Lattice QCD for b physics

Stefan Meinel

The University of Arizona

RIKEN BNL Research Center

2019 Lattice Workshop for US - Japan Intensity Frontier Incubation

What is special about b quarks?

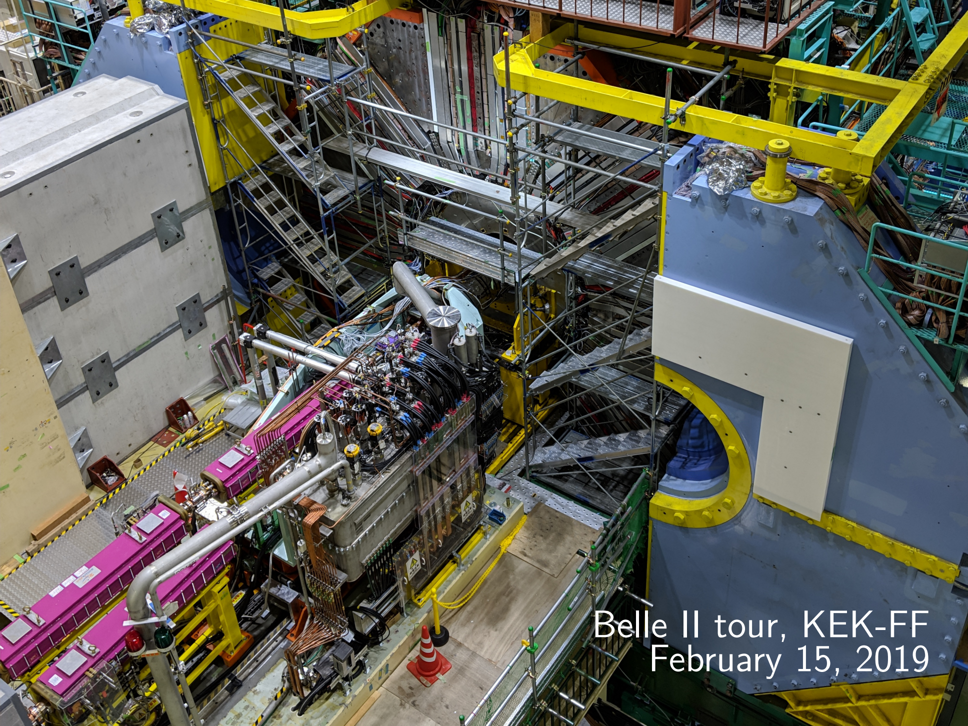
- The b is the heaviest quark that forms hadrons. Consequently there are many possible decay channels (also with τ leptons).

- The dominant decays are already CKM-suppressed:

$$|V_{cb}|^2 \approx 0.0017, \ \ |V_{ub}|^2 \approx 0.000014.$$

- CP-violating effects can be very large.

- Heavy-quark effective theory can be used.


Belle II tour, KEK-FF
February 15, 2019

b quarks on the lattice

Challenge: wide range of scales

$$m_\pi \approx 0.1\ \text{GeV}, \quad m_b \approx 5\ \text{GeV}$$

Approaches:

Special heavy-quark lattice action for the b

- Lattice HQET
- Lattice NRQCD/mNRQCD
- Wilson-like actions with m_Q-dependent, anisotropic coefficients

Same action for b as for light quarks

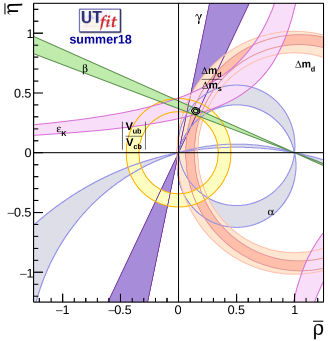
- Use very fine lattice spacings and/or extrapolate/interpolate in m_b
- Main advantage: renormalization/matching simplified or unnecessary $\rightarrow$ smaller systematic uncertainty

(See extra slides for some details and references.)

$|V_{ub}|$ and $|V_{cb}|$

$$\begin{pmatrix} V_{ud}^* & V_{cd}^* & V_{td}^* \\ V_{us}^* & V_{cs}^* & V_{ts}^* \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} + 1 + \frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cd}} = 0$$

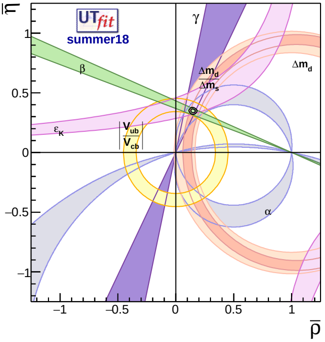


[http://www.utfit.org/UTfit/ResultsSummer2018SM]

$|V_{ub}|$ and $|V_{cb}|$

$$\begin{pmatrix} V_{ud}^* & V_{cd}^* & V_{td}^* \\ V_{us}^* & V_{cs}^* & V_{ts}^* \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

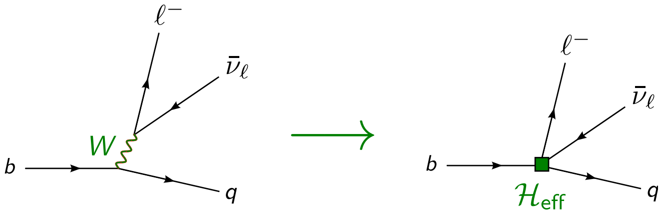
$$\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} + 1 + \frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cd}} = 0$$



[http://www.utfit.org/UTfit/ResultsSummer2018SM]

- V_{ub} is the smallest and least well known CKM matrix element. The determination of $|V_{ub}|$ constrains the length opposite to the well-measured angle β.
- V_{cb} is used for normalization of the triangle, and is also the dominant source of uncertainty in the SM calculation of ϵ_K (indirect CP violation in neutral kaons). [J. Bailey *et al.*, arXiv:1808.09657/PRD 2018]

Effective weak Hamiltonian for $b \to q\ell^-\bar{\nu}_\ell$ decays



$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{qb}\ \bar{q}\gamma^\mu(1-\gamma_5)b\ \bar{\ell}\gamma_\mu(1-\gamma_5)\nu$$

+additional Dirac structures beyond the SM

Purely leptonic $B^- \to \ell^- \bar{\nu}_\ell$ decays

In the SM:

$$\mathcal{B}(B^- \to \ell^- \bar{\nu}_\ell) = \tau_{B^-} \Gamma(B^- \to \ell^- \bar{\nu}_\ell) = \tau_{B^-} \frac{G_F^2 |V_{qb}|^2}{8\pi} m_B \, m_\ell^2 \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2 f_B^2,$$

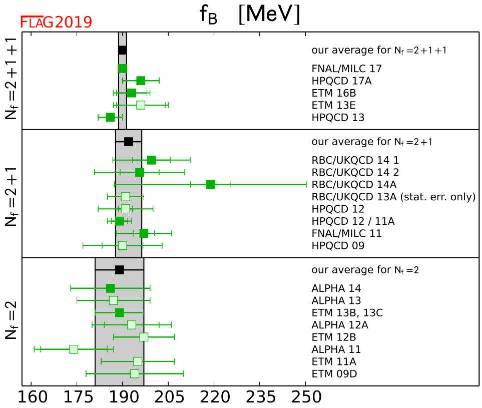
$$\langle 0 | \bar{q} \gamma^\mu \gamma_5 b | B(p) \rangle = i f_B p^\mu$$

Purely leptonic $B^- \to \ell^- \bar{\nu}_\ell$ decays

In the SM:

$$\mathcal{B}(B^- \to \ell^- \bar{\nu}_\ell) = \tau_{B^-} \Gamma(B^- \to \ell^- \bar{\nu}_\ell) = \tau_{B^-} \frac{G_F^2 |V_{qb}|^2}{8\pi} m_B \, m_\ell^2 \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2 f_B^2,$$

$$\langle 0 | \bar{q} \gamma^\mu \gamma_5 b | B(p) \rangle = i f_B p^\mu$$



Sub-percent precision thanks to relativistic action (HISQ) for b and light quarks on ultrafine lattices. [A. Bazavov *et al.* (Fermilab and MILC collaborations), arXiv:1712.09262/PRD 2018].

