π^0 exchange in the long-distance HLbL contribution to $g_\mu - 2$

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Outline

1. Construct formula of π^0 exchange in the long-distance HLbL contribution
2. Point-point-wall correlator
3. Preliminary results
4. Conclusion

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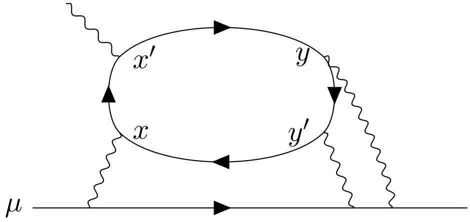
The formula that obtain the connected hadronic light-by-light contribution to the $g_\mu - 2$ is given by [Blum et al., 2016]:

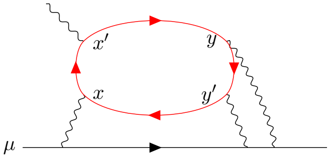
$$F_2 = 2m_\mu e^6 \sum_{y,y'} 3 \sum_{x'} \frac{1}{2}\epsilon_{ij\mu'} x'_j \times 6i^4 H_{\nu\nu'\mu\mu'}(y,y',x,x') M_{i\nu\nu'\mu}(y,y',x)/3$$

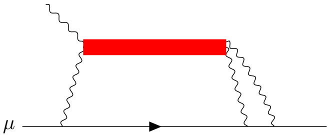


π^0 exchange

π^0 exchange will dominate the long-distance part of the HLbL contribution. The four-point hadronic function can be replaced by long distance π^0 four-point function:

$$6i^4 H_{\nu,\nu',\mu,\mu'}(y,y',x,x') \to A^{\pi^0}_{\nu,\nu',\mu,\mu'}(y,y',x,x') \qquad (1)$$





π^0 intermediate state

Apply the Källén-Lehmann representation but keep only the single π^0 intermediate state:

$$A^{\pi^0}_{\mu,\mu',\nu,\nu'}(x,x',y,y') = \frac{1}{(2\pi)^3}\int \frac{d^3p}{2E_\pi(p)} \left\langle 0|T\{J_\mu(x)J_{\mu'}(x')\}|\pi^0(\vec{p})\right\rangle \left\langle \pi^0(\vec{p})|T\{J_\nu(y)J_{\nu'}(y')\}|0\right\rangle \quad (2)$$

Then use four-dimensional translational invariance to remove the variables x and y from the current-current-π^0 amplitudes:

$$\begin{aligned}\left\langle 0|T\{J_\mu(x)J_{\mu'}(x')\}|\pi^0(\vec{p})\right\rangle &= \left\langle 0|T\{J_\mu(0)J_{\mu'}(\tilde{x})\}|\pi^0(\vec{p})\right\rangle e^{i\vec{p}\cdot\vec{x}-E_p x_0} \\ &= \mathcal{F}_{\mu\mu'}(\tilde{x}, -i\vec{\nabla}_x)e^{i\vec{p}\cdot\vec{x}-E_p x_0}\end{aligned} \quad (3)$$

Combine these results to obtain:

$$A^{\pi^0}_{\mu,\mu',\nu,\nu'}(x,x',y,y') = \mathcal{F}_{\mu\mu'}(\tilde{x}, -i\vec{\nabla}_x)\mathcal{F}_{\nu\nu'}(\tilde{y}, i\vec{\nabla}_y)\Delta_F(x-y, M_\pi), \quad (4)$$

where

$$\Delta_F(x-y, M_\pi) = \frac{1}{(2\pi)^4}\int d^4p \frac{e^{ip(x-y)}}{p_0^2+\vec{p}^2+M_\pi^2} \quad (5)$$

Long-distance Approximation

Now we will evaluate the derivatives with respect to x and y which appear in these equations but keep only the leading term in an expansion powers of 1/L:

$$\prod_{i=1}^{N}\left(\frac{\partial}{\partial x_{\rho_i}}\right)\Delta_F(x-y,M_\pi) \approx \left\{\prod_{i=1}^{N}\left(-M_\pi\frac{(x-y)_{\rho_i}}{|x-y|}\right)\right\}\left(\frac{M_\pi}{2\pi|x-y|}\right)^{(3/2)} e^{-|x-y|M_\pi} \tag{6}$$

