# Lattice QCD and Neutrino Oscillations

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BROOKHAVEN NATIONAL LABORATORY

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- Nucleons with Distillation
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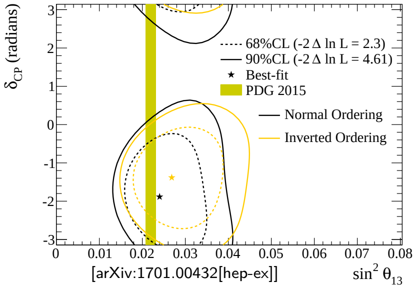
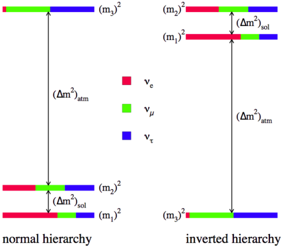
- Concluding Remarks

# Introduction

# Neutrino Oscillation Experiment Goals




[arXiv:1701.00432[hep-ex]]




[arXiv:1307.5487[hep-ex]]

Neutrino oscillation experiments are flagship experiments for the upcoming decade

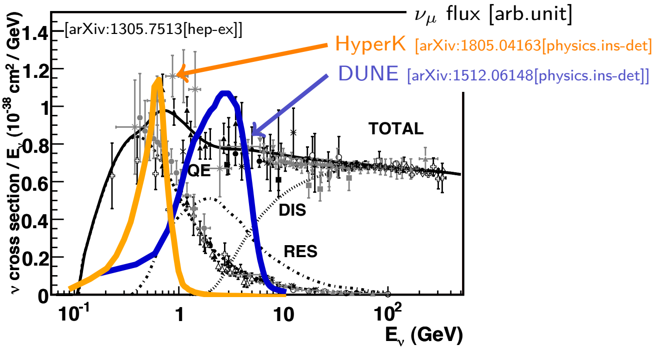
Experiments seek precision measurements of oscillation parameters:

- precision measurements of $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ and $\theta_{ij}$
- determine value of $\delta_{CP}$
- determine sign of $\Delta m_{31}^2$; i.e. mass hierarchy

Experiments will also probe: supernova neutrinos, proton decay, non-standard interactions

To maximize potential for physics discoveries in oscillation experiments,
need precise supporting theoretical predictions

# Flux ⊗ Cross Section



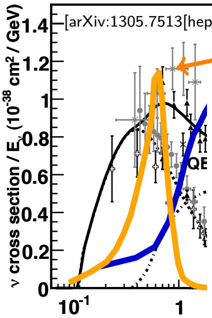

Neutrino event rate is the product of many pieces:

$$\underbrace{N(\vec{x})}_{\text{rate}} = \underbrace{\Phi(E_\nu)}_{\text{flux}} \times \underbrace{\sigma(E_\nu, \vec{x})}_{\text{xsec}} \times \underbrace{\epsilon(\vec{x})}_{\text{eff.}} \times \underbrace{P(\nu_A \to \nu_B)}_{\text{osc.prob.}}$$

In ideal situation, $\Phi$ & $\epsilon$ would exactly cancel in ratio of near/far detector

Flux spans range of energies, knowledge of individual interaction topologies important

# Flux ⊗ Cross Section




| Source of Uncertainty | MINOS $\nu_e$ | T2K $\nu_e$ | DUNE $\nu_e$ | Comments |
|---|---|---|---|---|
| Beam Flux after N/F extrapolation | 0.3% | 3.2% | 2% | See "Flux Uncertainties" in Section 3.6.2 |
| Interaction Model | 2.7% | 5.3% | $\sim 2\%$ | See "Interaction Model Uncertainties" in Section 3.6.2 |
| Energy scale ($\nu_\mu$) | 3.5% | included above | (2%) | Included in 5% $\nu_\mu$ sample normalization uncertainty in DUNE 3-flavor fit. |
| Energy scale ($\nu_e$) | 2.7% | 2.5% includes all FD effects | 2% | See "$\nu_e$ Energy-Scale Uncertainties" in Section3.6.2 |
| Fiducial volume | 2.4% | 1% | 1% | Larger detectors = smaller uncertainty. |
| Total | 5.7% | 6.8% | 3.6 % | |
| Used in DUNE Sensitivity Calculations | | | $5\% \oplus 2\%$ | Residual $\nu_e$ uncertainty: 2% |