← 2019 FLAG review: arXiv:1902.08191

Purely leptonic $B^- \to \ell^- \bar{\nu}_\ell$ decays

In the SM:

$$\mathcal{B}(B^- \to \ell^- \bar{\nu}_\ell) = \tau_{B^-} \Gamma(B^- \to \ell^- \bar{\nu}_\ell) = \tau_{B^-} \frac{G_F^2 |V_{qb}|^2}{8\pi} m_B \, m_\ell^2 \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2 f_B^2,$$

$$\langle 0 | \bar{q} \gamma^\mu \gamma_5 b | B(p) \rangle = i f_B p^\mu$$

Experiment:

$$\mathcal{B}(B^- \to \tau^- \bar{\nu}) = (1.09 \pm 0.24) \times 10^{-4}$$ [PDG average of BaBar and Belle]

$$\mathcal{B}(B^- \to \mu^- \bar{\nu}) = (6.5 \pm 2.7) \times 10^{-7}$$ [Belle, arXiv:1712.04123/PRL 2018]

$$\tau_{B^-} = (1.638 \pm 0.004) \times 10^{-12} \text{ s}$$ [PDG average, dominated by LHCb]

Hadronic matrix elements for semileptonic $b \to q\ell^-\bar{\nu}_\ell$ decays

Exclusive $H_b \to H_q\ell^-\bar{\nu}_\ell$:

$$\langle H_q(p')|J_\mu|H_b(p)\rangle$$

Inclusive $H_b \to X_q\ell^-\bar{\nu}_\ell$:

$$\mathrm{Im}\left[-i\int \mathrm{d}^4x\, e^{-iq\cdot x}\langle H_b(p)|\, \mathrm{T} J_\mu^\dagger(x) J_\nu(0)\, |H_b(p)\rangle\right]$$

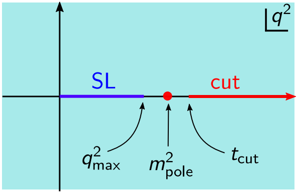
where $J_\mu = \bar{q}\gamma_\mu(1-\gamma_5)b$

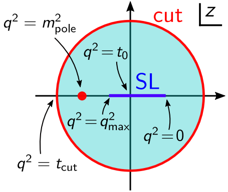
The z expansion for exclusive semileptonic decays

To fit the q^2 $[= (p - p')^2]$ dependence of form factors in a model-independent way, it is convenient to consider them as functions of a new dimensionless variable z, defined as

$$z = \frac{\sqrt{t_{\mathrm{cut}} - q^2} - \sqrt{t_{\mathrm{cut}} - t_0}}{\sqrt{t_{\mathrm{cut}} - q^2} + \sqrt{t_{\mathrm{cut}} - t_0}}$$

This maps the complex q^2 plane, cut along the real axis for $q^2 > t_{\mathrm{cut}}$, to the interior of the unit disk:



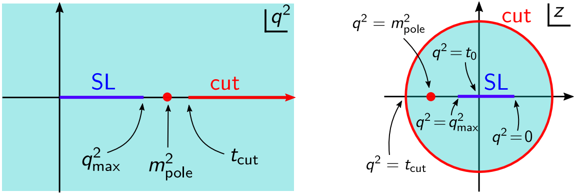


The BCL "simplified" z expansion for a form factor with a single pole below t_{cut} reads

$$f(q^2) = \frac{1}{1 - q^2/m^2_{\mathrm{pole}}} \sum_{k=0}^{\infty} a_k\, z^k$$

[C. Bourrely, I. Caprini, L. Lellouch, arXiv:0807.2722/PRD 2009]

The z expansion for exclusive semileptonic decays



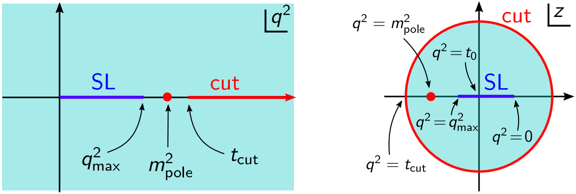
$$f(q^2) = \frac{1}{1 - q^2/m^2_{\mathrm{pole}}} \sum_{k=0}^{\infty} a_k\, z^k$$

Analyticity guarantees convergence. Unitarity provides bounds on the sizes of the coefficients a_k. It is sufficient to keep only the first few terms of the series.

The unitarity bounds take a simple form in the original BGL variant of the z expansion, which however requires a complicated "outer function".

[C. Boyd, B. Grinstein, R. Lebed, arXiv:hep-ph/9412324/PRL 1995]

The z expansion for exclusive semileptonic decays



Example: the form factor $f_+(\Lambda_b \to p)$

- t_{cut} is set to the onset location of the two-particle branch cut created by the current $J^\mu = \bar{u}\gamma^\mu b$,

$$t_{\text{cut}} = (m_B + m_\pi)^2$$

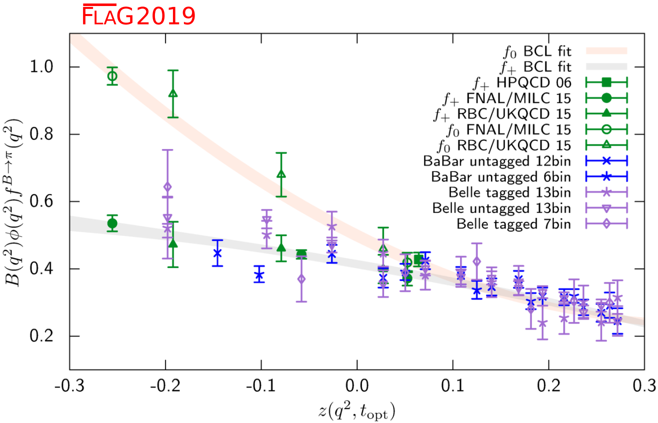
- m_{pole} is set to the mass of the $J^P = 1^-$ bound state created by the current $J^\mu = \bar{u}\gamma^\mu b$,

$$m_{\text{pole}} = m_{B*}$$

- t_0 determines which value of q^2 gets mapped to $z = 0$. I used

$$t_0 = q^2_{\text{max}} = (m_{\Lambda_b} - m_p)^2$$

$B \to \pi \ell \bar{\nu}$



New preliminary lattice QCD results:

B. Colquhoun (JLQCD collaboration), talk at KEK-FF 2019

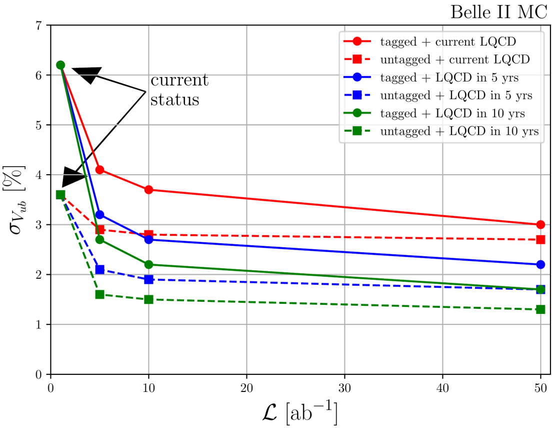
C. Bouchard (HPQCD collaboration), talk at Lattice 2018

Z. Gelzer (FNAL/MILC collaboration), talk at Lattice 2018

L. Riggio (ETM collaboration), talk at Lattice 2018

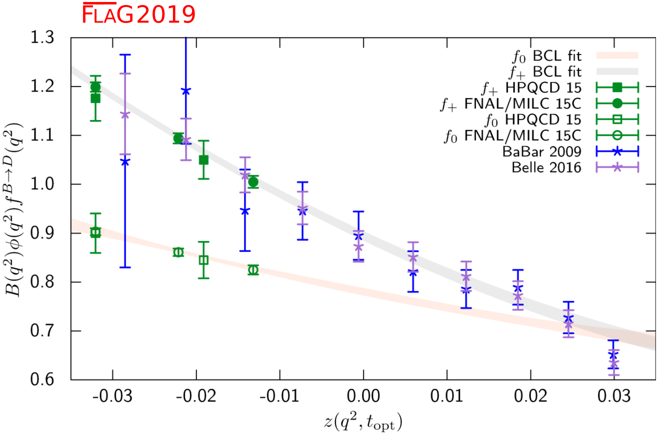
$B \to \pi \ell \bar{\nu}$

Belle II prospects



["Belle II Physics Book", E. Kou *et al.*, `arXiv:1808.10567`]

