Calculation of the mass difference between the long- and short-lived K mesons with physical quark masses on lattice

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Outline

1. Introduction to Δm_K
   - Kaon Mixing in the Standard Model
   - Kaon Mixing on Lattice
   - Discussion: Short-distance Effect in Δm_K
2. Calculating Δm_K on Lattice
   - From Double Integrated Correlator to Δm_K^{lat}
   - From Δm_K^{lat} to physical Δm_K
3. Measurement Methods
   - Sample AMA Correction
4. Results

The Standard Model

Three types of interactions

1. Electromagnetic(QED)
2. Strong(QCD)
3. Weak: least understood; good checks for new physics:
    - Unitarity of CKM matrix
    - CP violation

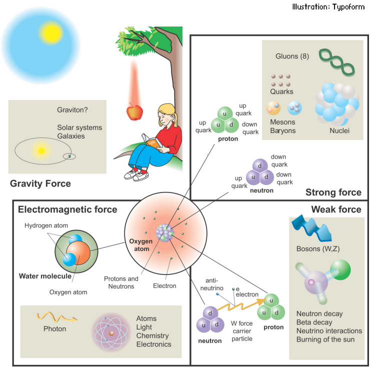


Figure: from https://www.nobelprize.org/prizes/physics/2004/popular-information/

Although weak interaction itself can be treated precisely with perturbation theory, many interesting weak interaction processes involve mesons and baryons(QCD related).

$K^0 - \overline{K^0}$ Mixing and Δm_K

$K^0(S=-1)$ and $\overline{K}^0(S=+1)$ mix through second order weak interactions:

$$i\frac{d}{dt}\begin{pmatrix}K^0(t)\\ \overline{K}^0(t)\end{pmatrix} = (M - \frac{i}{2}\Gamma)\begin{pmatrix}K^0(t)\\ \overline{K}^0(t)\end{pmatrix}, \quad (1)$$

Long-lived (K_L) and short-lived (K_S) are the two eigenstates:

$$K_S \approx \frac{K^0 - \overline{K}^0}{\sqrt{2}}, \quad K_L \approx \frac{K^0 + \overline{K}^0}{\sqrt{2}}. \quad (2)$$

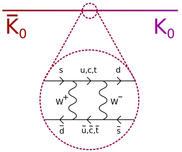


Figure: from wikipedia

Physics Motivation

$$\Delta m_K \equiv m_{K_L} - m_{K_S} = 2ReM_{0\bar{0}}$$

To second order of the weak Hamiltonian:

$$M_{0\bar{0}} = \langle \overline{K}^0 | H_W^{\Delta S=2} | K^0 \rangle + \mathcal{P} \sum_n \frac{\langle \overline{K}^0 | H_W^{\Delta S=1} | n \rangle \langle n | H_W^{\Delta S=1} | K^0 \rangle}{m_K - E_n}$$

- This quantity is:
  1. sensitive to new physics: 2nd order weak interaction, precisely measured
     $$\Delta m_{K,exp} = 3.483(6) \times 10^{-12} \text{ MeV}$$
  2. highly non-perturbative: contributions from distance as large as $\frac{1}{m_\pi}$
- Prediction based on the standard model?
  - Perturbation theory
  - Lattice QCD

Calculate Δm_K: Lattice QCD

Pros

1. Solves QCD problems non-perturbatively
2. From first principles

Challenges

1. Lattice artifacts:
   - Finite volume
   - Finite lattice spacing: short distance cutoff
2. "High" computational cost

Δm_K calculation

1. Long-distance dominating(GIM mechanism)
2. Non-perturbative

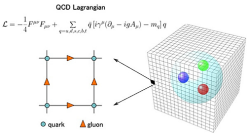


Figure: from wikipedia

Δm_K is one of RBC-UKQCD collaboration's calculations of long-distance contributions in kaon physics. It is closely related to other kaon physics calculations like ϵ_K and rare kaon decays.