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In ideal situation, $\Phi$ & $\epsilon$ would exactly cancel in ratio of near/far detector
Flux spans range of energies, knowledge of individual interaction topologies important

In practice, need control of cross sections and energies for precision measurement
Nucleon and nuclear uncertainties are a large part of error budget

# Neutrino-Nuclear Cross Sections

Intranuclear rescattering effects can be problematic

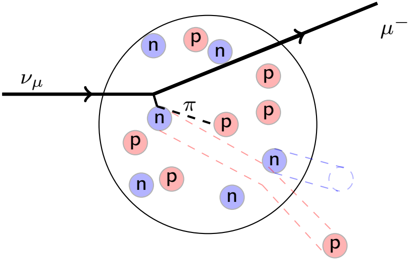
- Nuclear rescattering can change particle energy
- Topologies altered by absorption or emission of other particles

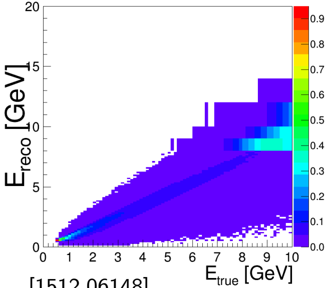
Resulting event-level data is subject to interpretation

$\Longrightarrow$ Neutrino energies cannot be determined on an event-by-event basis

$\Longrightarrow$ Energy spectrum must be reconstructed at the statistical level

$\Longrightarrow$ Reconstruction depends on the assumed nuclear model




[1512.06148]

# Dangers of Imprecision

| | $\delta = 0$ | $\delta = \pi/2$ | $\delta = \pi$ | $\delta = 3\pi/2$ | observed |
|---|---|---|---|---|---|
| $\nu_e$ | 24.2 | 19.6 | 24.1 | 28.7 | 32 |
| $\overline{\nu}_e$ | 6.9 | 7.7 | 6.8 | 6.0 | 4 |
| $P(N_{\nu_e} \geq 32\|\lambda)$ | 7.0% | 0.6% | 7.4% | 29.3% | – |
| $P(N_{\bar{\nu}_e} \leq 4\|\lambda)$ | 18.2% | 11.8% | 19.2% | 28.5% | – |

Event cts. from: [Iwamoto, ICHEP 2016]

Preference for maximal CPV in lepton sector driven by $\nu_e$ appearance measurements
Observed event count higher than highest prediction
$\implies$ Statistics? Underestimated predictions? Bad flux estimate?

$\nu N$ & $\bar{\nu} N$ cross sections are not same, $\nu_\mu$ & $\nu_e$ cross sections not same
$\implies$ could have different nuclear responses, results that mimic CPV signal

$\implies$ Care will be needed to ensure proper treatment of cross sections

# Focus

Need precise & robust determinations of form factors for input into nuclear models
Of the free nucleon form factors, the axial form factor is most relevant

Take two approaches to constrain nucleon axial form factor:

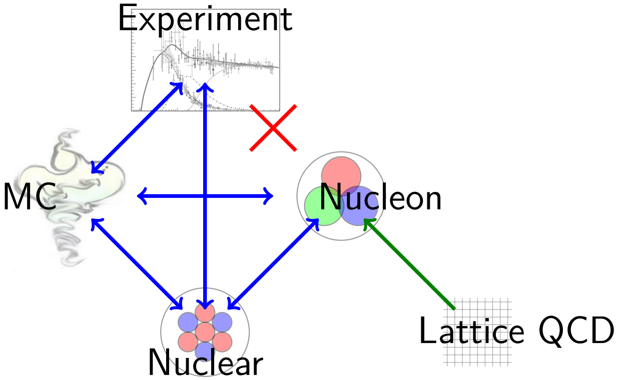
- Reanalysis of deuterium bubble chamber data
  - use model-independent $z$ expansion parametrization
    to study systematic uncertainties
- Lattice QCD calculation
  - compute the axial matrix element from first principles

First step is to compute axial charge: $g_A = F_A(Q^2)\Big|_{Q^2=0}$

Future extensions of this work will compute $Q^2$ dependence,
fit to $z$ expansion parametrization

Lattice data shown in this talk are preliminary nucleon mass calculations,
projects will eventually compute nucleon form factors

# Lattice QCD Checklist




Lattice QCD is ideal tool for filling in missing pieces

To have the greatest impact, must satisfy the checklist:

- Process is important for meeting experimental goals ✓
- Current precision not sufficient ✓
- Difficult/impractical to measure experimentally ✓
- Accessible to Lattice QCD ✓

# Deuterium Bubble Chamber Reanalysis

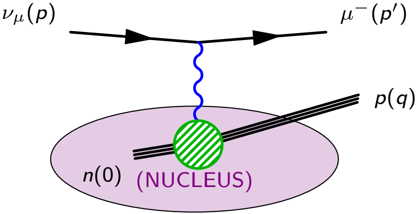
w/ M.Betancourt, R.Gran, R.Hill

# Quasielastic scattering

Quasielastic scattering good starting place understanding neutrino scattering:

$\nu$ interacts with single nucleon in nucleus
$\Longrightarrow$ QE is relatively easy measurement,
relatively theoretically clean

QE is primary signal measurement process
for neutrino oscillation experiments




In absence of intranuclear rescattering,
can infer incident neutrino energy from lepton kinematics alone:

$$E_\nu^{QE} = \frac{2(M_n - E_b)E_\ell - ((M_n - E_b)^2 - M_p^2 + m_\ell^2)}{2(M_n - E_b - E_\ell + p_\ell \cos\theta_\ell)}$$

Assumed to be single nucleon interaction, accesses free nucleon amplitudes

$\Longrightarrow$ Use amplitudes from QE as building block for more sophisticated interactions

Uncertainty dominated by nucleon axial form factor
$\Longrightarrow$ need to assess world best estimate of form factor

# Dipole Form Factor Parametrization

Most analyses assume the dipole axial form factor (Llewellyn-Smith):

$$F_A^{\text{dipole}}(Q^2) = \frac{g_A}{\left(1 + \frac{Q^2}{m_A^2}\right)^2}$$

[Phys.Rept.3 (1972),261]

Dipole is an ansatz:

- inconsistent with QCD
- unmotivated in interesting energy region

$\Longrightarrow$ uncontrolled systematics and therefore underestimated uncertainties

Large variation in $m_A$ over many experiments
(dubbed the "axial mass problem"):

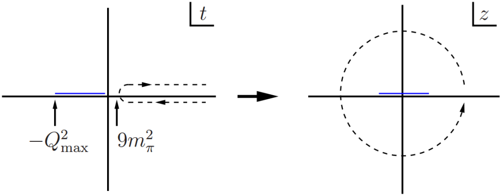
- $m_A = 1.026 \pm 0.021$ (Bernard *et al.*, [arXiv:0107088[hep-ph]])
- $m_A^{\text{eff}} = 1.35 \pm 0.17$ (MiniBooNE, [arXiv:1002.2680[hep-ex]])

Essential to use model-independent parameterization of $F_A$ instead

# $z$ Expansion Parametrization

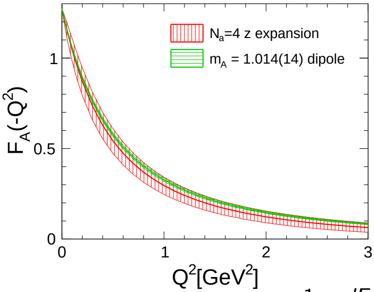
The $z$ Expansion [arXiv:1108.0423[hep-ph]] is a conformal mapping which takes kinematically allowed region ($t = -Q^2 \leq 0$) to within $|z| < 1$

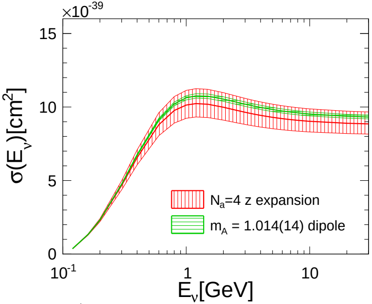
$$z(t; t_0, t_c) = \frac{\sqrt{t_c - t} - \sqrt{t_c - t_0}}{\sqrt{t_c - t} + \sqrt{t_c - t_0}} \qquad F_A(z) = \sum_{n=0}^{\infty} a_n z^n \qquad t_c = 9m_\pi^2$$




- Model independent: motivated by analyticity arguments from QCD
- Only few parameters needed: unitarity bounds
- Sum rules regulate large-$Q^2$ behavior