Then the four-point correlation reads:

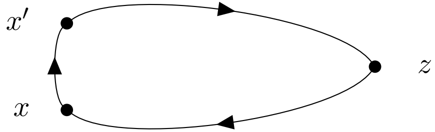
$$A^{\pi^0}_{\mu,\mu',\nu,\nu'}(x,x',y,y') = \mathcal{F}_{\mu\mu'}(\tilde{x},-iM_\pi\hat{n})\mathcal{F}_{\nu\nu'}(\tilde{y}-iM_\pi\hat{n})\Delta_F(x-y,M_\pi), \tag{7}$$

where $\hat{n}$ is a unit Euclidean four-vector.

$\gamma\gamma - \pi$ correlator in position space

To find to amplitude $\mathcal{F}_{\mu\mu'}(\tilde{x}, -iM_\pi \hat{n})$, consider the $\gamma\gamma - \pi$ correlator in position space:

$$\mathcal{B}_{\mu\mu'}(x, x', z) = \left\langle 0 | T\{J_\mu(x) J_{\mu'}(x') \pi^0(z)\} | 0 \right\rangle \tag{8}$$



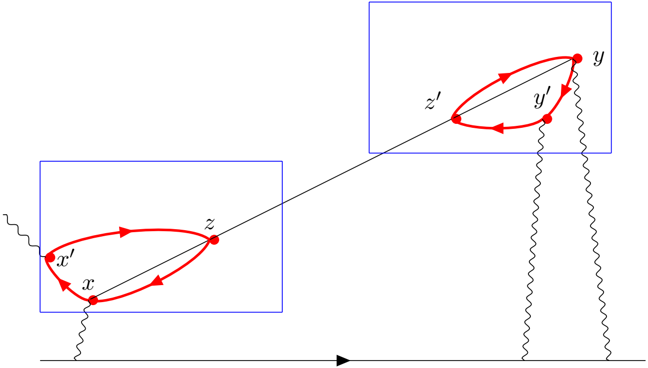
By using the same steps as the four-point function, we can find the same $\gamma\gamma - \pi$ vertex:

$$\mathcal{B}_{\mu\mu'}(x, x', z) = \mathcal{F}_{\mu\mu'}(\tilde{x}, iM_\pi \hat{n}) Z_{\pi^0}^{1/2} \Delta_F(x - z, M_\pi), \tag{9}$$

where $Z_{\pi^0}^{1/2} = \left\langle \pi^0(\vec{p} = 0) | \pi^0(0) | 0 \right\rangle$.

Combining all results, we get the hadronic light-by-light amplitude from π^0 contribution, which associate with two three-point correlators and pion propagators:

$$A^{\pi^0}_{\mu,\mu',\nu,\nu'}(x,x',y,y') = \Delta_F(x-y,M_\pi)\frac{1}{Z_{\pi^0}}\frac{\mathcal{B}_{\mu\mu'}(x,x',z)}{\Delta_F(x-z,M_\pi)}\frac{\mathcal{B}_{\nu\nu'}(y,y',z')}{\Delta_F(z'-y,M_\pi)} \tag{10}$$



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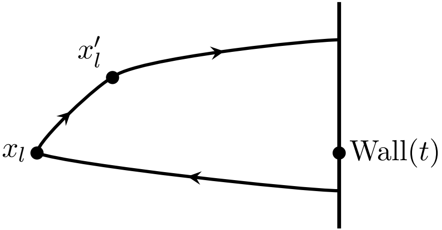
Point-point-wall correlator

$$\frac{B_{\mu\mu'}(0,x,Z=(\vec{0},t))}{\langle\pi(0)\pi^{\rm lat}(Z)\rangle} = \frac{B^W_{\mu\mu'}(0,x,t)}{\left\langle\pi(0)\pi^{\rm lat}_W(t)\right\rangle}, \tag{11}$$

where $B^W_{\mu\mu'}(0,x,t) = \left\langle J_\mu(0)J_{\mu'}(x)\pi^{\rm lat}_W(t)\right\rangle$,
$\pi_{\rm w}(t) = \sum_{\vec{x},\vec{y}} \bar{\psi}(\vec{x},t)\gamma_5\psi(\vec{y},t)$.

