Neutron Electric Dipole Moment from Beyond the Standard Model on the lattice

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Introduction
Standard Model CP Violation

Two sources of CP violation in the Standard Model.

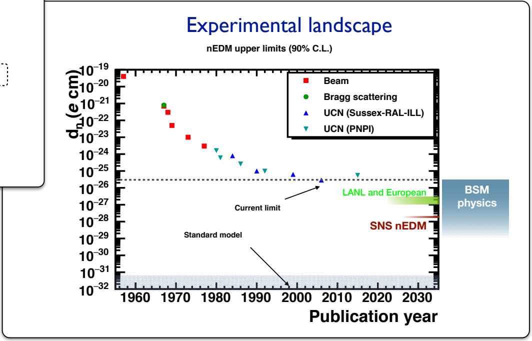
- Complex phase in CKM quark mixing matrix.
  - Too small to explain baryon asymmetry
  - Gives a tiny ($\sim 10^{-32}$ e-cm) contribution to nEDM
- CP-violating mass term and effective $\Theta G\tilde{G}$ interaction related to QCD instantons
  - Effects suppressed at high energies
  - nEDM limits constrain $\Theta \lesssim 10^{-10}$

Contributions from beyond the standard model

- Needed to explain baryogenesis
- May have large contribution to EDM

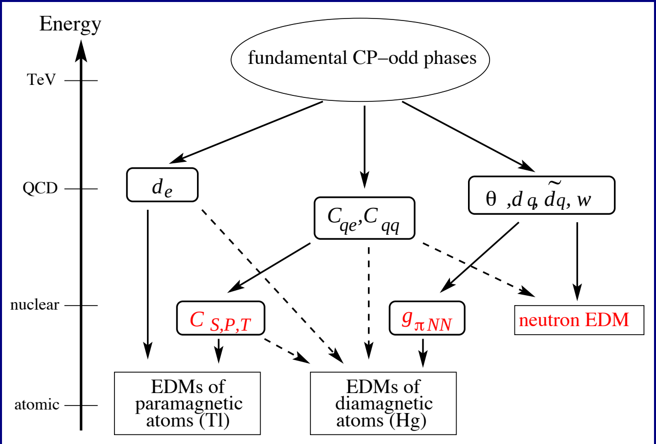
Introduction
Experimental situation

	Current	SM
e	10^{-29}	10^{-38}
μ	10^{-19}	10^{-35}
τ	10^{-16}	10^{-34}
n	10^{-26}	10^{-31}
p	10^{-23}	10^{-31}
^{199}Hg	10^{-29}	10^{-33}
^{129}Xe	10^{-27}	10^{-33}
^{225}Ra	10^{-23}	10^{-33}



Introduction

Effective Field Theory



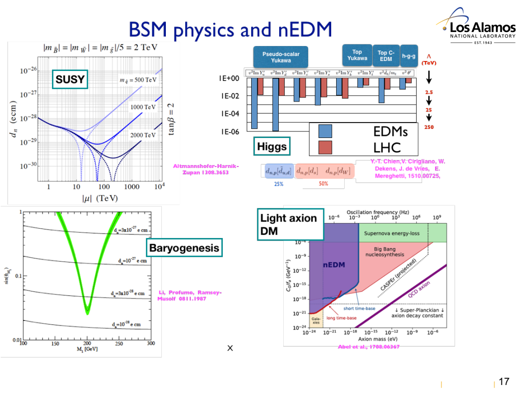
Pospelov and Ritz, *Ann. Phys.* **318** (2005) 119.

Introduction
BSM Operators

Standard model CP violation in the weak sector.
Strong CP violation from dimension 3 and 4 operators anomalously small.

- Dimension 3 and 4:
  - CP violating mass $\bar{\psi}\gamma_5\psi$.
  - Toplogical charge $G_{\mu\nu}\tilde{G}^{\mu\nu}$.
- Suppressed by $v_{\rm EW}/M^2_{\rm BSM}$:
  - Electric Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{F}^{\mu\nu}\psi$.
  - Chromo Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{G}^{\mu\nu}\psi$.
- Suppressed by $1/M^2_{\rm BSM}$:
  - Weinberg operator (Gluon chromo-electric moment): $G_{\mu\nu}G_{\lambda\nu}\tilde{G}_{\mu\lambda}$.
  - Various four-fermi operators.

