2019 Lattice Workshop for US - Japan Intensity Frontier
July 25-27 2019, Upton, NY, USA

Towards non-perturbative matching of 3/4-flavor Wilson coefficients with a position-space procedure

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N_f in Weak Hamiltonian

$$H_W = \Sigma_i\, w_i^{4f}(\mu)\, O_i^{4f}(\mu)$$

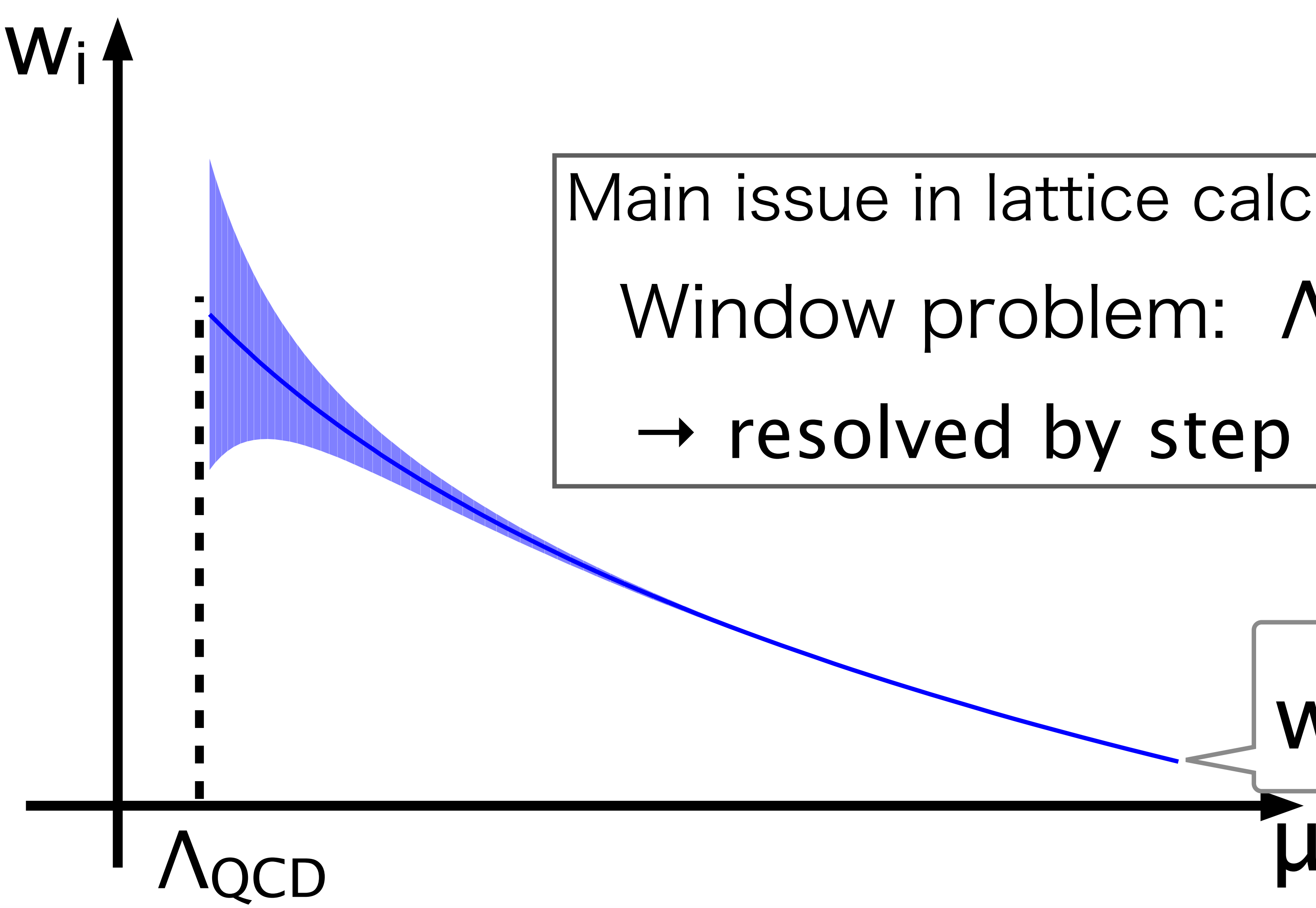
$$= \Sigma_i\, w_i^{3f}(\mu)\, O_i^{3f}(\mu)$$

$$= \ldots$$

We can use either of 3f or 4f for WMEs

WMEs w/ 4-flavor operators

$$\langle f|H_W|i\rangle = \Sigma_i \underbrace{w_i^{4f}(\mu)}_{\text{pQCD}} \underbrace{\langle f|O_i^{4f}(\mu)|i\rangle}_{\text{LQCD}}$$



Main issue in lattice calculation is

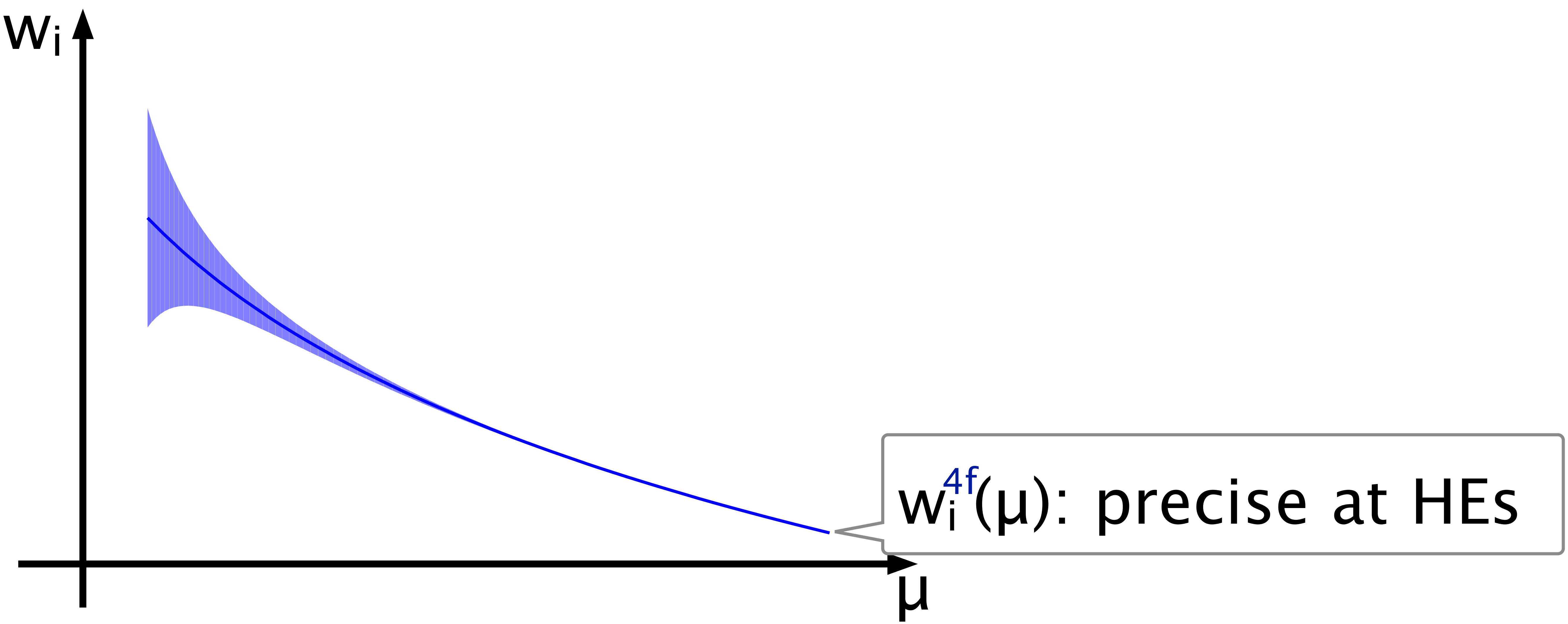
Window problem: $\Lambda_{QCD} \ll \mu \ll a^{-1}$

→ resolved by step scaling, etc…

$w_i^{4f}(\mu)$: precise at HEs

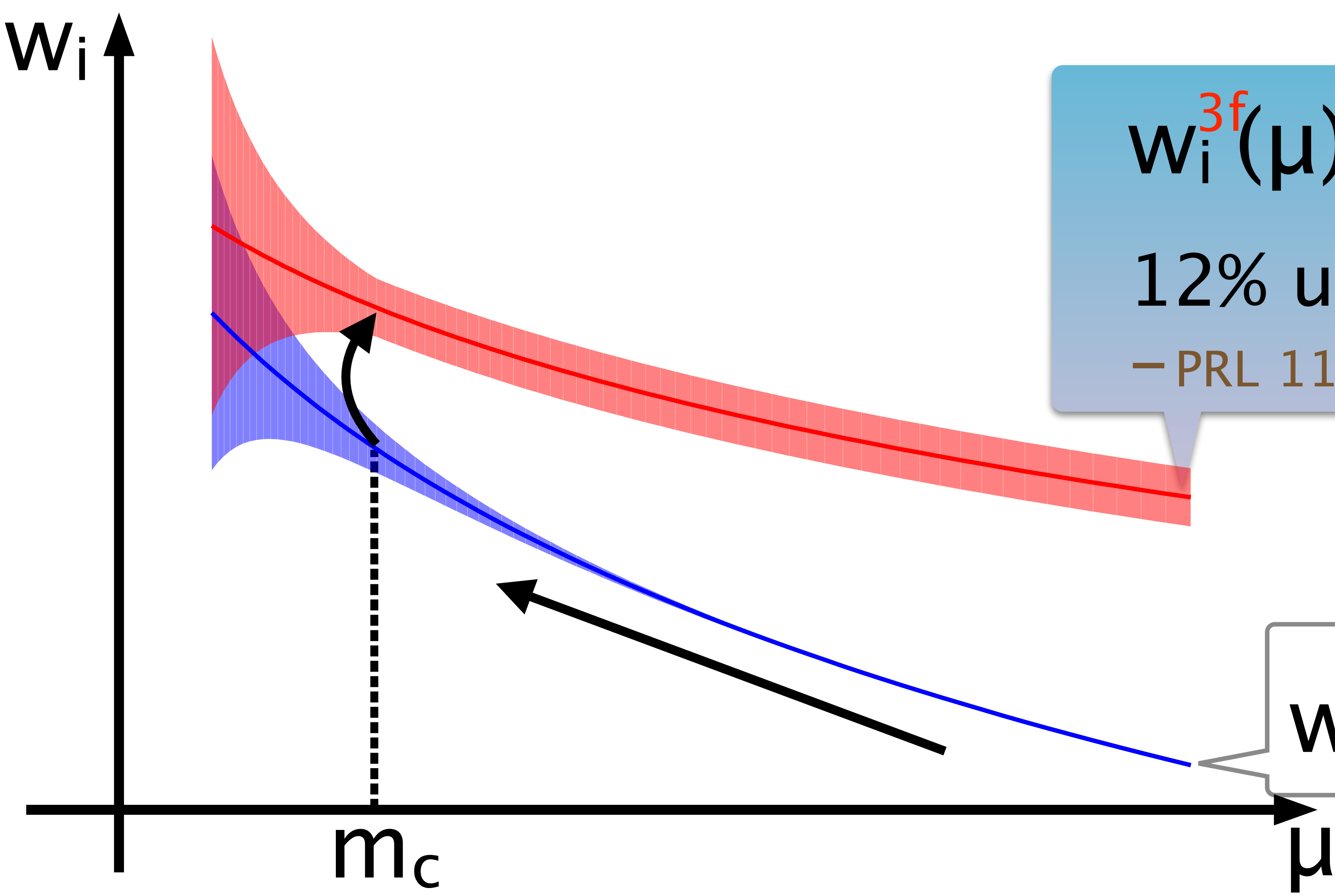
WMEs w/ 3-flavor operators

$$\langle f|H_W|i\rangle = \Sigma_i \underbrace{w_i^{3f}(\mu)}_{\text{pQCD}} \underbrace{\langle f|O_i^{3f}(\mu)|i\rangle}_{\text{LQCD}}$$



