Electroweak Boxes & Dispersion Relations

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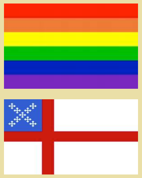


AMHERST CENTER FOR FUNDAMENTAL INTERACTIONS
Physics at the interface: Energy, Intensity, and Cosmic frontiers
University of Massachusetts Amherst

http://www.physics.umass.edu/acfi/

My pronouns: he/him/his

Collaborators: M. Gorchtein, H. Patel, C. Seng

PRL 121 (2018) 241804 [1807.10197], 1812.03352

Intensity Frontier Incubation Workshop,
BNL, March 2019

Goals For This Talk

- *Provide a brief BSM context & basic formalism*
- *Introduce the dispersion relation framework*
- *Apply the DR framework to nucleon & nuclear contributions to the $W\gamma$ box correction*
- *Discuss possible tests*

Outline

I. Context

Weak Decays: CKM Unitarity

$$d \to u\, e^- \,\bar{\nu}_e$$

$$s \to u\, e^- \,\bar{\nu}_e$$

$$b \to u\, e^- \,\bar{\nu}_e$$

$$\begin{pmatrix} u & c & t \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\Delta_{\mathrm{CKM}} = (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2)_{\mathrm{exp}} - 1$$

$$\boxed{0.94906 \pm 0.00041} \qquad \boxed{0.05031 \pm 0.00022} \qquad \boxed{0.00002}$$

$$\boxed{\Delta_{\mathrm{CKM}} = -0.0006 \pm 0.0005}$$

Precision ~ BSM Mass Scale

Precision Goal:

$$\delta\, \Delta_{CKM} \sim O(10^{-4})$$

Heavy BSM Physics:

$$\Delta_{CKM} \sim C \; (v/\Lambda)^2$$

$\Lambda \sim 10$ TeV (tree)

$\Lambda < 1$ TeV (loop)

Ultralight BSM Physics:

$$\Delta_{CKM} \sim \varepsilon^2 \; (\alpha /4\pi)$$

$\varepsilon < 1$ (loop)

Error Budget

$$\Delta_{\mathrm{CKM}} = (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2)_{\mathrm{exp}} - 1$$

0.94906 ± 0.00041

0.05031 ± 0.00022

0.00002

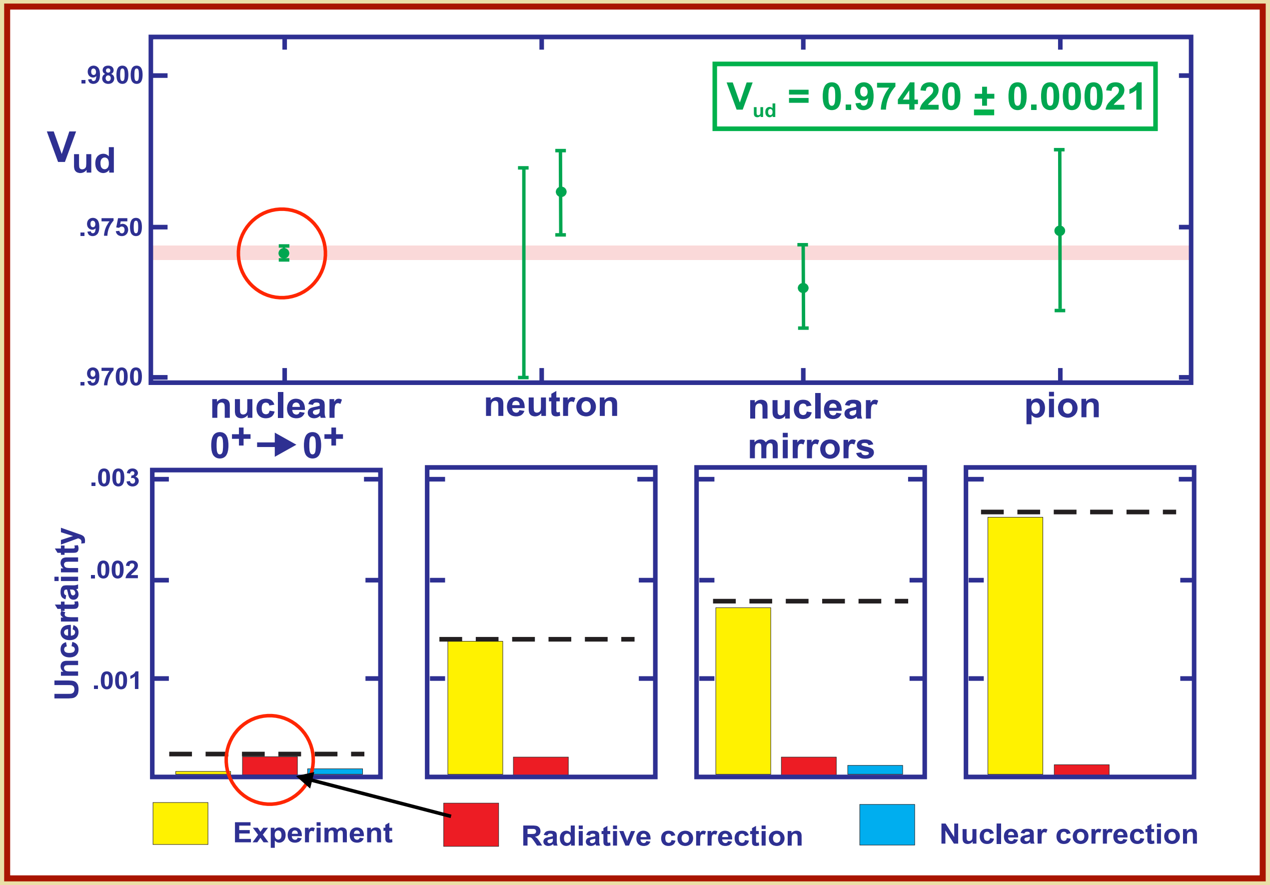
Error Budget

$$\Delta_{\mathrm{CKM}} = (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2)_{\mathrm{exp}} - 1$$

$\boxed{0.94906 \pm 0.00041}$

Radiative Correction

Factor of 2 reduction using disp relations

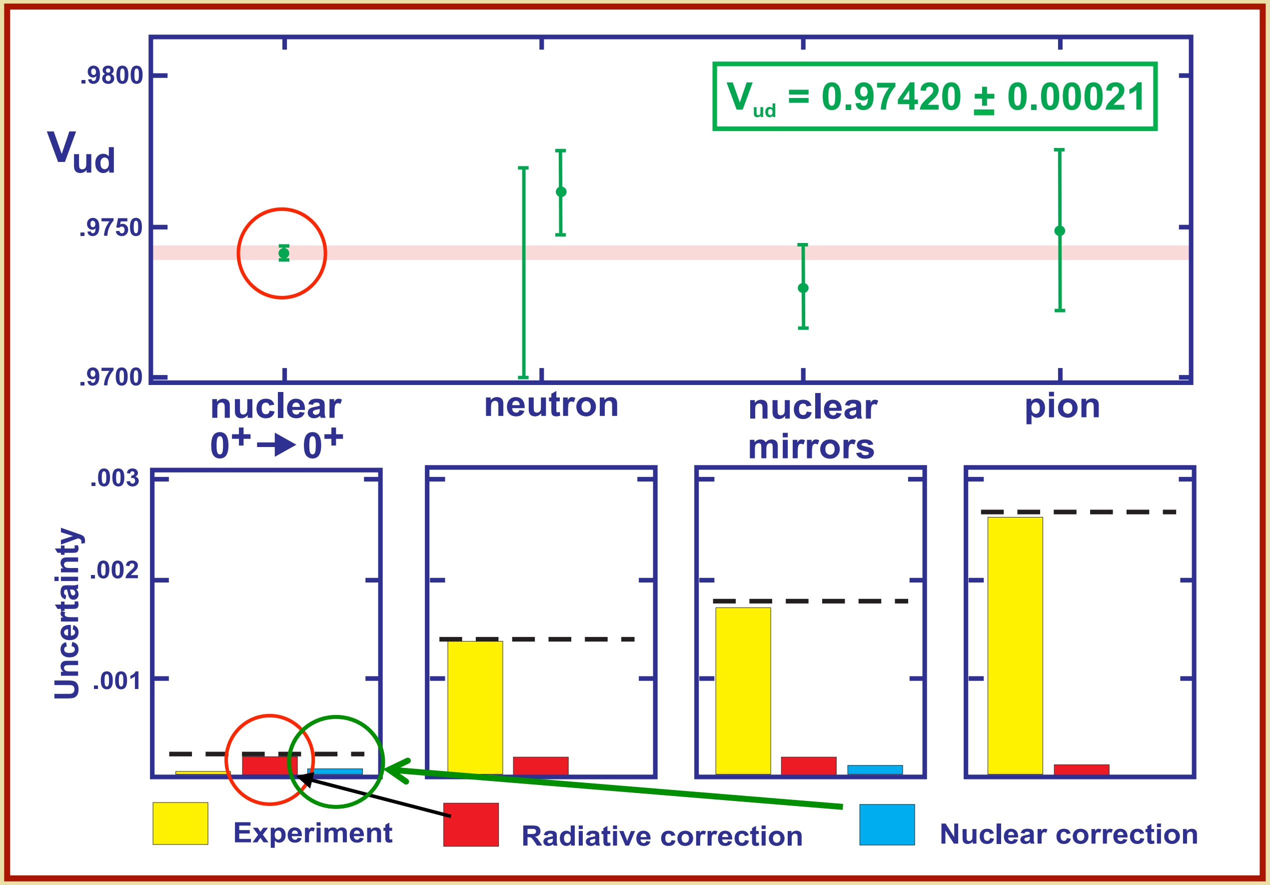


Thanks: J. Hardy

Error Budget

$$\Delta_{\mathrm{CKM}} = (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2)_{\mathrm{exp}} - 1$$

$$\boxed{0.94906 \pm 0.00041}$$



Radiative Correction

Factor of 2 reduction using disp relations

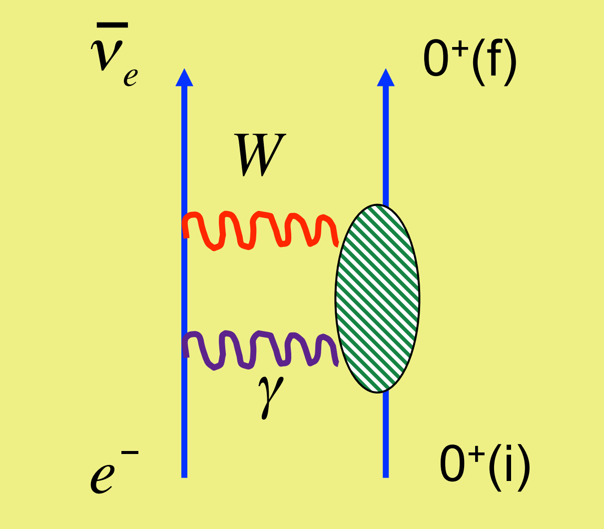
Nuclear Correction

Increase due to previously omitted contributions

Thanks: J. Hardy

Radiative Corrections

Dominant source of uncertainty:



$$M_{\gamma W} = \frac{G_F V_{ud}}{\sqrt{2}} \frac{\alpha}{8\pi} \left[\ln\left(\frac{M_Z^2}{\Lambda^2}\right) + C_{\gamma W}(\Lambda) \right]$$

Short distance **Long distance**

Long distance

Sensitive to hadronic & nuclear dynamics

Radiative Corrections & Ft Values

Corrected ft values:

$$\mathcal{F}t = ft\,(1+\delta_R')\,(1+\delta_{NS}-\delta_C)\,(1+\Delta_E^{NS})$$

