The nucleon electromagnetic form factors at high momentum transfer from Lattice QCD

based on C. Kallidonis et al. [arXiv: 1810.04294]

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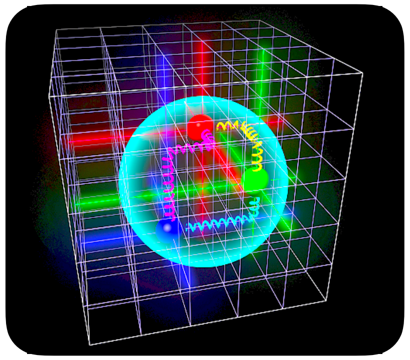

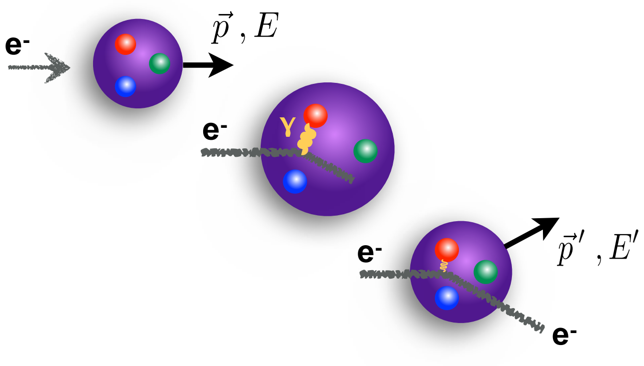

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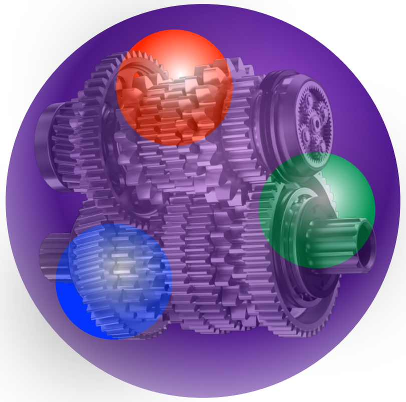
Outline

- Introduction - Motivation
- Methodology - Matrix elements
- Results
    - F_2^p / F_1^p
    - G_E^p / G_M^p
    - G_E^n / G_M^n
    - $F_{1,2}^{p,n}, F_{1,2}^{u,d}$
- Systematics
- Conclusions

Introduction - Motivation

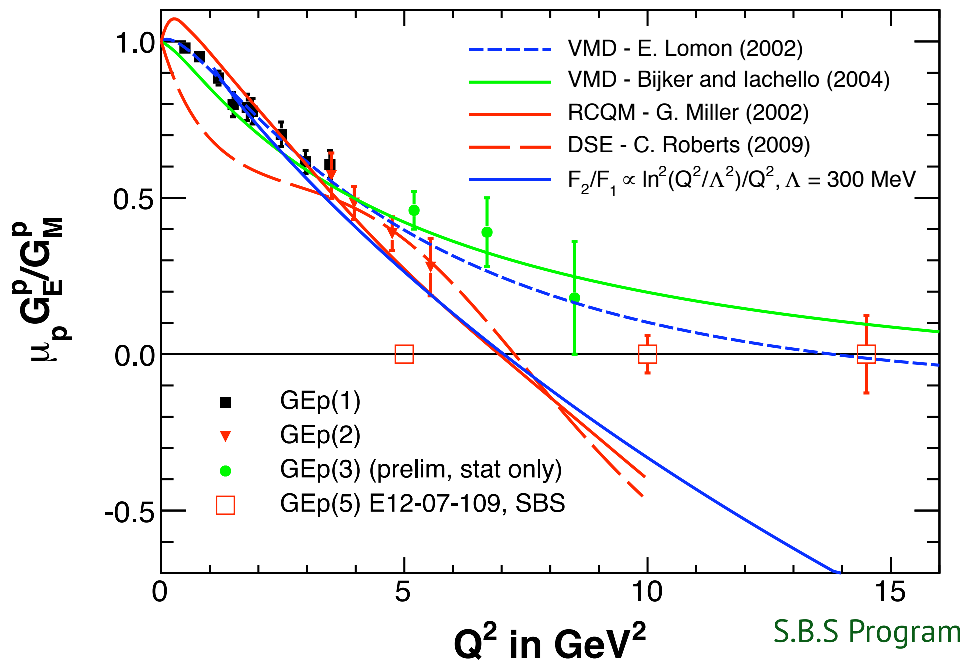
Nucleon electric and magnetic form factors are important probes of its internal structure

- description of the spatial distributions of electric charge and magnetization
- value at $G_E(Q^2 = 0)$, $G_M(Q^2 = 0)$: electric charge, magnetic moment
- slope of G_E, G_M at $Q^2 = 0$: electric and magnetic radius
- high-momentum regime required in effort for complete picture



High-momentum transfer calculation on the lattice:

- "bypass" experimental difficulties
- role of pQCD predictions in understanding of FFs?
- are various quark models, phenomenology believable?
- input to DVCS measurements, probing GPDs
- nucleon FFs: good framework to test high-momentum region on the lattice

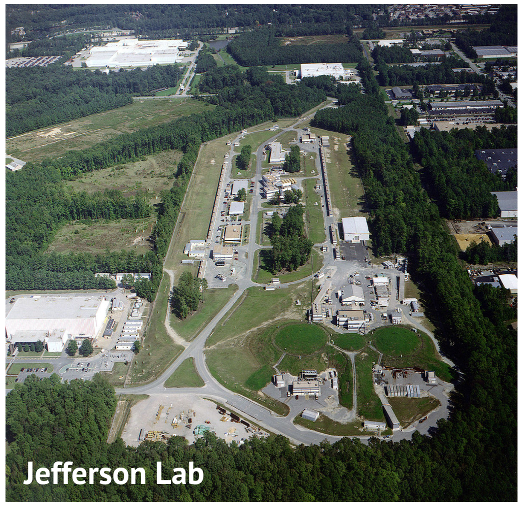


S.B.S Program report

Introduction - Motivation

Current experimental status:

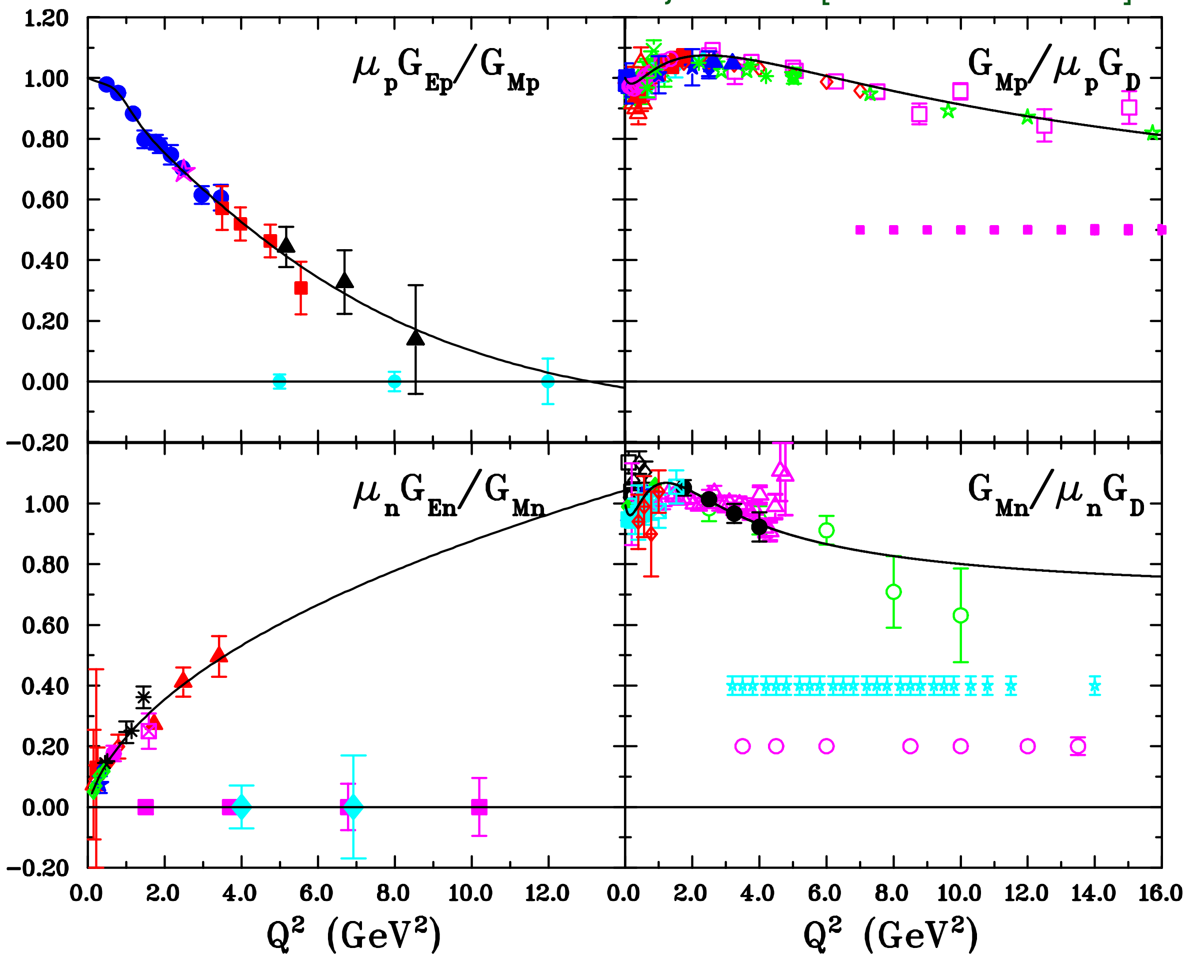
- proton: up to $Q^2 \sim 8.5$ GeV^2
- neutron: up to $Q^2 \sim 3.4$ GeV^2



Rich experimental activity @ CEBAF, JLab

- high-resolution spectrometer, Hall A
  - G_M^p up to $Q^2 = 17.5$ GeV^2
- Halls B, C
  - G_M^n up to $Q^2 = 14$ GeV^2
  - G_E^n/G_M^n up to $Q^2 = 6.9$ GeV^2
- new Super-BigBite Spectrometer, Hall A
  - G_E^p/G_M^p up to $Q^2 = 15$ GeV^2
  - G_E^n/G_M^n up to $Q^2 = 10.2$ GeV^2
  - G_M^n up to $Q^2 = 18$ GeV^2