WMEs w/ 3-flavor operators

$$\langle f|H_W|i\rangle = \Sigma_i \underbrace{w_i^{3f}(\mu)}_{\text{pQCD}} \underbrace{\langle f|O_i^{3f}(\mu)|i\rangle}_{\text{LQCD}}$$



$w_i^{3f}(\mu)$: large uncertainty

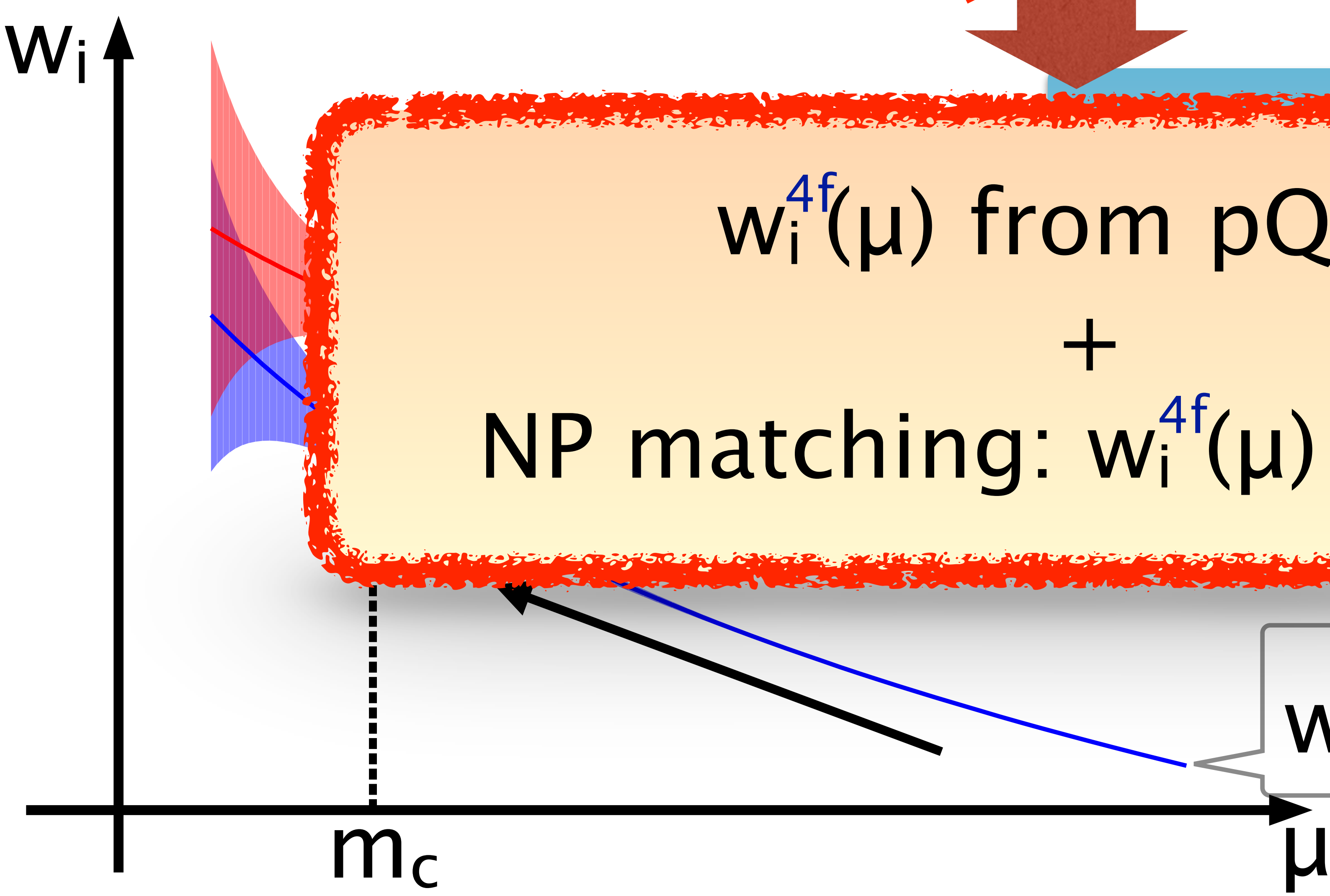
12% uncertainty for $K \to \pi\pi$

– PRL 115,212001(2015)

$w_i^{4f}(\mu)$: precise at HEs

WMEs w/ 3-flavor operators

$$\langle f|H_W|i\rangle = \Sigma_i\, \underbrace{w_i^{3f}(\mu)}_{\cancel{\text{pQCD}}}\, \underbrace{\langle f|O_i^{3f}(\mu)|i\rangle}_{\text{LQCD}}$$



...rtainty

$\rightarrow \pi\pi$

$w_i^{4f}(\mu)$ from pQCD

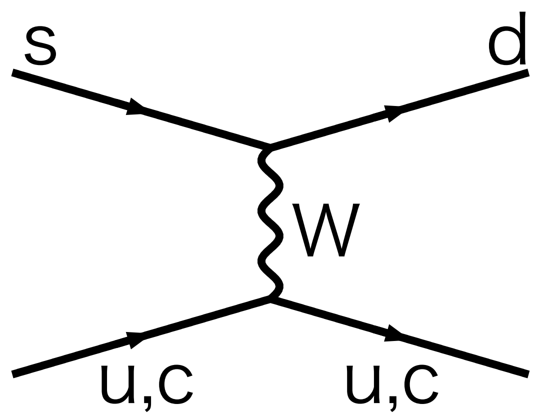
+

NP matching: $w_i^{4f}(\mu) \rightarrow w_i^{3f}(\mu)$

$w_i^{3f}(\mu) \neq w_i^{4f}(\mu)$?

- Of coarse sea charm effects $\Rightarrow w_i^{3f}(\mu) \neq w_i^{4f}(\mu)$
  - Maybe small difference → neglect in this talk
- If O_i^{4f} involves charm…

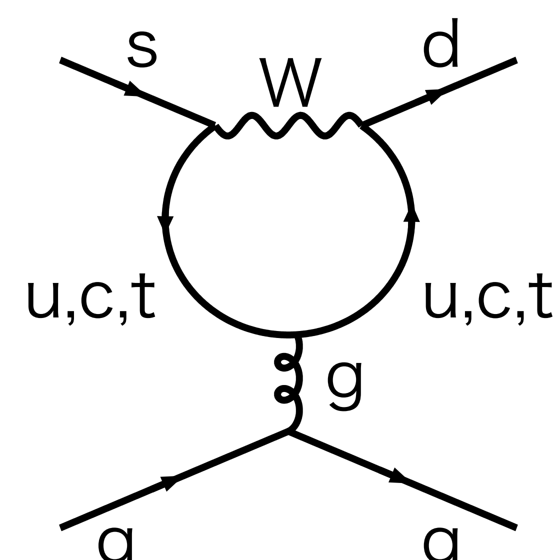
Ex)



current–current

$O_i^u = (\bar{s}d)_{V-A}\,(\bar{u}u)_{V-A}$

$O_i^c = (\bar{s}d)_{V-A}\,(\bar{c}c)_{V-A}$



QCD penguin

$O_i = (\bar{s}d)_{V-A}\,\Sigma_q\,(\bar{q}q)_{V\pm A}$

  - Corresponding w_i's in 3f & 4f different
  - w_i^{3f} necessary if MEs calculated with O_i^{3f}

$K \to \pi\pi$ by RBC/UKQCD (2015)

- 2+1 DWF / Iwasaki + DSDR gauge action
- a^{-1} = 1.38 GeV

  ⇒ too coarse to introduce charm

  ⇒ used 3-flavor operators for MEs
  & perturbative 4/3-flavor matching

  ⇒ 12% systematic uncertainty from this

- NP matching obtained from finer lattices desired

Contents

- Introduction
- New strategy in a gauge invariant procedure
  - Two-point functions in position space
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NP 4f-3f matching in position Sp.

$$H_W = \Sigma_i\, w_i^{4f}(\mu)\, O_i^{4f}(\mu) = \Sigma_i\, w_i^{3f}(\mu')\, O_i^{3f}(\mu')$$

- This means:

  $$\Sigma_i <\bar{O}(x)\, O_i^{4f}(\mu;y)^\dagger> w_i^{4f}(\mu) = \Sigma_i <\bar{O}(x)\, O_i^{3f}(\mu';y)^\dagger> w_i^{3f}(\mu')$$

  for any operator $\bar{O}(x)$

  at LDs: $1/|x-y| \ll m_c$
- Relation b/w w_i^{4f} & w_i^{3f} can be obtained by choosing appropriate number of $\bar{O}(x)$'s

  ⇒ We choose

  $$\bar{O}(x) = O_i^{3f}(a^{-1};x)$$

NP 4f–3f matching

$$\left[\begin{array}{l} H_W = \Sigma_i\, w_i^{4f}(\mu)\, O_i^{4f}(\mu) = \Sigma_i\, w_i^{3f}(\mu)\, O_i^{3f}(\mu) \\ \langle O_j^{3f}(\mu;x)\, H_W(y)^\dagger \rangle \end{array}\right.$$

$$\Sigma_j\, G_{ij}^{3f-4f}(\mu;x-y)\, w_j^{4f}(\mu) = \Sigma_j\, G_{ij}^{3f-3f}(\mu;x-y)\, w_j^{3f}(\mu)$$

$$G_{ij}^{nf-n'f}(\mu;x-y) = \langle O_i^{nf}(\mu;x)\, O_j^{n'f}(\mu;y)^\dagger \rangle$$

$$\boxed{w_i^{3f}(\mu) = \Sigma_{jk}\, (G^{3f-3f}(\mu;x-y))_{ij}^{-1}\, G_{jk}^{3f-4f}(\mu;x-y)\, w_k^{4f}(\mu)}$$

Previous effort in mom Sp.