$B \to D\ell\bar{\nu}$



New preliminary lattice QCD results:

Y.-C. Jang (LANL/SWME collaboration), talk at KEK-FF 2019

T. Kaneko (JLQCD collaboration), talk at KEK-FF 2019

$B \to D^* \ell \bar{\nu}$

The published lattice QCD results are for zero recoil ($q^2 = q^2_{\mathrm{max}} \;\Leftrightarrow\; v \cdot v' = 1$) only:

	$\mathcal{F}^{B \to D^*}(1)$
FNAL/MILC [1]	0.906(4)(12)
HPQCD [2]	0.895(10)(24)
FLAG average	0.904(12)

[1] J. Bailey *et al.* (FNAL/MILC collaboration), `arXiv:1403.0635`/PRD 2014

[2] J. Harrison, C. Davies, M. Wingate (HPQCD collaboration), `arXiv:1711.11013`/PRD 2018

$B \to D^* \ell \bar{\nu}$

The published lattice QCD results are for zero recoil ($q^2 = q^2_{\rm max} \;\; \Leftrightarrow \;\; v \cdot v' = 1$) only:

	$\mathcal{F}^{B\to D^*}(1)$
FNAL/MILC [1]	0.906(4)(12)
HPQCD [2]	0.895(10)(24)
FLAG average	0.904(12)

[1] J. Bailey *et al.* (FNAL/MILC collaboration), arXiv:1403.0635/PRD 2014

[2] J. Harrison, C. Davies, M. Wingate (HPQCD collaboration), arXiv:1711.11013/PRD 2018

$\Rightarrow$ to extract $|V_{cb}|$, the experimental data need to be extrapolated to $v \cdot v' = 1$:

	$\eta_{EW} \lvert V_{cb} \rvert \mathcal{F}^{B\to D^*}(1)$
HFLAV average [3], CLN parametrization [5]	$35.61(43) \times 10^{-3}$
Belle [4], CLN parametrization [5]	$35.06(15)(54) \times 10^{-3}$
Belle [4], BGL parametrization [6]	$38.73(25)(60) \times 10^{-3}$

[3] HFLAV, arXiv:1612.07233/EPJC 2017

[4] Belle collaboration, arXiv:1809.03290

[5] I. Caprini, L. Lellouch, M. Neubert, arXiv:hep-ph/9712417/NPB 1998

[6] C. Boyd, B. Grinstein, R. Lebed, arXiv:hep-ph/9412324/PRL 1995; arXiv:hep-ph/9705252/PRD 1997

$B \to D^* \ell \bar{\nu}$

New preliminary lattice QCD results for $B \to D^* \ell \bar{\nu}$, also at nonzero recoil:

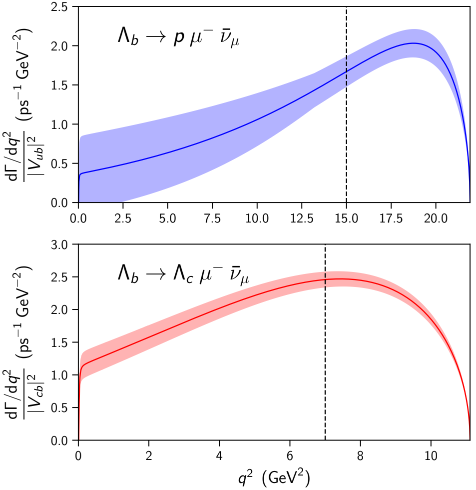
Y.-C. Jang (LANL/SWME collaboration), talk at KEK-FF 2019

T. Kaneko (JLQCD collaboration), talk at KEK-FF 2019

A. Vaquero (FNAL/MILC collaboration), talk at KEK-FF 2019

$\Lambda_b \to p\ell\bar{\nu}$ and $\Lambda_b \to \Lambda_c\ell\bar{\nu}$

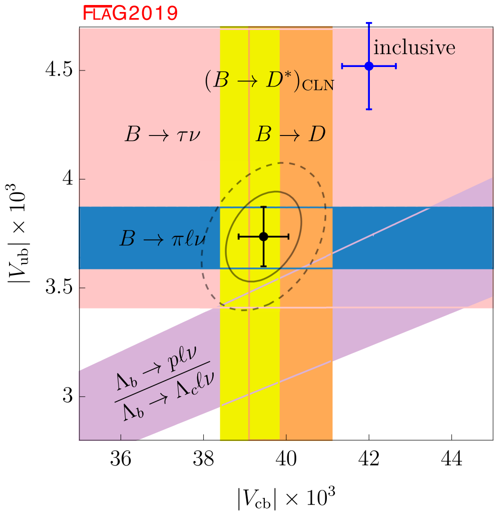
Predictions from lattice QCD: W. Detmold, C. Lehner, S. Meinel, arXiv:1503.01421/PRD 2015



Measurements by LHCb: Ratio of rates: arXiv:1504.01568/Nature Physics 2015

Shape of $\Lambda_b \to \Lambda_c\ell\bar{\nu}$: arXiv:1709.01920/PRD 2017

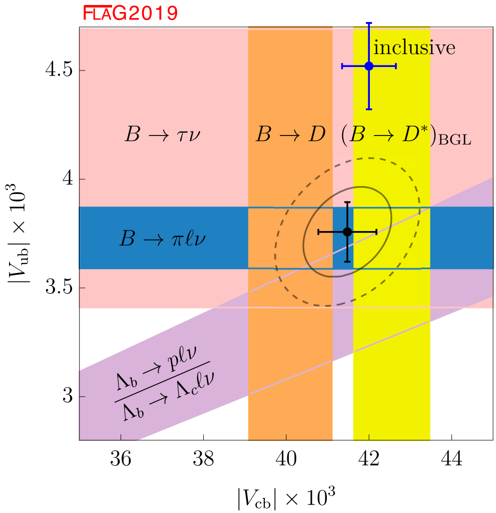
$|V_{ub}|$ and $|V_{cb}|$



Note: FLAG did not include the Λ_b result in the exclusive average

[Flavour Lattice Averaging Group, arXiv:1902.08191]

$|V_{ub}|$ and $|V_{cb}|$



Note: FLAG did not include the Λ_b result in the exclusive average

[Flavour Lattice Averaging Group, arXiv:1902.08191]

Lattice calculations for other decay modes

$b \to u\ell\bar{\nu}$:

- $B_s \to K\ell\bar{\nu}$

  A. Bazavov *et al.* (FNAL/MILC collaboration), `arXiv:1901.02561`

  F. Bahr *et al.* (ALPHA collaboration), `arXiv:1903.05870`

  T. Tsang (RBC/UKQCD collaboration), talk at KEK-FF 2019

- $B \to \rho(\to \pi\pi)\ell\bar{\nu}$

  S. Meinel (with L. Leskovec, M. Petschlies, G. Rendon, G. Silvi, S. Paul, *et al.*), talk at KEK-FF 2019

- $B^+ \to \ell^+\nu\gamma$

  C. Kane, C. Lehner, S. Meinel, A. Soni, work in progress

$b \to c\ell\bar{\nu}$:

- $B_s \to D_s\ell\bar{\nu}$, $B_s \to D_s^*\ell\bar{\nu}$

  C. Monahan *et al.* (HPQCD collaboration), `arXiv:1703.09728`/PRD 2017

  T. Tsang (RBC/UKQCD collaboration), talk at KEK-FF 2019

  J. Koponen (HPQCD collaboration), poster at KEK-FF 2019

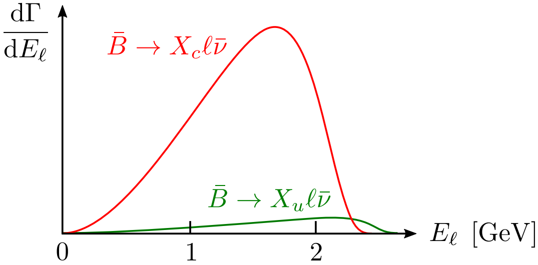
- $B_c \to \eta_c\ell\bar{\nu}$, $B_c \to J/\psi\ell\bar{\nu}$ [mainly for $R(J/\psi)$]

  A. Lytle (HPQCD collaboration), talk at Lattice 2016

- $\Lambda_b \to \Lambda_c^*(2595)\ell\bar{\nu}$, $\Lambda_b \to \Lambda_c^*(2625)\ell\bar{\nu}$ [mainly for $R(\Lambda_c^*)$]

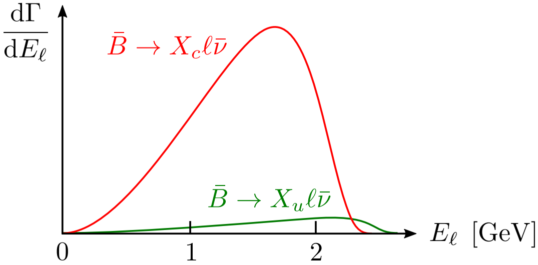
  S. Meinel (with G. Rendon), talk at CKM 2018

Inclusive $B \to X\ell\bar{\nu}$



(Not to scale. $\bar{B} \to X_u\ell\bar{\nu}$ rate is actually even lower.)