# Reanalysis Results Summary [arXiv:1603.03048[hep-ph]]




$$\frac{1}{F_A(0)} \frac{dF_A}{dQ^2}\bigg|_{Q^2=0} \equiv -\frac{1}{6} r_A^2$$

$$r_A^2 = 0.46(22)\ \mathrm{fm}^2\,, \quad \sigma_{\nu n \to \mu p}(E_\nu = 1\ \mathrm{GeV}) = 10.1(0.9) \times 10^{-39} \mathrm{cm}^2$$

compared to [Eur. Phys. J. C 53, 349]:

$$r_A^2 = 0.453(13)\ \mathrm{fm}^2\,, \quad \sigma_{\nu n \to \mu p}(E_\nu = 1\ \mathrm{GeV}) = 10.63(0.14) \times 10^{-39} \mathrm{cm}^2$$

Dipole model significantly underestimates error from nucleon form factor

Most theoretically clean data do not constrain form factor precisely

# Staggered Baryons

w/ Y.Lin, C.Hughes, A.Kronfeld, J.Simone, A.Strelchenko

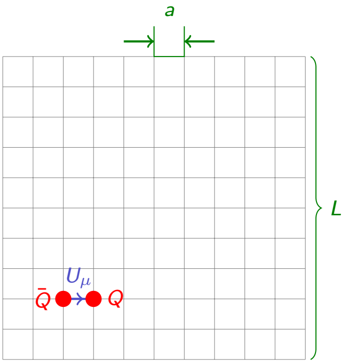
# Lattice QCD: Formalism

- Lattice QCD is a technique to numerically evaluate path integral

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \, \mathcal{D}U \, \exp(-S) \, \mathcal{O}_\psi \, [U]$$

- Discretize spacetime $\Longrightarrow$ #DOF finite
- Lattice spacing $a$ provides UV cutoff
- Lattice size $L$ provides IR cutoff

- Quark fields on sites $\Longrightarrow$ $Q(x)$
- Gauge fields between sites $\Longrightarrow$ $U_\mu(x)$
- Euclidean time $\Longrightarrow$ correlators $\propto e^{-Et}$




Typical strategy is to construct operators at "source," allow them to propagate through time, then annihilate at "sink"

Evaluate correlation functions on fixed background gauge field, compute on many gauge fields for Monte Carlo average

Correlation functions are products of matrix elements times exponentials, e.g.

$$C(t) = \sum_n |\langle \Omega | \, \mathcal{O} \, | n \rangle|^2 \, e^{-E_n t}$$

# Fermilab Lattice/MILC Effort

We are calculating the axial charge $g_A = F_A(Q^2)\big|_{Q^2=0}$ using staggered quarks on the MILC HISQ 2+1+1 gauge ensembles

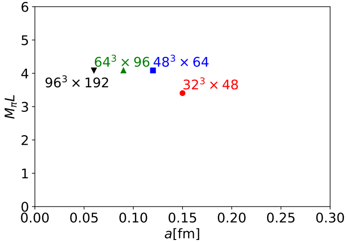
- no explicit chiral symmetry breaking in $m \to 0$ limit
- physical pion mass for multiple lattice spacings
- large volumes
- absolutely normalized
- high-statistics (computationally fast)

Effort is needed to handle:

- Formalism $\implies$ complicated group theory, difficult fitting

After completing the charge, we will continue to $F_A(Q^2)$ for $Q^2 > 0$

Data today will include nucleon mass calculation only



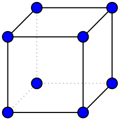
# Group Theory

Staggered lattice group is: $(((\mathcal{T}_M \times \mathbb{Q}_8) \rtimes W_3) \times D_4)/\mathbb{Z}_2$

Extra taste symmetry can be used to symmetrize baryon wavefunctions
$\implies$ Multiple baryon "tastes" show up in correlation functions

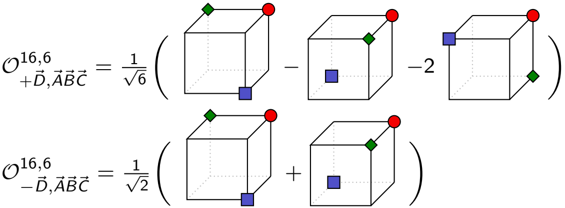
Spin and taste symmetries are mixed, spin components distributed over lattice sites

| Irrep | $I=\frac{3}{2}$ | $I=\frac{1}{2}$ |
|---|---|---|
| 8 | $3N+2\Delta$ | $5N+1\Delta$ |
| $8'$ | $0N+2\Delta$ | $0N+1\Delta$ |
| 16 | $1N+3\Delta$ | $3N+4\Delta$ |



Spin-taste components distributed over unit cube, alternate over every other site

Operators constructed from relative displacement of quarks within unit cube, e.g.