Outer correction

Nuclear struct part of $M_{\gamma W}$

E-independent

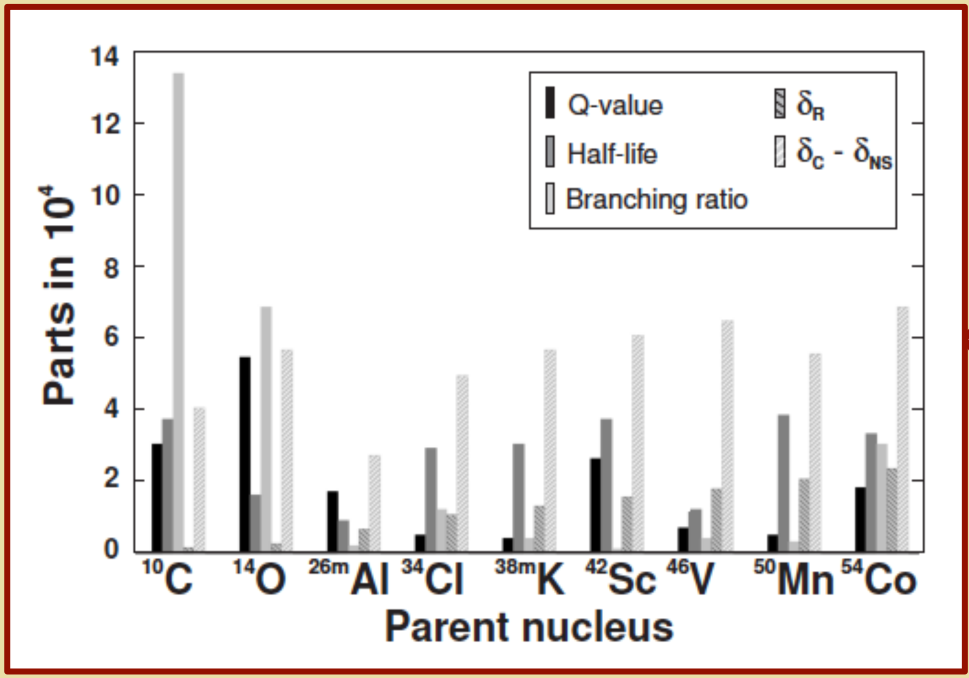
Nucl wavef'n

Not a RC

Nuclear struct part of $M_{\gamma W}$

E-dependent

$0^+ \to 0^+$ Dispersion Corrections: δ_{NS}



b_F : scalar currents

Input for V_{ud} & CKM unitarity test

Towner & Hardy, PRC 91 (2015) 2, 025501

Radiative Corrections & V_{ud}

Superallowed

$$|V_{ud}|^2 = \frac{2984.43s}{\mathcal{F}t(1+\Delta_R^V)}$$

Hadronic & short distance part of $M_{\gamma W}$

Neutron

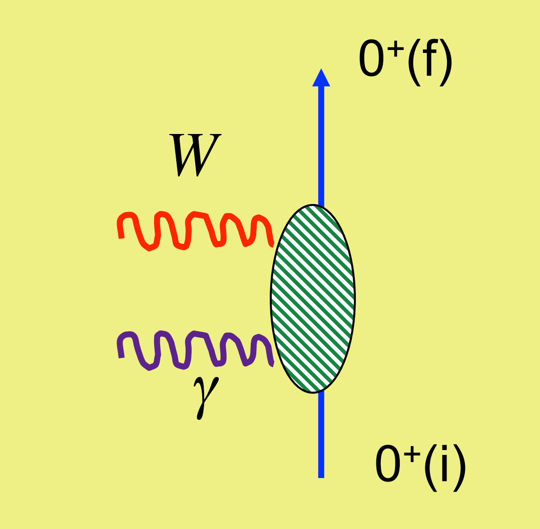
$$|V_{ud}|^2 = \frac{5099.34s}{\tau_n(1+3\lambda^2)(1+\Delta_R)}$$

$(g_A / g_V)^2$

Contains Δ_R^V

II. $W\gamma$ Box: Dispersion Relations

Dispersion Relations



Electroweak virtual Compton amplitude:

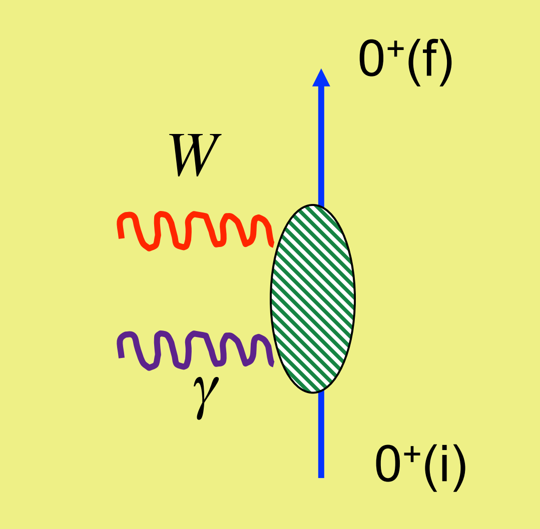
$$T^{\mu\nu}_{\gamma W} = \left[-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right] T_1 + \frac{\hat{p}^\mu \hat{p}^\nu}{(p\cdot q)} T_2 + \boxed{\frac{i\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2(p\cdot q)} T_3}$$

Radiative correction:

$$\Box^{VA}_{\gamma W} = 4\pi\alpha \mathrm{Re} \int \frac{d^4 q}{(2\pi)^4} \frac{M_W^2}{M_W^2 + Q^2} \frac{Q^2 + \nu^2}{Q^4} \frac{T_3(\nu, Q^2)}{M\nu}$$

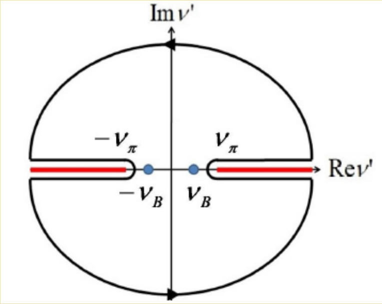
$$\Box^{VA}_{\gamma W} = \frac{1}{2} \left(\Delta^V_R\right)^{VA}_{\gamma W}$$

Dispersion Relations



Dispersion relation:

Write T_3 as integral over discontinuity along cut



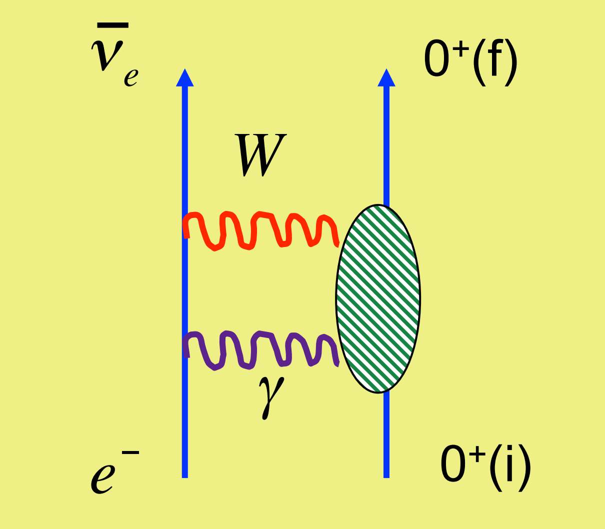
$$T_3^{(I)}(\nu, Q^2) = \frac{2}{i}\int_0^\infty d\nu' \left[\frac{1}{\nu' - \nu} + \frac{\xi^I}{\nu' + \nu}\right] \boxed{F_3^{(I)}(\nu', Q^2)}$$

Electroproduction structure functions:

$$\boxed{\begin{aligned} W_{\gamma W}^{(I)\mu\nu} &= \frac{1}{8\pi}\sum_X (2\pi)^4 \delta^4(p + q - p_X) \langle p | J_{em}^{(I)\mu} | X \rangle \langle X | J_W^\nu | n \rangle \\ &= \left[-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right] F_1^{(I)} + \frac{\hat{p}^\mu \hat{p}^\nu}{(p\cdot q)} F_2^{(I)} + \frac{i\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2(p\cdot q)} F_3^{(I)} \end{aligned}}$$

$$\boxed{d\sigma \propto L_{\mu\nu} W^{\mu\nu}}$$

Dispersion Relations



Radiative Correction:

$$\Box_{\gamma W}^{VA\,(0)} = \frac{\alpha}{\pi M}\int_0^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2}\int_0^\infty d\nu \frac{(\nu+2q)}{\nu(\nu+q)^2} F_3^{(0)}(\nu,Q^2)$$

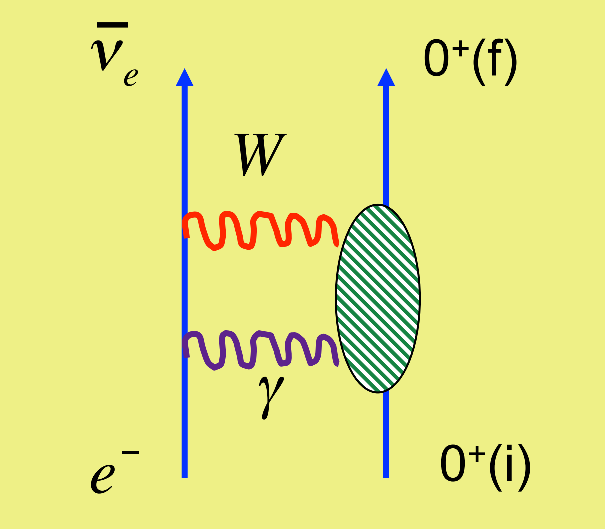
$$= \frac{3\alpha}{2\pi}\int_0^\infty \frac{dQ^2 M_W^2}{Q^2[M_W^2+Q^2]} M_3^{(0)}(1,Q^2)$$

Nachtmann Moments:

$$M_3^{(0)}(N,Q^2) = \frac{N+1}{N+2}\int_0^1 \frac{dx\xi^N}{x^2}\left[2x - \frac{N\xi}{N+1}\right] F_3^{(0)}$$

$$\xi = 2x\left(1+\frac{4M^2x^2}{Q^2}\right)^{-1}$$

Dispersion Relations



Radiative Correction:

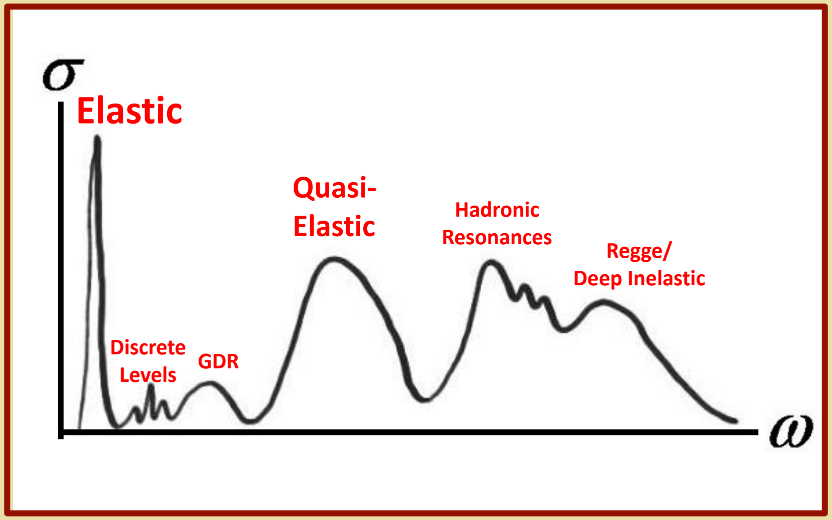
$$\Box_{\gamma W}^{VA\,(0)} = \frac{\alpha}{\pi M}\int_0^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2}\int_0^\infty d\nu\,\frac{(\nu+2q)}{\nu(\nu+q)^2}F_3^{(0)}(\nu,Q^2)$$

$$= \frac{3\alpha}{2\pi}\int_0^\infty \frac{dQ^2 M_W^2}{Q^2[M_W^2+Q^2]}M_3^{(0)}(1,Q^2)$$

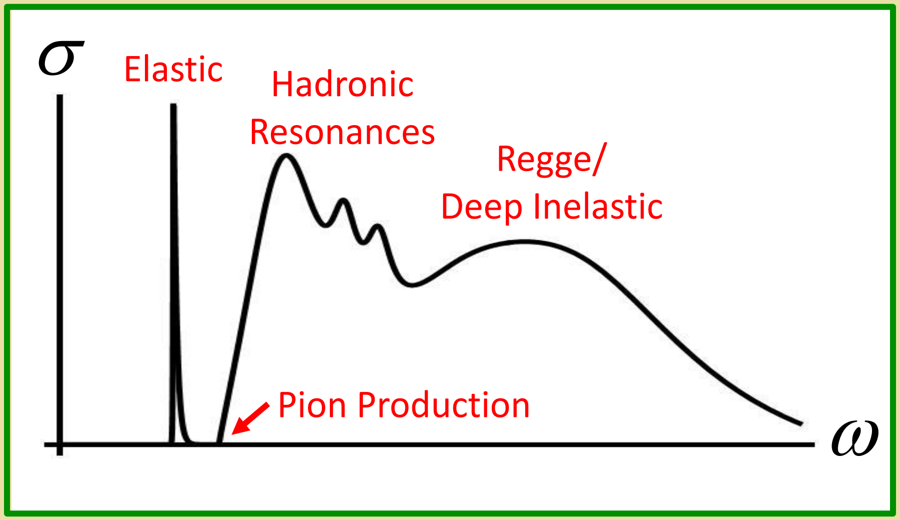
- ***Relate $F_3^{(0)}$ and $M_3^{(0)}$ to data and/or***
- ***Compute $F_3^{(0)}$ and $M_3^{(0)}$ using same methods used to describe semi-leptonic scattering processes with nucleon & nuclear targets***