Introduction
Impact on BSM physics



Introduction
Form Factors

Vector form-factors

Dirac F_1, Pauli F_2, Electric dipole F_3, and Anapole F_A

Sachs electric $G_E \equiv F_1 - (q^2/4M^2)F_2$ and magnetic $G_M \equiv F_1 + F_2$

$$\begin{aligned}\langle N|V_\mu(q)|N\rangle \quad = \quad & \overline{u}_N \Big[\gamma_\mu \; F_1(q^2) + i\frac{[\gamma_\mu,\gamma_\nu]}{2} q_\nu \; \frac{F_2(q^2)}{2m_N} \\ & + (2i\, m_N \gamma_5 q_\mu - \gamma_\mu\gamma_5 q^2) \; \frac{F_A(q^2)}{m_N^2} \\ & + \frac{[\gamma_\mu,\gamma_\nu]}{2} q_\nu \gamma_5 \; \frac{F_3(q^2)}{2m_N}\Big] \, u_N\end{aligned}$$

- The charge $G_E(0) = F_1(0) = 0$.
- $G_M(0)/2M_N = F_2(0)/2M_N$ is the (anomalous) magnetic dipole moment.
- $F_3(0)/2m_N$ is the electric dipole moment.
- F_A and F_3 violate P; F_3 violates CP.

Quark EDM

Tensor Charge

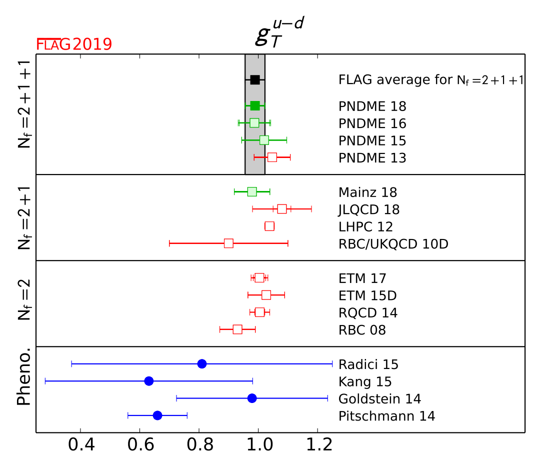
The only nEDM contribution known with certainty from the lattice is that due to the quark EDM. In this case, the ratio of nEDM to qEDM is just the tensor charge.

$$g_T^u = 0.784(28)(10) \qquad g_T^d = -0.204(11)(10) \qquad g_T^s = -0.027(16)$$

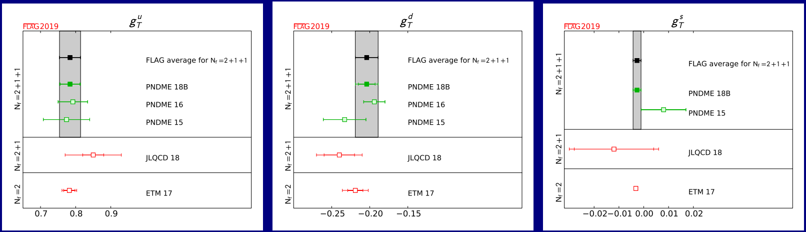
The isovector combinations are known better:

$$g_T^{u-d} = 0.989(32)(10)$$

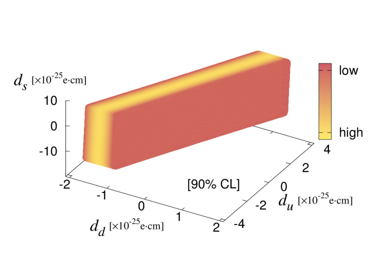
Quark EDM
FLAG: Isovector Tensor Charge

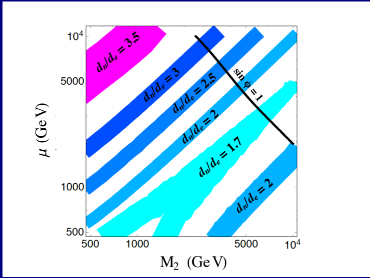


Quark EDM
FLAG: Isoscalar Tensor Charge



Quark EDM
Impact on BSM





Dirac Equation

Free neutrons

By Lorentz invariance, an on-shell spin-$\frac{1}{2}$ massive field ψ obeys

$$(e^{\beta\gamma_5}\not{p} - e^{i\alpha\gamma_5}m)\psi = 0\,.$$

This 'free' equation has space-time discrete symmetries:

$$\begin{aligned}
\mathcal{P}: &\quad \psi(\vec{x},t) \to\ e^{(\beta-i\alpha)\gamma_5}\ \gamma_0\ \psi(-\vec{x},t)\\
\mathcal{C}: &\quad \psi(\vec{x},t) \to i\ e^{\beta\gamma_5}\ \gamma_2\ \psi^*(\vec{x},t)\\
\mathcal{T}: &\quad \psi(\vec{x},t) \to -\ e^{-i\alpha\gamma_5}\ \gamma_1\gamma_3\ \psi^*(-\vec{x},t)
\end{aligned}$$

- Even when theory does not have these symmetries,
- asymptotic states always do,
- but operators have extra γ_5 phases.
- $e^{(-\beta+i\alpha)\gamma_5/2}\ \psi$ has standard phases.