Inclusive $B \to X\ell\bar{\nu}$



(Not to scale. $\bar{B} \to X_u\ell\bar{\nu}$ rate is actually even lower.)

Lattice QCD approaches:

- Calculate amplitude in unphysical kinematic region that is directly accessible on the lattice, use Cauchy formula to analytically continue physical amplitude to that region

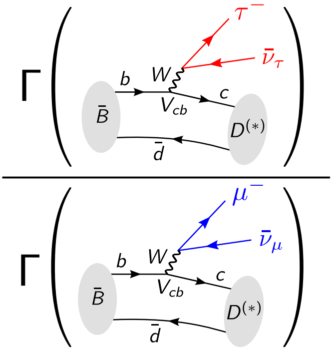
  S. Hashimoto, arXiv:1703.01881/PTEP 2017 – see his talk tomorrow morning!

- Compute amplitude at physical kinematics via "smeared" inverse Laplace transform (e.g. using Backus-Gilbert method)

  M. Hansen, H. Meyer, D. Robaina, arXiv:1704.08993/PRD 2017

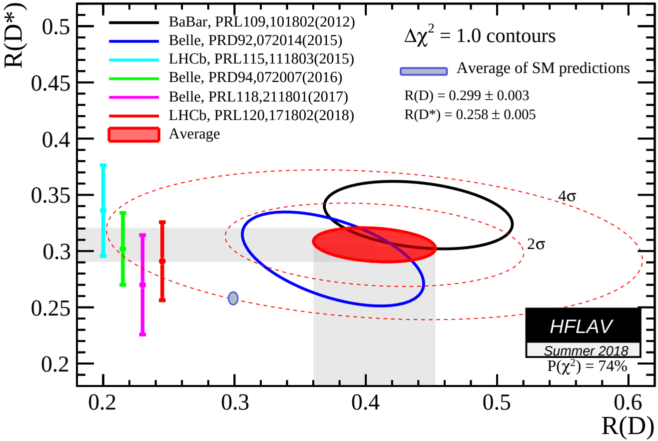
Test of lepton flavor universality with $b \to c\ell\bar{\nu}$ decays

$R[D^{(*)}] =$



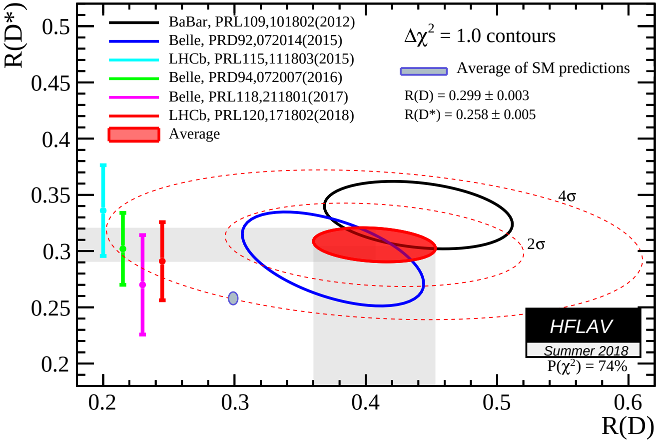
(Belle and Babar average over muons and electrons)

Test of lepton flavor universality with $b \to c\ell\bar{\nu}$ decays



[https://hflav-eos.web.cern.ch/hflav-eos/semi/summer18/RDRDs.html]

Test of lepton flavor universality with $b \to c\ell\bar{\nu}$ decays



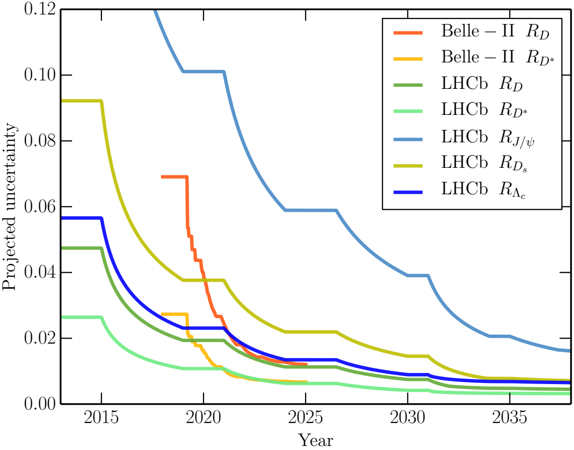
[https://hflav-eos.web.cern.ch/hflav-eos/semi/summer18/RDRDs.html]

Note: New Belle results at Moriond EW 2019 reduce tension of WA to 3.1σ

[Talk by Giacomo Caria on March 22, 2019]

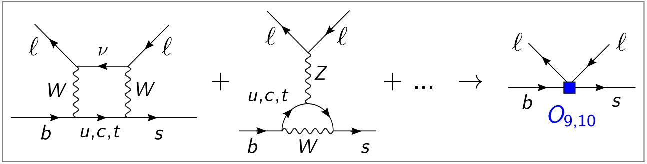
Test of lepton flavor universality with $b \to c\ell\bar{\nu}$ decays

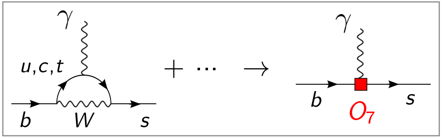
Belle II and LHCb prospects

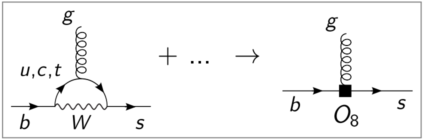


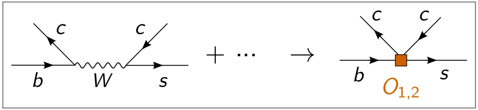
[S. Bifani, S. Descotes-Genon, A. Romero Vidal, M.-H. Schune, `arXiv:1809.06229`]

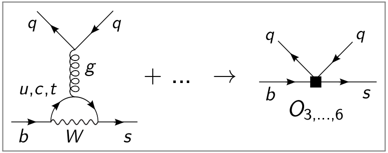
Effective weak Hamiltonian for $b \to s\ell^+\ell^-$ decays











($b \to d\ell^+\ell^-$ is analogous, but I will focus on $b \to s\ell^+\ell^-$ decays)

Effective weak Hamiltonian for $b \to s\ell^+\ell^-$ decays

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i O_i$$

with

$$\begin{aligned}
O_1 &= \bar{c}^b \gamma^\mu b_L^a \;\; \bar{s}^a \gamma_\mu c_L^b, \\
O_2 &= \bar{c}^a \gamma^\mu b_L^a \;\; \bar{s}^b \gamma_\mu c_L^b, \\
O_7 &= \frac{e\, m_b}{16\pi^2} \;\bar{s}\sigma^{\mu\nu} b_R \;\; F_{\mu\nu}^{(\text{e.m.})}, \\
O_9 &= \frac{e^2}{16\pi^2} \;\bar{s}\gamma^\mu b_L \;\; \bar{\ell}\gamma_\mu \ell, \\
O_{10} &= \frac{e^2}{16\pi^2} \;\bar{s}\gamma^\mu b_L \;\; \bar{\ell}\gamma_\mu \gamma_5 \ell, \\
&\dots
\end{aligned}$$

In the Standard Model, $\overline{\text{MS}}$ scheme, at $\mu = 4.2$ GeV,

C_1	C_2	C_7	C_9	C_{10}	...
-0.288	1.010	-0.336	4.275	-4.160	...