Leptoproduction: Had & Nuc Response

Nuclei



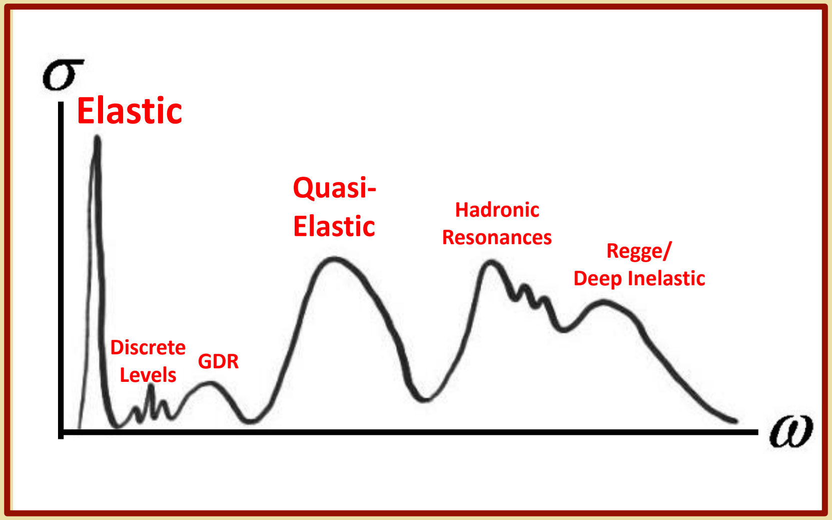
Free nucleons



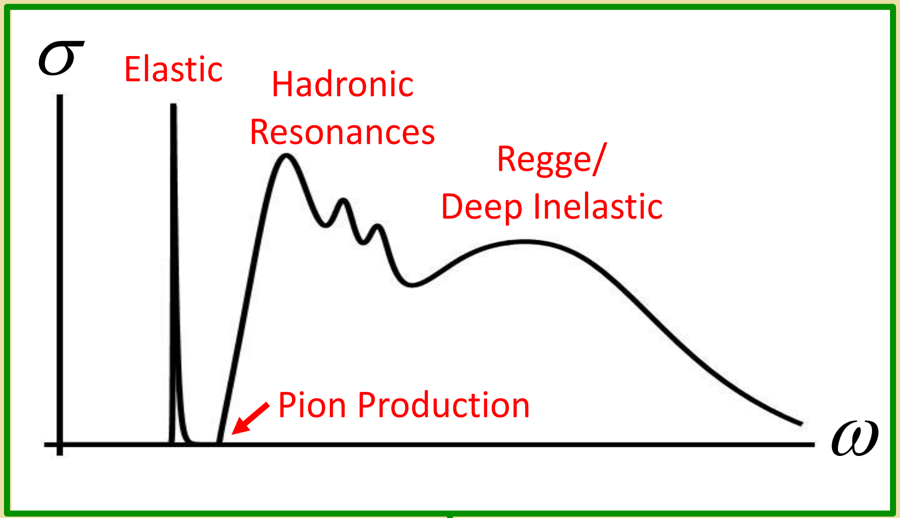
III. Wγ Box: Free Neutron

Leptoproduction: Had & Nuc Response

Nuclei



Free nucleons

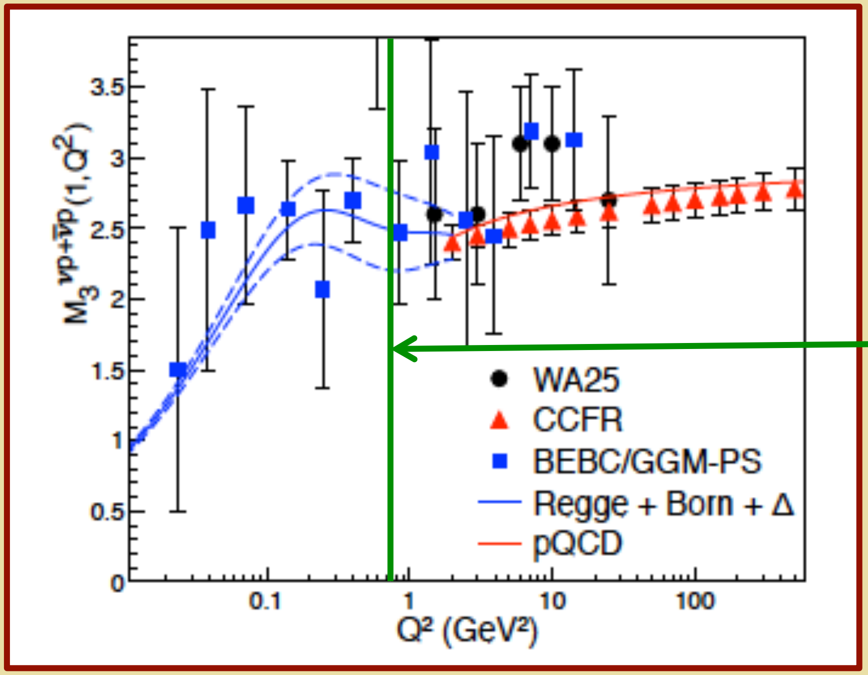


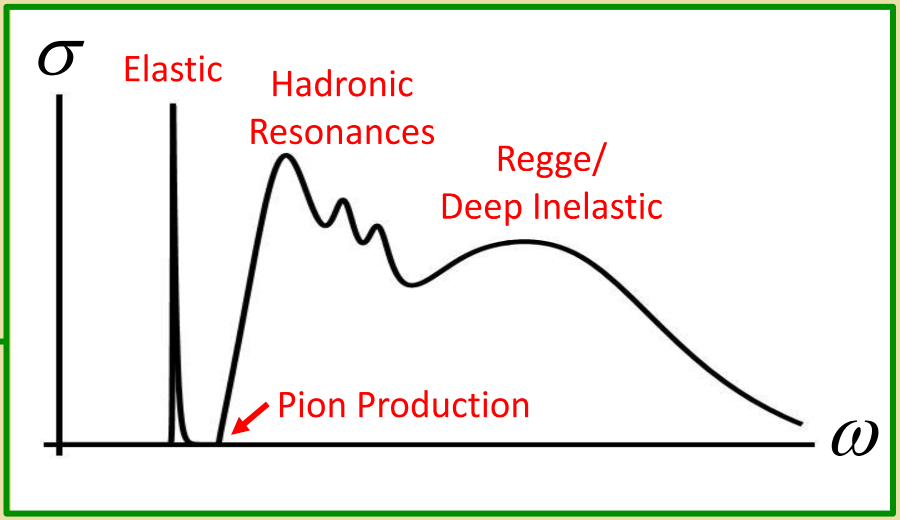
Single nucleon: PRL 121 (2008) 241804

$\Delta_R^V = 0.02361(38) \rightarrow 0.04267\ (22)$

Neutrino Scattering

Free nucleons

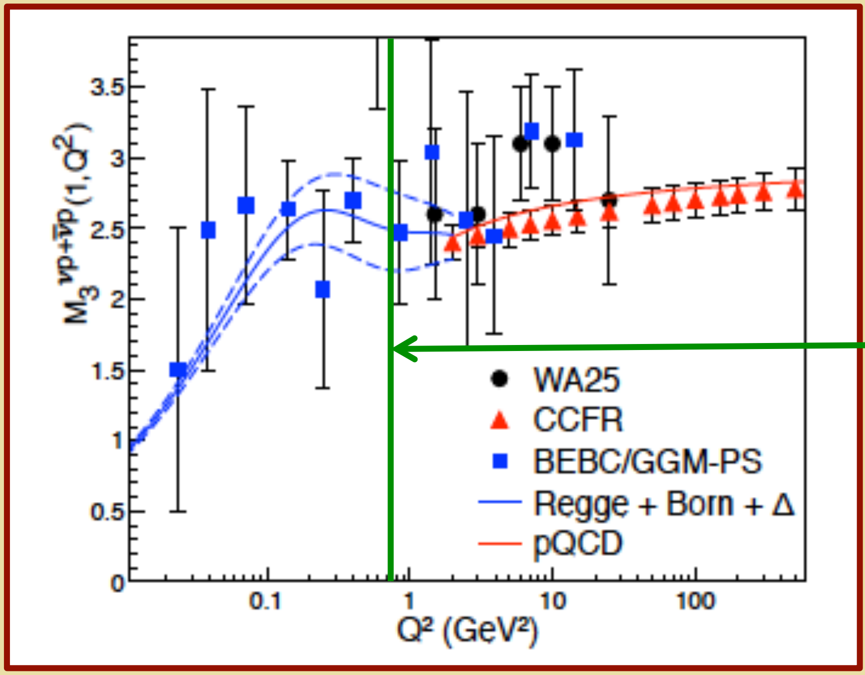


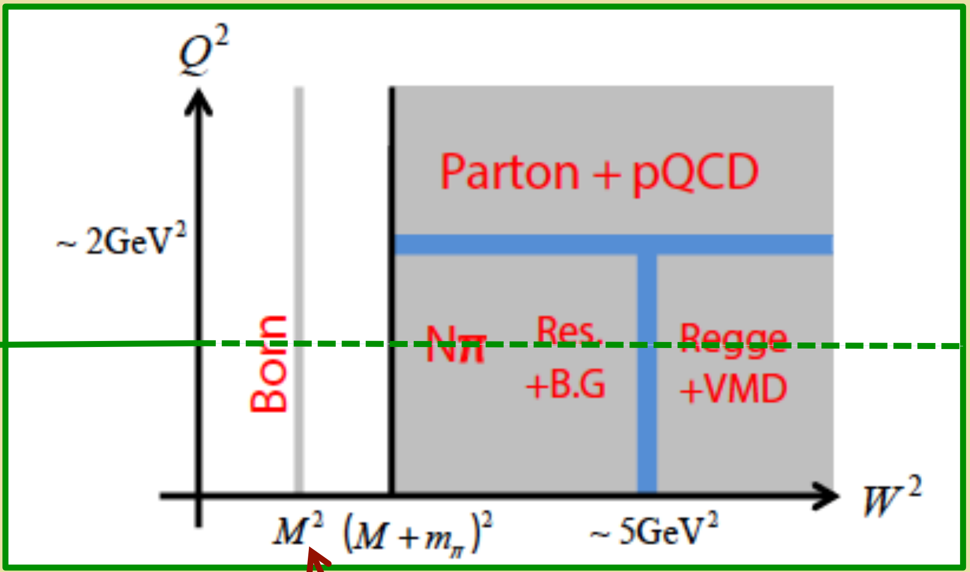


- ***Compute contributions to $M_3^{\nu p+\bar{\nu}p}$ at each Q^2 from different ω regions***
- ***Isospin rotate to $M_3^{(0)}$***

Neutrino Scattering

Free nucleons





Illustrate w/ Born

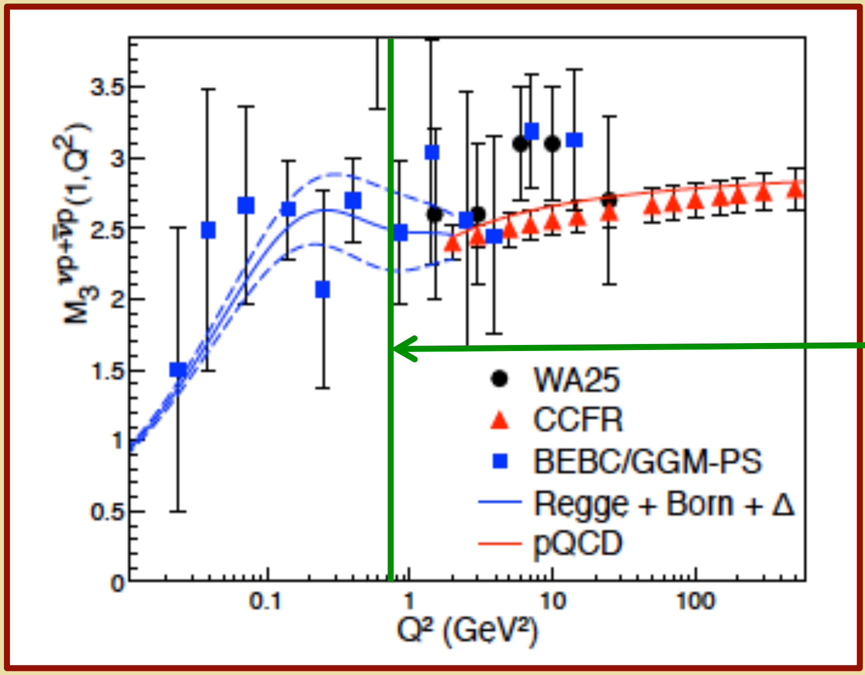
- ***Compute contributions to $M_3^{\nu p+\bar{\nu}p}$ at each Q^2 from different ω regions***
- ***Isospin rotate to $M_3^{(0)}$***