Z. Bai, N.H. Christ, X. Feng, A. Lawson, A. Portelli and C.T. Sachrajda, Phys. Rev. D **98**, 074509

Kaon Mixing: Long-distance Contribution on Lattice

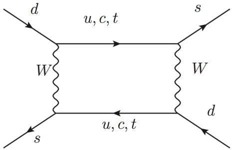
- No direct weak simulation on lattice:
  - From full weak Hamiltonian at W scale, integrate out W and Z, get effective $\Delta S = 1$ Hamiltonian:

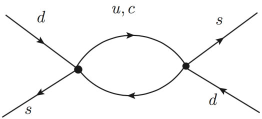
$$H_W = \frac{G_F}{\sqrt{2}} \sum_{q,q'=u,c} V_{qd} V^*_{q's} (C_1 Q_1^{qq'} + C_2 Q_2^{qq'}) \qquad (3)$$

where the $Q_i^{qq'}{}_{i=1,2}$ are current-current opeartors, defined as:

$$Q_1^{qq'} = (\bar{s}_i \gamma^\mu (1-\gamma_5) d_i)(\bar{q}_j \gamma^\mu (1-\gamma_5) q'_j),$$

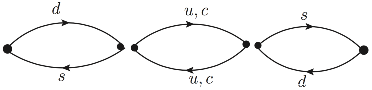
$$Q_2^{qq'} = (\bar{s}_i \gamma^\mu (1-\gamma_5) d_j)(\bar{q}_j \gamma^\mu (1-\gamma_5) q'_i),$$





Short Distance Effect: $V - A$ and GIM Mechanism

Short distance effect: Ultraviolet divergences as the two H_W approach each other:



GIM mechanism removes both quadratic and logarithmic divergences

- $$\frac{\not{p} - m_u}{\not{p}^2 + m_u^2} \sim \frac{1}{p} \rightarrow \left(\frac{\not{p} - m_u}{\not{p}^2 + m_u^2} - \frac{\not{p} - m_c}{\not{p}^2 + m_c^2} \right) \sim \frac{1}{p^2} \quad (4)$$

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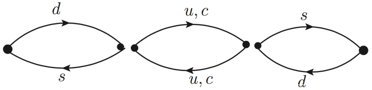

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- Due to the $(V - A)$ structure of the operator Q_i

$$\gamma^\mu(1-\gamma^5)\frac{\not p(m_c^2 - m_u^2)}{(\not p^2 + m_u^2)(\not p^2 + m_c^2)}\gamma^\nu(1-\gamma^5) \sim \frac{1}{p^3}. \qquad (5)$$

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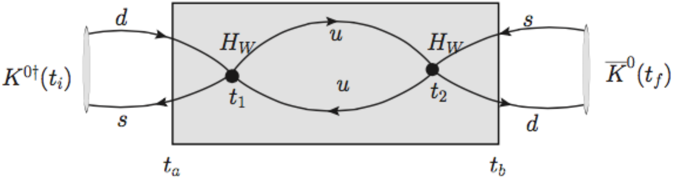
From Double Integrated Correlator to Δm_K^{lat}



- Δm_K is given by:

$$\begin{aligned}\Delta m_K &\equiv m_{K_L} - m_{K_S} \\ &= 2\mathcal{P}\sum_n \frac{\langle \overline{K}^0 | H_W | n\rangle\langle n | H_W | K^0\rangle}{m_K - E_n}\end{aligned} \tag{6}$$

- The integrated correlator is defined as:

$$\mathcal{A} = \frac{1}{2}\sum_{t_2=t_a}^{t_b}\sum_{t_1=t_a}^{t_b}\langle 0 | T\{\overline{K}^0(t_f) H_W(t_2) H_W(t_1) K^0(t_i)\} | 0\rangle \tag{7}$$

From Integrated Correlator to Δm_K^{lat}

- If we insert a complete set of intermediate states

$$\mathcal{A} = N_K^2 e^{-m_K(t_f - t_i)} \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{m_K - E_n} \{-T + \frac{e^{(m_K - E_n)T} - 1}{m_K - E_n}\} \tag{8}$$

  we identify the coefficient of the term linear in the size of integration box $T = t_b - t_a + 1$ as proportional to the expression for Δm_K

- Therefore, by fitting the coefficient of T from integrated correlators we can obtain:

$$\Delta m_K^{lat} \equiv 2 \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{m_K - E_n} \tag{9}$$

Subtract States with Lower energies

$$\mathcal{A}(T) = N_K^2 e^{-m_K(t_f - t_i)} \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{m_K - E_n} \{-T + \frac{e^{(m_K - E_n)T} - 1}{m_K - E_n}\}$$

- Before doing a linear fitting with respect to T, the second term in the curly bracket has to be removed.
- For an intermediate state $|n\rangle$ with energy E_n larger than m_K, for large enough T, the contribution from the second term is negligible.
- For a state $|n\rangle$ with energy E_n smaller than or close to m_K, we need to subtract its contribution.
  In our case of physical quark masses, $|0\rangle$, $|\pi\pi\rangle$, $|\eta\rangle$ and $|\pi\rangle$ states need to be subtracted.