V. Punjabi et al. [arXiv: 1503.01452]



Simulation details

- two Nf=2+1 Wilson-clover ensembles, produced by JLab/W&M lattice group
- different lattice volumes, similar lattice spacing

D5-ensemble: $\beta = 6.3$, $a = 0.094$ fm, $a^{-1} = 2.10$ GeV		
$32^3 \times 64$, $L = 3.01$ fm	$a\mu_l$	-0.2390
	$a\mu_s$	-0.2050
	κ	0.132943
	$C_{\rm sw}$	1.205366
	m_π (MeV)	280
	$m_\pi L$	4.26
	Statistics	86144
D6-ensemble: $\beta = 6.3$, $a = 0.091$ fm, $a^{-1} = 2.17$ GeV		
$48^3 \times 96$, $L = 4.37$ fm	$a\mu_l$	-0.2416
	$a\mu_s$	-0.2050
	κ	0.133035
	$C_{\rm sw}$	1.205366
	m_π (MeV)	170
	$m_\pi L$	3.76
	Statistics	50176

- Computational resources: BNL Institutional Cluster, USQCD 2017 allocation
- Calculation: Qlua interface: QUDA-MG for propagators, contractions on GPU within QUDA

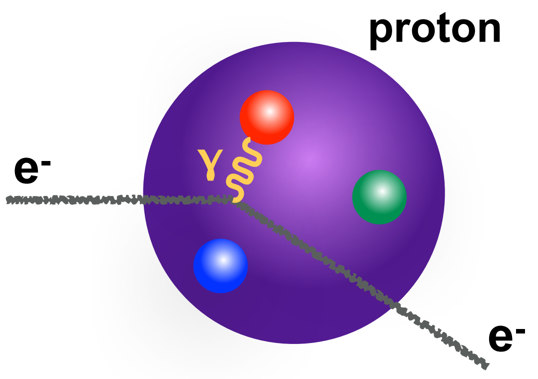
A.V. Pochinksy S. Syritsyn, C.K.

Correlation functions

Matrix element of the vector current: $\mathcal{V}_\mu(x) = \bar{\psi}(x)\gamma_\mu\psi(x)$

$$\langle N(p',s)|\mathcal{V}_\mu|N(p,s)\rangle = \bar{u}_N(p',s)\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2(q^2)\right]u_N(p,s)$$

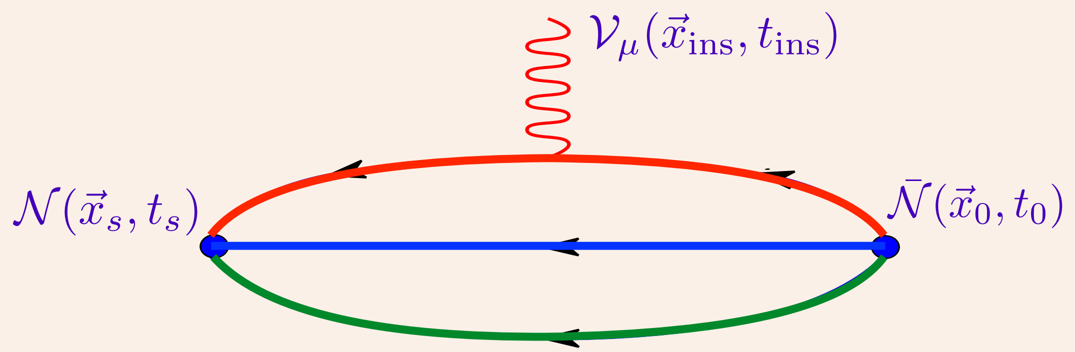
Dirac FF (F_1) — Pauli FF (F_2)



Sachs Form Factors:

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{(2m_N)^2}F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

Three-point function

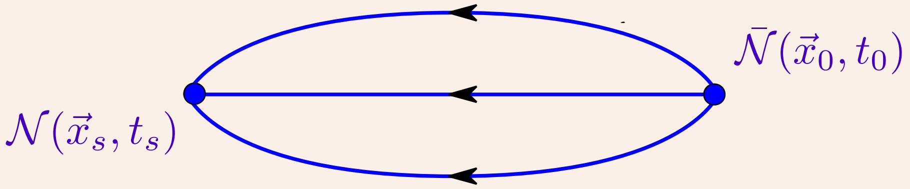


- seq. propagators: inversion through sink
- $(t_s - t_0) \sim 0.55$ fm - 0.95 fm
- consider only connected contributions

$$G_\mu(\Gamma, \vec{p}', \vec{q}, t_s, t_{\rm ins}) = \sum_{\vec{x}_s, \vec{x}_{\rm ins}} e^{-i\vec{p}'\cdot(\vec{x}_s - \vec{x}_0)} e^{i\vec{q}\cdot(\vec{x}_{\rm ins} - \vec{x}_0)} \Gamma_{\beta\alpha} \langle \mathcal{N}_\alpha(\vec{x}_s, t_s) \mathcal{V}_\mu(\vec{x}_{\rm ins}, t_{\rm ins}) \bar{\mathcal{N}}_\beta(\vec{x}_0, t_0)\rangle$$

Two-point function

$$C(\vec{p}', t_s) = \sum_{\vec{x}_s} e^{-i\vec{p}'\cdot(\vec{x}_s - \vec{x}_0)} (\Gamma_4)_{\beta\alpha} \langle \mathcal{N}_\alpha(\vec{x}_s, t_s) \bar{\mathcal{N}}_\beta(\vec{x}_0, t_0)\rangle$$



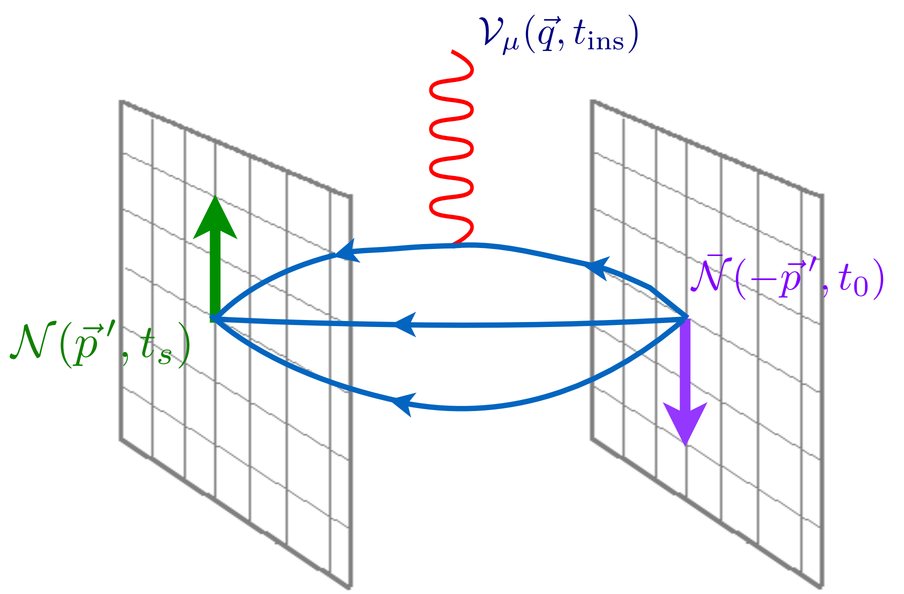
we incorporate **boosted** nucleon states to access the high-Q^2 region, keeping the energy of the states as low as possible

$$\mathcal{N}_\alpha(\vec{p}\,', t) = \sum_{\vec{x}} \epsilon^{abc} \left[u^a_\mu(x)(C\gamma_5)_{\mu\nu} d^b_\nu(x)\right] u^c_\alpha(x) e^{-i\vec{p}\,'\cdot\vec{x}}$$

$$Q^2 = (\vec{p} - \vec{p}\,')^2 - (E - E')^2$$

Breit frame: $\vec{p} = -\vec{p}\,'$, $E = E'$ → $Q^2 = 4\vec{p}^{\,2}$

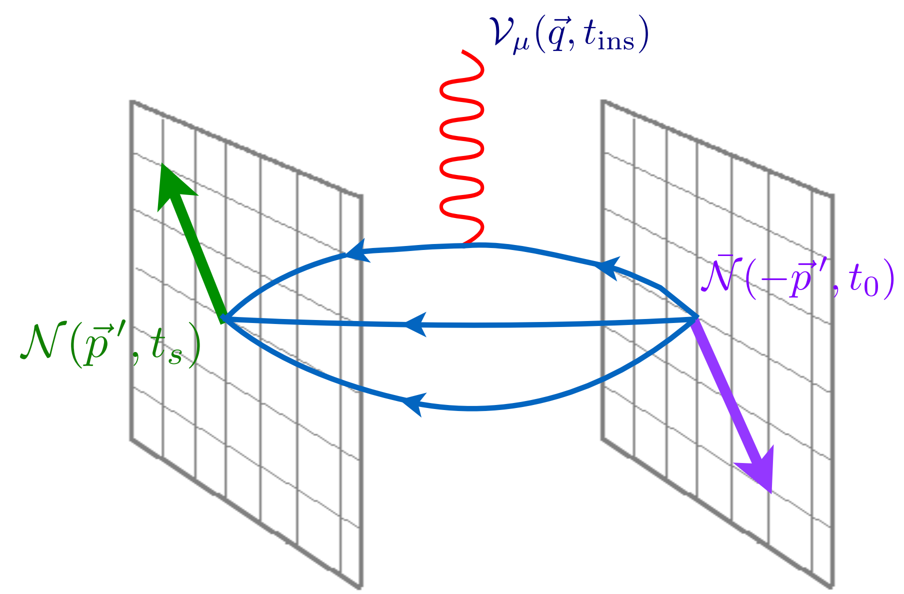
- boosting in single direction



D5 → $\vec{P}\,' = (-4,0,0) \to Q^2 \sim 10.9\ \mathrm{GeV}^2$

D6 → $\vec{P}\,' = (-5,0,0) \to Q^2 \sim 8.1\ \mathrm{GeV}^2$

- diagonal boosting in x-y plane



D5 → $\vec{P}\,' = (-3,-3,0) \to Q^2 \sim 12.2\ \mathrm{GeV}^2$

Gaussian "momentum" smearing:

$$\mathcal{S}_{\vec{k}_b}\psi(x) \equiv \frac{1}{1+6\alpha}\left[\psi(x) + \alpha \sum_{\mu=\pm1\ldots}^{3} U_\mu(x) e^{i\vec{k}_b\cdot\hat{\mu}}\psi(x+\hat{\mu})\right]$$

