	0.039	0.17	3	0	0
2	(-2,0,3)	1.50	(2,0,0)	0.154	0.69	3	0	0
3	(-1,-1,3)	1.38	(1,1,0)	0.077	0.35	3	0	0
4	(-2,-1,3)	1.56	(2,1,0)	0.193	0.87	3	0	0
5	(-2,-2,3)	1.72	(2,2,0)	0.308	1.39	3	0	0
31	(-3,0,3)	1.77	(3,0,0)	0.347	1.56	3	0	0
18	(0,0,5)	2.08	(0,0,-2)	0.154	0.69	4	-2	1/4
19	(-1,0,5)	2.12	(1,0,-2)	0.193	0.87	4	-2	1/4
20	(-2,0,5)	2.24	(2,0,-2)	0.308	1.39	4	-2	1/4
21	(-1,-1,5)	2.17	(1,1,-2)	0.231	1.04	4	-2	1/4
22	(-2,-1,5)	2.28	(2,1,-2)	0.347	1.56	4	-2	1/4
23	(-2,-2,5)	2.39	(2,2,-2)	0.463	2.08	4	-2	1/4
24	(0,0,1)	0.42	(0,0,2)	0.154	0.69	2	2	-1/2
25	(-1,0,1)	0.59	(1,0,2)	0.193	0.87	2	2	-1/2
26	(-2,0,1)	0.93	(2,0,2)	0.308	1.39	2	2	-1/2
27	(-1,-1,1)	0.72	(1,1,2)	0.231	1.04	2	2	-1/2
28	(-2,-1,1)	1.02	(2,1,2)	0.347	1.56	2	2	-1/2
29	(-2,-2,1)	1.25	(2,2,2)	0.463	2.08	2	2	-1/2

Preliminary study includes:

33 distinct classes from 786 combinations of source and sink momentum

Preliminary results for quasi-GPDs



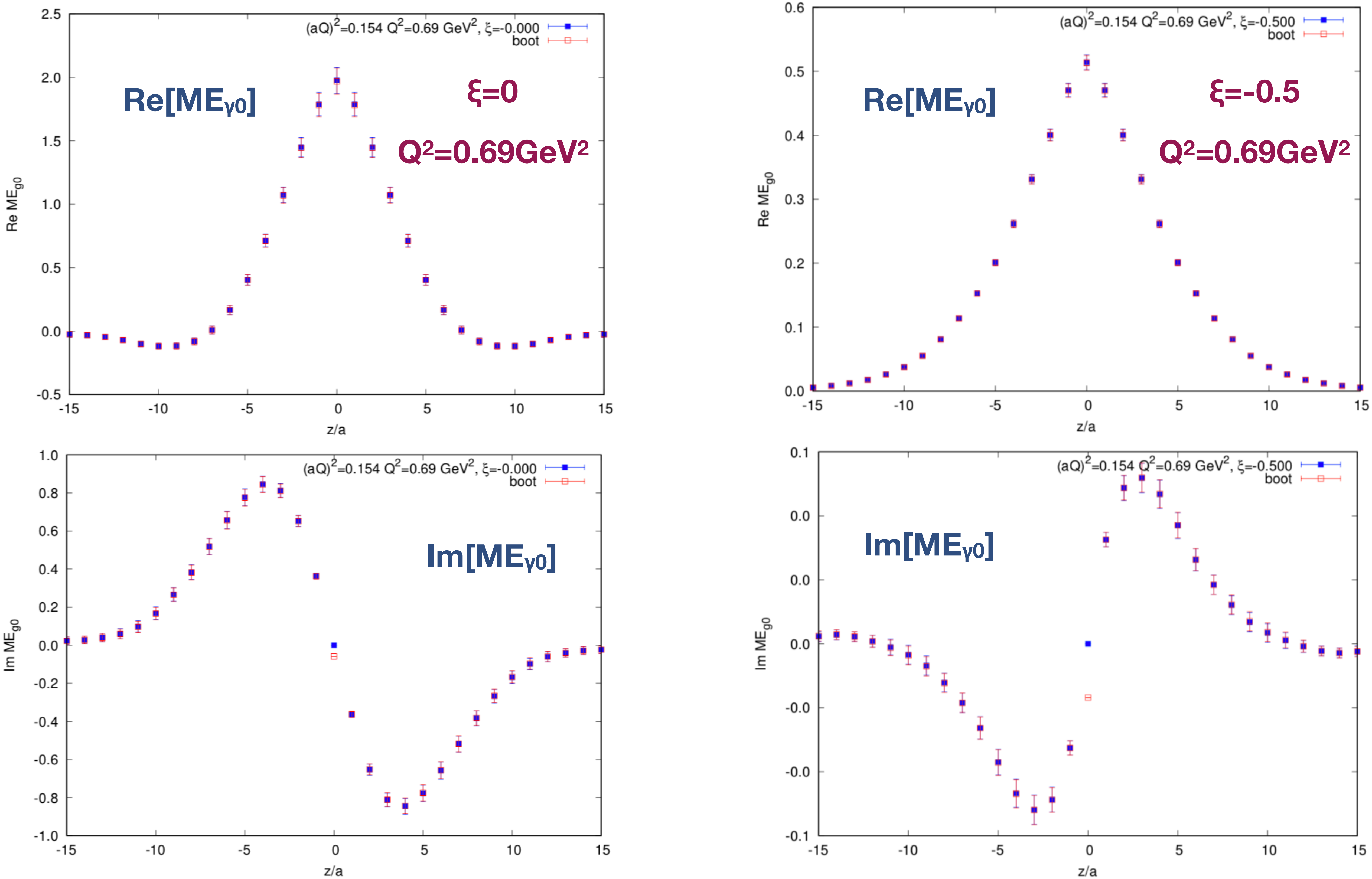


$$\xi = -\frac{P_3'' - P_3'}{P_3'' + P_3'} = -\frac{Q_3}{2P_3}$$

- Data on **negative** quasi-skewness ξ have better signal: produce smaller source momentum
- Relative error for Imaginary part not conclusive as the signal is suppressed

Preliminary results for quasi-GPDs

P_3=1.25GeV



Signal quality for momentum transfer is very promising!

TMD Collaboration

Project Readiness
&
Requested Resources

Benchmark - Requested Resources

- Codes benchmarked on Haswell CPUs (performance similar to Skylake)

- Cost of a run for one T_{sink}, one source position and one P3: 222 CPU-h

  We will analyze 350 configurations on average, three T_{sink}, 16 src positions and three values of the nucleon momentum
  Total cost: 11.2 M CPU-h

- Each run for the renormalization functions requires 7.26 CPU-h

  We will evaluate the Z-factors on 5 ensembles, 20 configurations each, 15 values of RI' scale, and we require 14K CPU-h

- Request in storage: 15 TB of disc space and 20 TM of storage space

To succeed our scientific goals we request a total of 11.22 M Sky-core-hours and 35 TB of storage

Future computational plan

Question from USQCD Scientific Program Committee:

"The final results of your project (extraction of GPDs) seem like they will require additional calculations beyond the ensemble studied in this proposal. What is the status of your overall calculation, and what other resources are already committed to it?"

- **Analysis of more ensembles once the current study is completed**
  physical point, larger volume, finer lattices
  R. Briceño, J. Guerrero, M. Hansen, C. Monahan, Phys. Rev. D98 (2018) 014511, arXiv:1805.01034

- **Parameters of next calculation will be decided based on acquire knowledge:**
  Signal quality for momentum transfer in boosted frame
  Non-zero skewness

- **In future more demanding calculations (exceeding USQCD support) we will seek computational resources from other facilities (Summit, Diz Daint, Jureca, SuperMUC, Prometheus, Stampede)**

Summary

Summary

- Substantial progress in 1-D hadron structure with quasi-PDFs:
  - Simulations at the physical point
  - Renormalization and Matching
  - Investigation of certain systematic uncertainties

- Extension of expertise to first investigations of more complicated quantities: quasi-GPDs

- International collaboration valuable for access to state-of-the-art configurations at the physical pion mass (e.g. ETMC). Access is immediate and unrestricted.

- Acquiring resources will be crucial for the physics goals of the TMD Collaboration (PI is a core member). Bridge between phenomenology and lattice QCD

Thank you

Funding Acknowledgements:

U.S. DEPARTMENT OF ENERGY | Office of Science

NSF

TMD Collaboration **Topical Collaboration**

Grant No. PHY-1714407