Introduction of $\bar{s}d$ and $\bar{s}\gamma_5 d$ Operators

- With the freedom of adding the operators $\bar{s}d$ and $\bar{s}\gamma_5 d$ to the weak Hamiltonian with properly chosen coefficients c_s and c_p, we are able to remove two of the contributions.
- If we choose c_s and c_p to satisfy:

$$\langle 0|H_W - c_p\bar{s}\gamma_5 d|K^0\rangle = 0, \quad \langle \eta|H_W - c_s\bar{s}d|K^0\rangle = 0.$$

- As a result, the original $\Delta S = 1$ effective weak Hamiltonian and therefore the current-current operators should be modified to be :

$$Q_i' = Q_i - c_{pi}\bar{s}\gamma_5 d - c_{si}\bar{s}d$$

with c_{pi} and c_{si} are calculated on lattice:

$$c_{si} = \frac{\langle \eta|Q_i|K^0\rangle}{\langle \eta|\bar{s}d|K^0\rangle}, \quad c_{pi} = \frac{\langle 0|Q_i|K^0\rangle}{\langle 0|\bar{s}\gamma_5 d|K^0\rangle}.$$

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Calculation of Δm_K^{lat}

$$\mathcal{A} = N_K^2 e^{-m_K(t_f - t_i)} \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{m_K - E_n} \{-T + \frac{e^{(m_K - E_n)T} - 1}{m_K - E_n}\}$$

Recall

$$H'_W = \frac{G_F}{\sqrt{2}} \sum_{q,q'=u,c} V_{qd} V^*_{q's} (C_1 Q'^{qq'}_1 + C_2 Q'^{qq'}_2)$$

The fitting of the integrated correlator further breaks into fitting of the integrated correlator with Q_1 and Q_2:

$$\mathcal{A}_{ij}(T) = N_K^2 e^{-m_K(t_f - t_i)} \sum_n \frac{\langle \bar{K}^0 | Q_i | n \rangle \langle n | Q_j | K^0 \rangle}{m_K - E_n} \{-T + \frac{e^{(m_K - E_n)T} - 1}{m_K - E_n}\}.$$

We therefore have:

$$\mathcal{A}(T) = \lambda_u^2 \sum_{i,j=1,2} C_i C_j \mathcal{A}_{ij}(T), \quad \lambda_u = V_{ud} V^*_{us}$$

Renormalization

We fit each $\mathcal{A}_{ij}(T)$ separately and obtain the k_{ij}, coefficient of the linear term of T. The value of Δm_K from the lattice should be:

$$\Delta m_K^{lat} = \frac{G_F^2}{2}\lambda_u^2 \sum_{i,j=1,2} (-2) \times C_i^{lat} C_j^{lat} k_{ij}. \tag{10}$$

Renormalization of lattice operator $Q_{1,2}$:

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C. Lehner, C. Sturm, Phys. Rev. D 84(2011), 0140

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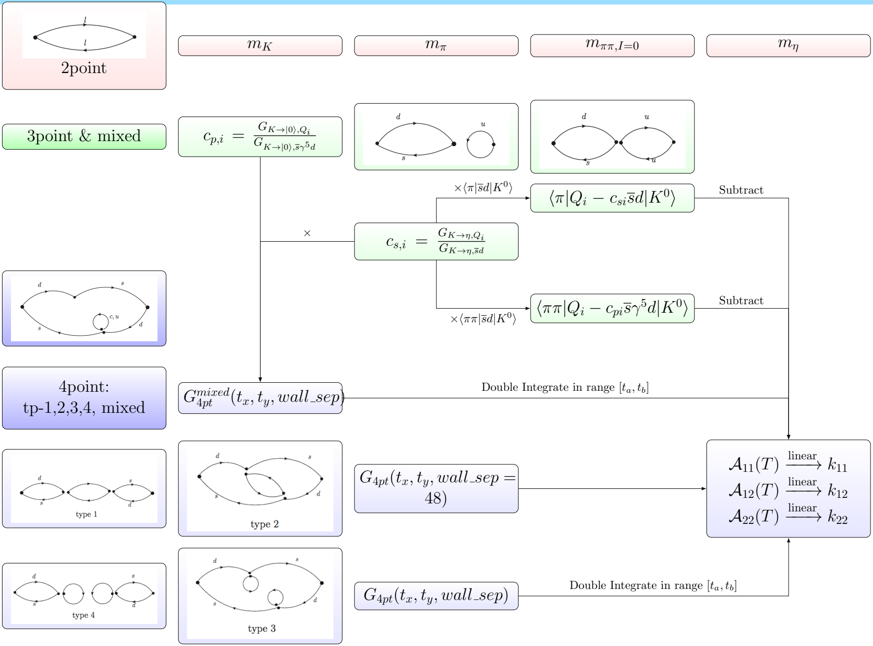
  C. Lehner, C. Sturm, Phys. Rev. D 84(2011), 0140