[Computed using EOS, `https://eos.github.io/`]

$B_s \to \ell^+\ell^-$

$$\overline{\mathcal{B}}(B_s \to \ell^+\ell^-) = \frac{1}{1-y_s}\,\frac{G_F^2\,\alpha_e^2\,m_{B_s}^3\,\tau_{B_s}}{64\,\pi^3}\left|V_{tb}V_{ts}^*\right|^2\sqrt{1-\frac{4\,m_\ell^2}{m_{B_s}^2}}\;\frac{4\,m_\ell^2}{m_{B_s}^2}\left|C_{10}\right|^2 f_{B_s}^2\,,$$

$$y_s = \frac{\Delta\Gamma_s}{2\,\Gamma_s}$$

[K. De Bruyn *et al.*, arXiv:1204.1737/PRL 2012]

	Experiment [1]	SM Theory [2]
$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)$	$(2.7^{+0.6}_{-0.4}) \times 10^{-9}$	$(3.64 \pm 0.11) \times 10^{-9}$
$\overline{\mathcal{B}}(B_d \to \mu^+\mu^-)$	$(16^{+16}_{-14}) \times 10^{-11}$	$(1.00 \pm 0.03) \times 10^{-11}$

[1] 2018 PDG average of LHCb, CMS, ATLAS

[2] A. Bazavov *et al.* (Fermilab and MILC collaborations), arXiv:1712.09262/PRD 2018

Hadronic matrix elements for exclusive $b \to s\ell^+\ell^-$ decays

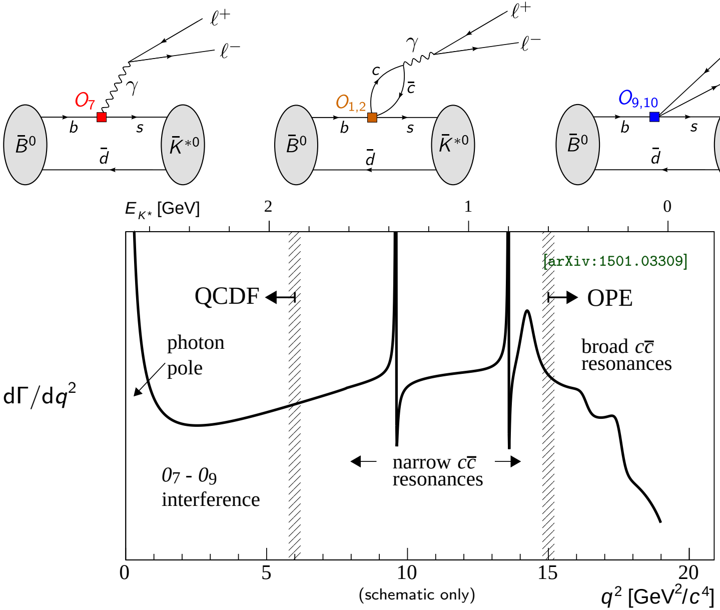
$O_7,\ O_9,\ O_{10}$:

$$\langle H_s(p')|\, \bar{s}\Gamma b\, |H_b(p)\rangle$$

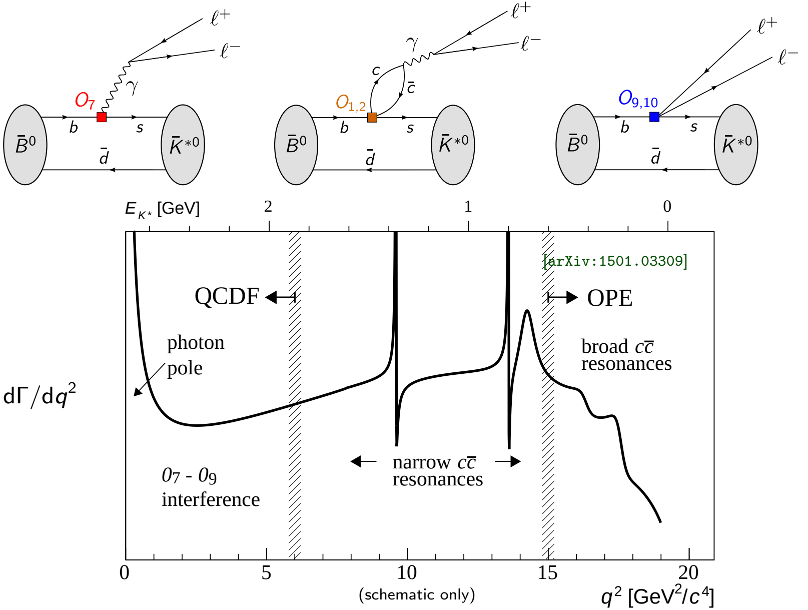
$O_{1,\ldots,6},\ O_8$:

$$\int \mathrm{d}^4x\ \ e^{iq\cdot x}\, \langle H_s(p')|\, \mathrm{T}\ O_i(0)\ J^{\mu}_{\mathrm{e.m.}}(x)\, |H_b(p)\rangle$$

Hadronic matrix elements for exclusive $b \to s\ell^+\ell^-$ decays

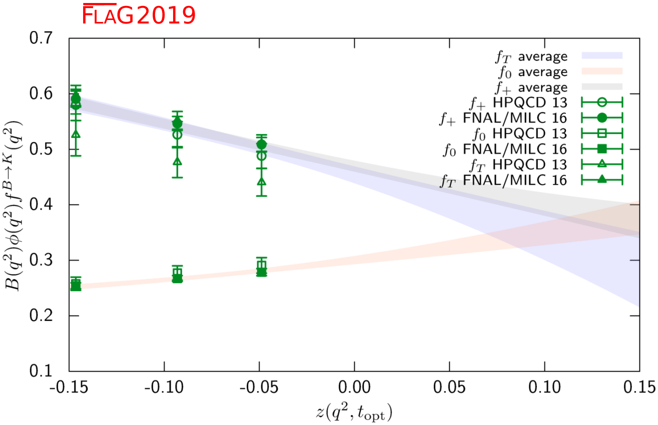


Hadronic matrix elements for exclusive $b \to s\ell^+\ell^-$ decays



For the charm contributions, see the talk by Katsumasa Nakayama tomorrow morning!

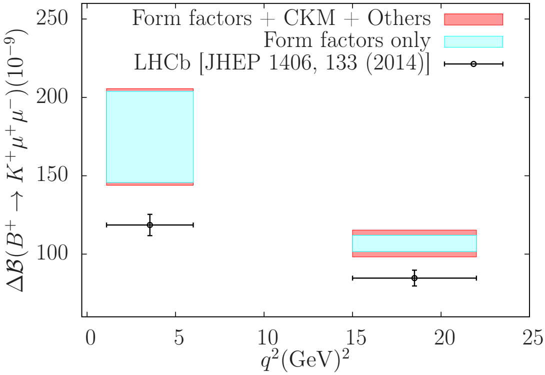
$B \to K$ form factors



New preliminary lattice QCD results:

Z. Gelzer (FNAL/MILC collaboration), talk at Lattice 2018

$B^+ \to K^+\mu^+\mu^-$ differential branching fraction



Contributions from $O_{1\ldots6;8}$ treated with OPE and QCDF.

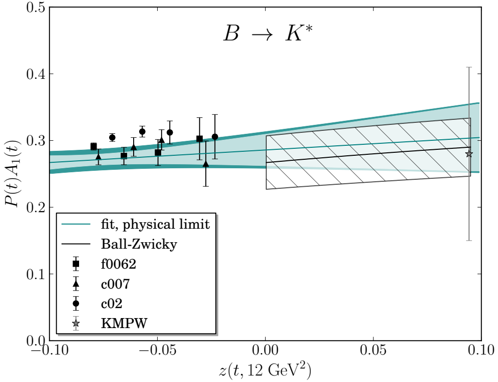
[D. Du *et al.* (FNAL/MILC collaboration), arXiv:1510.02349/PRD 2016]

$B \to K^*$ form factors

R. R. Horgan, Z. Liu, S. Meinel, M. Wingate, `arXiv:1310.3722`/PRD 2014

The K^* was treated as if it were stable under the strong interactions.