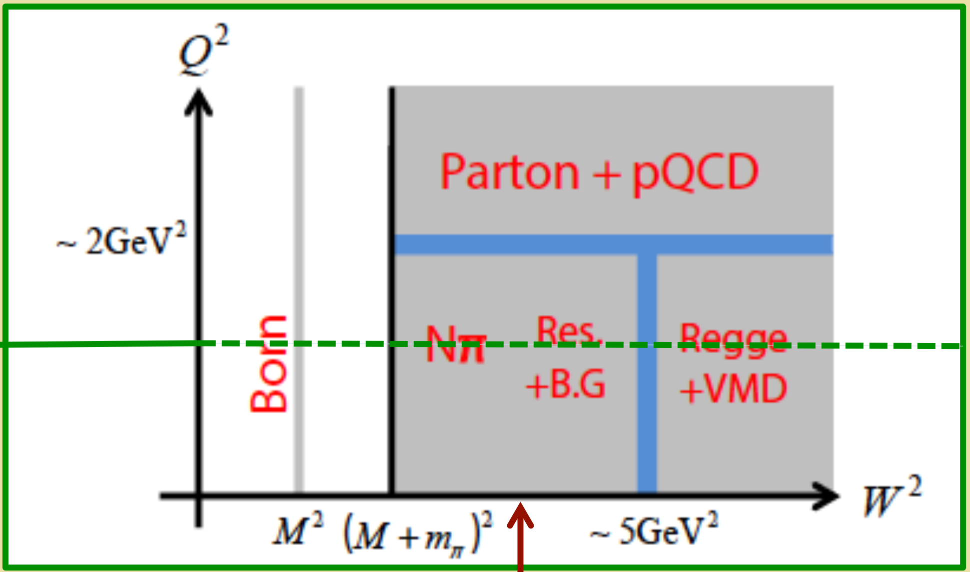
Born Contribution

$$\begin{aligned} F_{3,\,\mathrm{Born}}^{\nu p+\bar{\nu} p} &= -G_A(Q^2)G_M^V(Q^2)\delta(1-x), \\ F_{3,\,\mathrm{Born}}^{(0)} &= -\frac{1}{4}G_A(Q^2)G_M^S(Q^2)\delta(1-x), \end{aligned}$$

Neutrino Scattering

Free nucleons



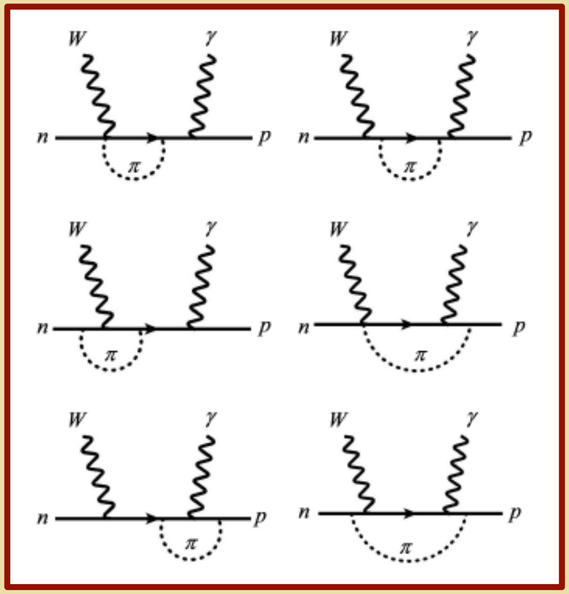


π N + Resonance

- ***Compute contributions to $M_3^{\nu p+\bar{\nu}p}$ at each Q^2 from different ω regions***
- ***Isospin rotate to $M_3^{(0)}$***

π N + Resonance

Non-resonant



$$F^{(0)}_{3,\,\pi N} = \sum_{i=1}^{6} F^{(0)\pi N}_{3,i}$$

Resonant

Δ(1232): ν p only

Form factors

$$F_{3,\Delta}^{\nu p+\bar{\nu} p} = -\frac{2\nu}{M}\frac{m_\Delta \Gamma_\Delta}{\pi}\frac{1}{(W^2-m_\Delta^2)^2+m_\Delta^2\Gamma_\Delta^2}\frac{V_3}{3}$$

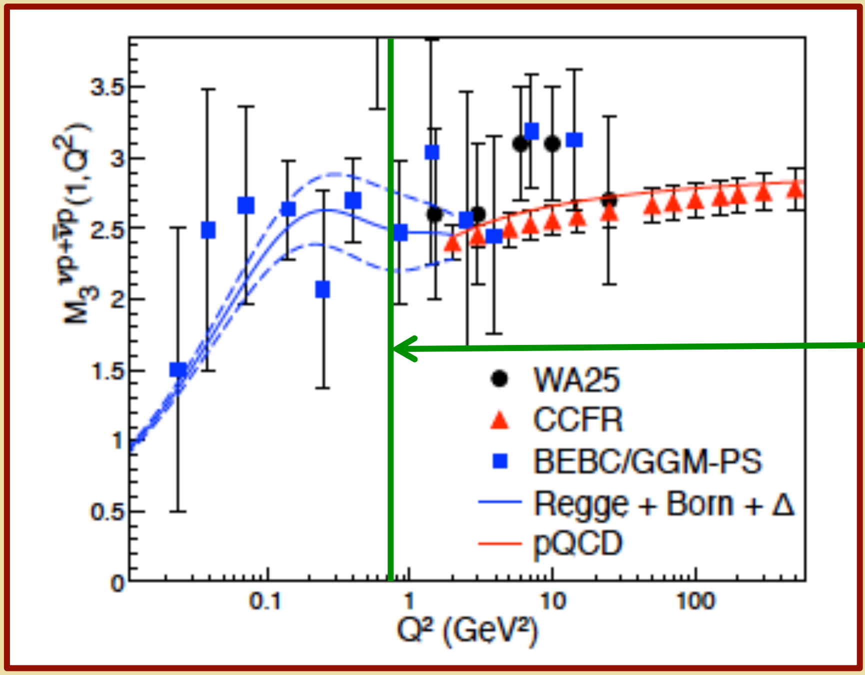
$$F_{3,\,\mathrm{res}} = \sum_R \frac{\frac{\nu}{q}\Gamma_R m_R}{(W^2-m_R^2)^2+\Gamma_R^2 m_R^2}\sqrt{\frac{M(m_R^2-M^2)}{16\pi^3\alpha}} \times \sum_{J_z=1/2,3/2} (A^{R,p}_{em,J_z}+A^{R,n}_{em,J_z})^* A^{R}_{w,J_z}$$

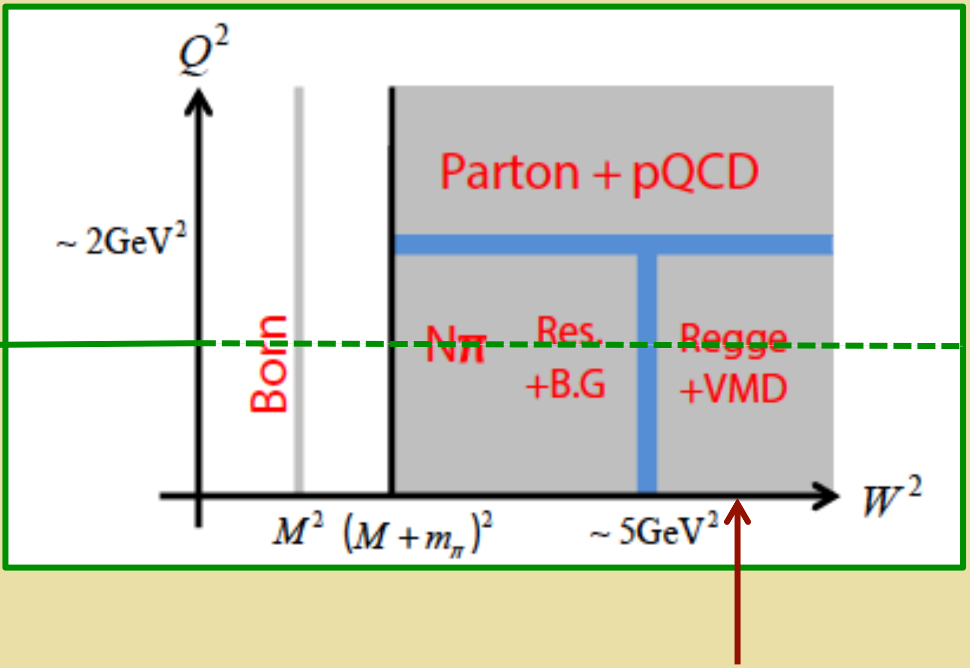
$F_3^{\nu p}$: both 1/2 , 3/2

$F_3^{(0)}$: only 1/2

Neutrino Scattering

Free nucleons



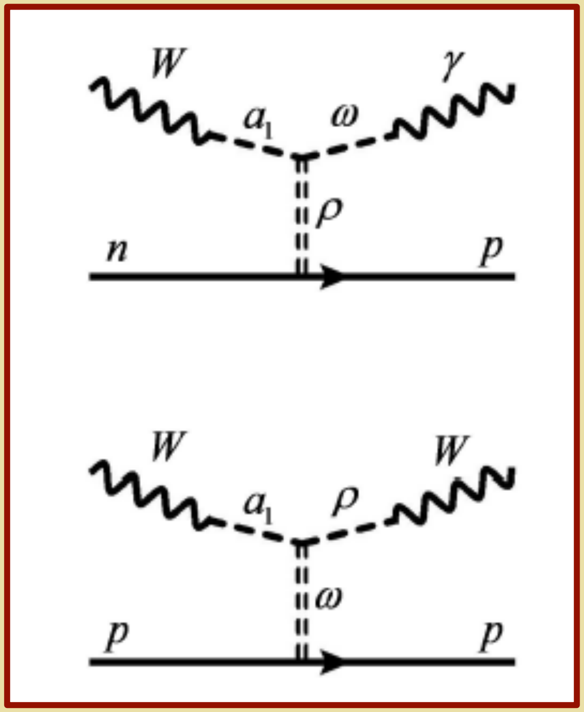


Regge

- ***Compute contributions to $M_3^{\nu p+\bar{\nu}p}$ at each Q^2 from different ω regions***