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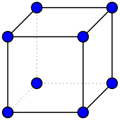
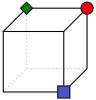
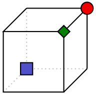
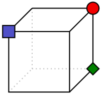
| 16 | $1N + 3\Delta$ | $3N + 4\Delta$ |

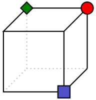
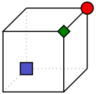
Focus on this irrep



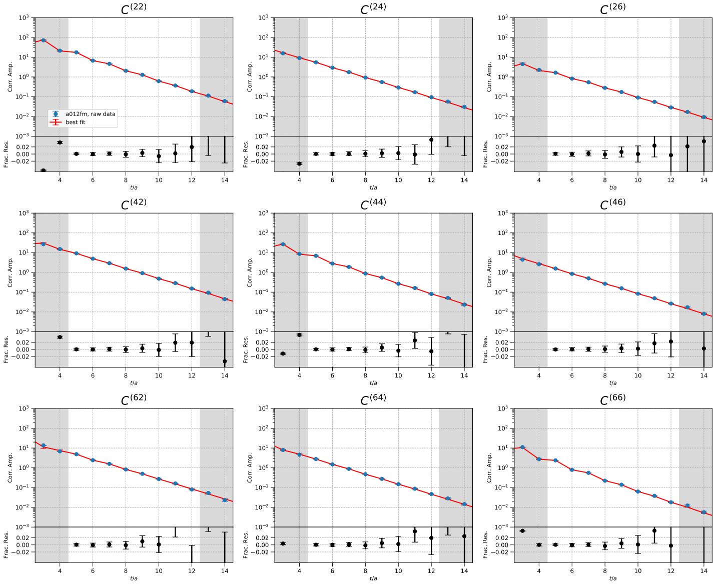
Spin-taste components distributed over unit cube, alternate over every other site

Operators constructed from relative displacement of quarks within unit cube, e.g.

$\mathcal{O}^{16,6}_{+\vec{D},\vec{A}\vec{B}\vec{C}} = \frac{1}{\sqrt{6}}\Big($  $-$  $-2$  $\Big)$

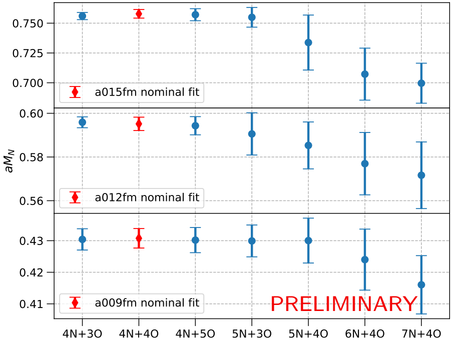
$\mathcal{O}^{16,6}_{-\vec{D},\vec{A}\vec{B}\vec{C}} = \frac{1}{\sqrt{2}}\Big($  $+$  $\Big)$

# Correlation Functions & Residuals




3 × 3 basis of correlation functions, simultaneous fits to all

# Systematics Checks - $N_{\text{state}}$ Stability



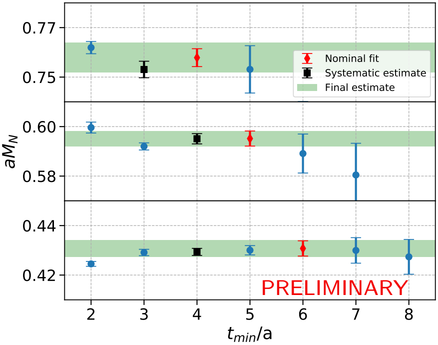

Correlated fits with priors in principle can take arbitrary number of fit states

In practice, chaning number of priors can reassociate contributions to fit posteriors

Check stability with increasing number of non-oscillating ($N$) and oscillating ($O$) states

Fits stable for small shifts in number of fit states

# Systematics Checks - $t_{\text{min}}$ Stability



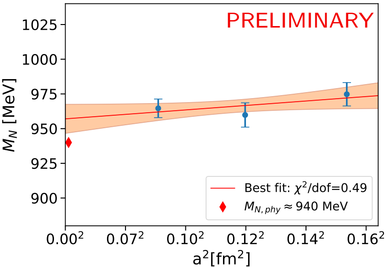

Correlators $\propto e^{-E_n t} \implies$ tests for sensitivity to larger $E_n$

Nominal fit fixed with roughly same physical source-sink separation, $\sim 0.6\ \mathrm{fm}$