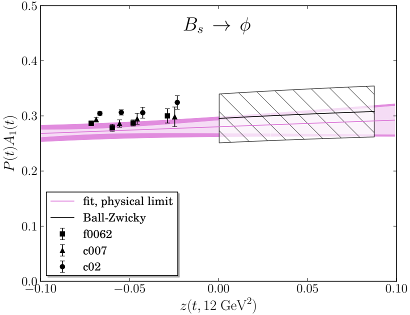
Example for one of the seven form factors:

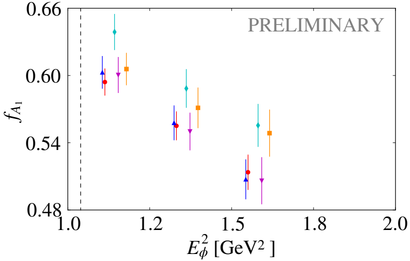


A new calculation in which the K^* is treated properly as a resonance is underway [S. Meinel (with L. Leskovec, M. Petschlies, G. Rendon, G. Silvi, *et al.*), talk at KEK-FF 2019].

$B_s \to \phi$ form factors

(ϕ also treated as stable)



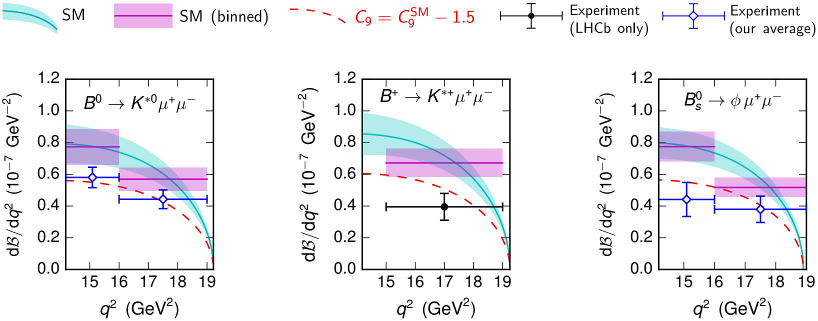


[R. R. Horgan, Z. Liu, S. Meinel, M. Wingate, arXiv:1310.3722/PRD 2014]

[J. Flynn *et al.* (RBC/UKQCD collaboration), arXiv:1612.05112]

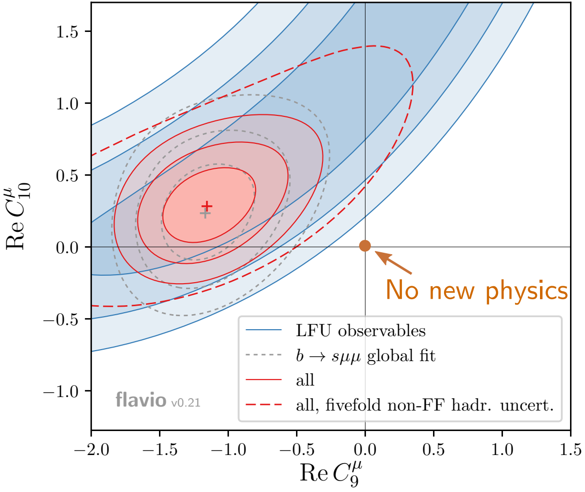
$B \to K^* \mu^+\mu^-$ and $B_s \to \phi\mu^+\mu^-$ differential branching fractions at high q^2

Contributions from $O_{1\ldots6;8}$ treated with OPE.



[R. R. Horgan, Z. Liu, S. Meinel, M. Wingate, arXiv:1310.3887/PRL 2014]

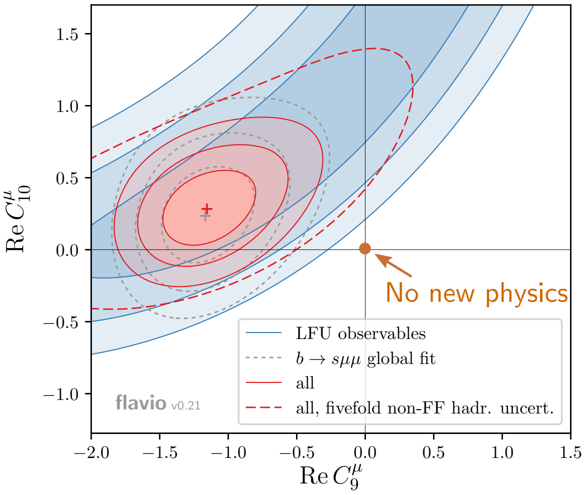
$b \to s\ell^+\ell^-$: Fit of C_9^μ and C_{10}^μ to experimental data (mesons only, 2017)



LFU observables:

$$R_{K^{(*)}} = \frac{\int_{q^2_{\text{start}}}^{q^2_{\text{stop}}} \frac{d\Gamma(B \to K^{(*)}\mu^+\mu^-)}{dq^2} dq^2}{\int_{q^2_{\text{start}}}^{q^2_{\text{stop}}} \frac{d\Gamma(B \to K^{(*)}e^+e^-)}{dq^2} dq^2}$$

$b \to s\ell^+\ell^-$: Fit of C_9^μ and C_{10}^μ to experimental data (mesons only, 2017)



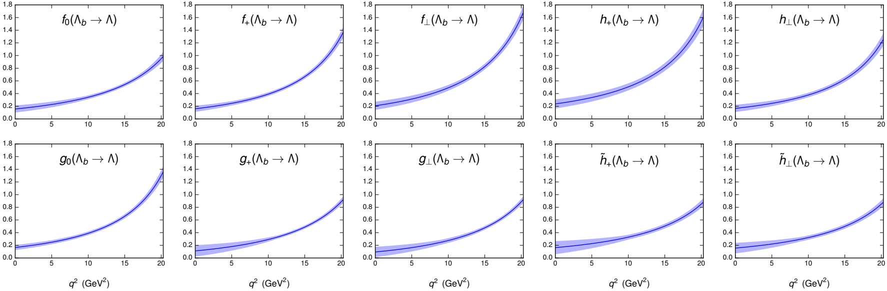
LFU observables:

$$R_{K^{(*)}} = \frac{\int_{q^2_{\mathrm{start}}}^{q^2_{\mathrm{stop}}} \frac{\mathrm{d}\Gamma(B\to K^{(*)}\mu^+\mu^-)}{\mathrm{d}q^2}\mathrm{d}q^2}{\int_{q^2_{\mathrm{start}}}^{q^2_{\mathrm{stop}}} \frac{\mathrm{d}\Gamma(B\to K^{(*)}e^+e^-)}{\mathrm{d}q^2}\mathrm{d}q^2}$$

New results from LHCb (R_K update) and Belle (R_{K^*}) at Moriond EW 2019!
See talk by David Straub (March 22, 2019) for updated fit results.

[W. Altmannshofer, P. Stangl, D. M. Straub, arXiv:1704.05435/PRD 2017]

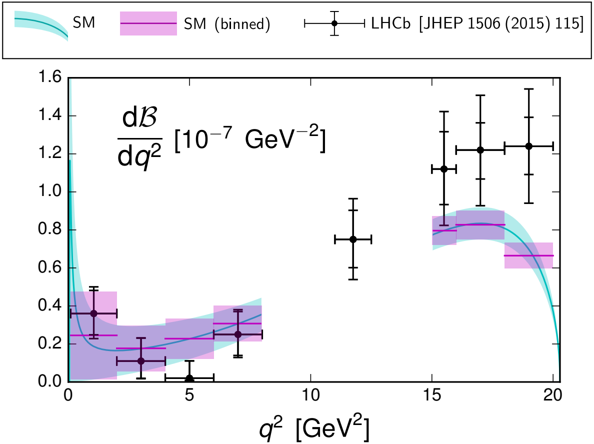
$\Lambda_b \to \Lambda$ form factors



[W. Detmold and S. Meinel, `arXiv:1602.01399`/PRD 2016]

$\Lambda_b \to \Lambda\mu^+\mu^-$ differential branching fraction

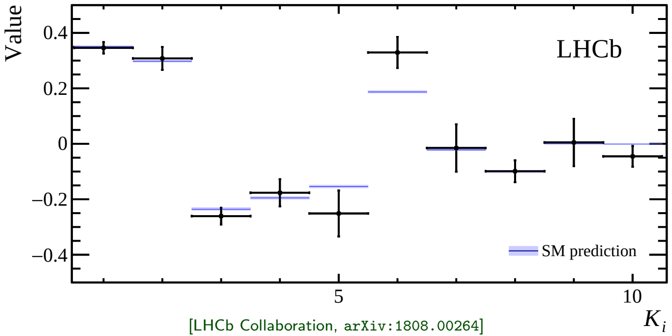
Contributions from $O_{1\ldots6;8}$ treated with OPE.