$\vec{k}_b = 0.5\,\vec{p}\,'$

G. Bali et al. [arXiv: 1602.05525]

Need to make sure that we get the nucleon ground state

Ratio of 2pt and 3pt functions

$$R^{\mu}(\Gamma,\vec{q},\vec{p}';t_s,t_{\rm ins}) = \frac{G_\mu(\Gamma,\vec{p}',\vec{q},t_s,t_{\rm ins})}{C(\vec{p}',t_s-t_0)} \times \sqrt{\frac{C(\vec{p},t_s-t_{\rm ins})C(\vec{p}',t_{\rm ins}-t_0)C(\vec{p}',t_s-t_0)}{C(\vec{p}',t_s-t_{\rm ins})C(\vec{p},t_{\rm ins}-t_0)C(\vec{p},t_s-t_0)}}$$

Projectors:

polarized $\Gamma_k = i\gamma_5\gamma_k\Gamma_4$

unpolarized $\Gamma_4 = \frac{1+\gamma_4}{4}$

1. Plateau method: $R^{\mu} \xrightarrow[t_s-t_{\rm ins}\gg 1]{t_{\rm ins}-t_0\gg 1} \Pi^{\mu}(\Gamma,\vec{q})$

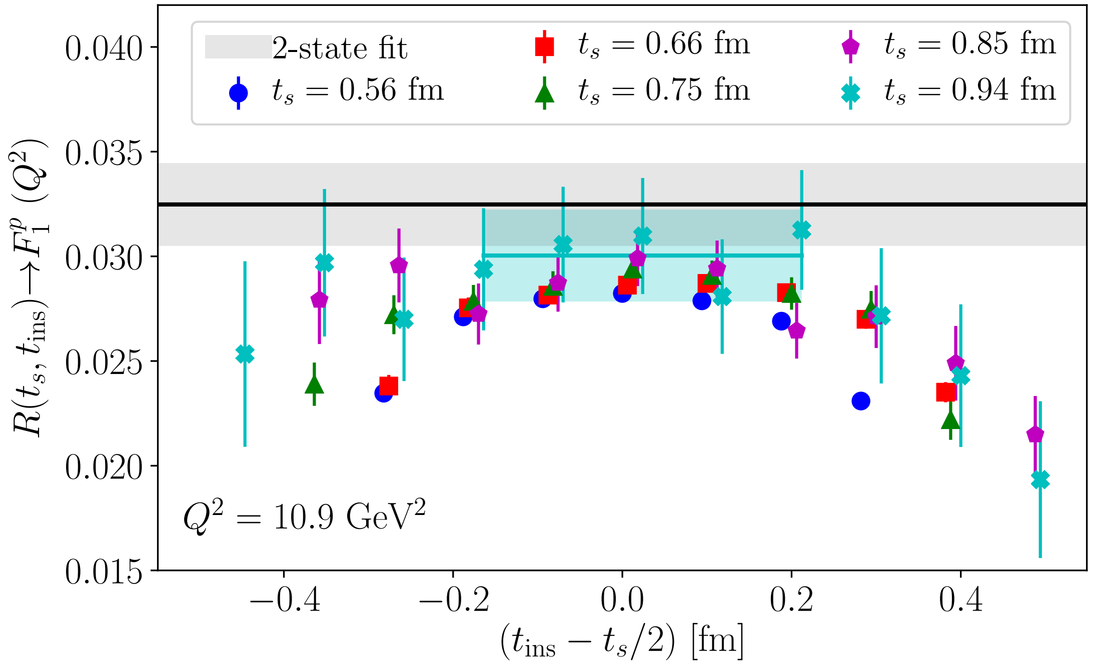
2. Two-state fit method:

$$C(\vec{p}',t_s) \simeq e^{-E(\vec{p}')t_s}\left[c_0(\vec{p}') + c_1(\vec{p}')e^{-\Delta E_1(\vec{p}')t_s}\right]$$

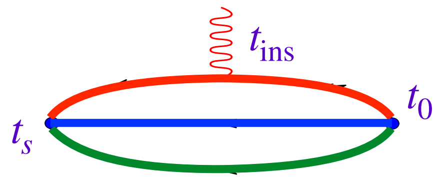
$$\begin{aligned} G_\mu(\Gamma,\vec{p}',\vec{p},t_s,t_{\rm ins}) &\simeq e^{-E_0(\vec{p}')(t_s-t_{\rm ins})}e^{-E_0(\vec{p})(t_{\rm ins}-t_0)} \times \\ \times\,[A_{00}(\vec{p},\vec{p}') &+ A_{01}(\vec{p},\vec{p}')e^{-\Delta E_1(\vec{p})(t_{\rm ins}-t_0)} + \\ &+ A_{10}(\vec{p},\vec{p}')e^{-\Delta E_1(\vec{p}')(t_s-t_{\rm ins})} + \\ &+ A_{11}(\vec{p},\vec{p}')e^{-\Delta E_1(\vec{p}')(t_s-t_{\rm ins})}e^{-\Delta E_1(\vec{p})(t_{\rm ins}-t_0)}] \end{aligned}$$

$$c_n(\vec{p}') = |\langle\mathcal{N}|n,\vec{p}'\rangle|^2/2E_n(\vec{p}')$$

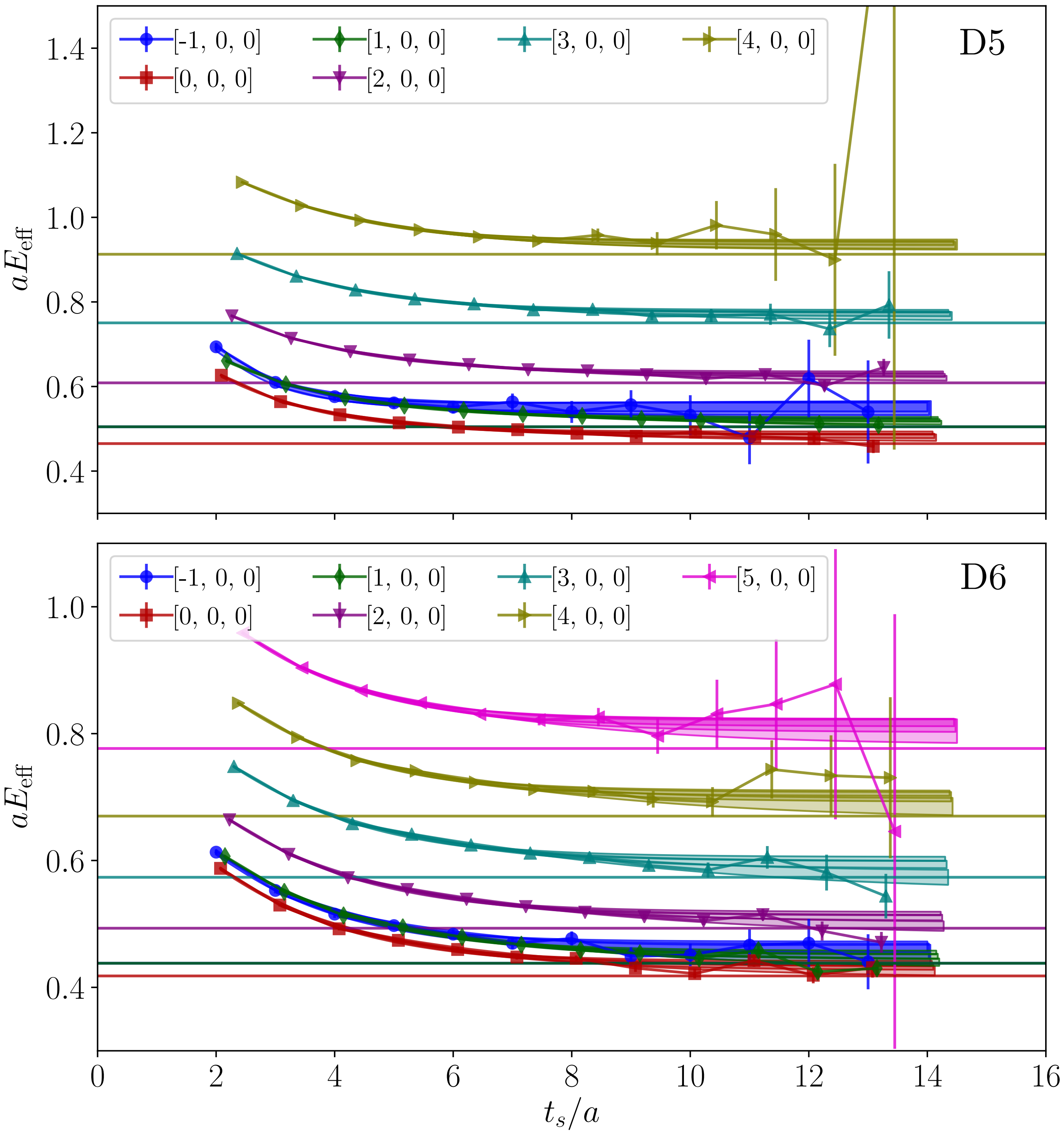
$$A_{nm}(\vec{p},\vec{p}') = \langle\mathcal{N}|n,\vec{p}'\rangle\langle m,\vec{p}|\mathcal{N}\rangle\langle n,\vec{p}'|\mathcal{V}_\mu|m,\vec{p}\rangle/[2\sqrt{E_n(\vec{p})E_n(\vec{p}')}]$$

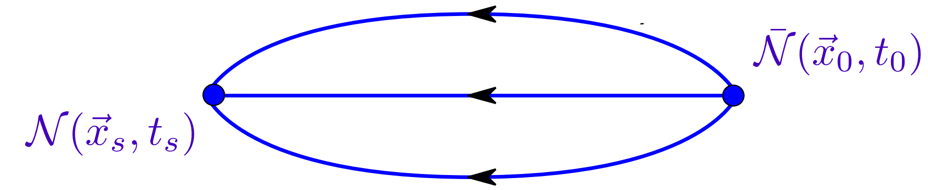


$$\langle 0,\vec{p}'|\mathcal{V}_\mu|0,\vec{p}\rangle = \frac{A_{00}(\vec{p},\vec{p}')}{\sqrt{c_0(\vec{p})c_0(\vec{p}')}}$$