- ***Isospin rotate to $M_3^{(0)}$***

Regge Contribution



$$F_{3,\mathbb{R}}^{\nu p+\bar{\nu}p} = \frac{C(Q^2)f_{th}(W)}{\left[1+Q^2/m_\rho^2\right]\left[1+Q^2/m_{a_1}^2\right]}\left(\frac{\nu}{\nu_0}\right)^{\alpha_0}$$

$$\downarrow$$

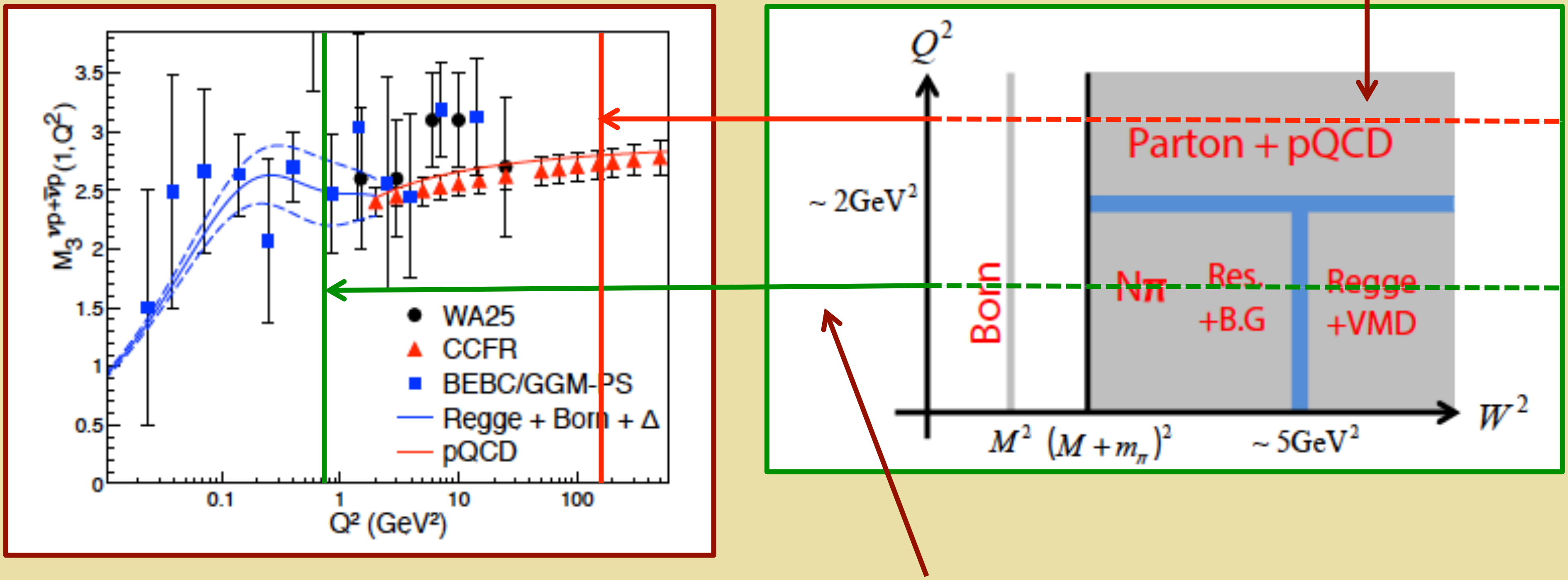
$$F_{3,\mathbb{R}}^{(0)} = \frac{1}{36}\frac{C_{\gamma W}(Q^2)f_{th}(W)}{\left[1+Q^2/m_\rho^2\right]\left[1+Q^2/m_{a_1}^2\right]}\left(\frac{\nu}{\nu_0}\right)^{\alpha_0}$$

- ***Matching at Q^2 = 0 and Q^2 = 2 (GeV)^2 [pQCD regime] → $C_{\gamma W}$ (Q^2) = C(Q^2)***
- ***Factor of 1/36: matching at pQCD scale***

Neutrino Scattering

DIS

Free nucleons



Born + (π N + Res) + Regge

- Compute contributions to $M_3^{\nu p+\bar{\nu}p}$ at each Q^2 from different ω regions

- Isospin rotate to $M_3^{(0)}$

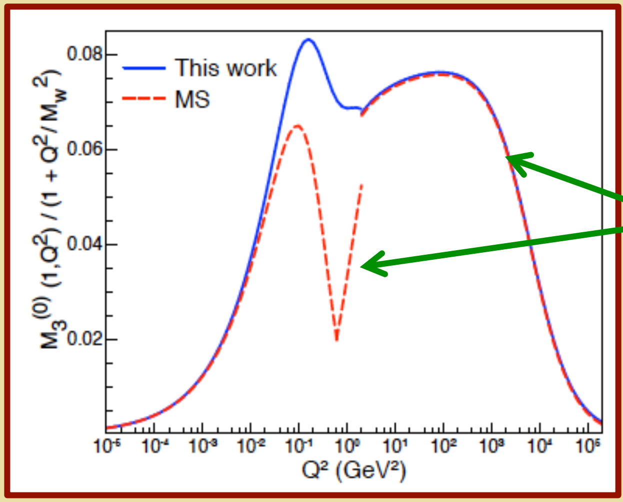
Wγ Box: Update from 2006

$$\Box_{\gamma W}^{VA} = \frac{\alpha}{2\pi}[C_{DIS} + C_B + C^{Regge} + C^{\pi N} + C^{Res}]$$

$$C_{DIS}^{MS} = 1.84 \ \rightarrow \ C_{DIS}^{new} = 1.87$$

$$C_B^{MS} = 0.829(83) \ \rightarrow \ C_B^{new} = 0.91(5)$$

$$C_{INT}^{MS} = 0.14(14) \ \rightarrow \ C^{Regge} + C^{\pi N} + C^{Res} = 0.48(7)$$



$$F_{\text{MS}}(Q^2) = \frac{12}{Q^2} M_3^{(0)}(1, Q^2)$$

See W. Marciano talk today

Future Tests

- *Lattice computation of $M_3^{(0)}$ (Q^2)*
- *PV electron scattering*

Isospin relation

$$\boxed{4F_3^{(0)} = F_{3,\gamma Z}^{p} - F_{3,\gamma Z}^{n}}$$

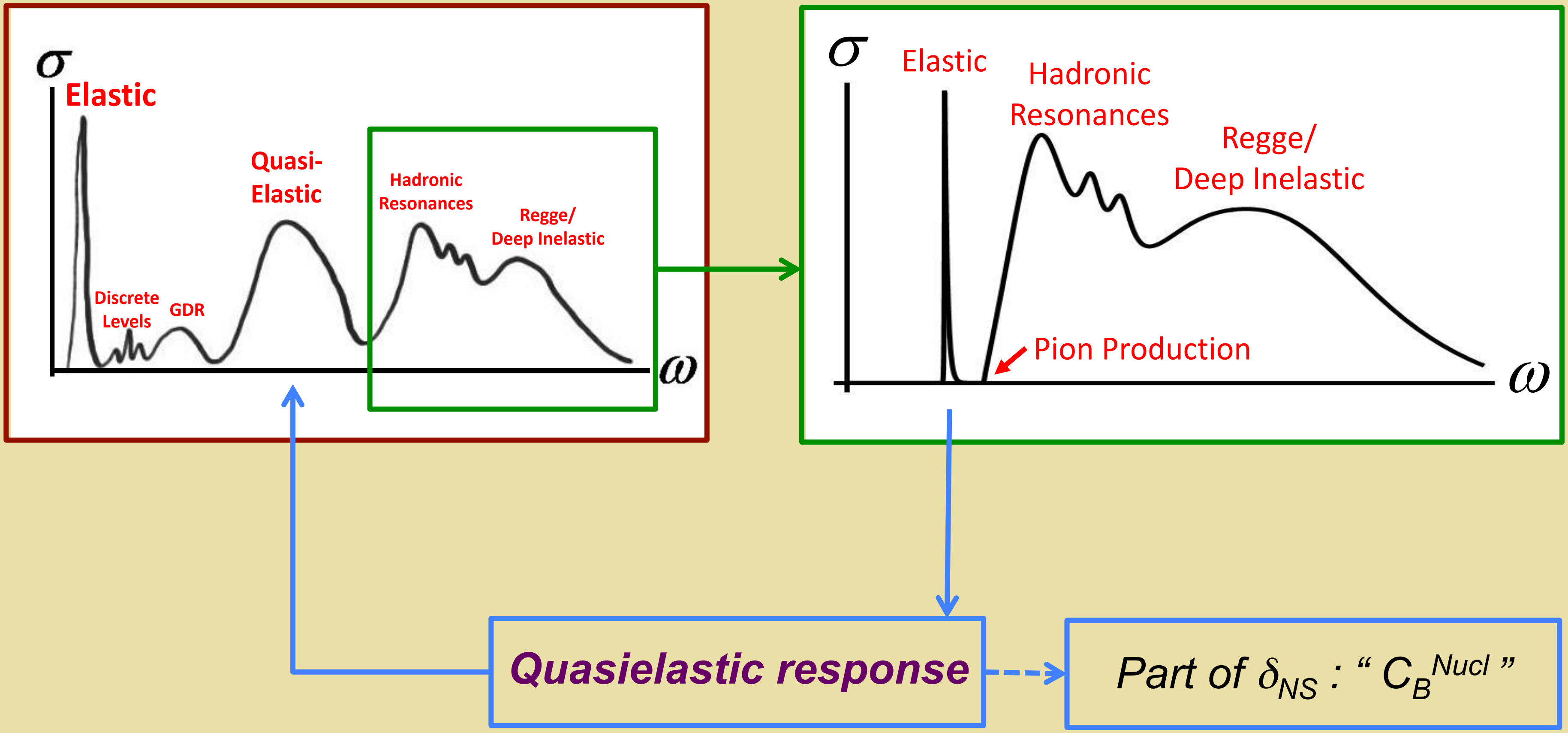
- *SoLID ?*
- *EIC ?*

IV. Wγ Box: Nuclei

Leptoproduction: Had & Nuc Response

Nuclei

Free nucleons

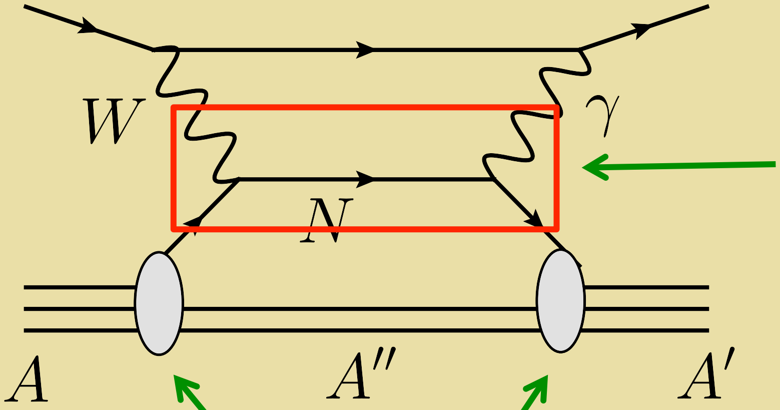


Quasielastic response

Part of δ_{NS} : “ C_B^{Nucl} ”

New work

Quasielastic Contribution to δ_{NS}



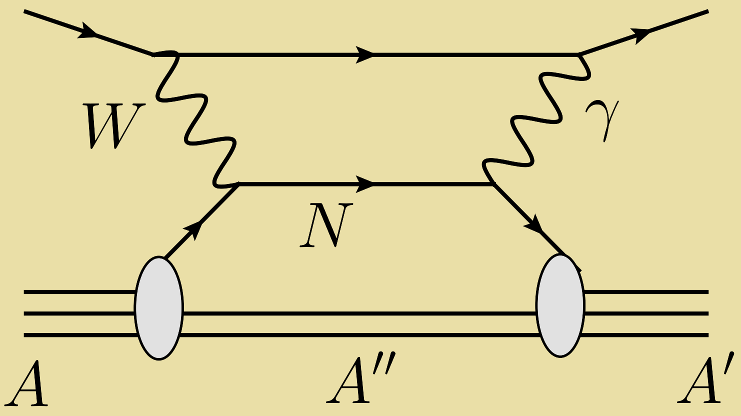
Free nucleon $W^{\mu\nu}$

Nucleon distribution functions

$$
\boxed{\begin{aligned} W^{\mu\nu} = \frac{3\mathcal{N} m_{\mathrm{N}}^2}{4\pi p_{\mathrm{F}}^3} \int \frac{\mathrm{d}^3 \mathbf{p}}{E(\mathbf{p})E(\mathbf{p}+\mathbf{q})}\, \delta\{\omega - [E(\mathbf{p}+\mathbf{q}) - E(\mathbf{p})]\} \\ \times \theta(p_{\mathrm{F}} - |\mathbf{p}|)\,\theta(|\mathbf{p}+\mathbf{q}| - p_{\mathrm{F}})\, f^{\mu\nu}(P+Q, P)\,, \end{aligned}}
$$

e.g. Relativistic Fermi Gas *Donnelly et al '91*

Quasielastic Contribution to δ_{NS}



$$\delta_{NS}^{QE} = \frac{\alpha}{\pi} C_{QE}$$

$$C_{QE} = 2 \int_0^\infty dQ^2 \int_{\nu_{min}}^{\nu_\pi} \frac{d\nu(\nu+2q)}{M\nu(\nu+q)^2} F_{3,\,\gamma W}^{(0),\,QE}(\nu, Q^2)$$

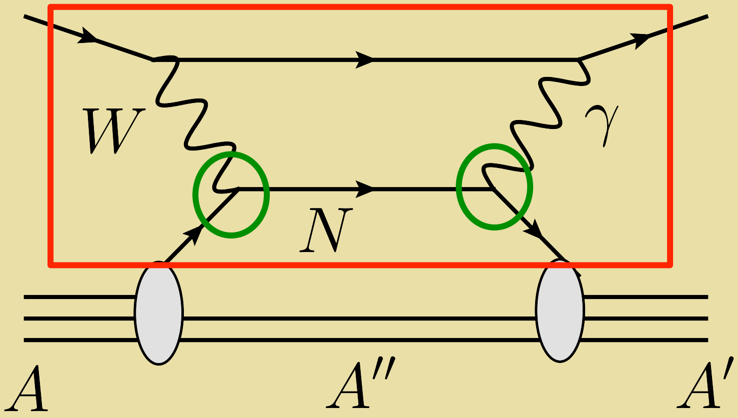
$$\frac{1}{Z} F_{3,\,\gamma W}^{(0),QE}(\nu, Q^2) = -G_A G_M^S \frac{3Q^2}{32q} F_P \frac{\left((\tilde{k}_+)^2 - (\tilde{k}_-)^2\right)}{k_F^3}$$