- Use Wilson coefficients in the $\overline{MS}$ scheme

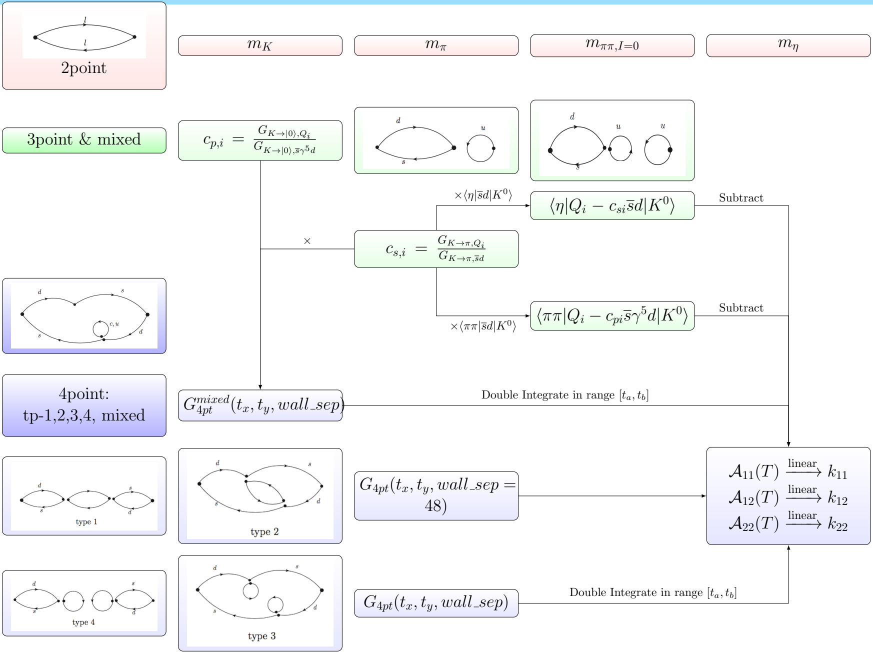
  G. Buchalla, A.J. Buras and M.E. Lautenbacher, arXiv:hep-ph/9512380

$$C_i^{lat} = C_a^{\overline{MS}}(1+\Delta r)_{ab}^{RI\to\overline{MS}} Z_{bi}^{lat\to RI}.$$

Full Calculation of Δm_K: Using $C_{s,\eta}$



Full Calculation of Δm_K: Using $C_{s,\pi}$



Data and Data Analysis: Sample AMA Correction

- We use Sample All Mode Averaging (AMA) to reduce the computational cost.

  T. Blum, T. Izubuchi, and E. Shintani, Phys. Rev. D88(9), 094503 (2013)

data type	CG stop residual
sloppy	$1e-4$
exact	$1e-8$

  The difference between the "exact" and the "sloppy" result for a same quantity(e.g. a strange propagator) is used as a correction.
- Usually AMA correction is performed on each configuration, among different time slices
- Our Sample AMA correction is applied among configurations
- We do only "sloppy" measurements on most configurations and do both "sloppy" and "exact" measurements on some other configurations to serve as corrections.

Super-jackknife Method

For a certain quantity Y, a pion correlator as an example

- N_s "sloppy" measurements $\{y_i\}_{i=1,\ldots,N_s}$
  N_c corrections $\{\Delta y_i\}_{i=1,\ldots,N_c}$

y_1	y_2	$\cdots$	y_{N_s-1}	y_{N_s}	Δy_1	Δy_2	$\cdots$	Δy_{N_c-1}	Δy_{N_c}

N_s measurements N_c measurements

- Jackknife the raw data to get two jackknife ensembles:
  $Y_i = \frac{1}{N_s-1}\sum_{j\neq i} y_j$, $\Delta Y_i = \frac{1}{N_c-1}\sum_{j\neq i}\Delta y_j$.

