Nucleon Electric Dipole Moment from θ_{QCD} on a Lattice

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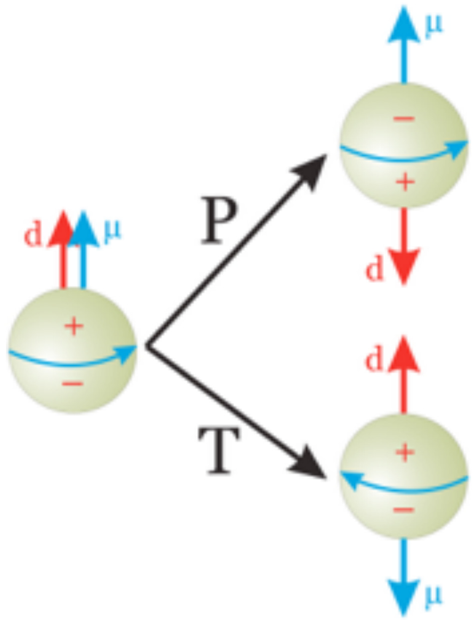
Outline

- Nucleon Electric Dipole Moments: lattice methodology
- Nucleon Electric Dipole Moments induced by θ_{QCD} term
- nEDM induced by chromo-EDM quark-gluon interaction

Nucleon Electric Dipole Moments



$$\mathcal{H} = -\vec{d}_N \cdot \vec{E} \qquad (\not{P}, \not{T}) \qquad \vec{d}_N = d_N \hat{S}$$

$$\langle N_{p'} | J^\mu | \bar{N}_p \rangle_{\cancel{CP}} = \bar{u}_{p'} \left[F_1 \gamma^\mu + (F_2 + i F_3 \gamma_5) \frac{\sigma^{\mu\nu}(p'-p)_\nu}{2m_N} \right] u_p$$

Dirac

Pauli (anom.magnetic)

Electric dipole

EDMs are the most sensitive probes of CPv:

- Prerequisite for Baryogenesis
- θ_{QCD}-induced EDM : Strong CP problem
- Signals for beyond SM physics (SM = 10^{-5} of the current exp.bound)

$$\mathcal{L}_{eff} = \sum_i \frac{c_i}{[\Lambda_{(i)}]^{d_i - 4}} \mathcal{O}_i^{[d_i]}$$

[J.Engel, M. Ramsey-Musolf, U. van Kolck, Prog.Part.Nucl.Phys. 71 (2013), pp. 21-74]

d=4 : θ_{QCD} (strong CP problem)

d=5(6) : quark EDM, quark-gluon chromo EDM

d=6 : 4-fermion CPv, 3-gluon (Weinberg)

$\Rightarrow$ $\boxed{d_{n,p} \quad F_3^{n,p}(Q^2)}$

Experimental Outlook

Current nEDM limits:

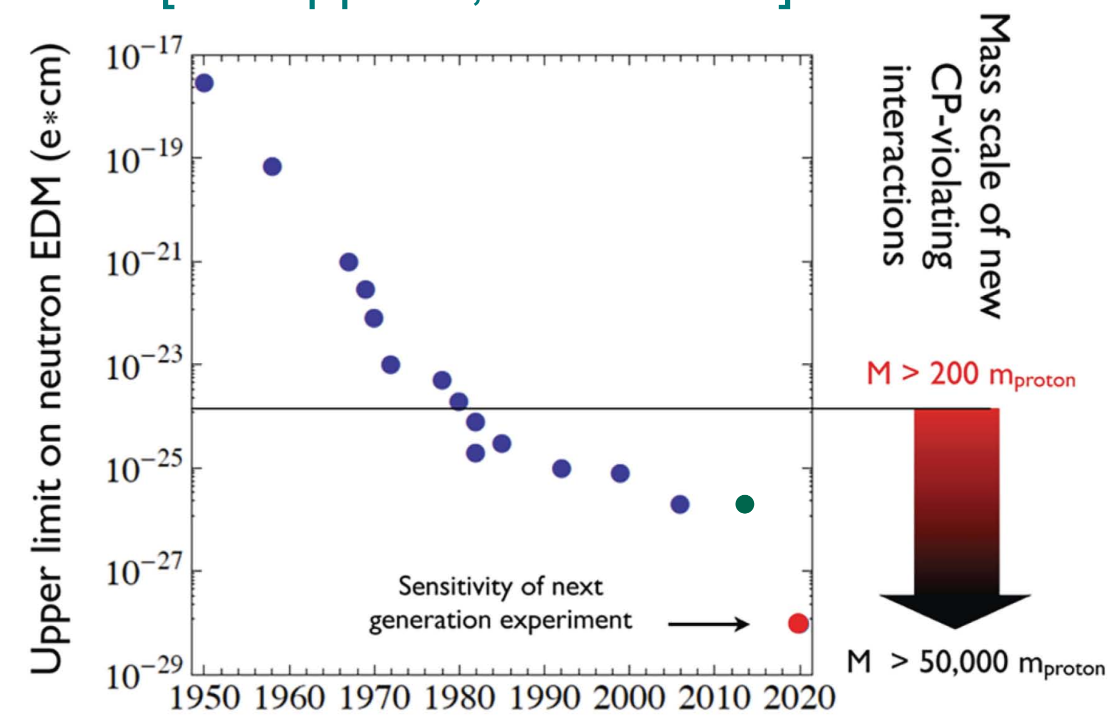
- $|d_n| < 2.9 \times 10^{-26}\, e \cdot \mathrm{cm}$
  [Baker et al, PRL97: 131801(2006)]
- $|d_n| < 1.6 \times 10^{-26}\, e \cdot \mathrm{cm}$
  [Graner et al, PRL116:161601(2016)]