- Condition

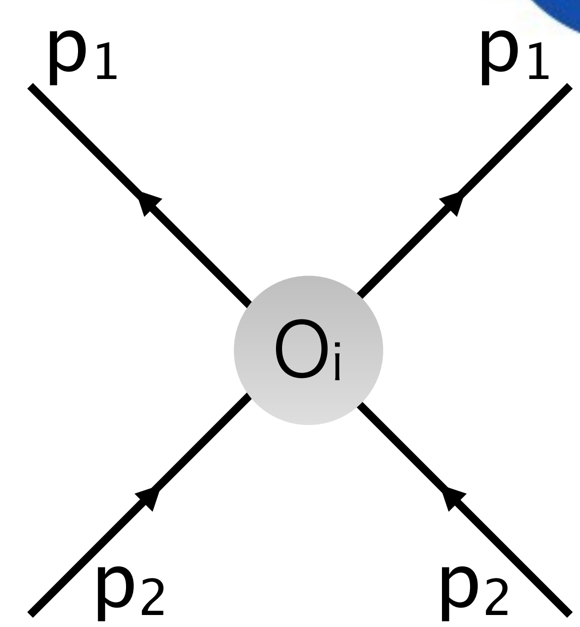
$$P^{abcd}_{\alpha\beta\gamma\delta}\ \Lambda^{abcd}_{\alpha\beta\gamma\delta}\ (O^{3f}_i(\mu);p_1,p_2)\ w^{3f}_i(\mu)$$

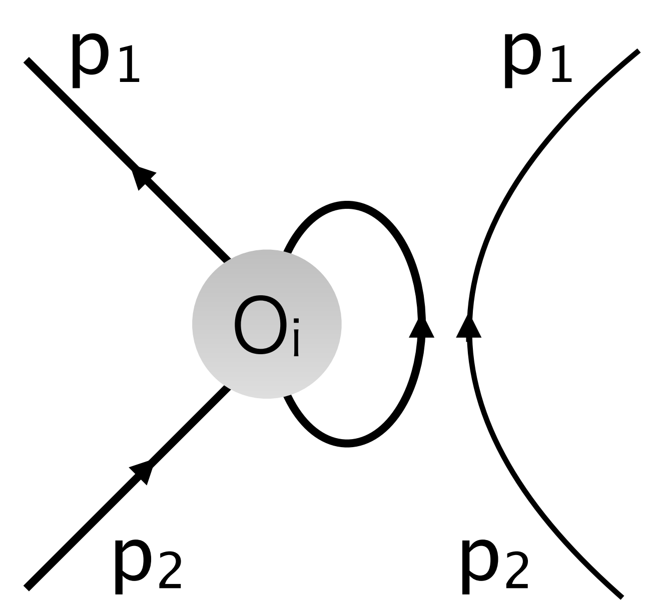
$$\shortparallel$$

$$\underbrace{P^{abcd}_{\alpha\beta\gamma\delta}}_{\text{flavor, color and spin projector}}\ \underbrace{\Lambda^{abcd}_{\alpha\beta\gamma\delta}\ (O^{4f}_i(\mu);p_1,p_2)}_{\text{G-fixed amputated Green's function}}\ w^{4f}_i(\mu)$$

G-fixed amputated Green's function

flavor, color and spin projector





- Condition valid in $|p_{1,2}| \ll mc$
- Statistical error
  - $|p_{1,2}| = 1.2$ GeV → 10%
  - $|p_{1,2}| = 0.6$ GeV → 50%
-

Why mom procedure so bad?

- Gauge fixing
  - Large Gribov noise
    - Gauge condition does not have a unique solution on the gauge orbit
    - Gauge-dependent quantities have some ambiguity
  - Mixing with gauge-noninvariant operators
- Off-shell condition
  - Mixing with operators that vanish by EoM

☆ All significant at small $p_{1,2}$

☆ Position-space procedure is free from all of these

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ΔS = 1 4-quark operators

3-flavor

Type	Q_i
current-current	$Q_1 = (\bar{s}_\alpha d_\alpha)_L (\bar{u}_\beta u_\beta)_L$ $Q_2 = (\bar{s}_\alpha d_\beta)_L (\bar{u}_\beta u_\alpha)_L$
QCD penguin	$Q_3 = (\bar{s}_\alpha d_\alpha)_L \sum_q^{3f} (\bar{q}_\beta q_\beta)_L$ $Q_4 = (\bar{s}_\alpha d_\beta)_L \sum_q^{3f} (\bar{q}_\beta q_\alpha)_L$ $Q_5 = (\bar{s}_\alpha d_\alpha)_L \sum_q^{3f} (\bar{q}_\beta q_\beta)_R$ $Q_6 = (\bar{s}_\alpha d_\beta)_L \sum_q^{3f} (\bar{q}_\beta q_\alpha)_R$
EW penguin	$Q_7 = \frac{3}{2}(\bar{s}_\alpha d_\alpha)_L \sum_q^{3f} e_q (\bar{q}_\beta q_\beta)_R$ $Q_8 = \frac{3}{2}(\bar{s}_\alpha d_\beta)_L \sum_q^{3f} e_q (\bar{q}_\beta q_\alpha)_R$ $Q_9 = \frac{3}{2}(\bar{s}_\alpha d_\alpha)_L \sum_q^{3f} e_q (\bar{q}_\beta q_\beta)_L$ $Q_{10} = \frac{3}{2}(\bar{s}_\alpha d_\beta)_L \sum_q^{3f} e_q (\bar{q}_\beta q_\alpha)_L$

7 independent operators

4-flavor

Type	P_i
current-current	$P_1 = (\bar{s}_\alpha d_\alpha)_L (\bar{u}_\beta u_\beta)_L$ $P_1^c = (\bar{s}_\alpha d_\alpha)_L (\bar{c}_\beta c_\beta)_L$ $P_2 = (\bar{s}_\alpha d_\beta)_L (\bar{u}_\beta u_\alpha)_L$ $P_2^c = (\bar{s}_\alpha d_\beta)_L (\bar{c}_\beta c_\alpha)_L$
QCD penguin	$P_3 = (\bar{s}_\alpha d_\alpha)_L \sum_q^{4f} (\bar{q}_\beta q_\beta)_L$ $P_4 = (\bar{s}_\alpha d_\beta)_L \sum_q^{4f} (\bar{q}_\beta q_\alpha)_L$ $P_5 = (\bar{s}_\alpha d_\alpha)_L \sum_q^{4f} (\bar{q}_\beta q_\beta)_R$ $P_6 = (\bar{s}_\alpha d_\beta)_L \sum_q^{4f} (\bar{q}_\beta q_\alpha)_R$
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9 independent operators

Color trivialization by Fierz trf.

- Def: $(\bar{s}d)_L(\bar{q}q)_{R/L} = \bar{s}\gamma_\mu(1-\gamma_5)d \cdot \bar{q}\gamma_\mu(1\pm\gamma_5)q$

- Left-Left operators

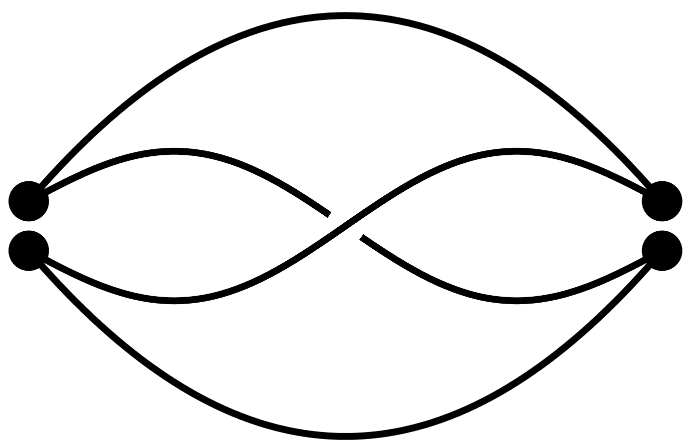
$$(\bar{s}_\alpha d_\beta)_L(\bar{q}_\beta q_\alpha)_L = (\bar{s}_\alpha q_\alpha)_L(\bar{q}_\beta d_\beta)_L$$

- Right-Right operators

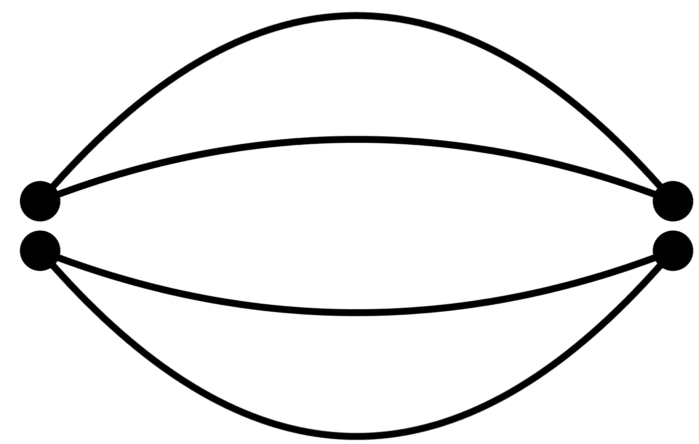
$$(\bar{s}_\alpha d_\beta)_L(\bar{q}_\beta q_\alpha)_R = -2\bar{s}_\alpha(1+\gamma_5)q_\alpha \cdot \bar{q}_\beta(1-\gamma_5)d_\beta$$

Contractions

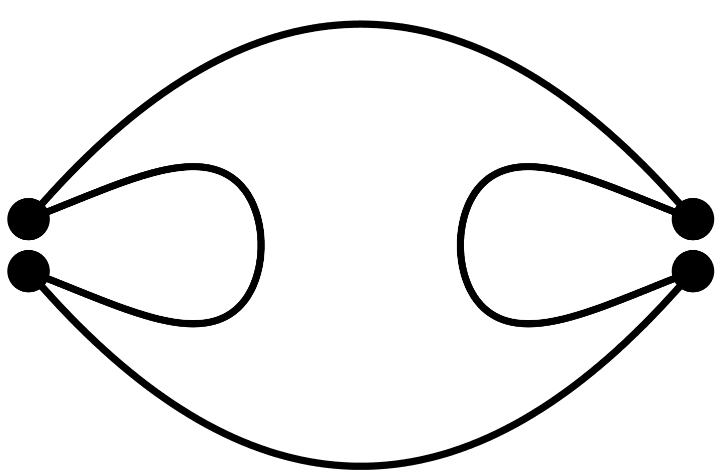
- 4/3-flavor matching should be independent of m_{ud} & m_s
  ⇒ Calculate w/ SU(3) valence quarks + 1 heavy quark