Y_1	Y_2	$\cdots$	Y_{N_s-1}	Y_{N_s}	ΔY_1	ΔY_2	$\cdots$	ΔY_{N_c-1}	ΔY_{N_c}

N_s jackknife elements N_c jackknife elements

- We then combine the two jackknife ensembles to form a super-jackknife ensemble with $N_s + N_c$ elements.

$Y_1+\overline{\Delta Y}$	$Y_2+\overline{\Delta Y}$	$\cdots$	$Y_{N-1}+\overline{\Delta Y}$	$Y_N+\overline{\Delta Y}$	$\overline{Y}+\Delta Y_1$	$\overline{Y}+\Delta Y_2$	$\cdots$	$\overline{Y}+\Delta Y_{N_2-1}$	$\overline{Y}+\Delta Y_{N_2}$

(N_s+N_c) super-jackknife elements

Status of RBC-UKQCD Calculations of Δm_k

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  All diagrams included on a $64^3 \times 128$ lattice with **physical mass** on 59 configurations: $\Delta m_k = (5.5 \pm 1.7) \times 10^{-12} MeV$

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- Here I present an update of the results extending Z. Bai's calculation from 59 to 152 configurations.

arXiv:1812.05302

Details of the Calculation

- The calculation was performed on a $64^3 \times 128 \times 12$ lattice with 2+1 flavors of Möbius DWF and the Iwasaki gauge action with physical pion mass (136 MeV) and inverse lattice spacing $a^{-1} = 2.36$ GeV.

β	am_l	am_h	$\alpha = b + c$	L_s
2.25	0.0006203	0.02539	2.00	12

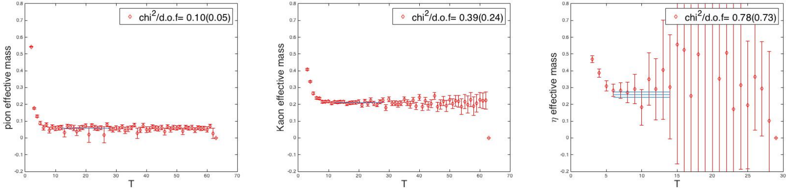
For valence charm quark, we used $am_c \simeq 0.31$.

- I will compare results presented in 2017 with our updated results.

Data Set	# of Sloppy	# of Correction	# of Type12
"old 59"	52	7	11
"new 152"	116	36	36

Results **preliminary**

2-point diagram



Data Set	K^0	π	η	$\pi\pi_{I=0}$
new 152	496.5(3)	135.4(3)	608(5)	268.5(1.2)
old 59	496.9(7)	135.9(3)	684(84)	268.3(1.5)

- These results are consistent within errors. As the statistics increase, the errors scale approximately as $\frac{1}{\sqrt{N}}$.

Results **preliminary**

3-point diagram: direct subtraction terms

Data Set	$\langle\pi\|Q_1\|K^0\rangle$	$\langle\pi\|Q_2\|K^0\rangle$	$\langle 0\|Q_1\|K^0\rangle$	$\langle 0\|Q_2\|K^0\rangle$
new 152	$-5.02(3)\times 10^{-4}$	$1.407(4)\times 10^{-3}$	$-1.284(3)\times 10^{-2}$	$2.449(4)\times 10^{-2}$
old 59	$-5.08(5)\times 10^{-4}$	$1.407(8)\times 10^{-3}$	$-1.289(4)\times 10^{-2}$	$2.454(7)\times 10^{-2}$

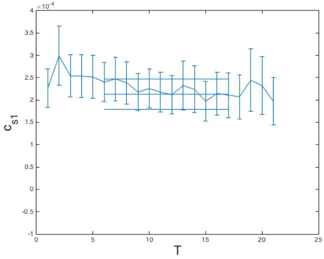
Table: The K^0 to π matrix element and the K^0 to vacuum matrix element, without subtracting the $\bar{s}d$ operator.