Dirac Equation

Phase conventions

Consider $N = (\bar{u}^c\gamma_5 d)u$ with usual $\alpha = \beta = 0$ phases for u and d.
If theory has $\mathcal{C}$, $\mathcal{P}$, $\mathcal{T}$, then N has the same phases.
Otherwise, by Lorentz invariance, we still have

$$\mathcal{P} = \left[e^{2\beta(p^2)\gamma_5}\Pi(p^2)\not{p} - e^{2i\alpha(p^2)\gamma_5}\Sigma(p^2)m\right]^{-1} ,$$

which means $e^{(-\beta(p^2)+i\alpha(p^2))\gamma_5}N$ has the standard propagator (and hence the standard asymptotic symmetry representation).

But $e^{(-\beta(p^2)+i\alpha(p^2))\gamma_5}$ is not local!

Dirac Equation
State-dependent phases

By Källen-Lehman spectral representation, the propagator is

$$\sum \rho(\mu^2) Z_N(\mu^2) \frac{e^{2\beta(\mu^2)\gamma_5} \not{p} + e^{-2i\alpha(\mu^2)\gamma_5} Z_m(\mu^2)\mu}{p^2 - (Z_m(\mu^2)\mu)^2} .$$

When there is no overall symmetry operator, phases are state-dependent.

$N_{\rm st} \equiv e^{(-\beta(m_N^2)+i\alpha(m_N^2))\gamma_5} N$ has standard transformations and equation of motion, but only for the neutron; the excited states have non-standard phases.

Note that nontrivial α violates CP, whereas nontrivial β violates PT.

Dirac Equation
Electric Dipole Moment

$e\sigma \cdot B$ is even under $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$,
$e\sigma \cdot E$ is odd under $\mathcal{P}$ and $\mathcal{T}$.

$$\Sigma \cdot F \propto \begin{pmatrix} \sigma \cdot B & i\sigma \cdot E \\ i\sigma \cdot E & \sigma \cdot B \end{pmatrix} ,$$

which is $\sigma \cdot B$ in the rest frame iff $\not{p} \pm m = 0$.
In general, we need to use $e^{i\alpha\gamma_5} \Sigma \cdot F$.
So, important to use $N_{\rm st}$ instead of N in analyses.
At the Green's function level, this is

$$\langle T N_{\rm st} O \bar{N}_{\rm st} \rangle = e^{(-\beta(m_N^2)+i\alpha(m_N^2))\gamma_5/2} \langle T N O \bar{N} \rangle e^{(\beta(m_N^2)+i\alpha(m_N^2))\gamma_5/2} .$$

Abramczyk, Aoki, Blum, Izubichi, Ohki, and Syritsyn arXiv:1701.07792 [hep-lat]

Operator Mixing

Divergent mixing

- The quark EDM and the CPV mass-term have no lower dimensional operators they can mix with.
- Anomaly relates the Θ-term to the CPV mass-term.
- The chromoEDM operator has a divergent mixing with the CPV mass-term.
- The Weinberg operator has a divergent mixing with the Θ-term.

We need to take $a \to 0$ limit to control discretization errors $O(\Lambda_{QCD}a)^n$ from the low scales inside the hadron.

Operator Mixing

Nonperturbative renormalization condition

The divergent mixing, e.g. $O_{CEDM} - (c/a^2)O_{CPV}$, means that in matrix elements, large $O(1/(\Lambda_{QCD}a)^2)$ cancel with the correct choice of c.

Can use nonperturbative renormalization to remove divergence and relate to $\overline{\mathrm{MS}}$ scheme.

- RI-sMOM mixes with gauge-variant and Equation-of-motion operators (about 60 for Weinberg).
- RI-sMOM with Background gauge-fixing still needs the EOM operators.
- Position space renormalization needs multiloop calculations (four-loop for cEDM to $O(\alpha_s)$).

Operator Mixing
Gradient flow

Regulate the theory before discretizing!
Gradient flow smears the point operators on a length scale $\sqrt{8t_{WF}}$ and commutes with the continuum limit along the line of constant physics.

$$\begin{aligned} Z_\psi^{-1}(a)\mathrm{CEDM}(a,t_{WF}) &\overset{a\to 0}{\to} \mathrm{CEDM}(t_{WF}) \\ \mathrm{Weinberg}(a,t_{WF}) &\overset{a\to 0}{\to} \mathrm{Weinberg}(t_{WF}) \end{aligned}$$

Continuum perturbation links these to $\overline{\mathrm{MS}}$ quantities. Power subtractions are proportional to $1/8\Lambda_{QCD}^2 t$.

Need a window $a^2 \ll 8t \ll \Lambda_{QCD}^{-2}$ for being able to do this perturbatively.

Operator Mixing
Lattice implementation

Use Runge-Kutta integrator for gradient flow.