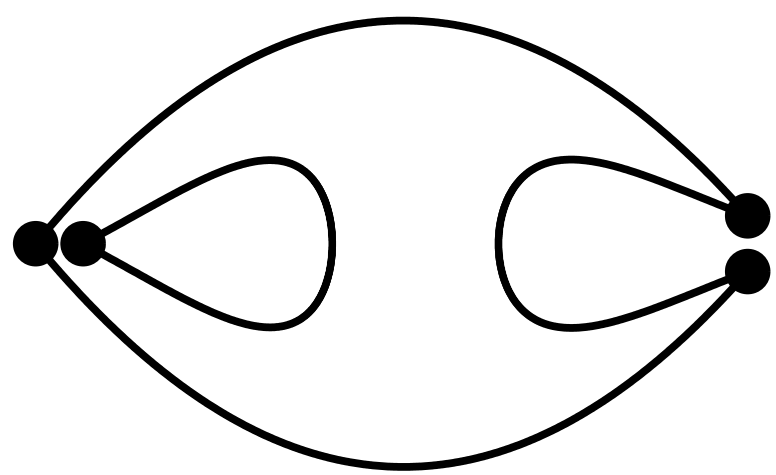
6 contractions



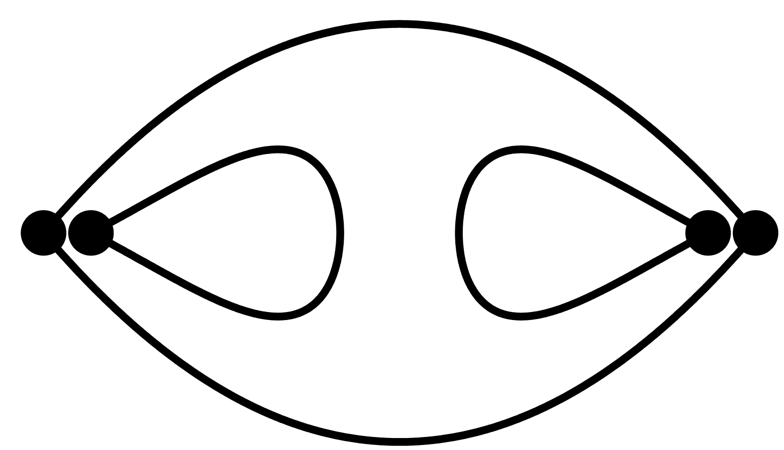
6 contractions



18 contractions



32 contractions



18 contractions

Subtraction of power divergence

- Loop diagram can contain power divergence
  - from power divergence of operators

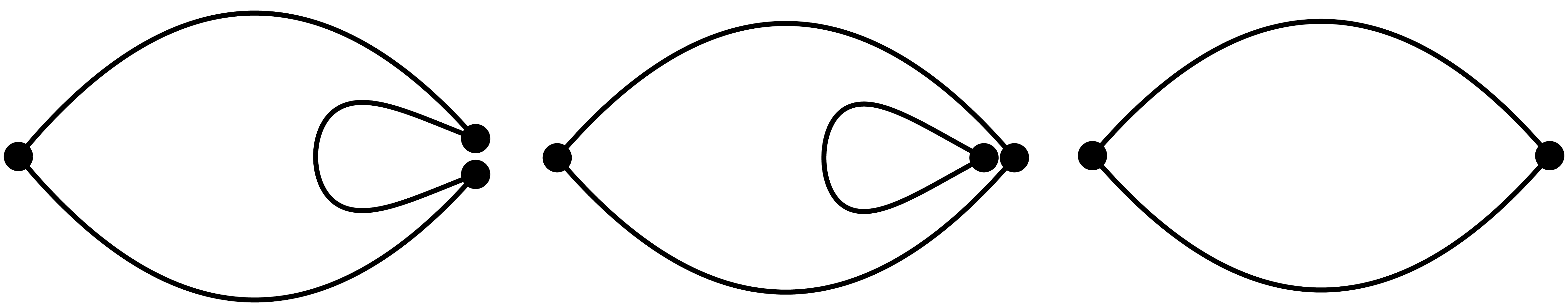
$$O_i \sim \frac{m_q}{a^2}$$

  - Eliminate by redefining

$$O_i' = O_i - C_- \bar{s}(1-\gamma_5)d - C_+ \bar{s}(1+\gamma_5)d$$

w/ a condition

$$\left\langle \bar{s}(1\pm\gamma_5)d(x) \cdot O_i'(y)^\dagger \right\rangle \Big|_{x-y=x_0} = 0$$



12 contractions 12 contractions 3 contractions

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More about source $\bar{O}(x)$

- It can take any form
  - T-slice sum
  - Smearing

$$\langle \bar{O}(\mu;x)\, H_W(y)^\dagger \rangle$$

- Our final goal: continuum limit of 4/3-flavor matching
  - to apply it to the result on 1.38 GeV lattice

- We propose an idea to take continuum limit in a more appropriate way
  - Averaging correlators over sphere to get O(4)-symmetric ones
  - Example for Z_m shown in the rest of talk

Quark mass renormalization

- $Z_m = Z_S^{-1}$
- Position-space renormalization of scalar current

$$Z_S^{\overline{MS}/lat}(\mu,1/a)^2\ \Pi_S^{lat}(1/a;x) = \Pi_S^{\overline{MS}}(\mu;x)$$

$$Z_S^{\overline{MS}/lat}(\mu,1/a) = \sqrt{\frac{\Pi_S^{\overline{MS}}(\mu;x)}{\Pi_S^{lat}(1/a;x)}}$$

  - ✦ Π_S: two-point function of scalar currents
  - ✦ $Z_S = Z_P$ if chirally symmetric lattice action (DWF in this work)
- We analyze

$$\widetilde{m}_q^{\overline{\mathrm{MS}}}(\mu;x;a) = m_q^{\mathrm{bare,phys}}(a)\sqrt{\frac{\frac{1}{2}(\Pi_S^{\mathrm{lat}}(1/a;x)+\Pi_P^{\mathrm{lat}}(1/a;x))}{\Pi_S^{\overline{\mathrm{MS}}}(\mu;x)}}$$

Position-space NPR

- 2pt functions used
  - Usually point-to-point correlators (no sum over t-slice)

- Discretization errors (DEs)
  - UV side of window problem
  - Significant even at LDs (~ Λ_{QCD})
  - Calculable only at lattice sites
    - Hard to take continuum limit for renormalized quantities

- We propose a new treatment for DEs (spherical average)

Lattice calculation

- Ensembles
  - 2+1 Domain-wall fermions
  - 3 lattice spacings: 1.7−3.1 GeV
  - Pion masses: 300−420 MeV

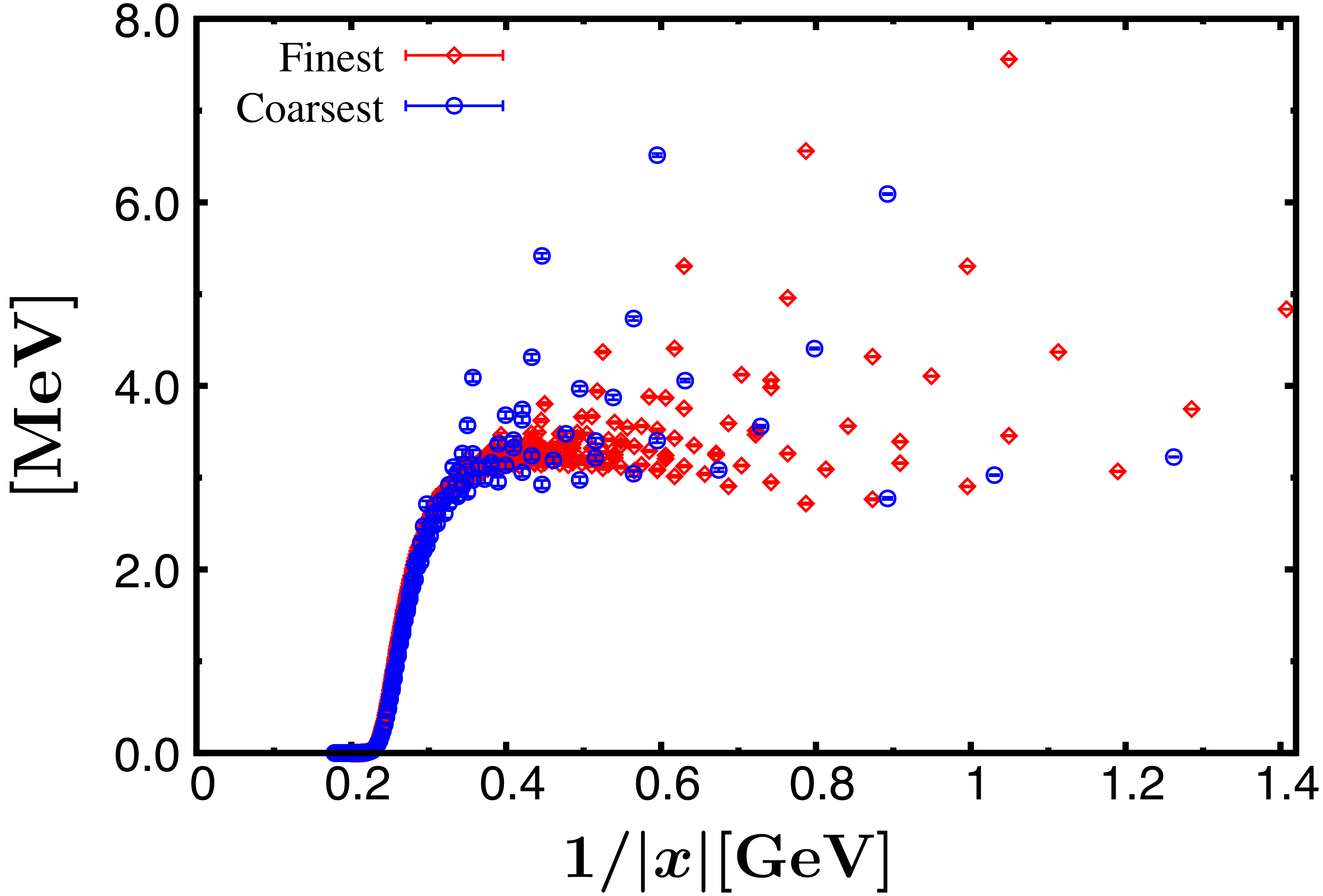
- For each ensemble, we analyze

$$\widetilde{m}_q^{\overline{\mathrm{MS}}}(\mu;x;a) = \underline{m_q^{\mathrm{bare,phys}}(a)}\,\underline{\sqrt{\frac{\frac{1}{2}(\Pi_S^{\mathrm{lat}}(1/a;x)+\Pi_P^{\mathrm{lat}}(1/a;x))}{\Pi_S^{\overline{\mathrm{MS}}}(\mu;x)}}}$$