[W. Detmold and S. Meinel, arXiv:1602.01399/PRD 2016]

$\Lambda_b \to \Lambda\mu^+\mu^-$ new angular analysis (2018)

$\frac{d^5\Gamma}{d\vec{\Omega}} = \frac{3}{32\pi^2} \sum_i^{34} K_i f_i(\vec{\Omega})$ $\qquad$ $15 < q^2 < 20\ \mathrm{GeV}^2$



[LHCb Collaboration, arXiv:1808.00264]

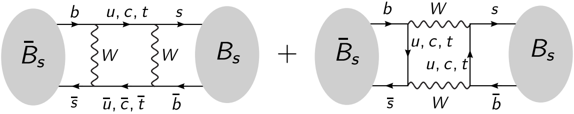
$$A_{\mathrm{FB}}^{\ell} = \frac{3}{2} K_3, \qquad A_{\mathrm{FB}}^{h} = K_4 + \frac{1}{2} K_5, \qquad A_{\mathrm{FB}}^{\ell h} = \frac{3}{4} K_6$$

Note: the 2015 LHCb result for A_{FB}^{ℓ}, which deviated 3.4σ from our SM prediction, was incorrect (it was actually the CP asymmetry in A_{FB}^{ℓ}).

→ Our Wilson coefficient fits [S. Meinel and D. van Dyk, arXiv:1603.02974/PRD 2016] need to be redone.

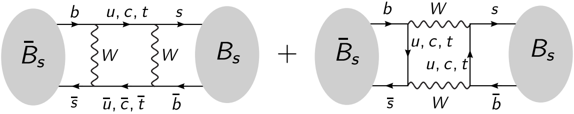
Neutral $B_{(s)}$ meson mixing

$B_{(s)}$-$\bar{B}_{(s)}$ mixing can be used to determine $|V_{td}|$ and $|V_{ts}|$, and provides important constraints on new physics in $b \to s$ and $b \to d$.



Neutral $B_{(s)}$ meson mixing

$B_{(s)}$-$\bar{B}_{(s)}$ mixing can be used to determine $|V_{td}|$ and $|V_{ts}|$, and provides important constraints on new physics in $b \to s$ and $b \to d$.



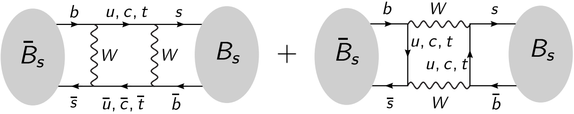
In the Standard Model,

- ΔM_s is dominated by the top-quark contribution $\Rightarrow$ well described by local dimension-6 operator $Q_1 = \left[\bar{b}\gamma_\mu(1-\gamma_5)s\right]\left[\bar{b}\gamma^\mu(1-\gamma_5)s\right]$.
  Lattice QCD input:
  $$\langle \bar{B}_s | Q_1 | B_s \rangle$$

  [Next slide]

Neutral $B_{(s)}$ meson mixing

$B_{(s)}$-$\bar{B}_{(s)}$ mixing can be used to determine $|V_{td}|$ and $|V_{ts}|$, and provides important constraints on new physics in $b \to s$ and $b \to d$.



In the Standard Model,

- ΔM_s is dominated by the top-quark contribution $\Rightarrow$ well described by local dimension-6 operator $Q_1 = \left[\bar{b}\gamma_\mu(1-\gamma_5)s\right]\left[\bar{b}\gamma^\mu(1-\gamma_5)s\right]$. Lattice QCD input:

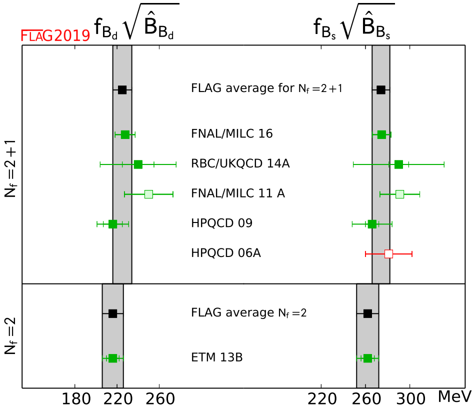
$$\langle \bar{B}_s | Q_1 | B_s \rangle$$

[Next slide]

- $\Delta\Gamma_s$ is dominated by the charm-quark contribution. An additional heavy-quark expansion is needed. The uncertainty is dominated by the matrix elements of dimension-7 operators.

[M. Wingate, talk at CKM 2018; C. Davies *et al.*, arXiv:1712.09934]

Neutral $B_{(s)}$ meson mixing



$$\langle \bar{B}^0_q \left| Q_1^q(\mu) \right| B^0_q \rangle = \frac{8}{3} m_B^2 f_{B_q}^2 B_{B_q}(\mu)$$

$$\hat{B}_{B_q} = \left(\frac{\bar{g}(\mu)^2}{4\pi} \right)^{-\gamma_0/(2\beta_0)} \left\{ 1 + \frac{\bar{g}(\mu)^2}{(4\pi)^2} \left[\frac{\beta_1\gamma_0 - \beta_0\gamma_1}{2\beta_0^2} \right] \right\} B_{B_q}(\mu) \quad ,$$

Neutral B_s meson mixing

Experiment is currently far more precise than theory!

	Experiment	SM Theory
ΔM_s	17.757 ± 0.021 ps^{-1} [1]	20.01 ± 1.25 ps^{-1} [2]
$\Delta \Gamma_s$	0.088 ± 0.006 ps^{-1} [1]	0.085 ± 0.015 ps^{-1} [3]

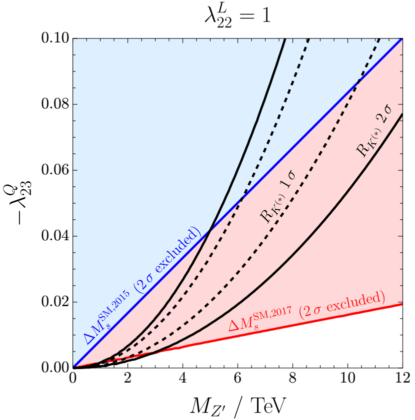
[1] HFLAV, `https://hflav-eos.web.cern.ch/hflav-eos/osc/PDG_2018/`

[2] L. Di Luzio, M. Kirk, A. Lenz, `arXiv:1811.12884`

[3] M. Artuso, G. Borissov, A. Lenz, `arXiv:1511.09466`/RMP 2016

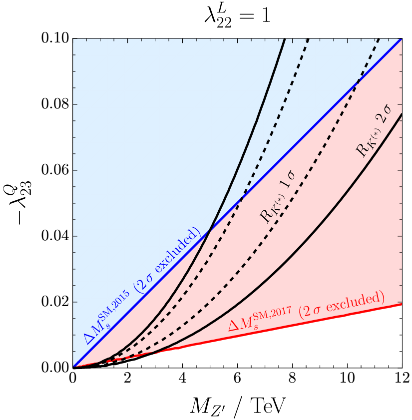
Interplay of B_s-$\bar{B}_s$ mixing and $b \to s\ell^+\ell^-$ anomalies

Example for a Z' model: [L. Di Luzio, M. Kirk, A. Lenz, arXiv:1811.12884]



Interplay of B_s-$\bar{B}_s$ mixing and $b \to s\ell^+\ell^-$ anomalies

Example for a Z' model: [L. Di Luzio, M. Kirk, A. Lenz, arXiv:1811.12884]



Improved lattice calculations of B_s-$\bar{B}_s$ mixing matrix elements can have a huge impact!