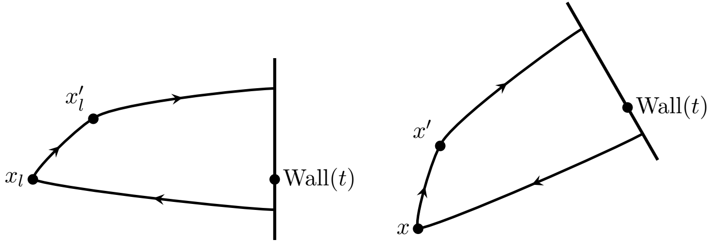


Rotational Matrix

The point-point-wall correlator $B^{l,W}_{\alpha\alpha'}(x_l, x'_l, t)$ can be computed before rotation.

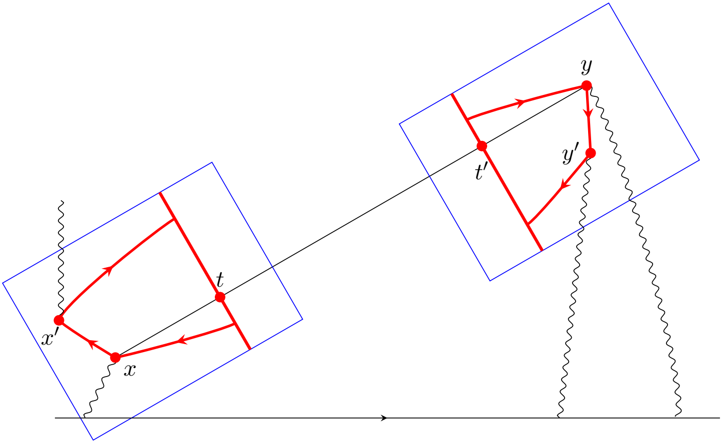
$$x = \Lambda x_l, \quad x' = \Lambda x'_l \tag{12}$$

$$B^{W}_{\mu\mu'}(x, x', t) = \Lambda_{\mu\alpha}\Lambda_{\mu'\alpha'}B^{l,W}_{\alpha\alpha'}(x_l, x'_l, t) \tag{13}$$



Lattice implementation with point-point-wall correlator

$$A^{\pi^0}_{\mu,\mu',\nu,\nu'}(x,x',y,y') = \Delta_F(x-y, M_\pi)\frac{B^W_{\mu\mu'}(x,x',t)}{\left\langle \pi(0)\pi^{\text{lat}}_W(t)\right\rangle}\frac{B^W_{\mu\mu'}(y,y',t')}{\left\langle \pi(0)\pi^{\text{lat}}_W(t')\right\rangle} \tag{14}$$



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Preliminary results: ensembles

Observable	Fit	% err.
$am'_{\text{res}}(m_l)$	0.0022824(70)	0.31
am_π	0.13975(10)	0.07
am_K	0.504154(89)	0.02
am_Ω	1.6726(25)	0.15
am'_Ω	2.040(63)	3.09
af_π	0.13055(11)	0.09
af_K	0.15815(13)	0.09
Z_A	0.73457(11)	0.02
Z_V^π	0.72672(35)	0.05
Z_V^K	0.7390(11)	0.15
$aE^2_{\pi\pi}$	0.28175(21)	0.08
$a\delta E^2_\pi$	0.002246(52)	2.31
ap	0.01775(21)	1.16
$\delta_0^2(p)$	-0.339(12)°	3.40
$m_\pi a_0^2$	-0.0464(10)	2.23