Functions of B.E., $M_A \cdots$

Nucleon form factors

Pauli blocking

TH: Nucleon Born Contribution to δ_{NS}

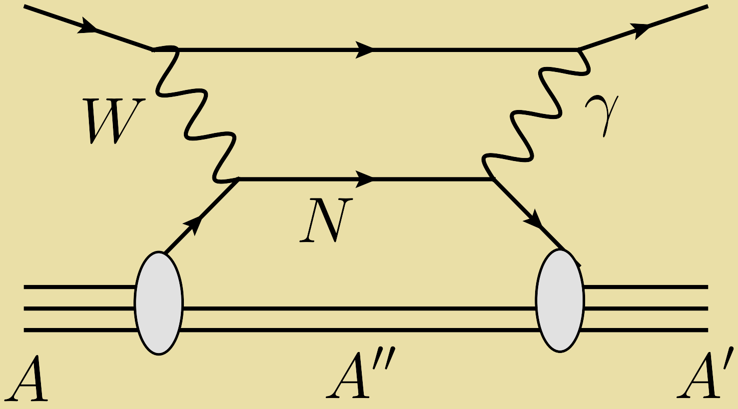


*TH** approach:* ***N x free nucleon loop computation but with "quenched" nucleon ff's***

$$\begin{aligned} G_M^S &\longrightarrow q_S^{(0)} G_M^S \\ G_A &\longrightarrow q_A G_A \end{aligned}$$

*** Towner & Hardy*

Impact on δ_{NS}



$$\Delta\,\delta_{NS} = \frac{\alpha}{\pi}\left(C_{QE} - q_S^{(0)} q_A C_B\right) = -(4.6 \pm 0.9) \times 10^{-4}$$

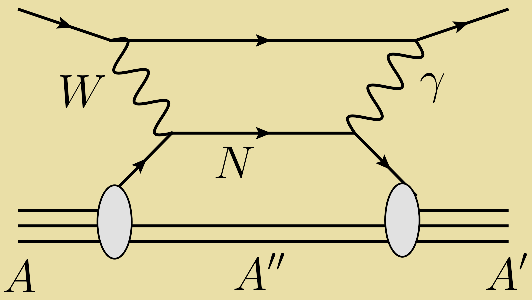
Neglects A-dep variations

Ave over 20 transitions

Optimistic:

- Correlations
- 2-body currents
- Rel corrections

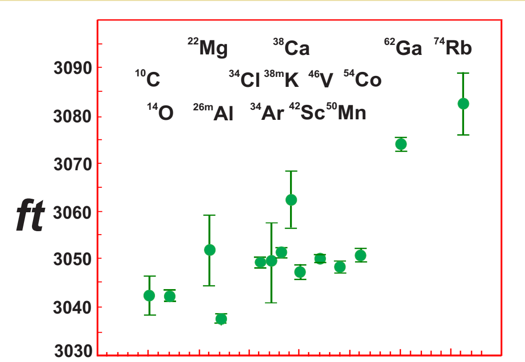
Refinements

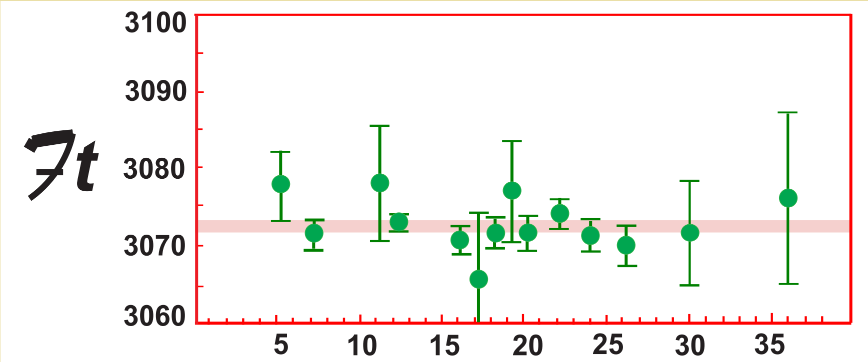


Apply state-of-art methods

- *Correlations*
- *2-body currents*
- *Rel corrections*

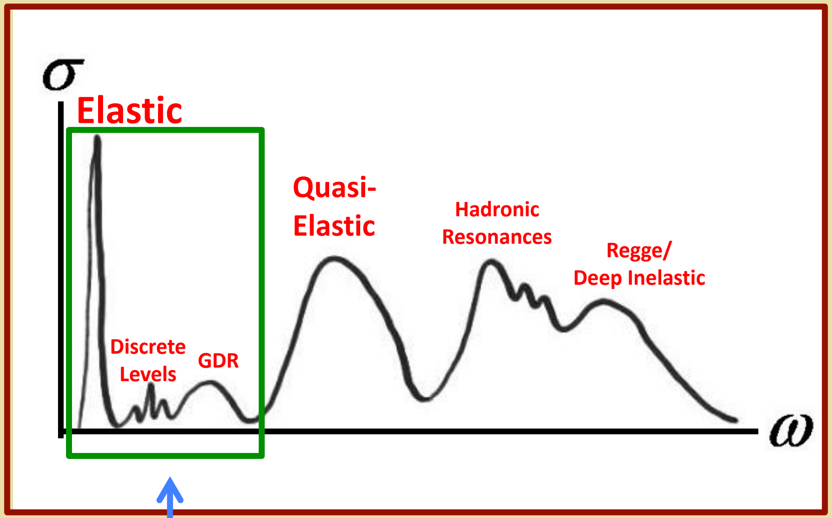
Consistency w/ CVC ?