[RBC/UKQCD (2016)]

Supposed to be a constant Z_m if in renormalization window

$\widetilde{m}_q^{\overline{MS}}(3\,\mathrm{GeV};x)$



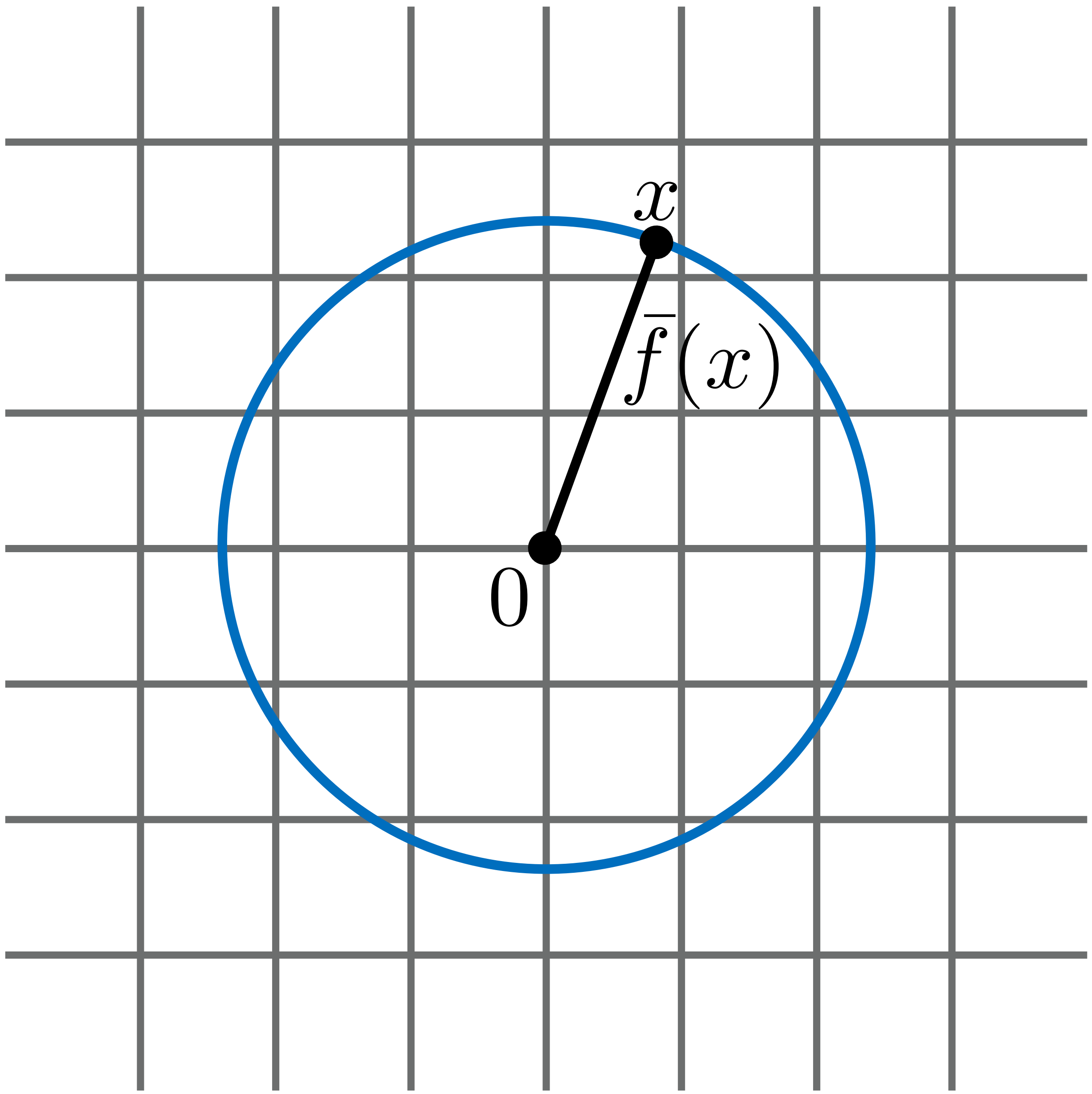
- Different lattice points distinguished ((1,1,1,1) ≠ (0,0,0,2))
- Large discretization errors

Average over spheres

- Evaluate the value of a quantity at each 4d point from values at lattice points, with a guideline

$$\bar{f}(x) = \eta(f^{\mathrm{lat}}; x)$$

※ details in next slides

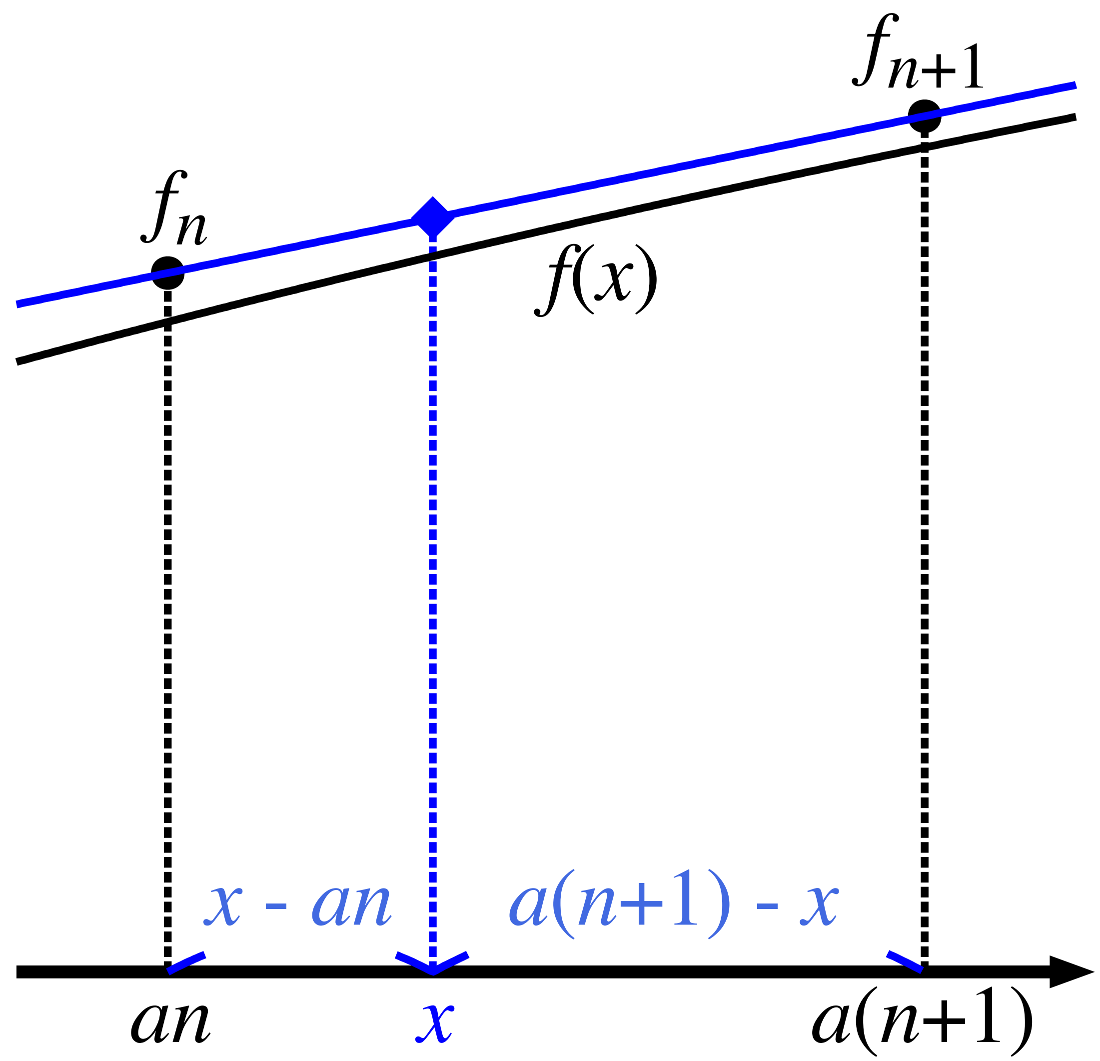


- Take the average over the sphere for each distance |x|

$$\hat{f}(|x|) = \frac{1}{2\pi^2} \oint_{S^3(|x|)} \mathrm{d}\Omega\, \bar{f}(x)$$

Evaluation of $\bar{f}(x)$ (1-dim)

- Linear interpolation

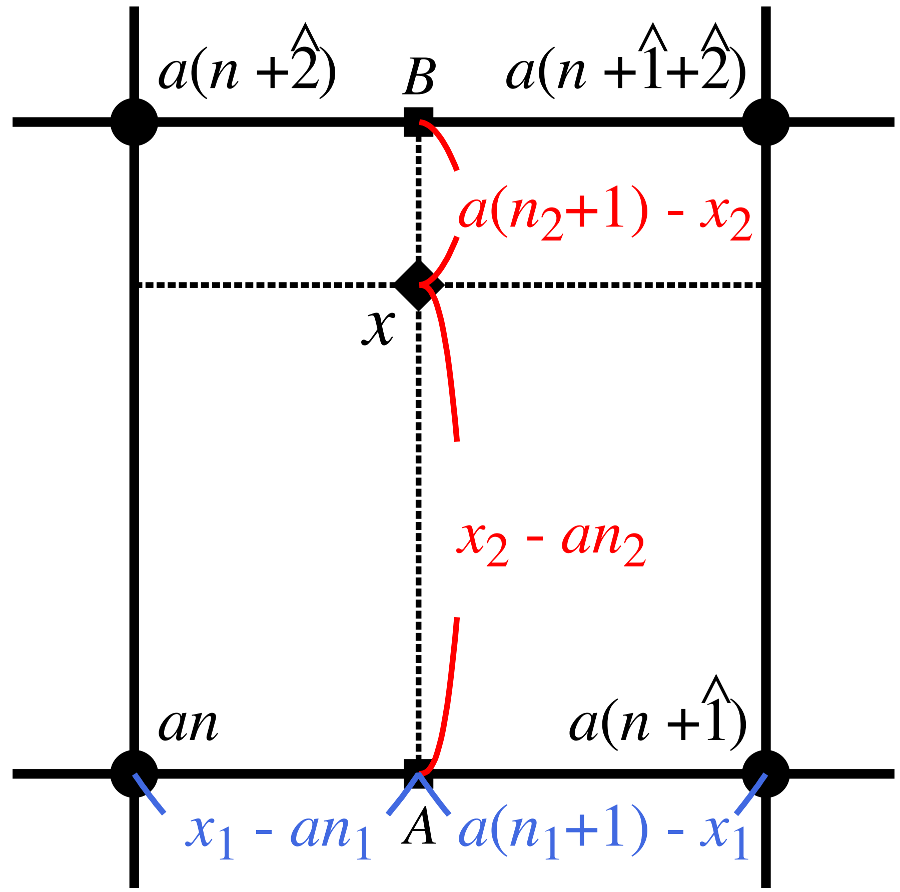


$$\bar{f}(x) = \frac{(a(n+1) - x)f_n + (x - an)f_{n+1}}{a} = f(x) \underline{+ O(a^2)}$$