Future nEDM sensitivity :

- 1–2 years : next best limit?
- 3–4 years : x10 improvement
- 7-10 years : x100 improvement

	10^{-28} *e* cm
CURRENT LIMIT	<300
Spallation Source @ORNL	**< 5**
Ultracold Neutrons @LANL	~30
PSI EDM	<50 (I), **<5 (II)**
ILL PNPI	<10
Munich FRMII	**< 5**
RCMP TRIUMF	<50 (I), **<5 (II)**
JPARC	**< 5**
Standard Model (CKM)	< *0.001*

[B.Filippone, KITP 2016]



- Other experiments: light nuclei in storage rings, octupole-deformed ^{225}Ra, etc

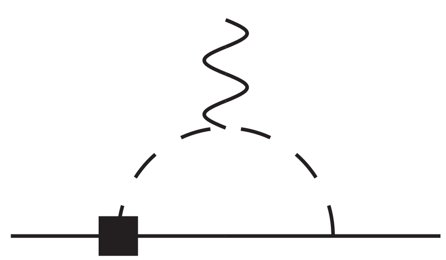
Nucleon EDMs: Task for Lattice QCD

Neutron (and proton) EDMs are particularly vital to CPv searches:

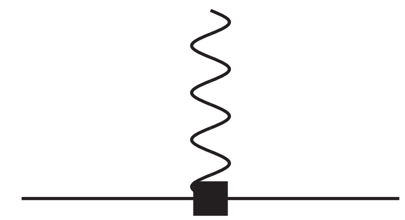
- Directly measured, also in upcoming experiments
- Independent of nuclear models – just QCD
- Contributions to ^{2}D, ^{3}H, ^{3}He $\Leftarrow$ storage ring experiments suggested

[Guo, Meissner '12 JHEP12:097]

nEDM in ChPT = pion loops + "counterterms"



predicted by ChPT



unknown LECs

$\Longrightarrow$ *Nonperturbative QCD calculations are required*

CP-odd Nucleon Structure on a Lattice

[M.Abramczyk, S.Aoki, SS., *et al* (2017) PRD96(2017)014501]:
To define $F_{2,3}$ correctly, one has to use positive-parity spinors

$$\gamma_4 u = +u$$
$$\bar{u}\gamma_4 = +\bar{u}$$

$$\langle N_{p'}|\bar{q}\gamma^\mu q|N_p\rangle_{\cancel{CP}} = \bar{u}_{p'}\left[F_1\gamma^\mu + (F_2 + i\boxed{F_3}\gamma_5)\frac{i\sigma^{\mu\nu}(p'-p)_\nu}{2m_N}\right]u_p$$

CPv interaction induces a chiral phase in fermion fields on a lattice (and EFT):

$$\langle \text{vac}|N|p,\sigma\rangle_{\cancel{CP}} = e^{i\alpha\gamma_5}u_{p,\sigma} = \tilde{u}_{p,\sigma}$$