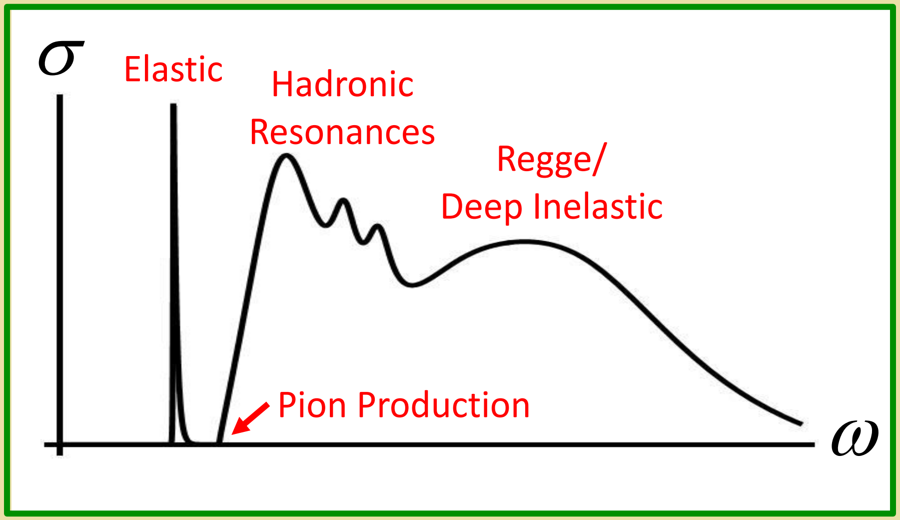


Other Nuclear Corrections

Nuclei



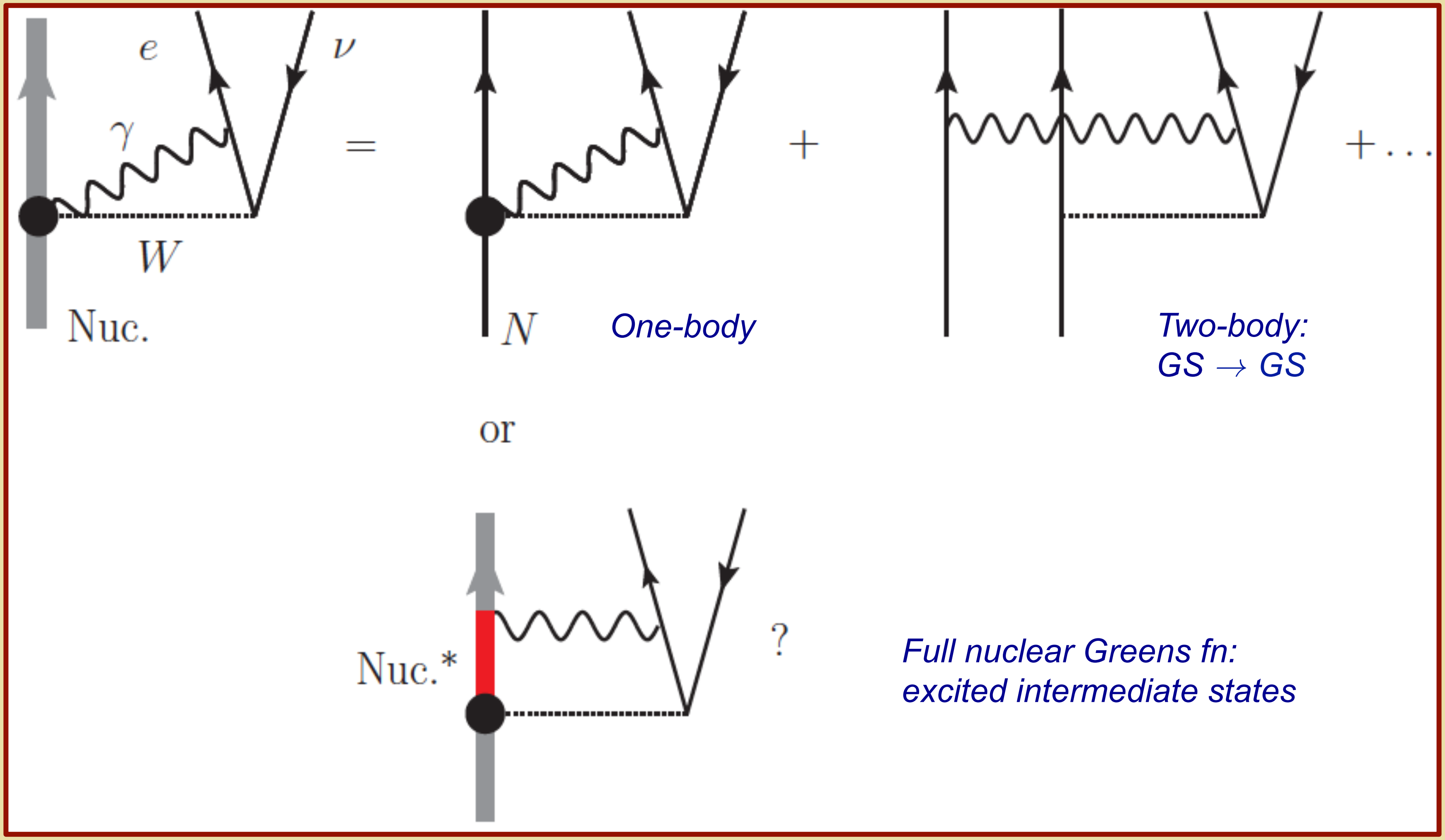
Free nucleons



Low-lying transitions

Part of δ_{NS}

$0^+ \rightarrow 0^+$ Decay: δ_{NS}

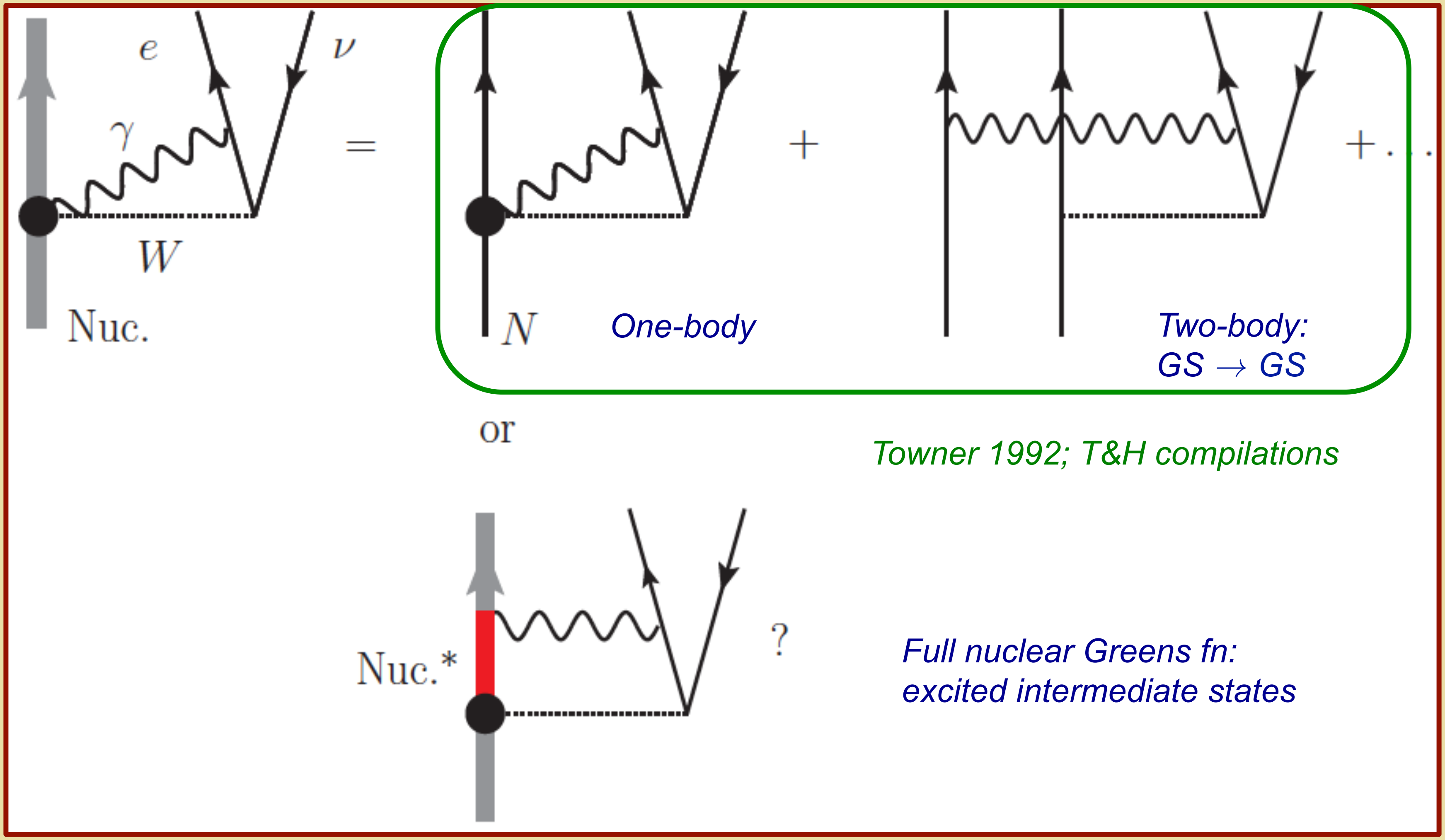


One-body

Two-body: GS → GS

Full nuclear Greens fn: excited intermediate states

$0^+ \rightarrow 0^+$ Decay: δ_{NS}



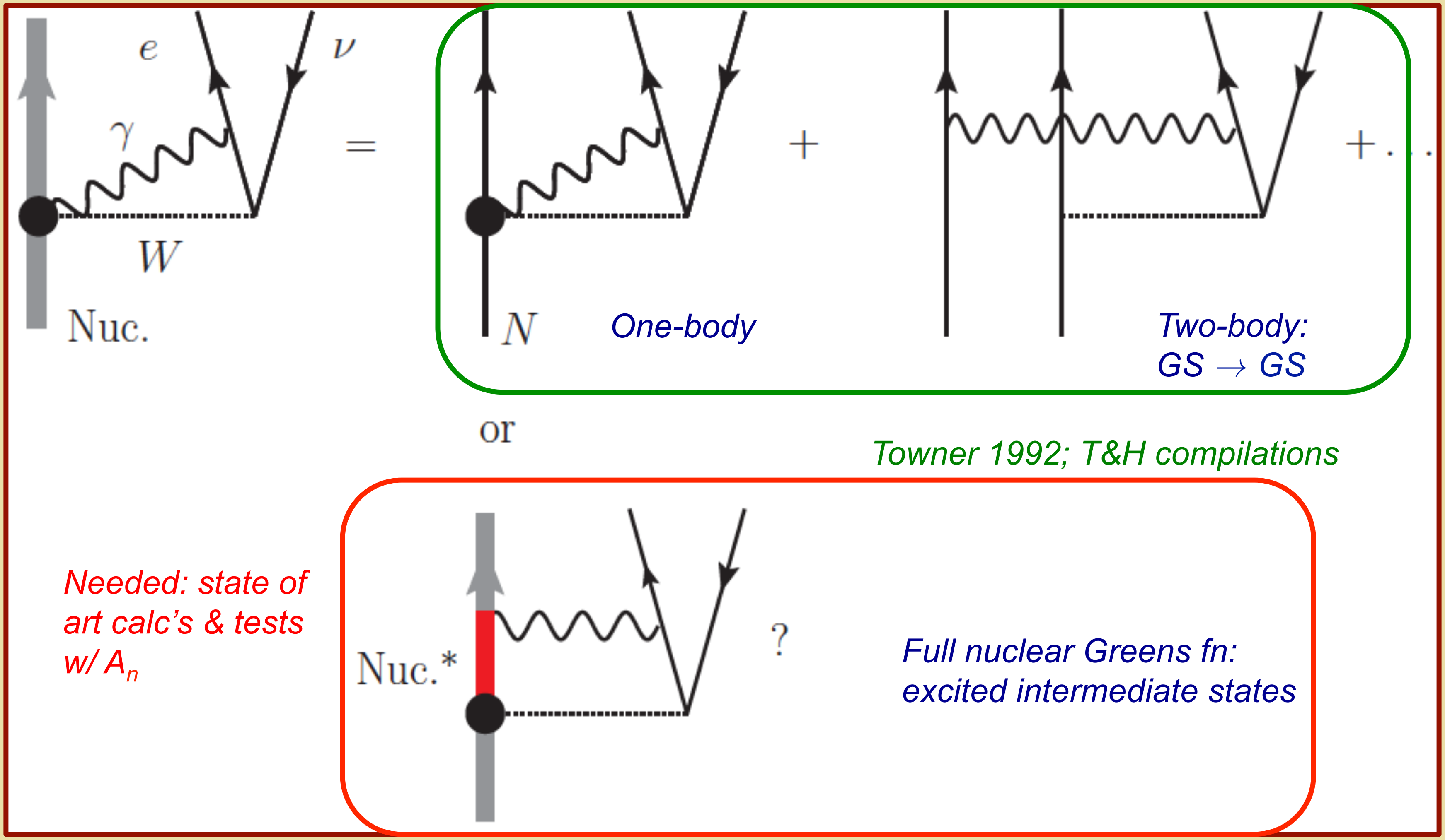
One-body

Two-body: GS → GS

Towner 1992; T&H compilations

Full nuclear Greens fn: excited intermediate states

$0^+ \to 0^+$ Decay: δ_{NS}



One-body

Two-body: GS → GS

Towner 1992; T&H compilations

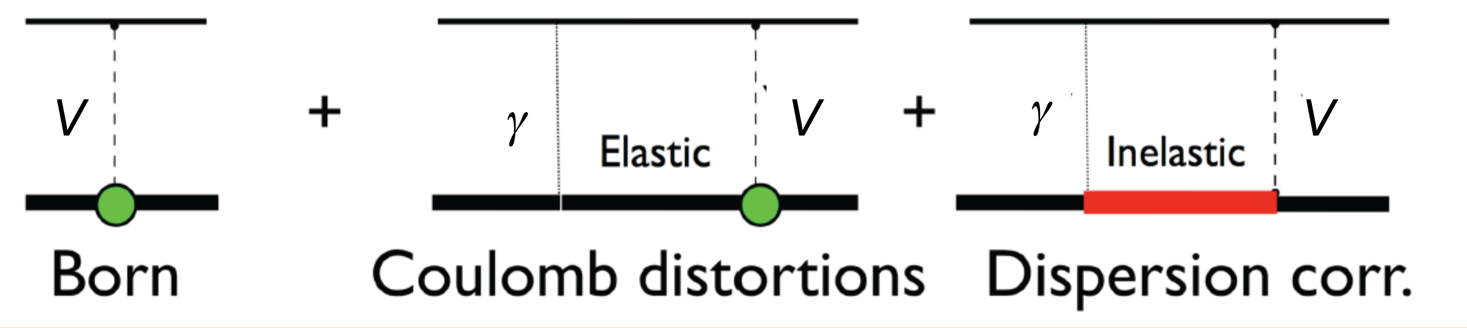
Needed: state of art calc's & tests w/ A_n

Full nuclear Greens fn: excited intermediate states

V. EW Boxes More Generally

Dispersion Corrections

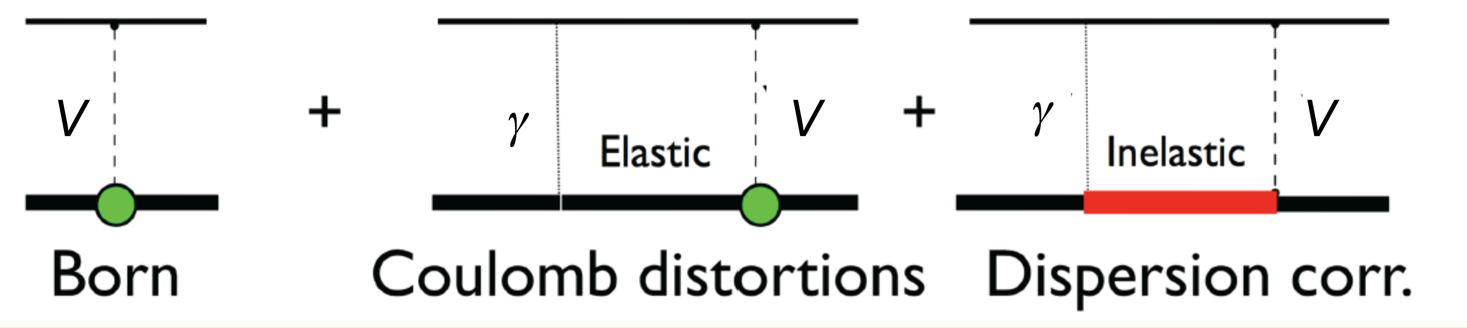
Two-boson exchange in semileptonic processes: important for elastic PV eN & eA scattering (^{12}C) & nuclear β-decay; beam normal asymmetry, Olympus… provide tests



$V = Z^0, W, \gamma$

Dispersion Corrections

Two-boson exchange in semileptonic processes: important for elastic PV eN & eA scattering (^{12}C) & nuclear β-decay; beam normal asymmetry, Olympus… provide tests

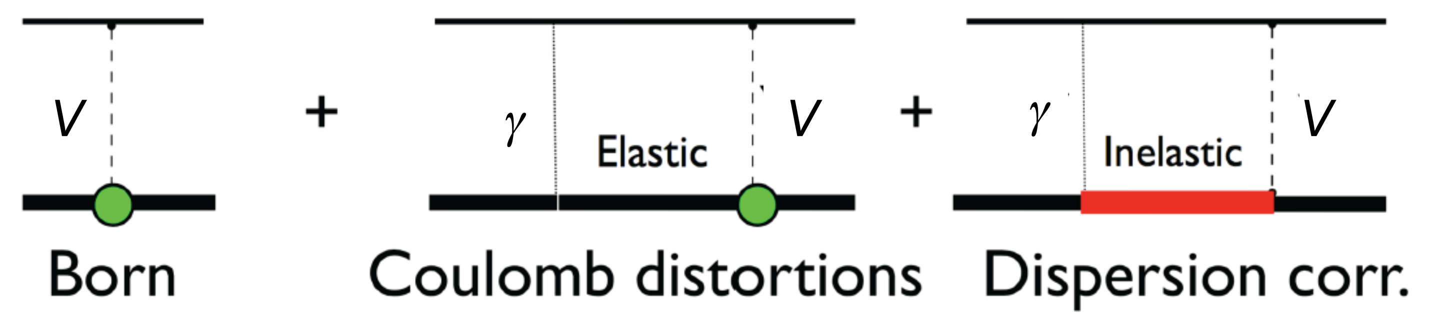


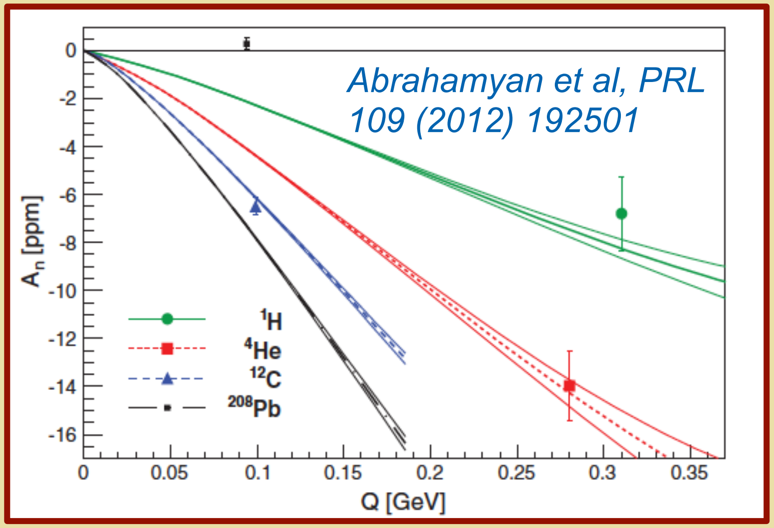
$V = Z^0, W, \gamma$

$V = \gamma$ ***Beam normal asymmetry***

Dispersion Corrections

Two-boson exchange in semileptonic processes: important for elastic PV eN & eA scattering (^{12}C) & nuclear β-decay; beam normal asymmetry, Olympus… provide tests