Accurate up to $O(a^2)$

Evaluation of $\bar{f}(x)$ (2-dim)

- Bilinear interpolation



$$\bar{f}(x) = \frac{(a(n_2+1) - x_2)\bar{f}(A) + (x_2 - an_2)\bar{f}(B)}{a}$$

$$= a^{-2}\begin{pmatrix} a(n_1+1) - x_1 & x_1 - an_1 \end{pmatrix}\begin{pmatrix} f_n & f_{n+\hat{2}} \\ f_{n+\hat{1}} & f_{n+\hat{1}+\hat{2}} \end{pmatrix}\begin{pmatrix} a(n_2+1) - x_2 \\ x_2 - an_2 \end{pmatrix}$$

$$= f(x) \underline{+ O(a^2)}$$

Evaluation of $\bar{f}(x)$ (4-dim)

- Quadrilinear interpolation

$$\bar{f}(x) = a^{-4} \sum_{i,j,k,l=0}^{1} \Delta_{1,i}\Delta_{2,j}\Delta_{3,k}\Delta_{4,l} f_{n+i\hat{1}+j\hat{2}+k\hat{3}+l\hat{4}}$$

$$\Delta_{\mu,i} = |a(n_\mu + 1 - i) - x_\mu|$$

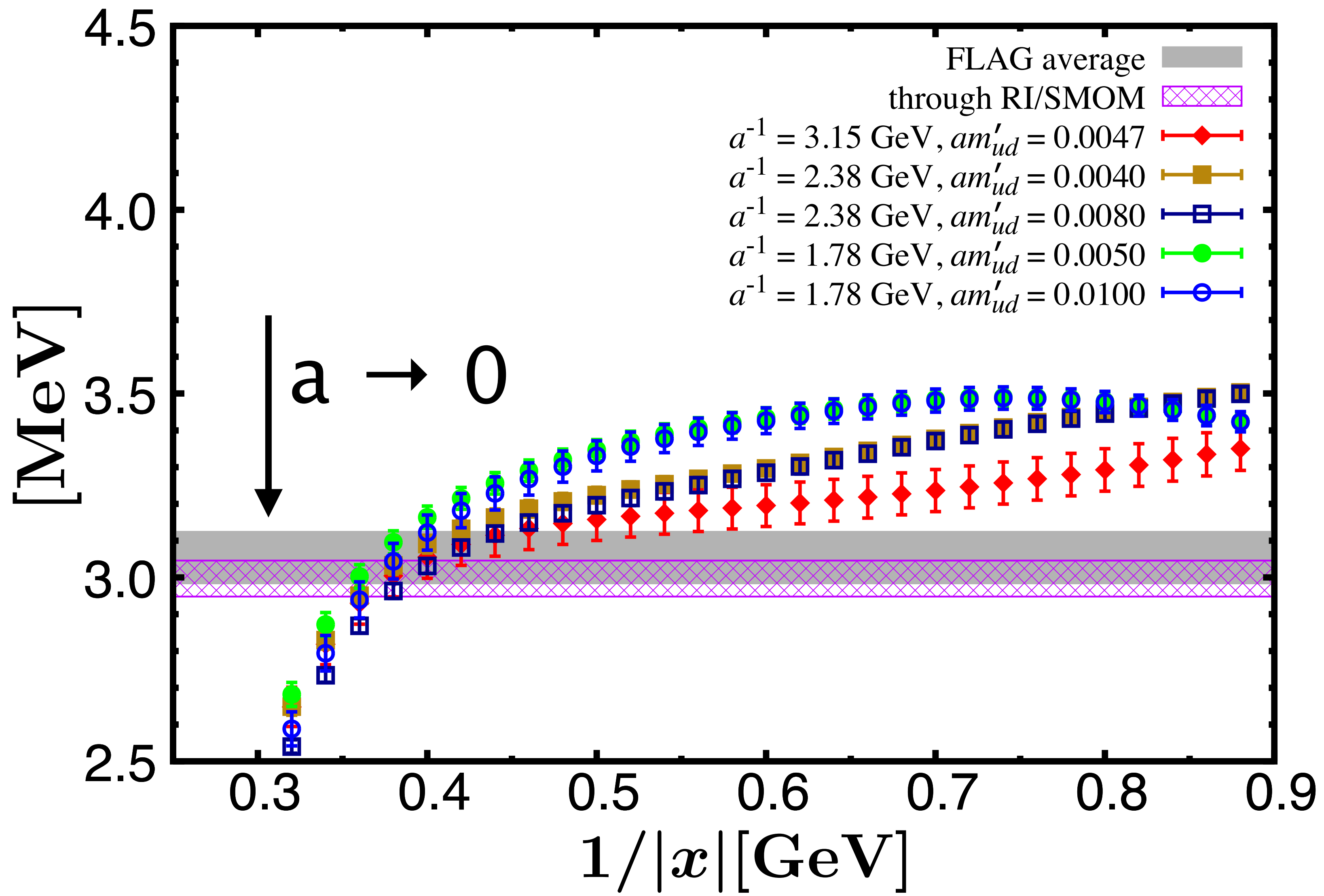
  - Accurate up to $O(a^2)$

Result for spherical average

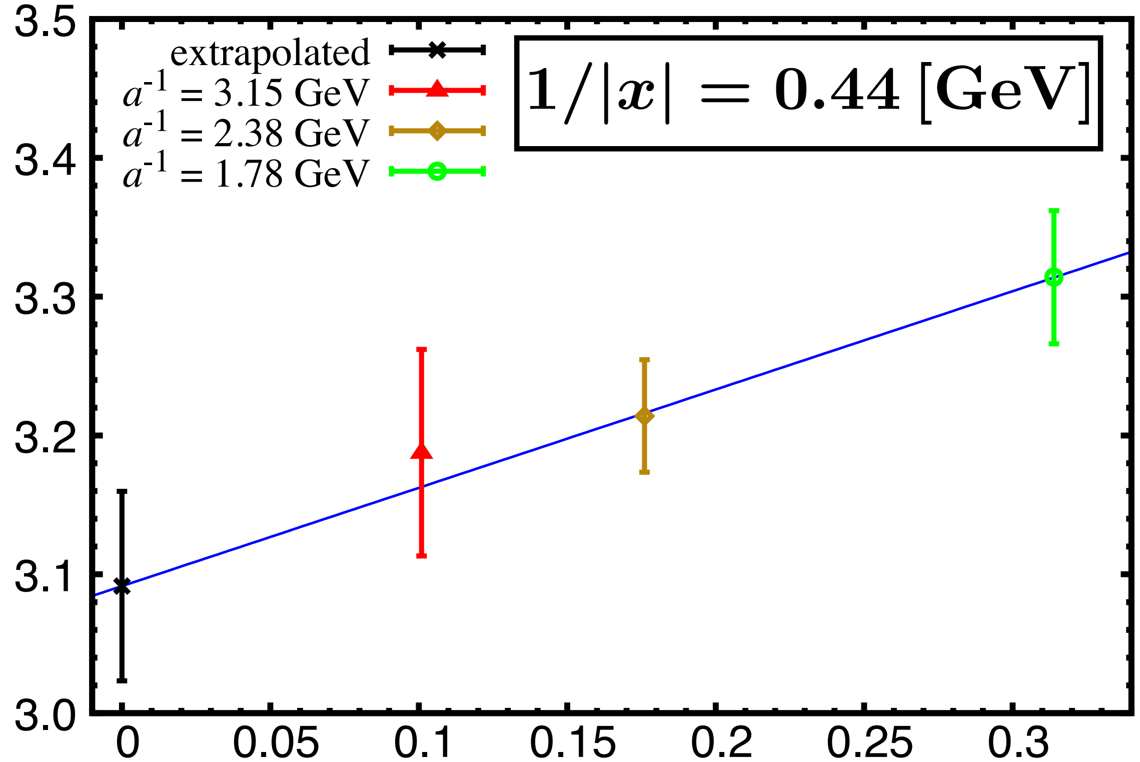
- Sphere average of

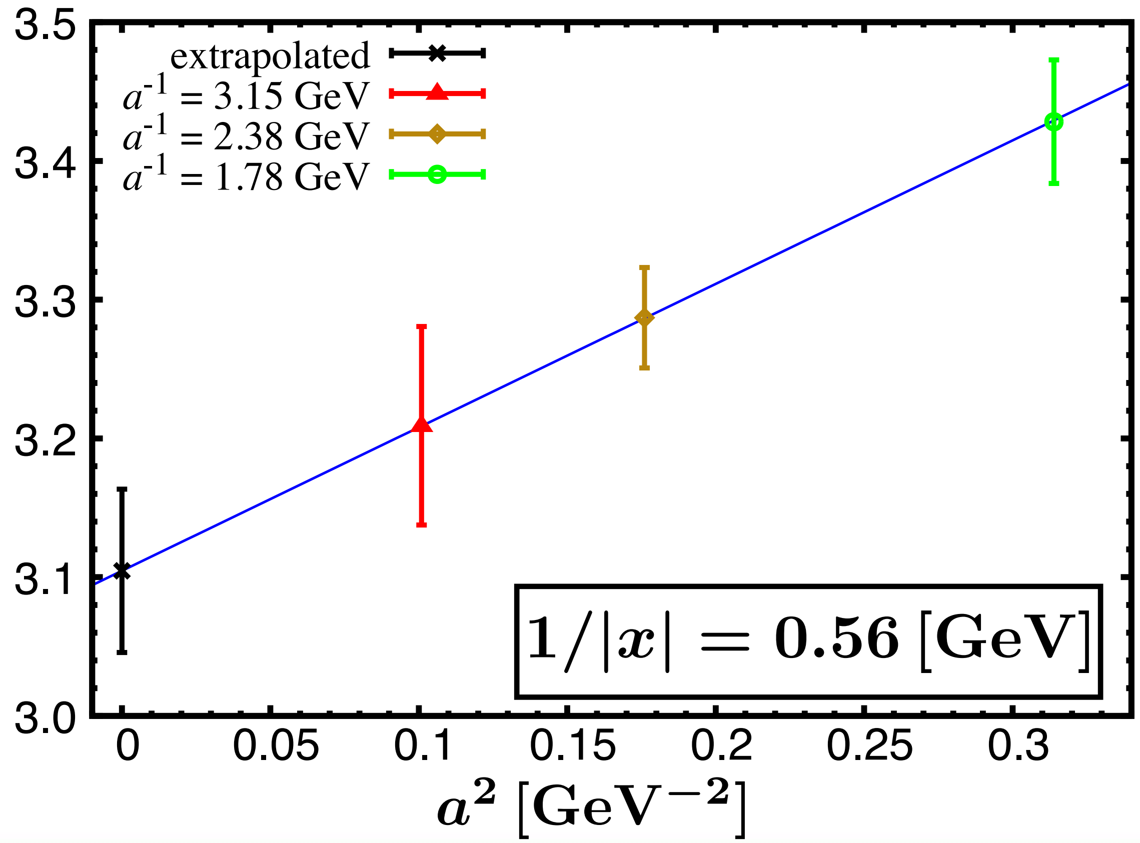
$$\widetilde{m}_q^{\overline{\mathrm{MS}}}(\mu;x;a) = m_q^{\mathrm{bare,phys}}(a)\sqrt{\frac{\frac{1}{2}(\Pi_S^{\mathrm{lat}}(1/a;x)+\Pi_P^{\mathrm{lat}}(1/a;x))}{\Pi_S^{\overline{\mathrm{MS}}}(\mu;x)}}$$