Effective Energy

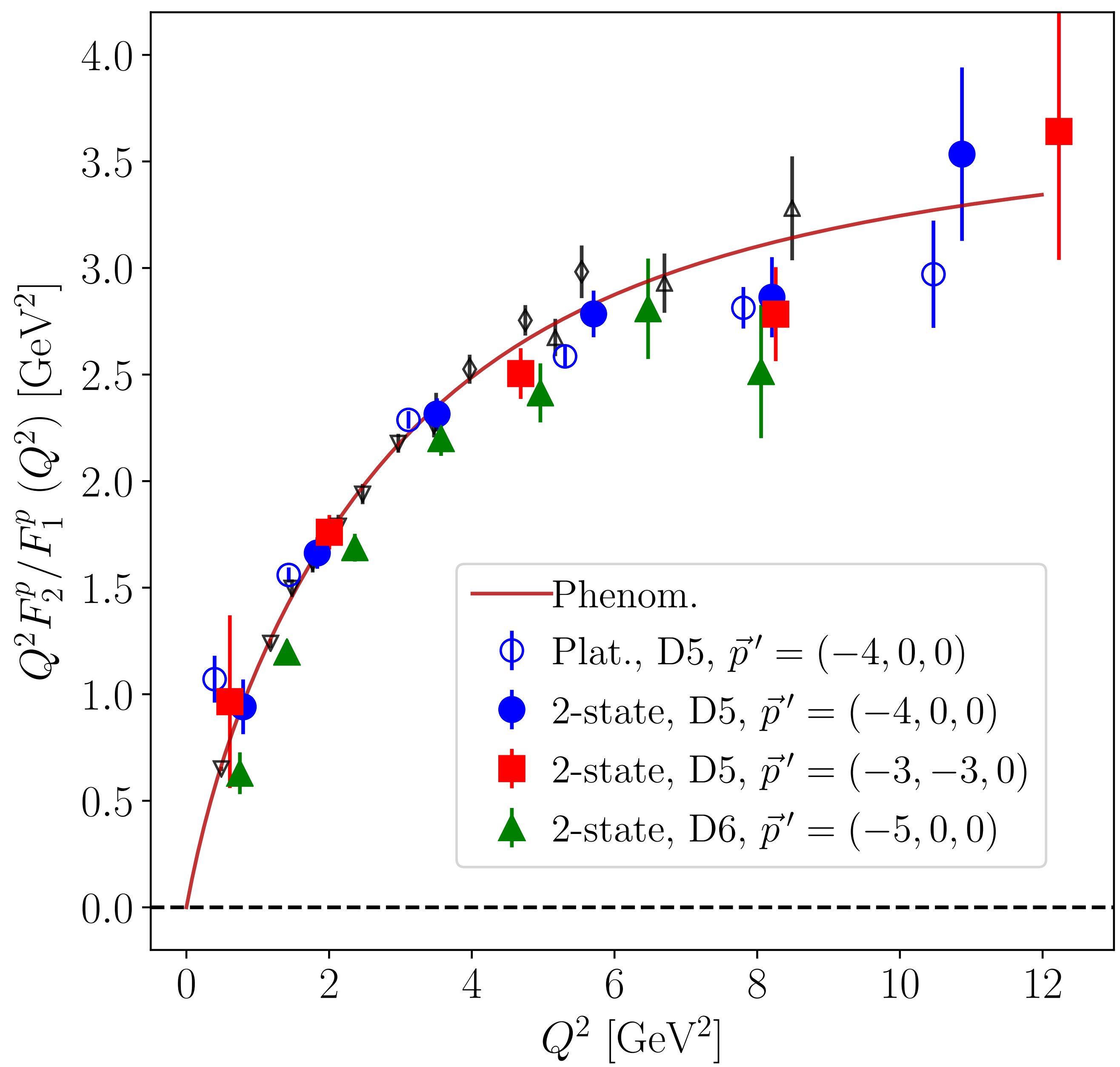




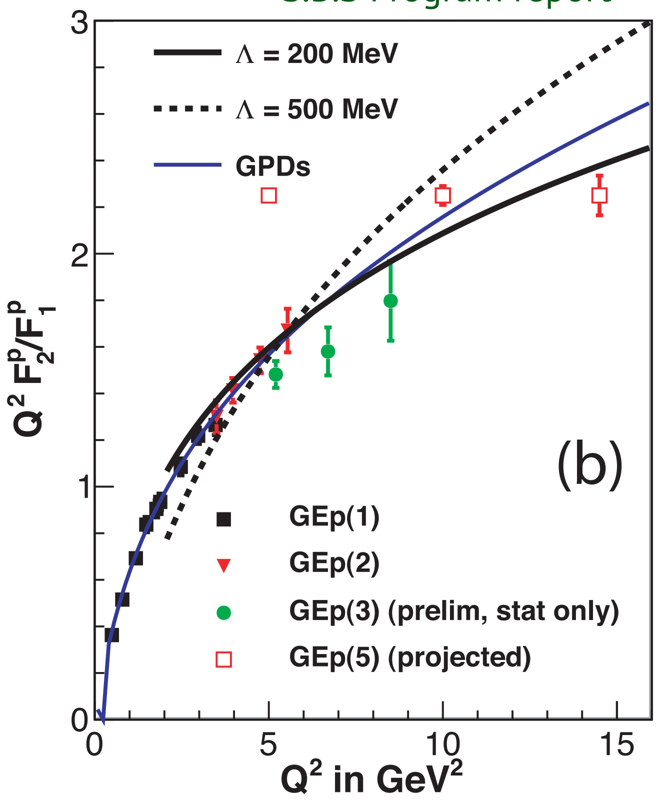
$$E^N_{\mathrm{eff}}(t_s) = \log\left(\frac{C(\vec{p}, t_s)}{C(\vec{p}, t_s + 1)}\right) \xrightarrow{t_s \gg 1} E_N$$

- two-state fits to our lattice data are of good quality
- horizontal line from $E = m_N^2 + p^2$ using lattice value of m_N
- ground state energy slightly overestimates cont. dispersion relation
- excited states faint after ~ $t_s/a = 9$