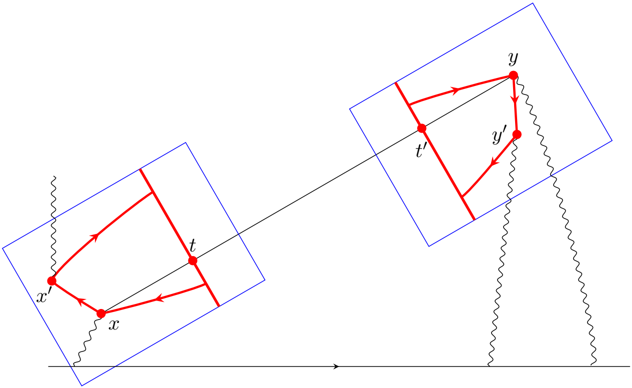
24D Ensemble

Observable	Fit	% err.
$am'_{\text{res}}(m_l)$	0.0020539(96)	0.47
am_π	0.13412(27)	0.20
am_K	0.31498(20)	0.06
am_Ω	1.3316(43)	0.32
am'_Ω	1.832(56)	3.04
af_π	0.12435(21)	0.17
af_K	0.13734(22)	0.16
Z_A	0.72959(25)	0.03
Z_V^π	0.7242(11)	0.15
Z_V^K	0.7270(12)	0.17
$aE^2_{\pi\pi}$	0.26916(61)	0.23
$a\delta E^2_\pi$	0.00092(14)	14.86
ap	0.01110(83)	7.45
$\delta_0^2(p)$	-0.199(44)°	22.16
$m_\pi a_0^2$	-0.0418(61)	14.61

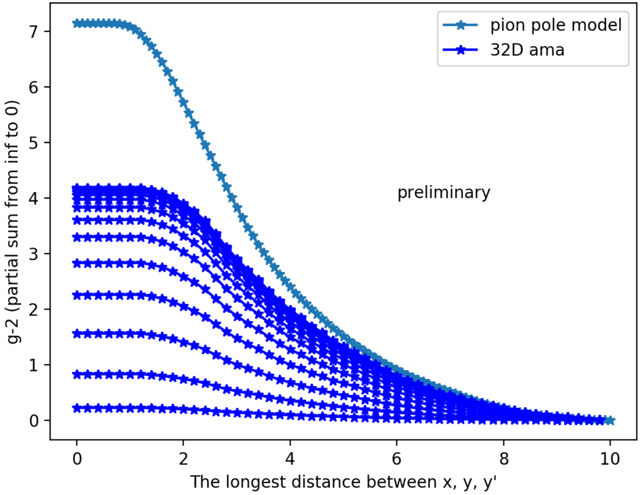
32D Ensemble

Preliminary results: parameters

t-min $= 10$
xx'-limit $= 10$



Preliminary results: plots



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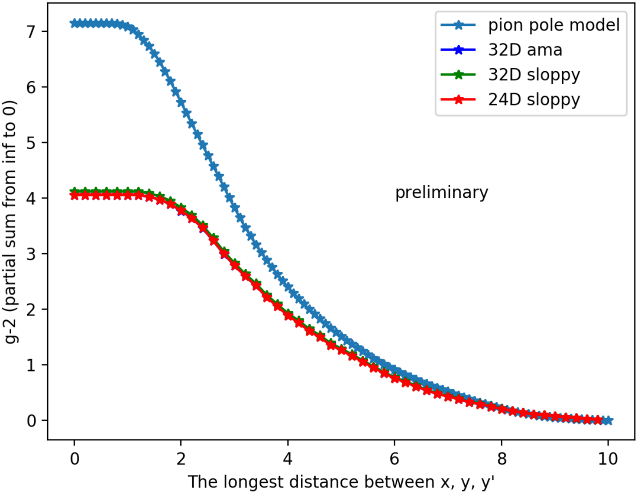

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Conclusion

- The π^0 exchange in the long-distance part of the HLbL contribution can be evaluated in a "QCD volume" of arbitrary size.
- The point-point-wall correlator is a better estimation of current-current-π^0 amplitude $\mathcal{F}_{\mu\mu'}(\hat{x}, -iM_\pi \hat{n})$.
- The statistical average of point-point-wall correlator can be evaluated in each lattice configuration before computing the hadronic amplitude.
- The point-point-wall correlator make the AMA correction much easier.