- Able to calculate at any distance
- Plateau seen better for finer lattices

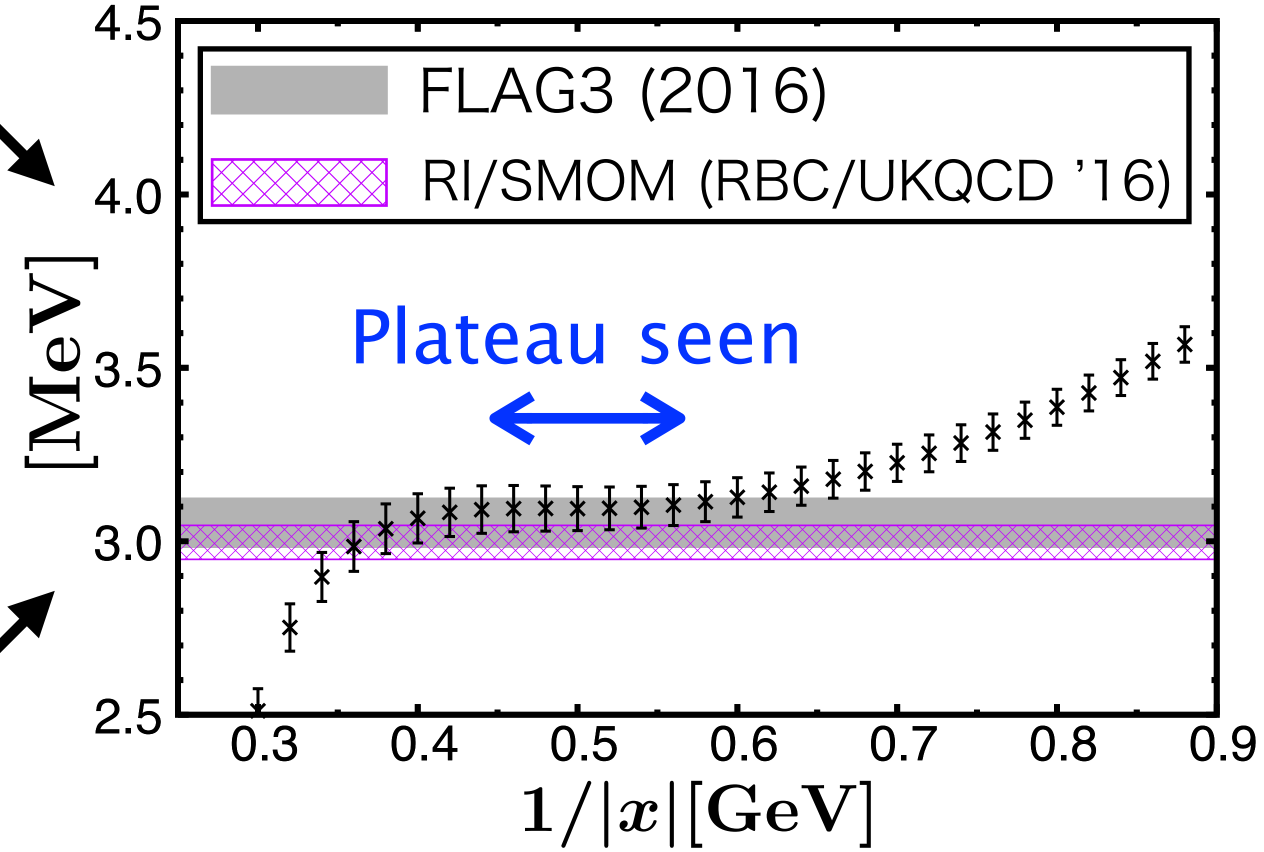


Continuum limit





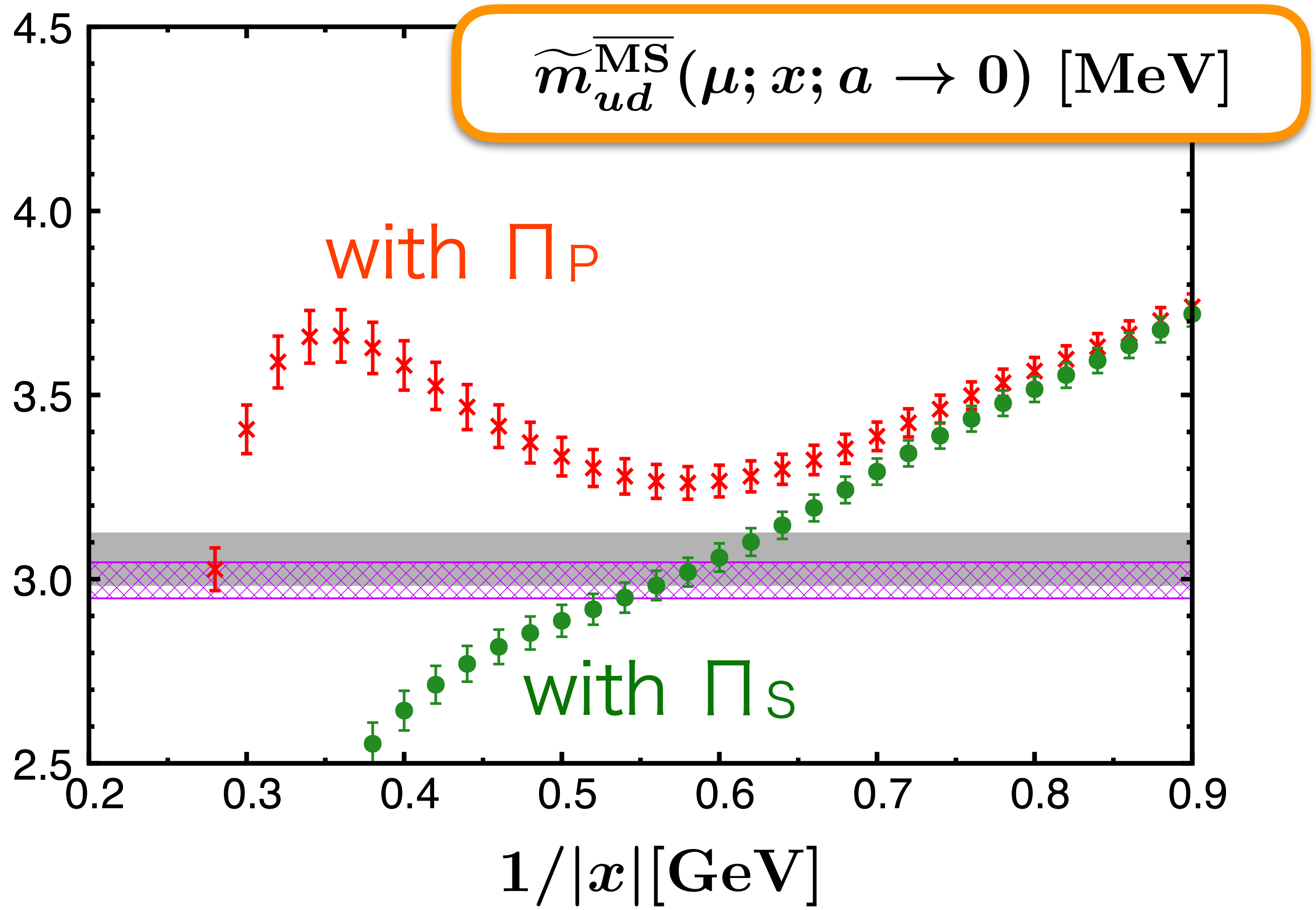
Chiral limit of Z_m is simultaneously taken



$m_{ud}^{\overline{MS}}$ (3 GeV)$|_{2+1f}$ = 3.11 $(4)_{stat}(5)_{sys}(2)_{pert}(7)_{\alpha_s}$

Why $\frac{1}{2}(\Pi_S + \Pi_P)$?