Save intermediate configurations in memory for the propagator adjoint-flow.

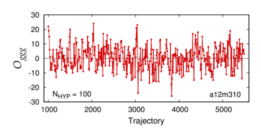
Use large step-size for much of the calculation.

Use bias-correction with a few small step-size calculations.

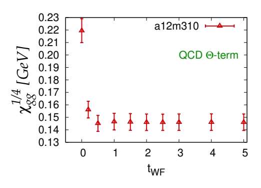
Compare results at multiple t_{WF}.

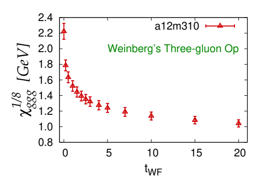
Lattice Results
Wilson flow on MILC lattices

Large fluctuations from configuration to configuration.



Smearing reduces the fluctuations; scale dependence remains.

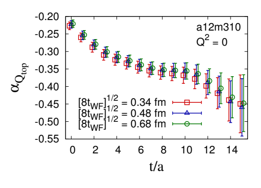


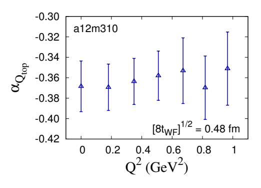


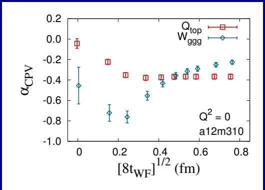
Lattice Results

Signal in α from Weinberg

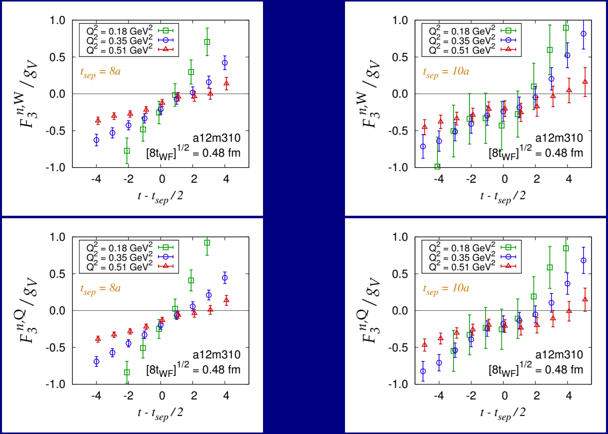
$$a \approx 0.12\text{fm};\, M_\pi \approx 310\text{MeV};\, 128 \text{ measurements} \times 1012 \text{ configurations}$$



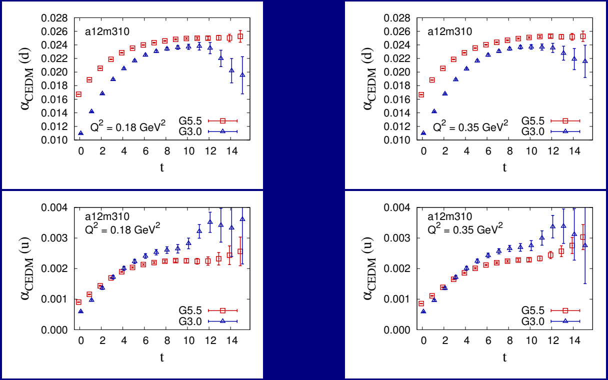


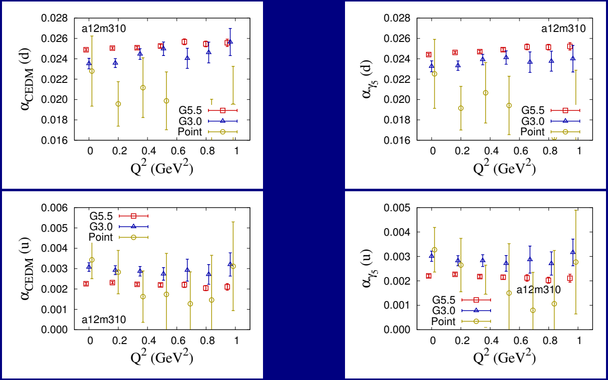


Lattice Results
Signal in F_3 from Weinberg



Lattice Results
Signal in α from CEDM




a12m310
G5.5
G3.0
Point
$\alpha_{\rm CEDM}$ (d)
α_{γ_5} (d)
$\alpha_{\rm CEDM}$ (u)
α_{γ_5} (u)
Q^2 (GeV^2)
0.028
0.026
0.024
0.022
0.020
0.018
0.016
0.006
0.005
0.004
0.003
0.002
0.001
0.000
0 0.2 0.4 0.6 0.8 1

Conclusions

Exciting future

Gradient Flow seems to be a good next thing to try.
Have to establish signal and window.
Have to check excited states and extrapolation.
Perturbative calculations in progress.
Next one needs the four-fermion operators