Form Factor Results I: F_2/F_1 for proton



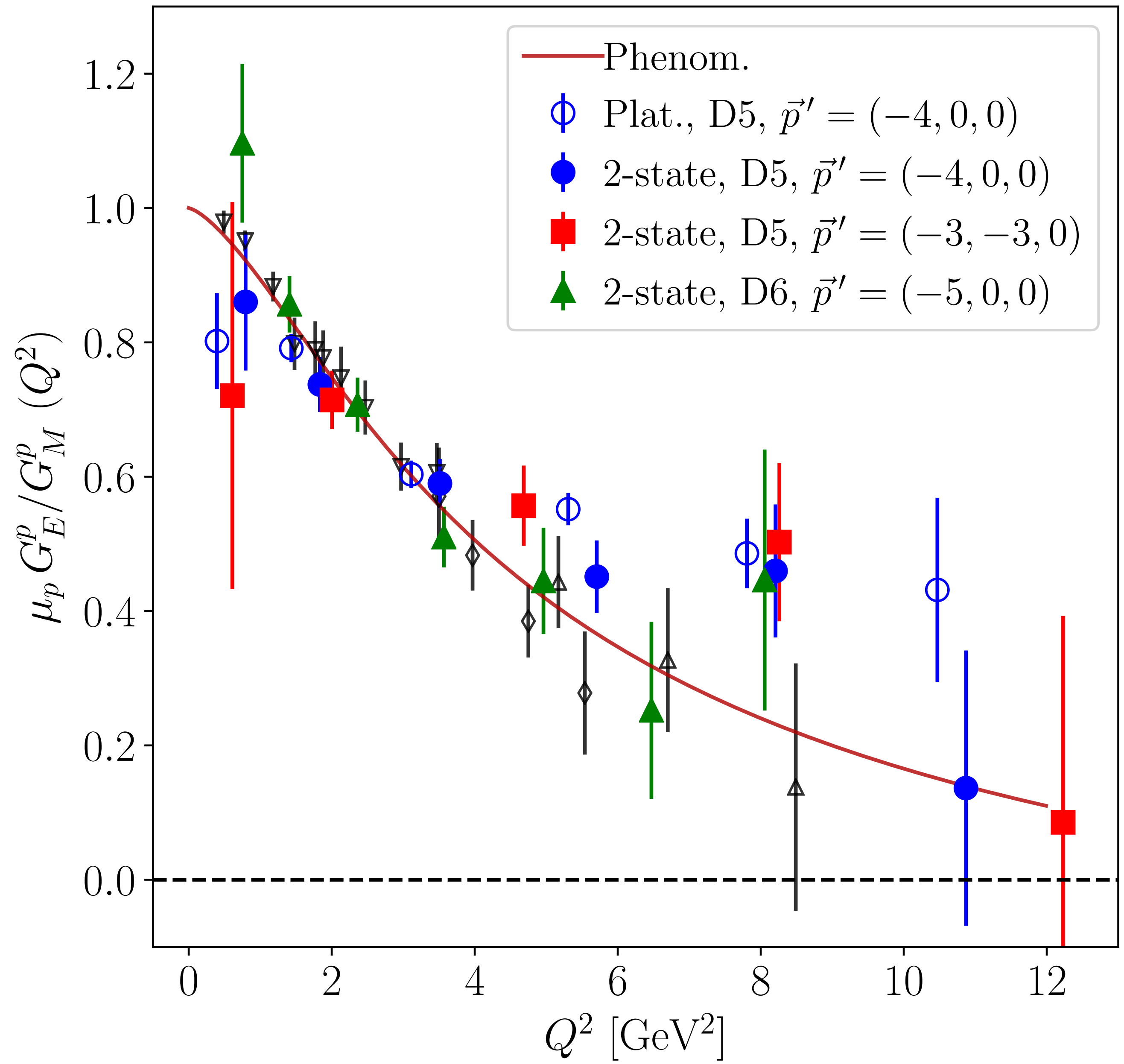
S.B.S Program report

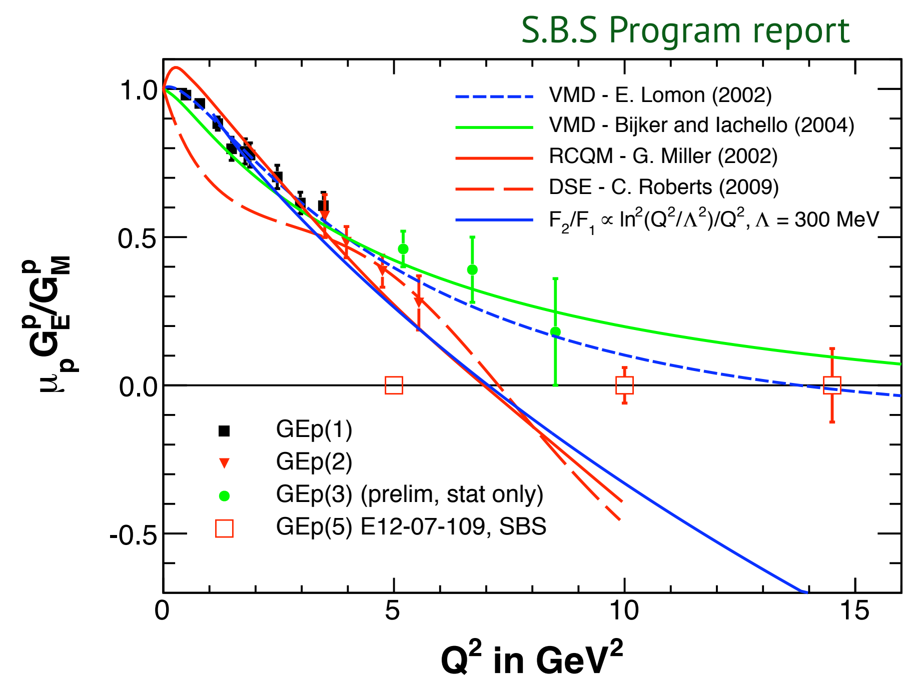


W. M. Alberico et al. [arXiv: 0812.3539]

- experimental data: up to Q^2 ~8.5 GeV^2
- Q^2 - dependence compares well with exp. data and phenom. parametrization
- $Q^2 F_2^p/F_1^p(Q^2) \sim \log^2[Q^2/\Lambda]$ scaling reproduced A.V. Belitsky et al. [arXiv: hep-ph/0212351]
- consistency between on-axis / x-y diagonal boost momentum for D5

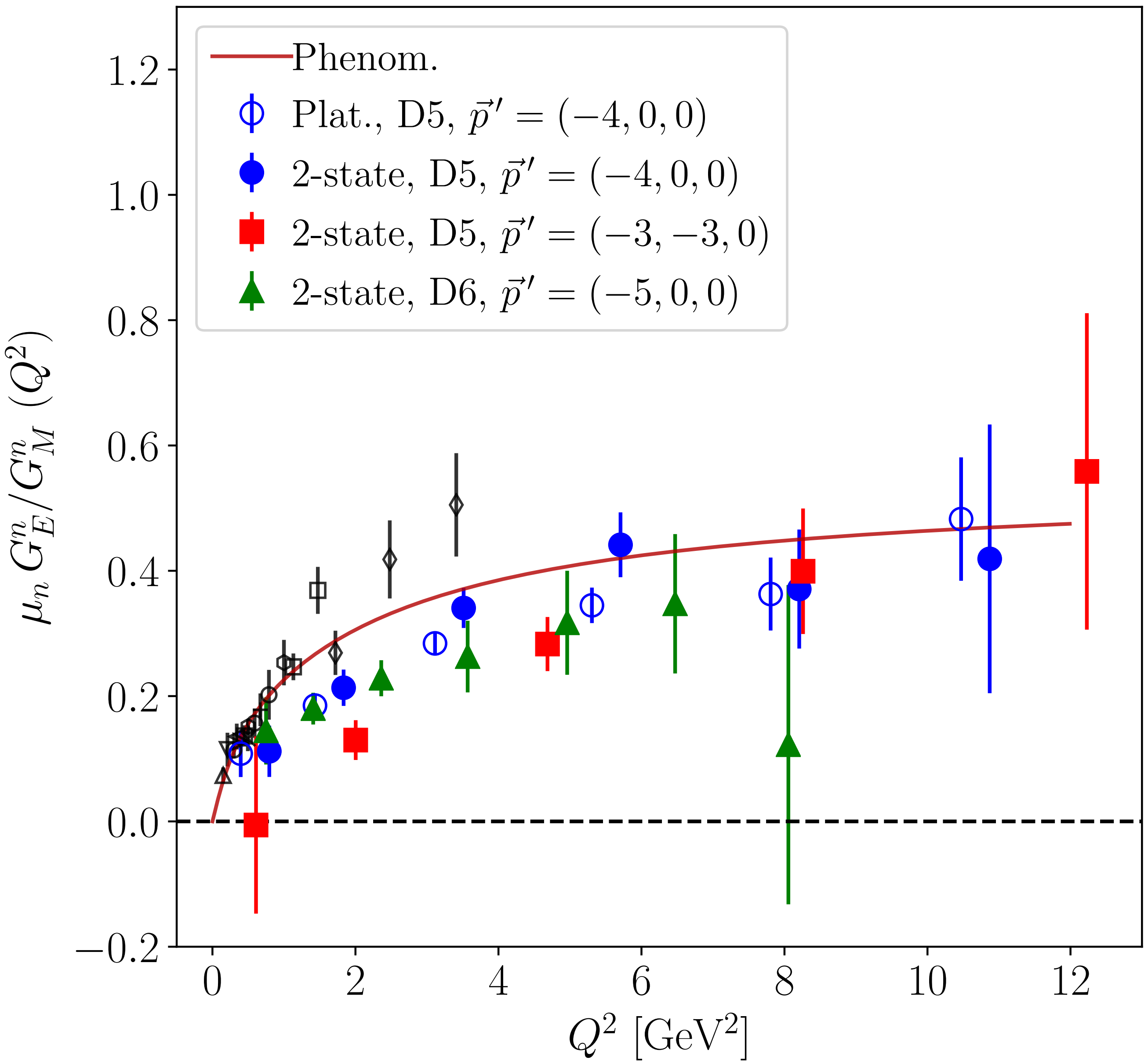
Form Factor Results II: G_E/G_M for proton

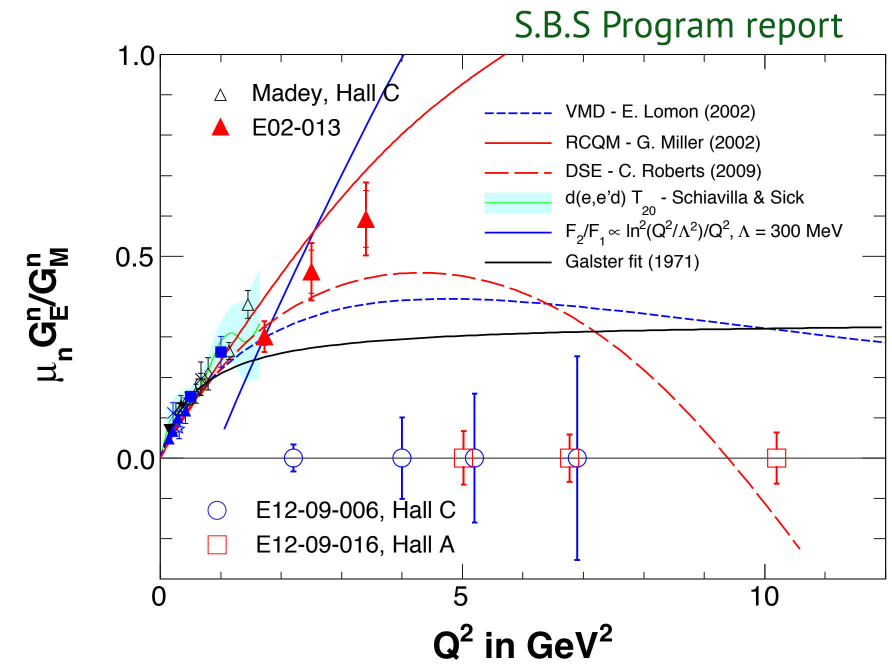




- experimental data: up to $Q^2 \sim 8.5\ \text{GeV}^2$
- consistency between our lattice data
- good agreement with experiment / phenomenology for proton up to $Q^2 \sim 6\ \text{GeV}^2$
- variety in theoretical predictions: lattice data support smoother approach towards zero

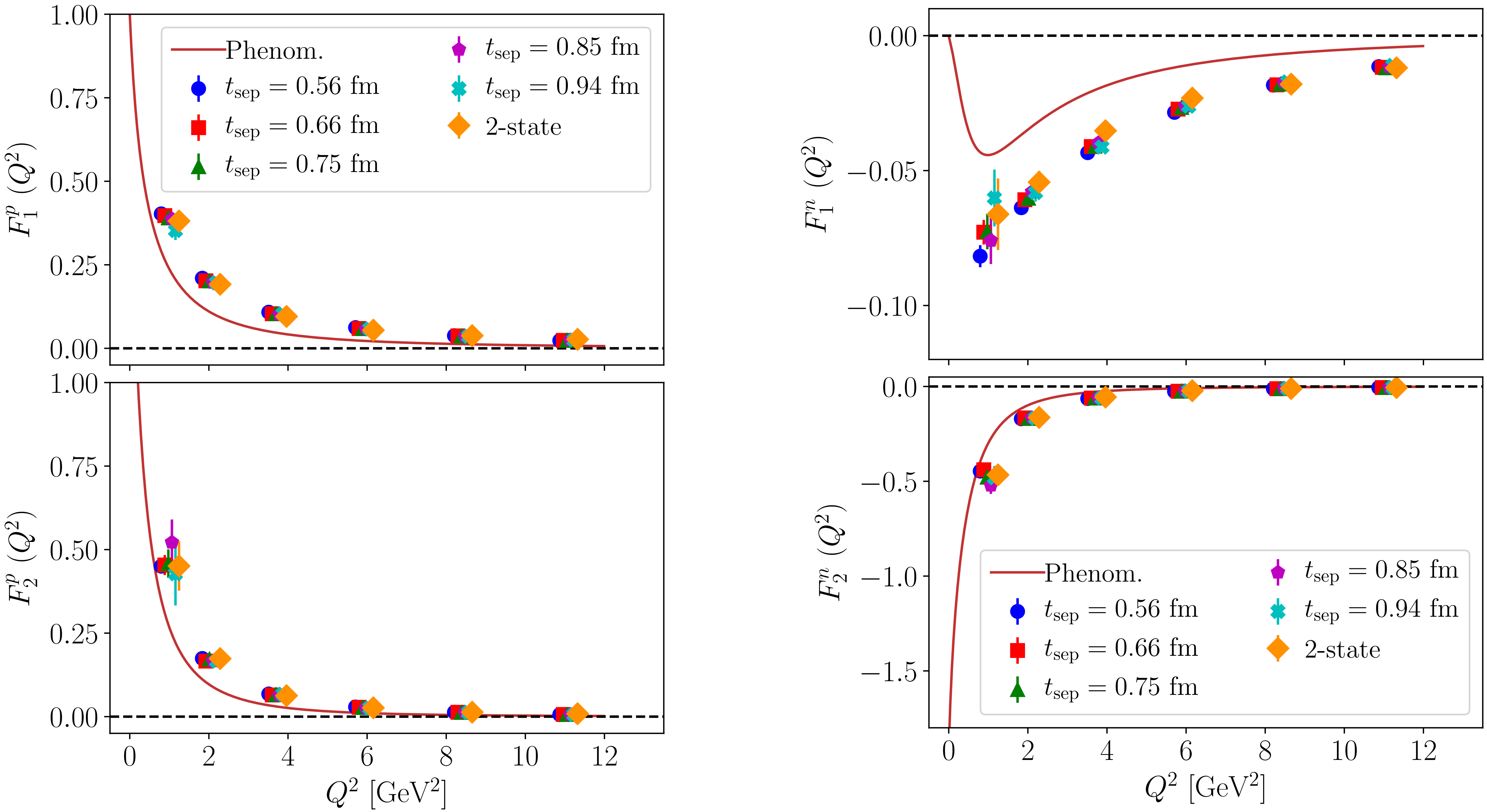
Form Factor Results III: G_E/G_M for neutron