Systematic uncertainty from excited states estimated by comparing
to timeslice at around $\sim 0.45\ \mathrm{fm}$

Difference of timeslice central values added in quadrature with statistical error

# Continuum Extrapolation




Fit assuming $\Lambda_{QCD} = 200$ MeV and form

$$M_N(a) = M_{N,\mathrm{phys}} \left(1 + o_2(\Lambda_{QCD} a)^2 + o_4(\Lambda_{QCD} a)^4\right)$$

| ensemble | a015fm | a012fm | a009fm |
|---|---|---|---|
| $aM_N$ | $0.7579(48)_{\mathrm{fit}}(36)_{\mathrm{stat}}$ | $0.5952(1)_{\mathrm{fit}}(31)_{\mathrm{stat}}$ | $0.4308(14)_{\mathrm{fit}}(31)_{\mathrm{stat}}$ |

| Parameter | Prior | Posterior |
|---|---|---|
| $M_{N,\mathrm{phys}}[\mathrm{MeV}]$ | 940(50) | 957(10) |
| $o_2$ | unconstrained | 0.65(69) |
| $o_4$ | 0.0(1.0) | 0.004(1.000) |

$\Longrightarrow$ $1.5\sigma$ consistency with nucleon mass

$\Longrightarrow$ Driven up by $a = 0.09$ fm ensemble: needed another for control!

# Distillation

w/ M.Bruno, T.Izubuchi, C.Lehner

# Distillation Nucleon Correlators [arXiv:0905.2160 [hep-lat]]

Projection matrices constructed from eigenvectors of Laplacian operator

$$\mathcal{P}^{ab}_{xy} = \sum_i \langle x|i^a\rangle \langle i^b|y\rangle$$

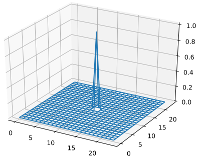
Project propagators down to perambulators: $M^{ji} = \langle j|\, D^{-1}\, |i\rangle$

To contract into baryons, construct endcaps from antisymmetrization:
$S_{ijk} = \sum_{abc,\vec{x}} \epsilon^{abc} \langle x|i^a\rangle \langle x|j^b\rangle \langle x|k^c\rangle$

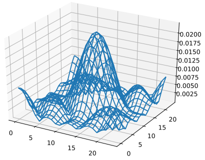
Two-point baryon correlation functions from contracting perambulators with endcaps:

$$C(t) = \sum_{ijk\overline{i}\overline{j}\overline{k}} S_{ijk}(t) \left(S_{\overline{i}\overline{j}\overline{k}}(0)\right)^* M_{i\bar{i}}(t,0) M_{j\bar{j}}(t,0) M_{k\bar{k}}(t,0)\,,$$

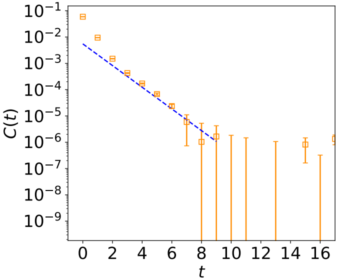
Orthogonality of eigenvectors used in place of lattice site index
$\Longrightarrow$ significantly reduced computational burden
$\Longrightarrow$ ideal for creating $N\pi$ multiparticle correlation functions




$\Longrightarrow$



# Distillation Nucleon Correlators




Initial study of baryon correlation functions on $24^3 \times 64$, $a \approx 0.20$ fm ensemble

Blue dashed line is to guide the eye, not fit

Proof of principle, more study is needed

# Summary

- Precise determinations of nucleon form factors are an essential part of the long-baseline neutrino oscillation program
- Dipole shape underestimates uncertainties in free-nucleon cross sections
- Need robust determination of nucleon amplitudes with realistic errors to determine impact on future neutrino oscillation experiments
- $z$ Expansion parameterization is a choice consistent with QCD and sufficiently general to give realistic uncertainty estimates

- Lattice QCD can access nucleon form factors from first principles
- Growing interest in neutrino physics within lattice community and vice versa, can expect many new results in upcoming years

- World-first staggered baryon mass result expected to appear next month, computation of $g_A$ with staggered baryons to follow
- Initiating study of nucleon matrix elements with distillation to study excited state contamination in correlation functions

Thanks for listening!