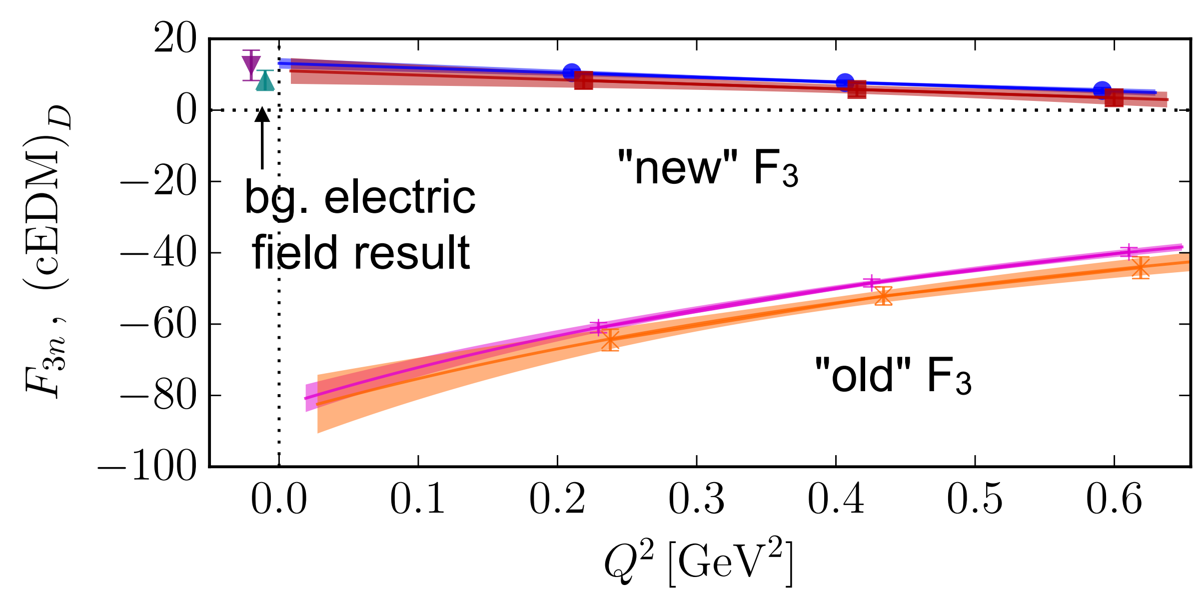
$$N \downarrow \; u\,[u^T C\gamma_5 d]$$

$$\tilde{u}_{p,\sigma} \downarrow \; (\not{\partial} + m_N e^{-2i\alpha\gamma_5})\tilde{u}_p = 0$$

$$\sum_\sigma \tilde{u}_{p,\sigma}\bar{\tilde{u}}_{p,\sigma} \sim \left(-i\not{p}_{\mathcal{E}} + m_N e^{2i\alpha\gamma_5}\right)$$

Prior to 2017, all lattice EDM results were biased by mixing with $F_{2,3}$

$$\text{“}F_3\text{”} \approx [F_3]_\text{true} - 2\alpha[F_2]_\text{true}$$
$$\text{“}d_{n,p}\text{”} \approx [d_{n,p}]_\text{true} - 2\alpha\frac{\kappa_{n,p}}{2m_N}$$



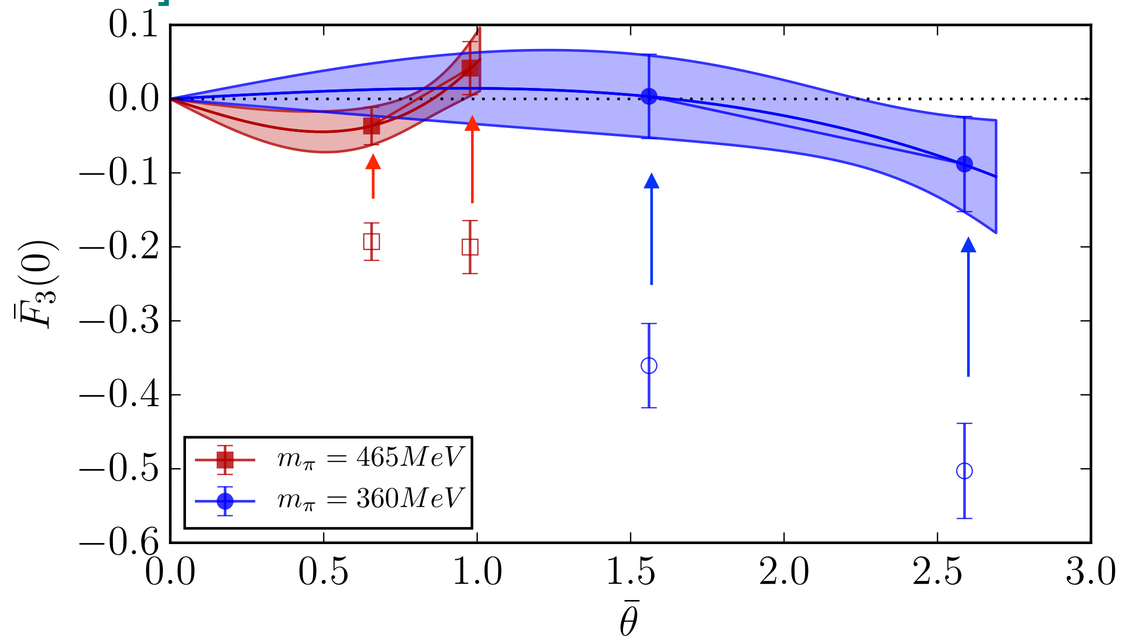
comparison of EDFF to EDM from background field method