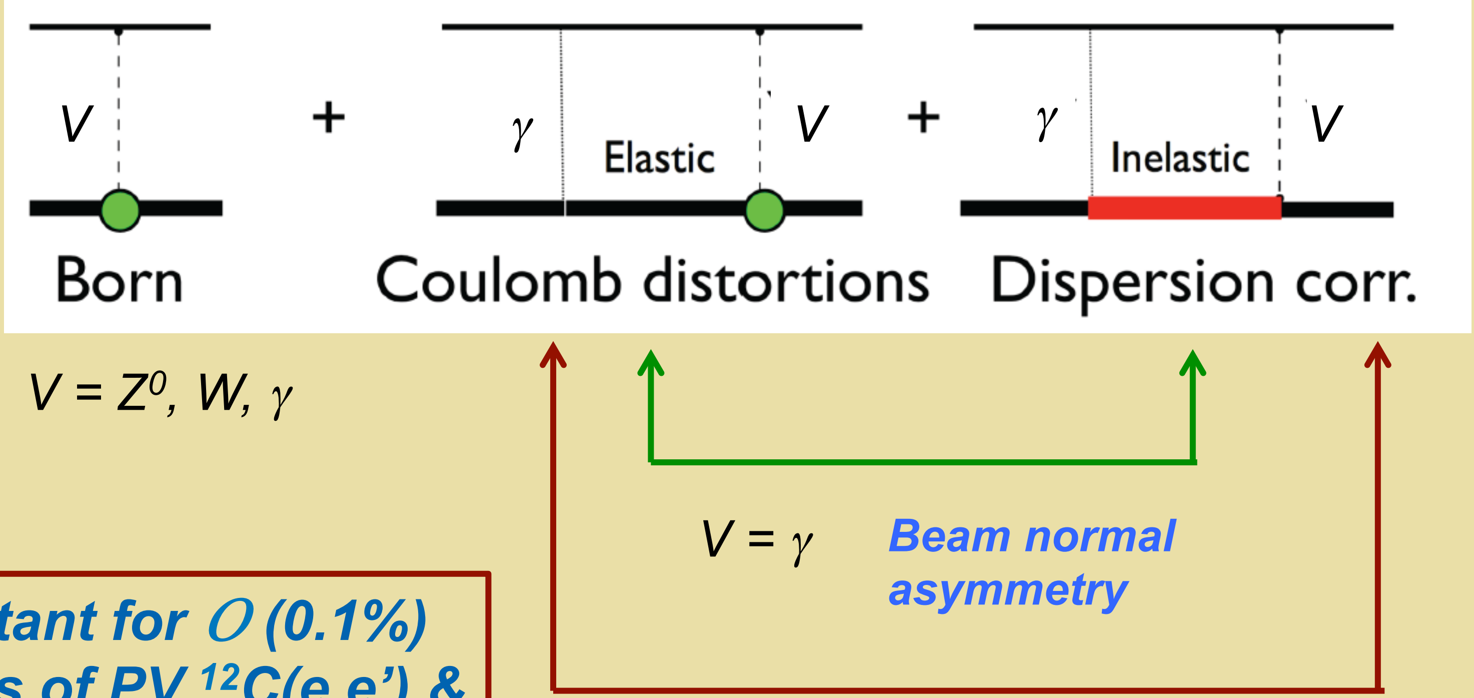
Abrahamyan et al, PRL 109 (2012) 192501

$V = \gamma$ ***Beam normal asymmetry***

- *J Lab Hall A*
- *Future: Mainz, J Lab*

Dispersion Corrections

Two-boson exchange in semileptonic processes: important for elastic PV eN & eA scattering (^{12}C) & nuclear β-decay; beam normal asymmetry provides, Olympus… provide tests



$V = Z^0, W, \gamma$

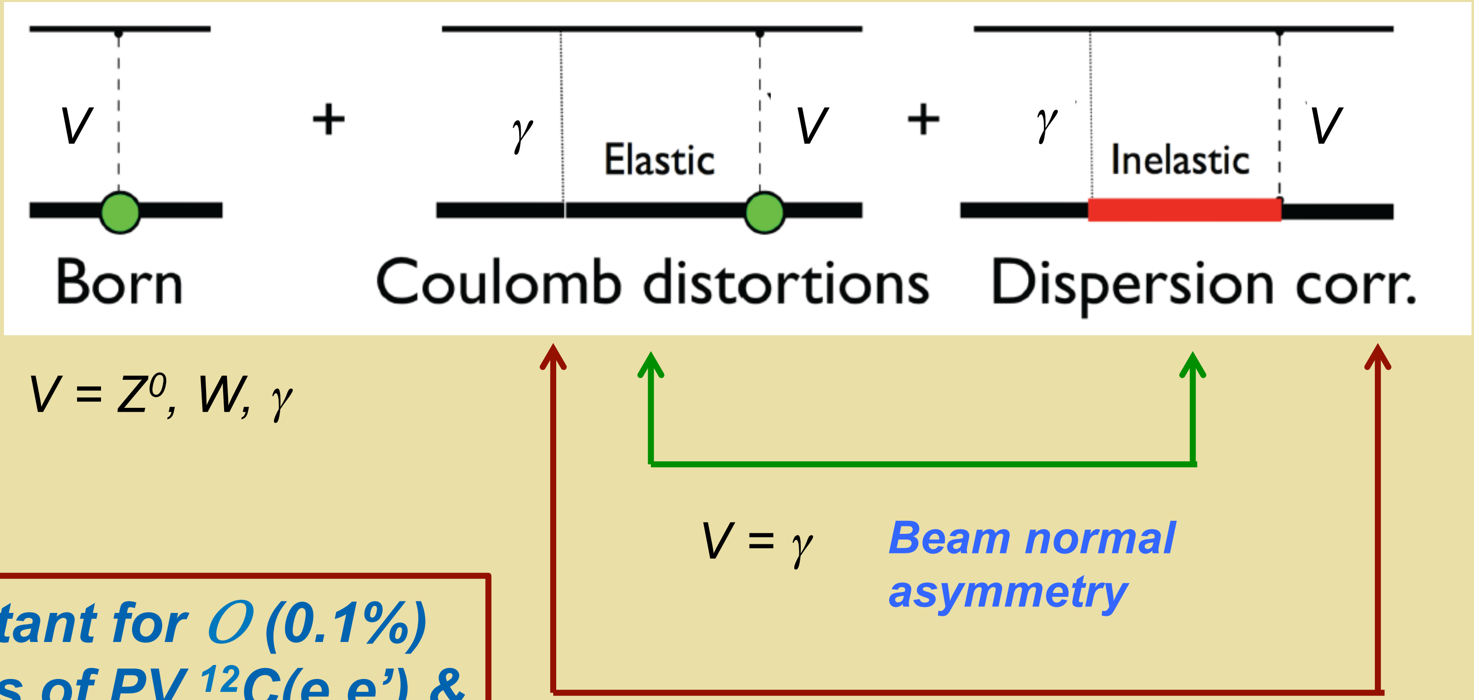
$V = \gamma$ ***Beam normal asymmetry***

$V = Z^0, W$ ***Nucleus-dependent QED & EW corrections***

Important for $\mathcal{O}$ (0.1%) probes of PV ^{12}C(e,e’) & superallowed β-decay

Dispersion Corrections

Proposal: (1) carry out a consistent set of computations for A_n , PV asymmetry, & δ_{NS} using different methods (2) develop a program of A_n measurements to test computations



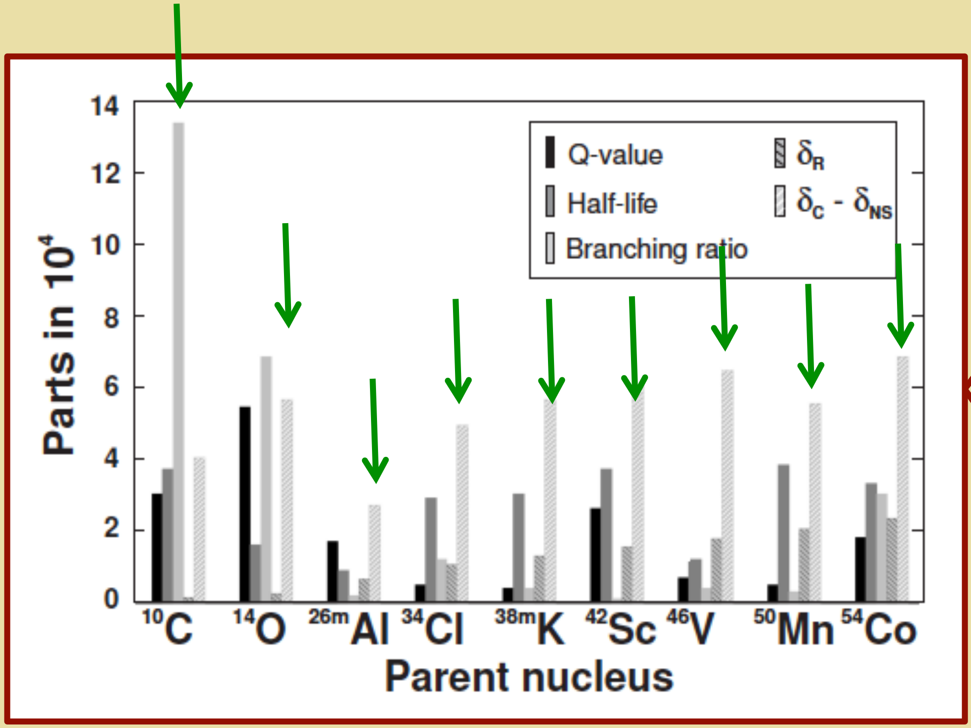
$V = Z^0, W, \gamma$

$V = \gamma$ ***Beam normal asymmetry***

$V = Z^0, W$ ***Nucleus-dependent QED & EW corrections***

Important for $\mathcal{O}$ (0.1%) probes of PV $^{12}C(e,e')$ & superallowed β-decay

$0^+ \rightarrow 0^+$ Dispersion Corrections: δ_{NS}



b_F : scalar currents

Input for V_{ud} & CKM unitarity test

Towner & Hardy, PRC 91 (2015) 2, 025501

- *Re-compute with state-of-the-art many-body methods*
- *Test w/ A_n predictions & expt for ^{10}B, ^{14}N, ^{26}Mg, ^{34}S, ^{38}Ar, ^{42}Ca, ^{46}Ti, ^{50}Cr, ^{54}Fe*
- *Investigate strategy for obtaining reduced error bars*

IV. Outlook

- *Studies of neutron and nuclear β-decay are heading to a new era of precision, with a goal $\delta\Delta_{CKM} \sim O(10^{-4})$*
- *Hadronic and nuclear uncertainties in computing the $W\gamma$ box radiative correction remain one of the key challenges to reaching this goal*
- *Recent developments using dispersion relations open a new path toward reducing this uncertainty with an opportunity for new experimental tests using leptoproduction & theoretical tests with lattice QCD*
- *There exists an exciting opportunity to implement a unified, comprehensive program EW box computations (β-decay, PV electron scattering) and experimental tests with polarized electron-nucleus scattering (A_n).*