- experimental data: up to $Q^2 \sim 3.4$ GeV^2
- neutron: out lattice data underestimate experiment / phenomenology
  - disconnected diagrams?
- same qualitative behavior

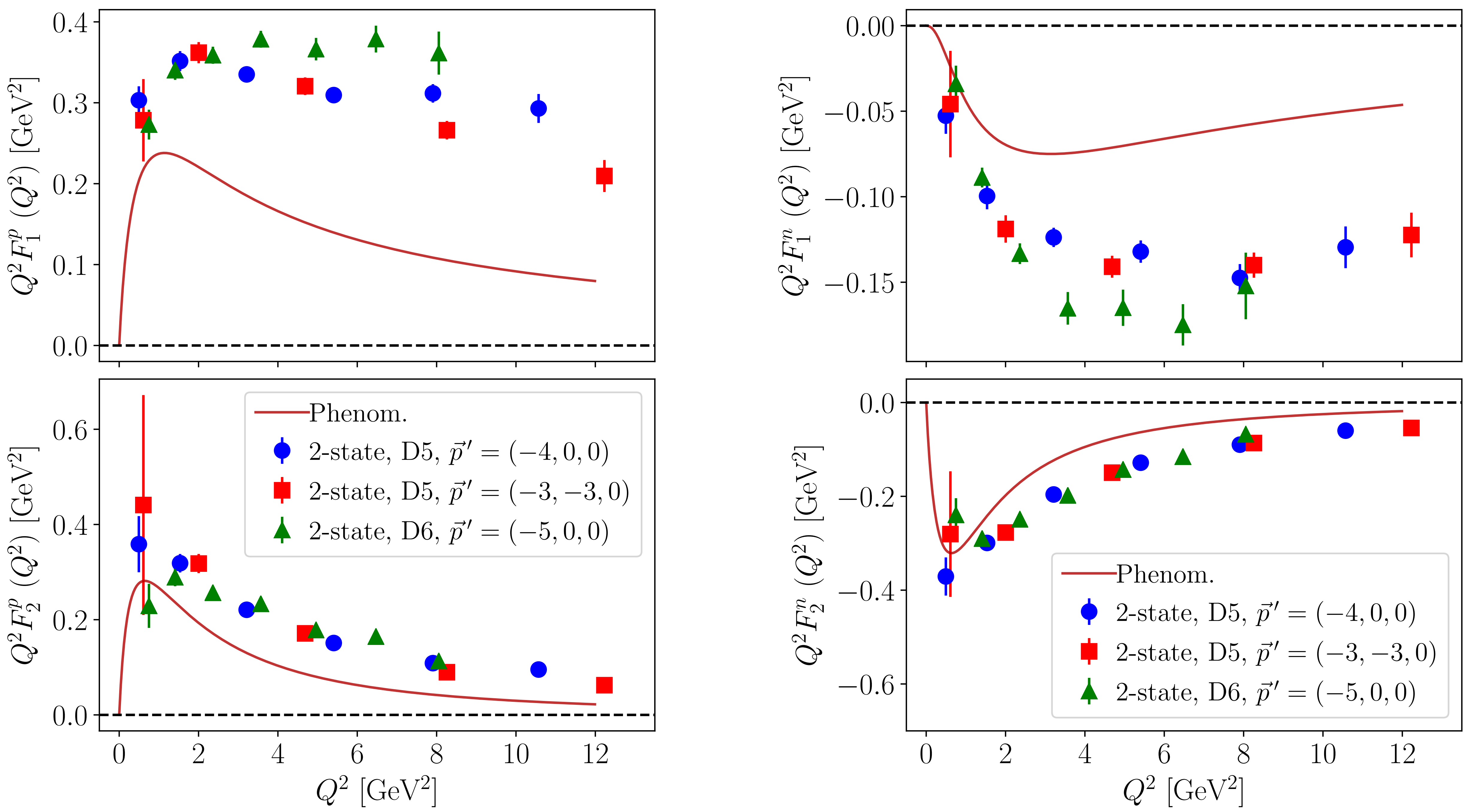
Form Factor Results IV: $F_1^{p,n}, F_2^{p,n}$



- shallow trend towards phenom. with increasing source-sink separation
- similar qualitative behavior, overestimation of phenom. prediction

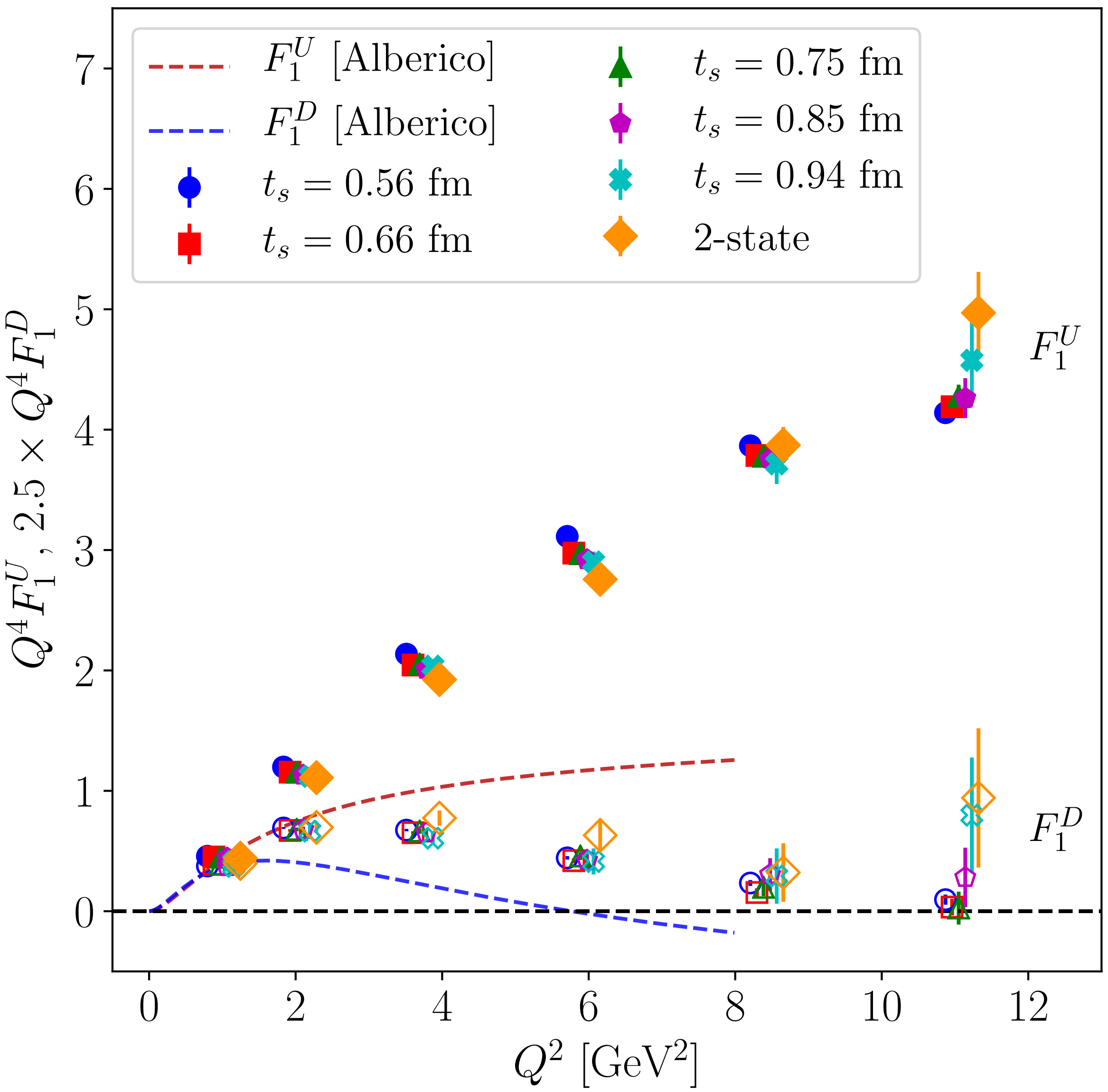
W. M. Alberico et al. [arXiv: 0812.3539]