θ_{QCD}-induced nEDM : Status

[M.Abramczyk, S.Aoki, SS., et al (2017) PRD96(2017)014501]

Correction to previous results:

$$[F_3]_{\mathrm{true}} = \text{“}F_3\text{”} + 2\alpha F_2$$



		m_π [MeV]	m_N [GeV]	F_2	α	$\tilde{F}_3$	F_3
[ETMC 2016]	n	373	1.216(4)	−1.50(16)[a]	−0.217(18)	−0.555(74)	0.094(74)
[Shintani et al 2005]	n	530	1.334(8)	−0.560(40)	−0.247(17)[b]	−0.325(68)	−0.048(68)
	p	530	1.334(8)	0.399(37)	−0.247(17)[b]	0.284(81)	0.087(81)
[Berruto et al 2006]	n	690	1.575(9)	−1.715(46)	−0.070(20)	−1.39(1.52)	−1.15(1.52)
	n	605	1.470(9)	−1.698(68)	−0.160(20)	0.60(2.98)	1.14(2.98)
[Guo et al 2015]	n	465	1.246(7)	−1.491(22)[c]	−0.079(27)[d]	−0.375(48)	−0.130(76)[d]
	n	360	1.138(13)	−1.473(37)[c]	−0.092(14)[d]	−0.248(29)	0.020(58)[d]

After removing the spurious contribution,

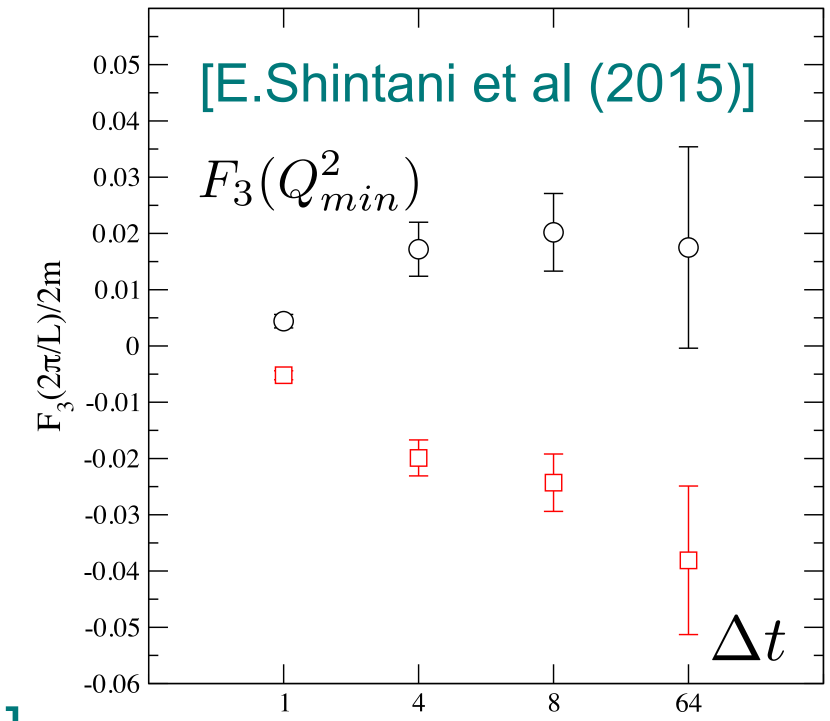
- *no lattice signal for θ_{QCD}-induced nEDM ⇨ d_N is very small, compatible with zero*
- *RESOLVED conflict with phenomenology values, lack of m_q scaling*

θ-Term Noise Reduction for EDM

Variance of lattice θ-induced nEDM signal ~ (*Volume*)$_{4d}$ *:*

$$d_N \sim \langle Q \cdot (N J_\mu \bar{N}) \rangle$$

Top. charge $Q \sim \int_{V_4} (G\tilde{G})$, with $\langle |Q|^2 \rangle \sim V_4$