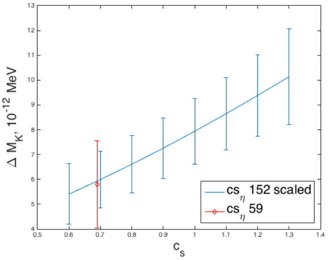
Data Set	$\langle\pi\pi_{I=2}\|Q_1\|K^0\rangle$	$\langle\pi\pi_{I=2}\|Q_2\|K^0\rangle$	$\langle\pi\pi_{I=0}\|Q_1\|K^0\rangle$	$\langle\pi\pi_{I=0}\|Q_2\|K^0\rangle$
new 152	$1.473(6)\times 10^{-5}$	$1.473(6)\times 10^{-5}$	$-8.7(1.5)\times 10^{-5}$	$9.5(1.5)\times 10^{-5}$
old 59	$1.471(10)\times 10^{-5}$	$1.471(10)\times 10^{-5}$	$-6.6(2.5)\times 10^{-5}$	$7.9(2.3)\times 10^{-5}$

Table: The K to $\pi\pi$ matrix element for Isospin 0 and 2. The I=2 matrix element for Q_1 and Q_2 are the same because they come from the same three point diagrams.

Results **preliminary**

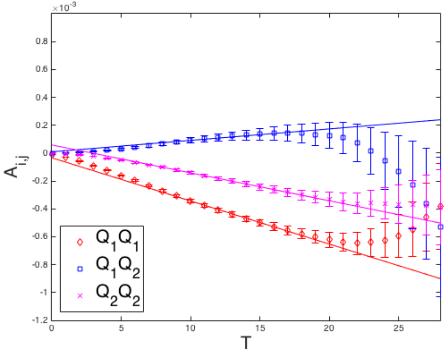
3-point diagram: c_s and c_p





Data Set	c_{s1}	c_{s2}	c_{p1}	c_{p2}
new 152	$2.13(33) \times 10^{-4}$	$-3.16(25) \times 10^{-4}$	$-1.472(2) \times 10^{-4}$	$2.807(2) \times 10^{-4}$
old 59	$1.53(64) \times 10^{-4}$	$-2.77(42) \times 10^{-4}$	$-1.476(3) \times 10^{-4}$	$2.811(3) \times 10^{-4}$

- **c_s from η are relatively noisy**: influences via mixed diagrams: $\langle \pi | \bar{s} d | K^0 \rangle$, type-4 mixed diagrams the integrate correlator $\mathcal{A}_{ij}$ fittings: All diagrams, uncorrelated



Data Set	Subtraction	Δm_K	$\Delta m_{K,11}$	$\Delta m_{K,12}$	$\Delta m_{K,22}$
new 152	$K \to \pi, \pi\pi$	8.0(1.3)	0.83(0.10)	1.41(0.50)	5.70(0.81)
new 152	$K \to \eta, \pi\pi$	**6.0(1.0)**	0.63(0.06)	0.58(0.37)	4.82(0.68)
old 59	$K \to \pi, \pi\pi$	5.8(1.8)	0.68(0.12)	0.69(0.17)	4.47(1.09)

Table: Results for Δm_K from uncorrelated fits in units of 10^{-12} MeV.

Results **preliminary**

Sample AMA statistical errors

Our use of the sample AMA method reduced the computational cost of the calculation by a factor of 2.3, while the statistical error on the correction will add to the total statistical error. $\sigma \sim \sqrt{\sigma_{slp}^2 + \sigma_{corr}^2}$

Data Set	type 3&4 error from "sloppy"	type 3&4 error from correction	type 3&4 error in total
new 152	0.9	0.6	1.1
old 59	1.1	0.6	1.2

The AMA correction does not contribute much to the error in our final answer.

- Finite-volume corrections: small compared to statistical errors
  "Effects of finite volume on the $K_L - K_S$ mass difference"
  N.H. Christ, X. Feng, G. Martinelli and C.T. Sachrajda, arXiv:1504.01170
  Calculation gives: $\Delta m_K(FV) = -0.22(7) \times 10^{-12} MeV$

$2f(mk)$	$h = \delta + \phi$	$coth$	dh/dE	$coth \times dh/dE$	$\Delta m_K(FV)$
−0.0035(10)	−0.49(6)	−1.85(27)	33.5(4)	−62(10)	−0.22(7)

Table: The $\pi\pi_{I=0}$ contribution to Δm_K, and the terms determining the corresponding finite volume correction. The last term is the finite volume correction to the $K_L - K_S$ mass difference Δm_K, in units of $10^{-12} MeV$.