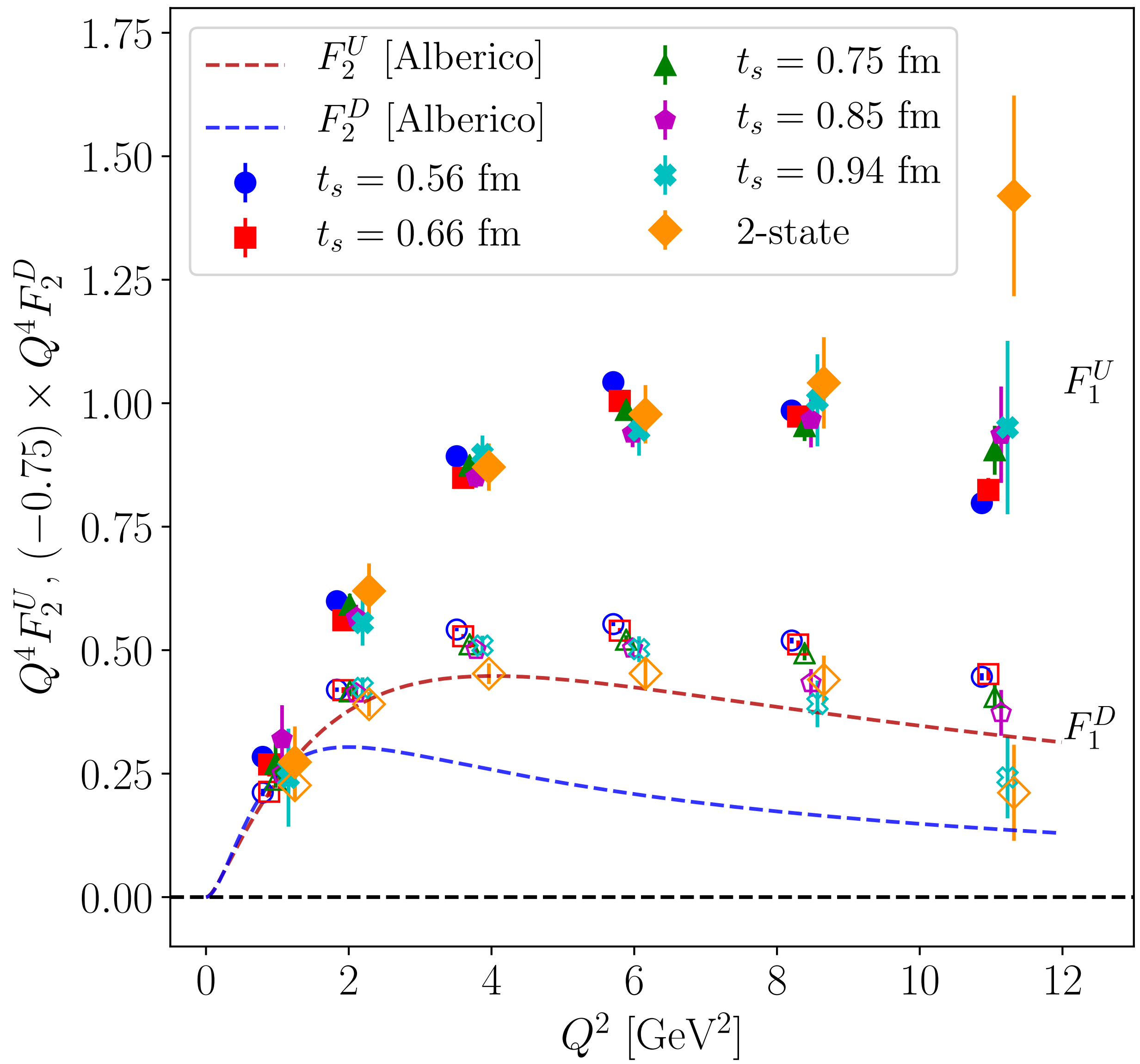
Form Factor Results IV: $F_1^{p,n}, F_2^{p,n}$



- discrepancies for individual form factors
- **a thorough investigation is needed**

Form Factor Results V: $F_1^{u,d}, F_2^{u,d}$





- discrepancies observed for form factors of up- and down- quarks

Bonus!

Bonus - Systematics I: Parity mixing for boosted states

- At non-zero momentum, correlators projected with $\Gamma^{\pm} \equiv \frac{1}{2}(\mathbb{1} + \gamma_4)$ include $\mathcal{O}((E-m)/2E)$ parity contaminations
- need to make sure that correlators from states at non-zero momentum correspond to the same zero-momentum states

F. M. Stokes et al. [arXiv: 1302.4152]

Parity-Expanded Variational Analysis (PEVA): Isolates parity of boosted hadron states

expand operator basis of correlation matrix $C_{ij}(\Gamma;\vec{p},t) = \mathrm{Tr}\left[\Gamma \sum_{\vec{x}} \langle \phi^i(x)\bar{\phi}^j(0)\rangle e^{-i\vec{p}\cdot\vec{x}}\right]$

$$\Gamma_p \equiv \frac{1}{4}(\mathbb{1}+\gamma_4)(\mathbb{1} - i\gamma_5\gamma_k\hat{p}_k)$$

$$\phi^i_p \equiv \Gamma_p \phi^i$$

$$\phi^{i'}_p \equiv \Gamma_p \gamma_5 \phi^i$$

$$\begin{aligned}
\mathcal{G}_{ij}(\vec{p},t) &= C_{ij}(\Gamma_p;\vec{p},t)\\
\mathcal{G}_{ij'}(\vec{p},t) &= C_{ij}(-\gamma_5\Gamma_p;\vec{p},t)\\
\mathcal{G}_{i'j}(\vec{p},t) &= C_{ij}(\Gamma_p\gamma_5;\vec{p},t)\\
\mathcal{G}_{i'j'}(\vec{p},t) &= C_{ij}(-\gamma_5\Gamma_p\gamma_5;\vec{p},t)
\end{aligned}$$

$$\left(\begin{array}{cccc|cccc}
\mathbf{0\bar{0}} & \mathbf{0\bar{1}} & \mathbf{0\bar{2}} & \mathbf{0\bar{3}} & 0\bar{0}' & 0\bar{1}' & 0\bar{2}' & 0\bar{3}'\\
\mathbf{1\bar{0}} & \mathbf{1\bar{1}} & \mathbf{1\bar{2}} & \mathbf{1\bar{3}} & 1\bar{0}' & 1\bar{1}' & 1\bar{2}' & 1\bar{3}'\\
\mathbf{2\bar{0}} & \mathbf{2\bar{1}} & \mathbf{2\bar{2}} & \mathbf{2\bar{3}} & 2\bar{0}' & 2\bar{1}' & 2\bar{2}' & 2\bar{3}'\\
\mathbf{3\bar{0}} & \mathbf{3\bar{1}} & \mathbf{3\bar{2}} & \mathbf{3\bar{3}} & 3\bar{0}' & 3\bar{1}' & 3\bar{2}' & 3\bar{3}'\\
\hline
0'\bar{0} & 0'\bar{1} & 0'\bar{2} & 0'\bar{3} & 0'\bar{0}' & 0'\bar{1}' & 0'\bar{2}' & 0'\bar{3}'\\
1'\bar{0} & 1'\bar{1} & 1'\bar{2} & 1'\bar{3} & 1'\bar{0}' & 1'\bar{1}' & 1'\bar{2}' & 1'\bar{3}'\\
2'\bar{0} & 2'\bar{1} & 2'\bar{2} & 2'\bar{3} & 2'\bar{0}' & 2'\bar{1}' & 2'\bar{2}' & 2'\bar{3}'\\
3'\bar{0} & 3'\bar{1} & 3'\bar{2} & 3'\bar{3} & 3'\bar{0}' & 3'\bar{1}' & 3'\bar{2}' & 3'\bar{3}'
\end{array}\right)$$

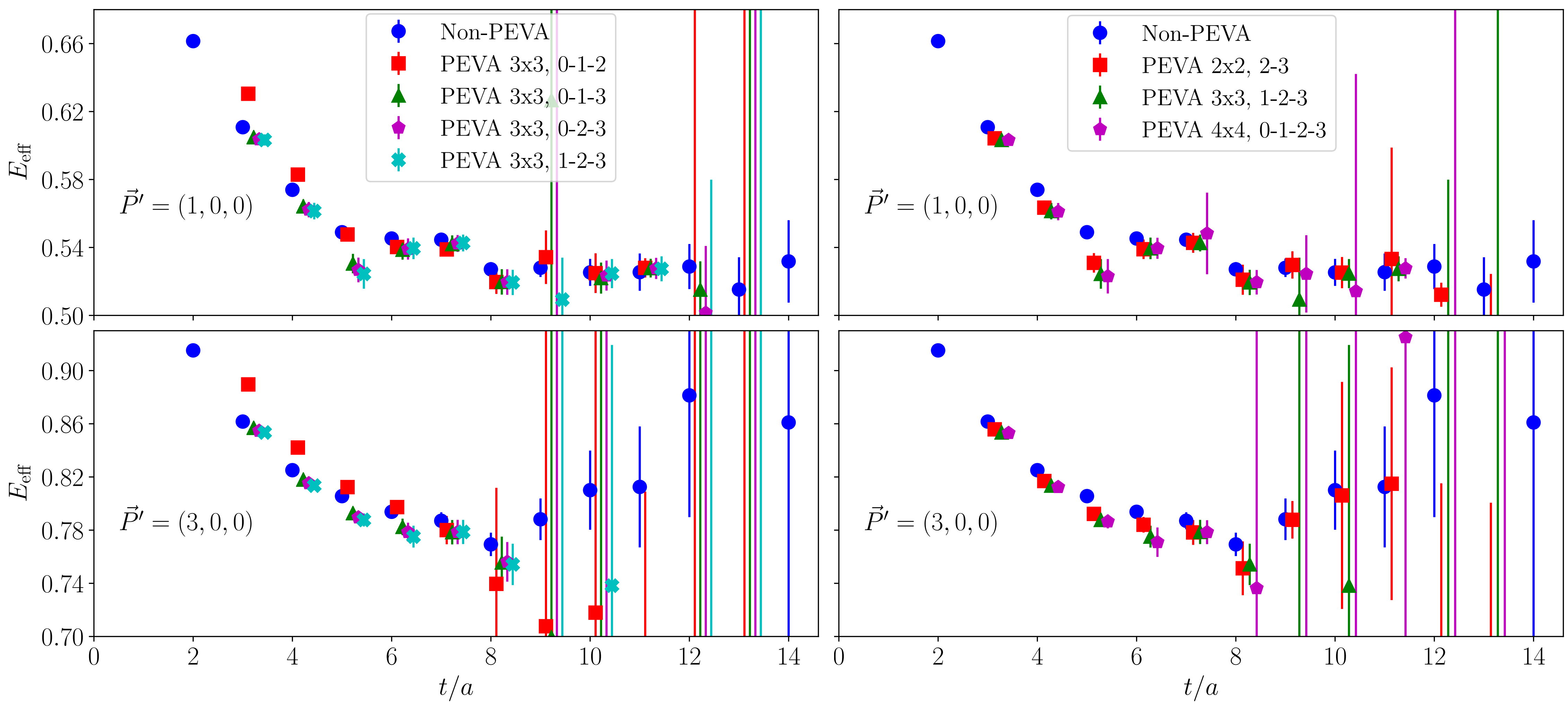
GEVP: $\mathcal{G}(\vec{p},t+\Delta t)\,\boldsymbol{u}^{\alpha}(\vec{p}) = e^{-E_\alpha(\vec{p})\Delta t}\mathcal{G}(\vec{p},t)\,\boldsymbol{u}^{\alpha}(\vec{p})$