- $\Pi_{S,P}$: directly affected by instantons [Novikov etal 1979] [Shuryak 1993]
  - Largely deviated from pQCD/OPE
  - Serious window problem



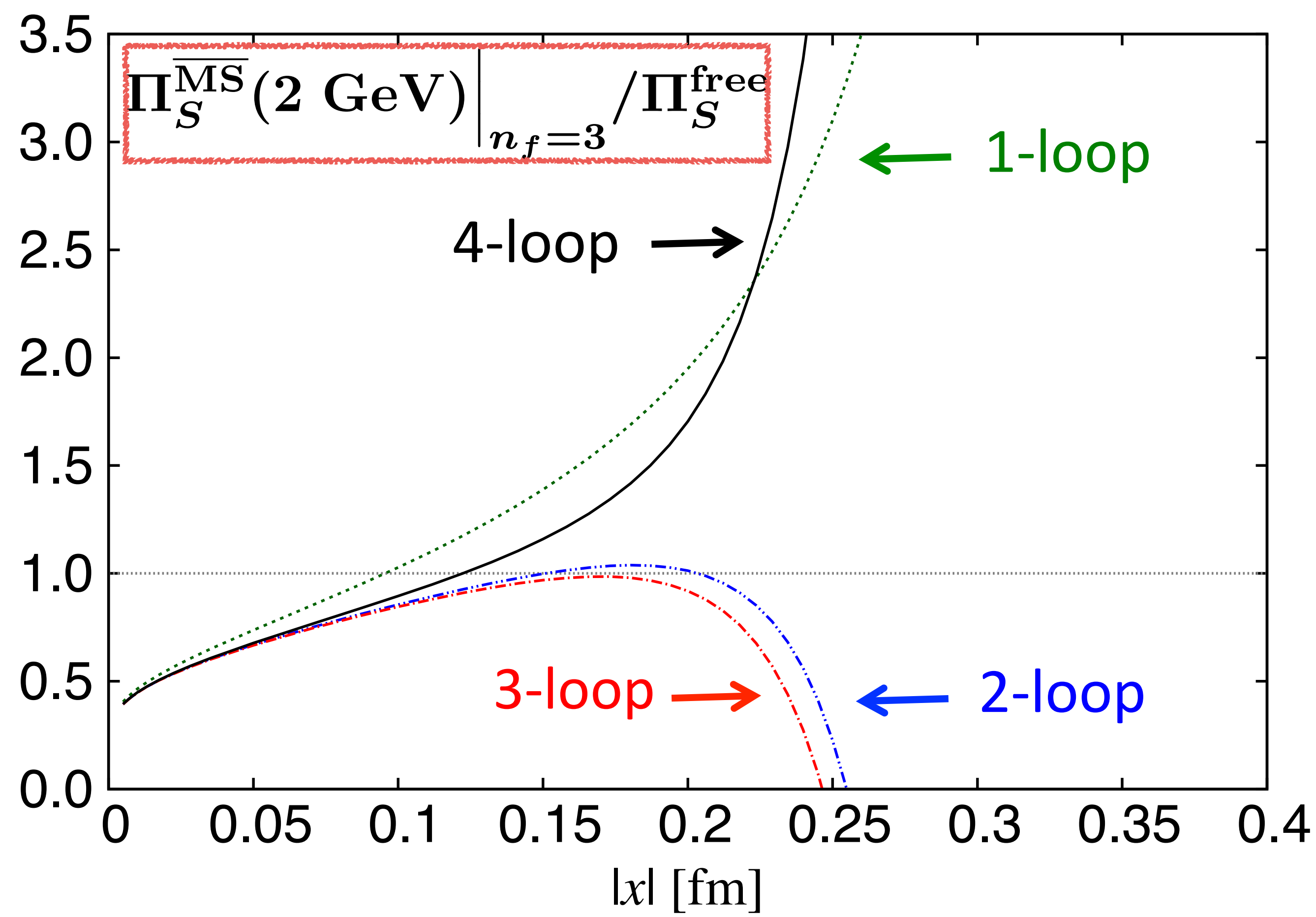
- Naïve average $\frac{1}{2}(\Pi_S + \Pi_P)$ free from single instanton interactions

$\overline{\text{MS}}$ correlator

- Available up to $O(\alpha_s^4)$ [Chetyrkin, Maier (2011)]

- Bad convergence of original perturbative series

$$\Pi_S^{\overline{\text{MS}}}(\mu_x = 1/x; x)\Big|_{n_f=3}$$
$$= \frac{3}{\pi^4 x^6}\left(1 + 0.20a_s - 19a_s^2 - 11a_s^3 + 579a_s^4\right)$$
$$(a_s = \alpha_s(\mu_x = 1/|x|)/\pi)$$



- large coef. at $O(a_s^4)$
- a_s blows up at ~ 0.3 fm

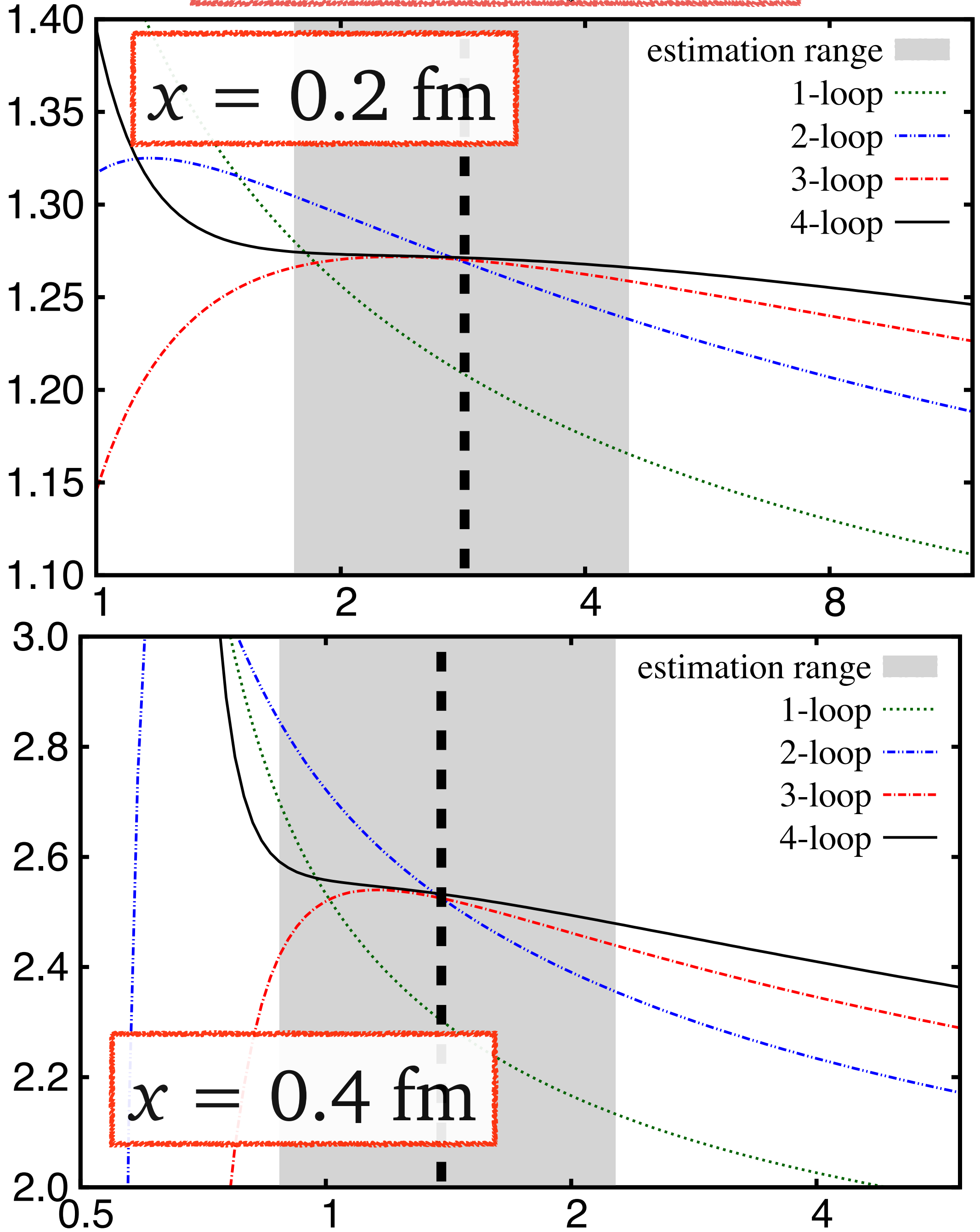
- No window for recently available lattice spacings

Scale of α_s

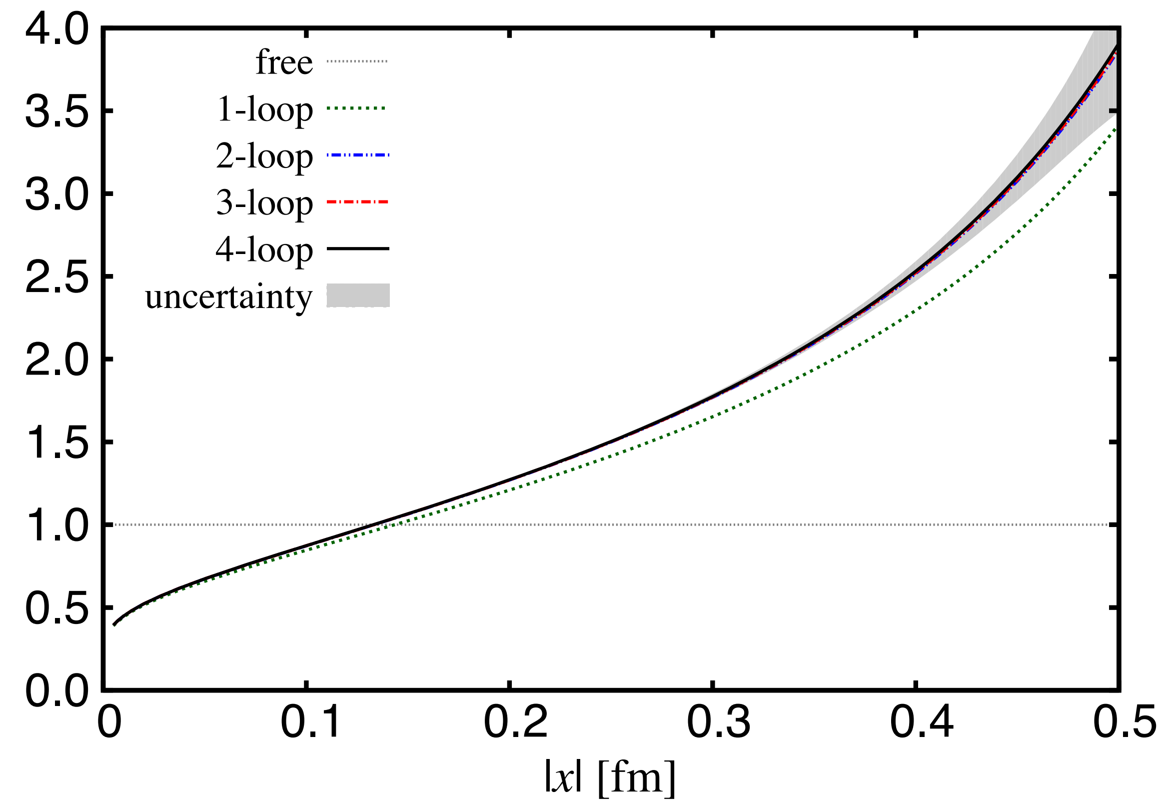
[MT etal (2015)]

- Dependence on the scale μ^* of α_s

$\Pi_S^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})\Big|_{n_f=3} / \Pi_S^{\mathrm{free}}$



- We use $\mu_x^* = 2.9/x$
- Truncation error estimated by gray band



Summary

- NP 4f/3f matching desired for $K \to \pi\pi$ calculation
  - Gauge invariant
  - Free from Gribov noise
  - Free from mixing w/ irrelevant operators
- New treatment for discretization errors is proposed
  - Interpolation + Spherical average
  - Good for continuum limit
  - Likely working well for quark mass renormalization (agrees with FLAG & our previous result through RI/SMOM)