Summary

These are exciting times for *b* physics. We have some interesting hints for deviations from the SM, and we now have **two** amazing *b*-physics experiments with complementary strengths that will continue to dramatically improve our knowledge.

Summary

These are exciting times for *b* physics. We have some interesting hints for deviations from the SM, and we now have **two** amazing *b*-physics experiments with complementary strengths that will continue to dramatically improve our knowledge.

Lattice QCD calculations are essential to make the connection between the experimental data and the fundamental short-distance processes. For many quantities, we need to improve the precision to keep up with the experiments. We also need to compute novel types of observables, which can now be accessed with lattice QCD thanks to groundbreaking developments of new methods.

Extra slides: heavy-quark lattice actions

Lattice HQET

Leading-order HQET in rest frame:

$$S_\psi = \delta m \int \mathrm{d}^4x \;\; \psi^\dagger \psi + \int \mathrm{d}^4x \;\; \psi^\dagger D_0 \, \psi$$

Lattice discretization:

$$S_{\psi,\mathrm{lat.}} = \sum_x \psi^\dagger(x) \big[(1+\delta m)\psi(x) - U_0^\dagger(x-\hat{0})\psi(x-\hat{0}) \big]$$

[E. Eichten, B. Hill, PLB **240**, 193 (1990)]

Higher-order $1/m$ corrections, starting with

$$\mathcal{L}_\psi^{(1)} = \psi^\dagger \left[-\frac{\mathbf{D}^2}{2m} - g\frac{\boldsymbol{\sigma}\cdot\mathbf{B}}{2m} \right] \psi,$$

are treated as insertions in correlation functions, with nonperturbative renormalization. This means that the theory remains renormalizable, and one can go to the continuum limit.

[J. Heitger, R. Sommer, `arXiv:hep-lat/0310035`/JHEP 2004]

Lattice HQET can only be used for singly-heavy hadrons.

Lattice NRQCD

Continuum action (with tree-level matching coefficients):

$$\mathcal{S}_\psi = \int \mathrm{d}^4x \; \psi^\dagger \Big[\underbrace{D_0 - \frac{\mathbf{D}^2}{2m}}_{\mathcal{O}(v^2)}$$
$$\underbrace{- \frac{g}{2m}\boldsymbol{\sigma}\cdot\mathbf{B} + \frac{g}{8m^2}\left(i\mathbf{D}^{\mathrm{ad}}\cdot\mathbf{E} - \boldsymbol{\sigma}\cdot(\mathbf{D}\times\mathbf{E} - \mathbf{E}\times\mathbf{D})\right) - \frac{\mathbf{D}^4}{8m^3}}_{\mathcal{O}(v^4)} + \ldots \Big]\psi$$

Here, the power counting indicated is based on v^2, the average heavy-quark velocity-squared inside heavy quarkonium. For bottomonium, $v^2 \approx 0.1$.

Lattice NRQCD is a discretization of this, where all terms are kept in the action [G. P. Lepage, L. Magnea, C. Nakhleh, U. Magnea, K. Hornbostel, arXiv:hep-lat/9205007/PRD 1992].
One must keep $am \gtrsim 1$ in the simulations.

Matching coefficients for the action and for currents have been computed using one-loop lattice perturbation theory.
[See for example C. Monahan, J. Shigemitsu, R. Horgan, arXiv:1211.6966/PRD 2013;
R. Dowdall, C. Davies, T. Hammant, R. Horgan, C. Hughes, arXiv:1309.5797/PRD 2014;
C. Davies *et al.*, arXiv:1812.11639/PRD 2019]

Lattice "moving NRQCD"

This is a Lorentz-boosted version of lattice NRQCD. The $\mathbf{v}$ below is the boost velocity, not the power-counting parameter discussed before. The continuum action of mNRQCD (with tree-level matching coefficients) is

$$\begin{aligned}
S \;=\; \int \mathrm{d}^4x\ \psi_v^\dagger \Bigg[& D_0 - i\mathbf{v}\cdot\mathbf{D} - \frac{\mathbf{D}^2 - (\mathbf{v}\cdot\mathbf{D})^2}{2\gamma m} - \frac{g}{2\gamma m}\boldsymbol{\sigma}\cdot\mathbf{B}' \\
& - \frac{i}{4\gamma^2 m^2}\left(\left\{\mathbf{v}\cdot\mathbf{D},\ \mathbf{D}^2\right\} - 2(\mathbf{v}\cdot\mathbf{D})^3\right) + \frac{g}{8m^2}\left(i\mathbf{D}^{\mathrm{ad}}\cdot\mathbf{E} + \mathbf{v}\cdot(\mathbf{D}^{\mathrm{ad}}\times\mathbf{B})\right) \\
& - \frac{g}{8\gamma m^2}\,\boldsymbol{\sigma}\cdot\left(\mathbf{D}\times\mathbf{E}' - \mathbf{E}'\times\mathbf{D}\right) + \frac{g}{8(\gamma+1)m^2}\left\{\mathbf{v}\cdot\mathbf{D},\ \boldsymbol{\sigma}\cdot(\mathbf{v}\times\mathbf{E}')\right\} \\
& - \frac{(2-\mathbf{v}^2)g}{16m^2}\left(D_0^{\mathrm{ad}} + i\mathbf{v}\cdot\mathbf{D}^{\mathrm{ad}}\right)(\mathbf{v}\cdot\mathbf{E}) - \frac{ig}{4\gamma^2 m^2}\left\{\mathbf{v}\cdot\mathbf{D},\ \boldsymbol{\sigma}\cdot\mathbf{B}'\right\} \\
& - \frac{1}{8\gamma^3 m^3}\left(\mathbf{D}^4 - 3\left\{\mathbf{D}^2,\ (\mathbf{v}\cdot\mathbf{D})^2\right\} + 5(\mathbf{v}\cdot\mathbf{D})^4\right) + \ldots \Bigg]\,\psi_v .
\end{aligned}$$

Lattice mNRQCD allows to give high momentum ($> a^{-1}$) to b-hadrons on the lattice while keeping discretization errors under control.

[R. R. Horgan, L. Khomskii, S. Meinel, M. Wingate, K. M. Foley, G. P. Lepage, G. M. von Hippel, A. Hart, E. H. Müller, C. T. H. Davies, A. Dougall, K. Y. Wong, arXiv:0906.0945/PRD 2009]

Wilson-like actions with m_Q-dependent, anisotropic coefficients

Wilson-like actions have a smooth heavy-quark limit. Cutoff effects can be understood and (partially) removed using HQET/NRQCD analysis.

[A. El-Khadra, A. Kronfeld, P. Mackenzie, arXiv:hep-lat/9604004/PRD 1997;

A. Kronfeld, arXiv:hep-lat/0002008/PRD 2000;

J. Harada, S. Hashimoto, K.-I. Ishikawa, A. Kronfeld, T. Onogi, N. Yamada, arXiv:hep-lat/0112044/PRD 2002;

J. Harada, S. Hashimoto, A. Kronfeld, T. Onogi, arXiv:hep-lat/0112045/PRD 2002]

- Fermilab approach: am tuned to yield correct heavy-light meson kinetic mass (the energy at zero momentum is irrelevant).
- Columbia approach: (also known as RHQ action): am, ν, and c_P tuned to yield correct heavy-light meson kinetic mass, rest mass, and hyperfine splitting [N. Christ, M. Li, H.-W. Lin, arXiv:hep-lat/0608006/PRD 2007]
- Oktay-Kronfeld action: adds dimension-6 and dimension-7 operators [M. Oktay, A. Kronfeld, arXiv:0803.0523/PRD 2008]
- Matching of currents is usually done with the "mostly nonperturbative method":

$$J_\Gamma = \underbrace{\sqrt{Z_V^{(qq)} Z_V^{(bb)}}}_{\text{nonperturbative}} \rho_\Gamma \Big[\bar{q}\Gamma b + \mathcal{O}(a) \text{ improvement terms}\Big]$$