Investigation:

- D5 ensemble, ~8000 statistics
- two-point functions from nucleon interpolating operators at four different values of Gaussian smearing ⟶ different overlap with nucleon ground state
- perform PEVA analysis with 2,3,4 operators

effect due to parity mixing is negligible within our statistics

$$\left(\begin{array}{cccc|cccc}
0\bar{0} & 0\bar{1} & 0\bar{2} & 0\bar{3} & 0\bar{0}' & 0\bar{1}' & 0\bar{2}' & 0\bar{3}' \\
1\bar{0} & 1\bar{1} & 1\bar{2} & 1\bar{3} & 1\bar{0}' & 1\bar{1}' & 1\bar{2}' & 1\bar{3}' \\
2\bar{0} & 2\bar{1} & 2\bar{2} & 2\bar{3} & 2\bar{0}' & 2\bar{1}' & 2\bar{2}' & 2\bar{3}' \\
3\bar{0} & 3\bar{1} & 3\bar{2} & 3\bar{3} & 3\bar{0}' & 3\bar{1}' & 3\bar{2}' & 3\bar{3}' \\
\hline
0'\bar{0} & 0'\bar{1} & 0'\bar{2} & 0'\bar{3} & 0'\bar{0}' & 0'\bar{1}' & 0'\bar{2}' & 0'\bar{3}' \\
1'\bar{0} & 1'\bar{1} & 1'\bar{2} & 1'\bar{3} & 1'\bar{0}' & 1'\bar{1}' & 1'\bar{2}' & 1'\bar{3}' \\
2'\bar{0} & 2'\bar{1} & 2'\bar{2} & 2'\bar{3} & 2'\bar{0}' & 2'\bar{1}' & 2'\bar{2}' & 2'\bar{3}' \\
3'\bar{0} & 3'\bar{1} & 3'\bar{2} & 3'\bar{3} & 3'\bar{0}' & 3'\bar{1}' & 3'\bar{2}' & 3'\bar{3}'
\end{array}\right)$$



Bonus - Systematics II: Momentum discretization

Common convention: $\vec{p} = \vec{\kappa}\ \ ,\ \vec{\kappa} = \frac{2\pi}{L}\vec{n}\ \ ,\ n_x, n_y, n_z = \frac{1}{a}\left[-\frac{L}{2}, \frac{L}{2}\right)$

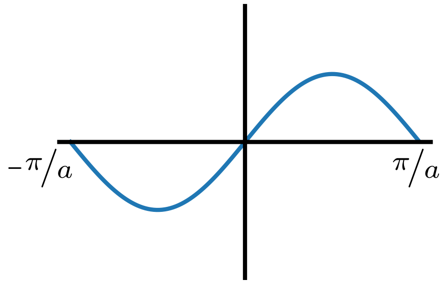
- take appropriate traces and ratios of two-point function to isolate momentum components

$$C(\vec{p},t) \overset{t\gtrsim 1}{=} |Z(\vec{p})|^2 \mathcal{S}(\vec{p}) e^{-E(\vec{p})t} \qquad \mathcal{S}(\vec{p}) = \frac{-i\not{p} + m}{2E(\vec{p})}$$

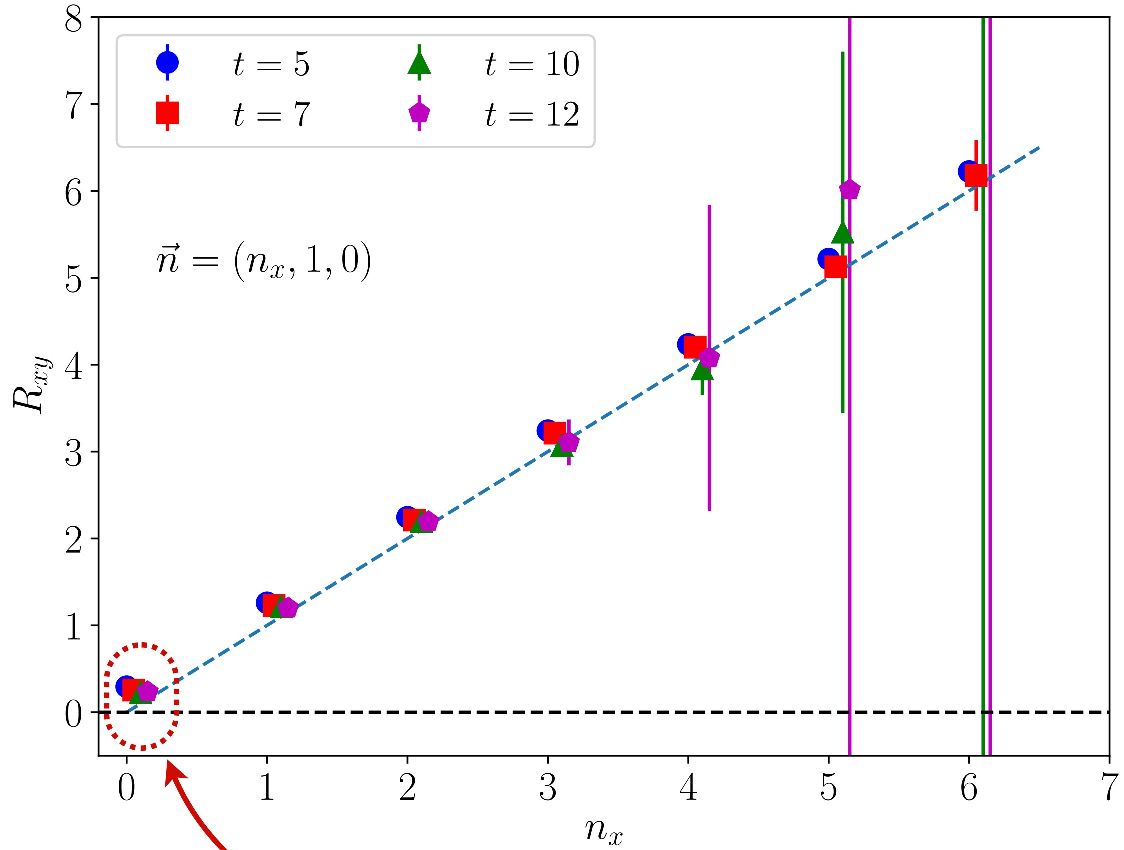
$$\mathrm{Im}\{\mathrm{Tr}[\gamma_k \mathcal{S}(\vec{p})]\} = -4p_k \rightarrow R_{xy}(\vec{p},t) \equiv \frac{\mathrm{Im}\{\mathrm{Tr}[\gamma_x C(\vec{p},t)]\}}{\mathrm{Im}\{\mathrm{Tr}[\gamma_y C(\vec{p},t)]\}} \xrightarrow{\text{cont.}} \frac{p_x}{p_y}$$

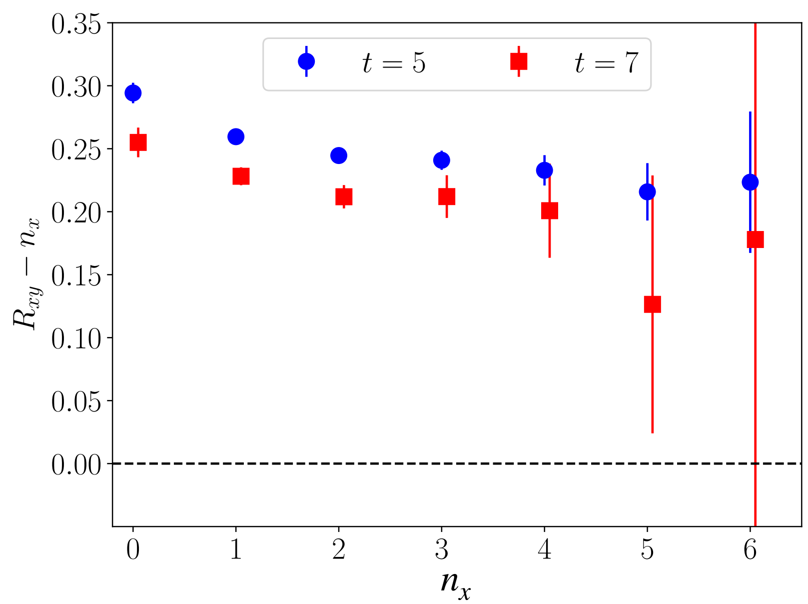
lattice momentum form:

- $\vec{p} \overset{?}{=} \vec{\kappa}$
- $\vec{p} \overset{?}{=} \vec{\kappa} - \frac{1}{6}\vec{\kappa}(a\vec{\kappa})^2$
- $\vec{p} \overset{?}{=} \frac{1}{a}\sin(a\vec{\kappa})$



$n_x = 6 \rightarrow \kappa_x = 3\pi/8a$





effect due to anisotropic quark (boosted) smearing??

Conclusions and Outlook

- high-Q^2 on the lattice: feasible, but: need to control systematics, noise-to-signal ratio

- our lattice results overestimate phenom. Q^2-dependence for F_1, F_2
- however: good agreement with experiment for F_2/F_1 and G_E/G_M ratios up to $Q^2 \sim 6\ \mathrm{GeV}^2$
- consistent results between $m_\pi = 170\ \mathrm{MeV}$ (D5), $m_\pi = 280\ \mathrm{MeV}$ (D6): small pion mass and volume effects

To-do:

- understand/resolve disagreement for individual form factors F_1, F_2
- complete investigation of excited state effects
- consider other systematic effects
  - $\mathcal{O}(a)$ **improvement**
  - physical pion mass
  - continuum extrapolation
- disconnected diagrams on the way

Thank you