- The lattice spacing in our calculation is $a^{-1} = 2.36 GeV$, which is only twice the charm quark mass. Discretization effects are estimated to be the largest source of systematic error: $\sim (m_c a)^2$ is $\sim 25\%$.

Conclusion and Outlook

- Our **preliminary** result based on 152 configurations is

$$\Delta m_K = 5.8(1.0)_{stat}(unknown)_{sys} \times 10^{-12} MeV$$

to be compared to the experimental value:

$$(\Delta m_K)^{exp} = 3.483(6) \times 10^{-12} MeV$$

- We view such a comparison as premature given the possibly large and poorly estimated finite lattice spacing error.

Outlook

- Continue the calculation of Δm_K on Summit at Oak Ridge National Lab in 2019
  - Systematic error estimate: on $24^3 \times 32$ and $32^3 \times 64$ lattice with the same $m_c a$
  - Finner lattice spacing
  - Improved solver, contraction code for GPU(QUDA)

Figure: from `https://www.olcf.ornl.gov/calendar/summit-training-workshop/`

- Reduce the statistical error with measurements on larger number of configurations
- Include other elements of our kaon physics program

Thanks for your attention!

- Wall source at time t with spin α and color a is defined as:

$$b(\vec{y}, t_y) = \begin{cases} \chi_{a\alpha}, & t_y = t \\ 0, & t_y \neq t_y, \end{cases} \tag{11}$$

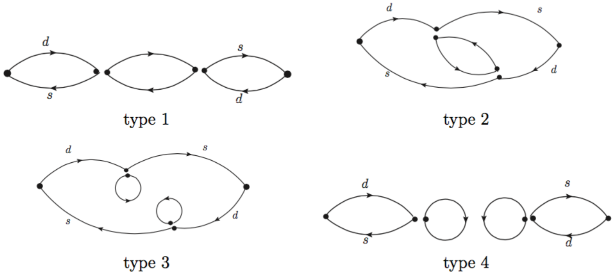
where $\chi_{a\alpha}$ is a 12-component vector with 1 at spin α and color a and 0 at anywhere else.

Contractions: 4-point Correlators

$$\mathcal{A}_{ij}(T) = \frac{1}{2}\sum_{t_2=t_a}^{t_b}\sum_{t_1=t_a}^{t_b}\langle 0|T\{\overline{K}^0(t_{snk})Q'_j(t_2)Q'_i(t_1)K^0(t_{src})\}|0\rangle, \quad (12)$$

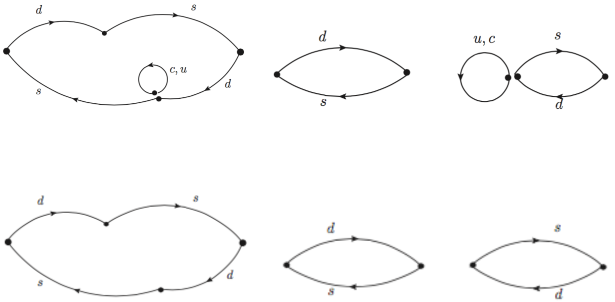
where $Q'_i = Q_i - c_{pi}\bar{s}\gamma_5 d - c_{si}\bar{s}d$.

- For contractions among Q_i, there are four types of diagrams to be evaluated.



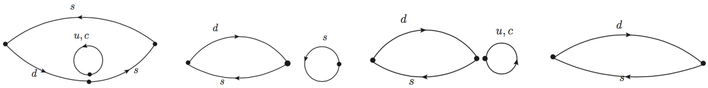
Contractions: 4-point Correlators

In addition, there are "mixed" diagrams from the contractions between the $c_s \bar{s} d$, $c_p \bar{s}\gamma^5 d$ operators and Q_i operators.