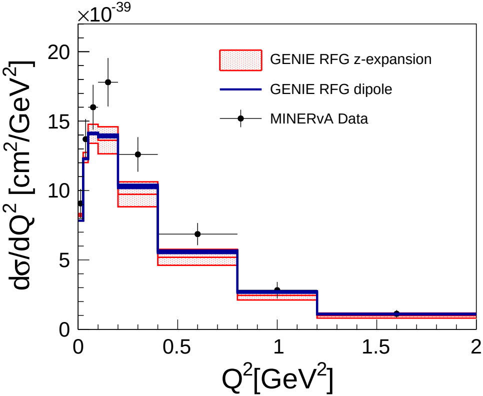
# BACKUP

# $z$ Expansion in GENIE

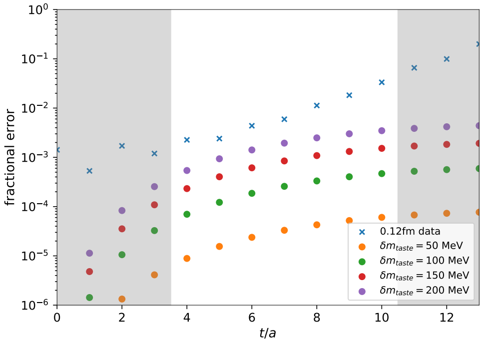
$z$ expansion coded into GENIE - may be turned on with configuration switch

Officially released in production version 2.12

Uncertainties on free-nucleon cross section as large as data-theory discrepancy
$\implies$ need to improve $F_A$ determination to make headway on nuclear effects




See tutorial: https://indico.fnal.gov/event/12824/



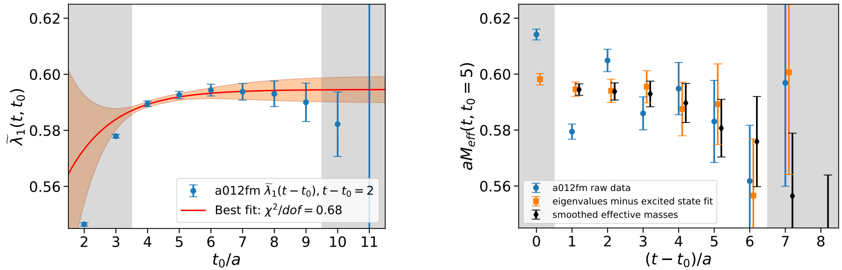

Not enough precision to resolve all taste states in spectrum

Assuming $\mathcal{O}(1)$ coefficients on all states, contributions absorbed into "effective" states

As a result, some priors neglected in favor of fewer parameters

$\Longrightarrow$ Improved fit stability when using fewer priors than states

# GEVP with Staggered Baryons




$$\frac{1}{4}\Big\{[\mathbf{C}(t_0-1)]^{-1}\mathbf{C}(t-1) + 2[\mathbf{C}(t_0)]^{-1}\mathbf{C}(t) + [\mathbf{C}(t_0+1)]^{-1}\mathbf{C}(t+1)\Big\} v_i^R(t) = \widetilde{\lambda}_i(t,t_0) v_i^R(t)$$

$$-\frac{\ln \widetilde{\lambda}_i(t,t_0)}{\tau} = M_i + C_i e^{-\widetilde{\delta M_i} t_0}$$

| ensemble | $\widetilde{\lambda}_1$ fit | $\lambda_1$ fit | Bayesian fit |
|---|---|---|---|
| a015fm | $0.7555(59)_{\text{fit}}(22)_{\text{stat}}$ | $0.7562(9)_{\text{fit}}(25)_{\text{stat}}$ | $0.7579(48)_{\text{fit}}(36)_{\text{stat}}$ |
| a012fm | $0.5946(22)_{\text{fit}}(48)_{\text{stat}}$ | $0.5945(13)_{\text{fit}}(29)_{\text{stat}}$ | $0.5952(1)_{\text{fit}}(31)_{\text{stat}}$ |
| a009fm | $0.4295(8)_{\text{fit}}(26)_{\text{stat}}$ | $0.4307(2)_{\text{fit}}(34)_{\text{stat}}$ | $0.4308(14)_{\text{fit}}(31)_{\text{stat}}$ |