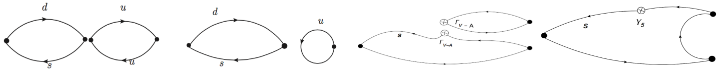
3-point Correlators

- Coefficients: $c_{si} = \frac{\langle \eta | Q'_i | K^0 \rangle}{\langle \eta | \bar{s} d | K^0 \rangle}, \quad c_{pi} = \frac{\langle 0 | Q'_i | K^0 \rangle}{\langle 0 | \bar{s} \gamma_5 d | K^0 \rangle}.$



- Explicit subtractions:

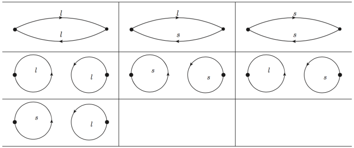
$$\frac{\langle \bar{K}^0 | Q'_i | \pi \rangle \langle \pi | Q'_j | K^0 \rangle}{m_K - E_\pi} \frac{e^{(m_K - E_\pi)T} - 1}{m_K - E_\pi} \text{ and } \frac{\langle \bar{K}^0 | Q'_i | \pi\pi \rangle \langle \pi\pi | Q'_j | K^0 \rangle}{m_K - E_{\pi\pi}} \frac{e^{(m_K - E_{\pi\pi})T} - 1}{m_K - E_{\pi\pi}}.$$

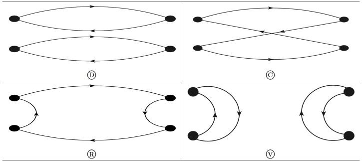


2-point Correlators

2-point correlators are used to compute the mass and normalization factor of π, K^0 η and $\pi\pi$ states.

$$C(t) \sim |\langle n|O_n^\dagger(0)|0\rangle|^2 e^{-E_n t}$$





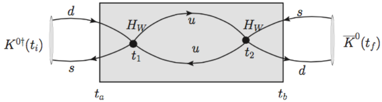
Calculate Propagators on Lattice

The contractions shown in previous section can be written as products of quark propagators.

- from the source point y to the sink point x:

$$S(x, y) = D^{-1}(x, y).$$

- There are $64^3 \times 128 \sim 10^7$ sites on lattice.



- For a certain source distribution $b(y)$, the propagator to x is given by:

$$S(x) = \sum_y S(x, y) b(y),$$

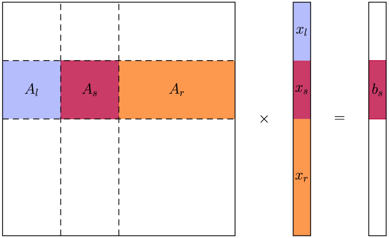
and $S(x)$ can be obtained by solving a $\mathbf{A}\vec{a} = \vec{b}$ problem using CG:

$$\sum_y D(x, y) S(y) = b(x).$$

Calculate Propagators on Lattice

$S(x)$ can be obtained by solving a $\mathbf{A}\vec{a} = \vec{b}$ problem using CG:

$$\sum_y D(x, y) S(y) = b(x).$$



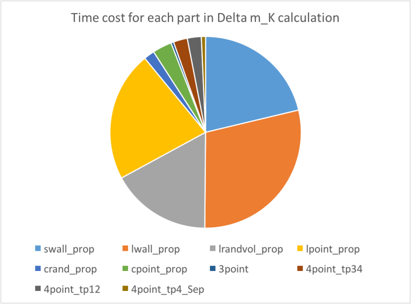
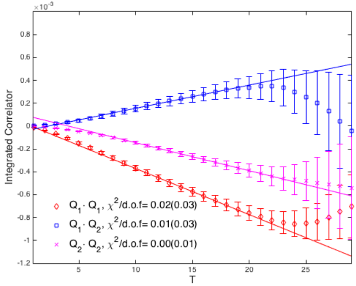


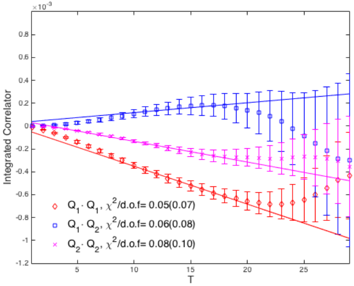
Figure on the left: from Jiqun Tu

Results **preliminary**

the integrate correlator $\mathcal{A}_{ij}$ fittings: All diagrams, uncorrelated



(a) All diagrams fitting: 152 configurations



(b) All diagrams fitting: 59 configurations

Data Set	Δm_K	Δm_K(tp12)	Δm_K(tp34)	Δm_K(tp3)	Δm_K(tp4)
new 152	8.2(1.3)	8.3(0.6)	0.1(1.1)	1.58(31)	-1.28(94)
old 59	5.8(1.8)	7.0(1.3)	-1.1(1.2)	1.17(43)	-2.16(1.20)

Table: Results for Δm_K from uncorrelated fits in units of 10^{-12} MeV with fitting range 10:20.