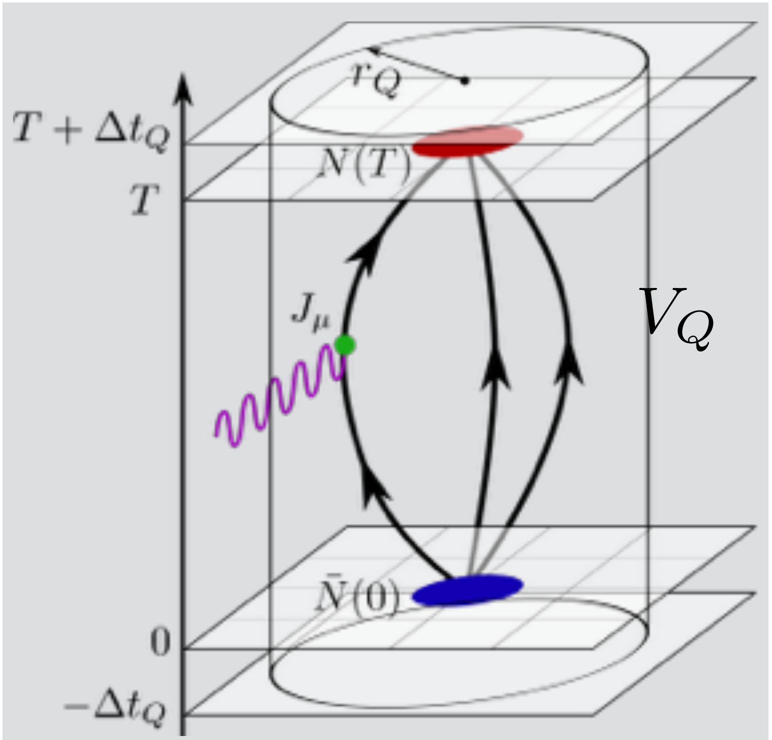
Constrain Q sum to the "fiducial" volume

- in time, around current, $|t_Q - t_J| < \Delta t$ [E.Shintani et al (2015)]
- 4-d sphere, $|x_Q - x_{sink}| < R$ [K.-F.Liu et al (2017)]
- in time, around source, $|t_Q - t_{source}| < \Delta t$ [J. Dragos, Lattice'18]

$$\tilde{Q} \sim \int_{V_Q} d^4 z \, \mathrm{Tr}[G\tilde{G}]$$

Proper treatment of nucleon parity mixing is critical for correct determination of F_3
$\Longrightarrow$ *nucleon must "settle" in the new θ≠0 vacuum*

$$N^{(+)} \to \tilde{N}^{(+)} \approx N^{(+)} + i\alpha N^{(-)}$$
$$N^{(-)} \to \tilde{N}^{(-)} \approx N^{(-)} - i\alpha N^{(+)}$$



$\Longrightarrow$ *constrain time and space(*) differently :*

4d "cylinder" $V_Q \; : \; |\vec{z}| < r_Q, \quad -\Delta t_Q < z_0 < T + \Delta t_Q$

(*) space cut may interfere with out-state momentum projection

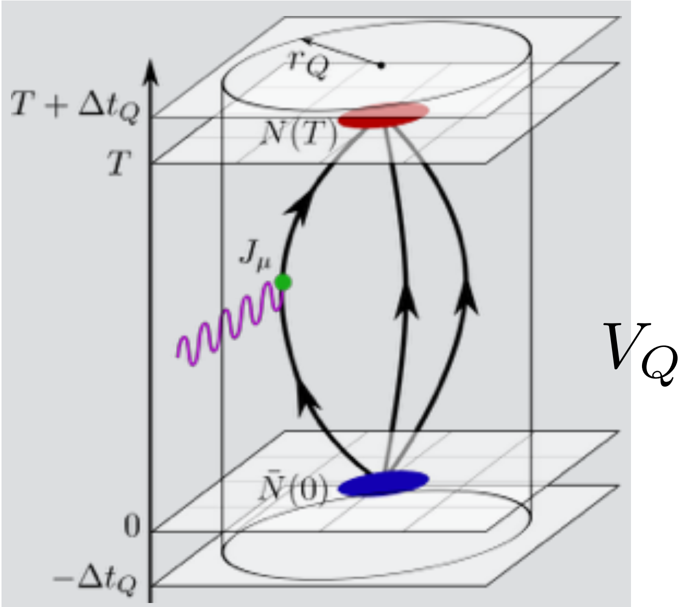
Test at m_π=340 MeV : Parity Mixing

Time Δt_Q required for transition to CP-broken vacuum *after* $L_{int} = \theta G\tilde{G}$ *is "turned on" :*

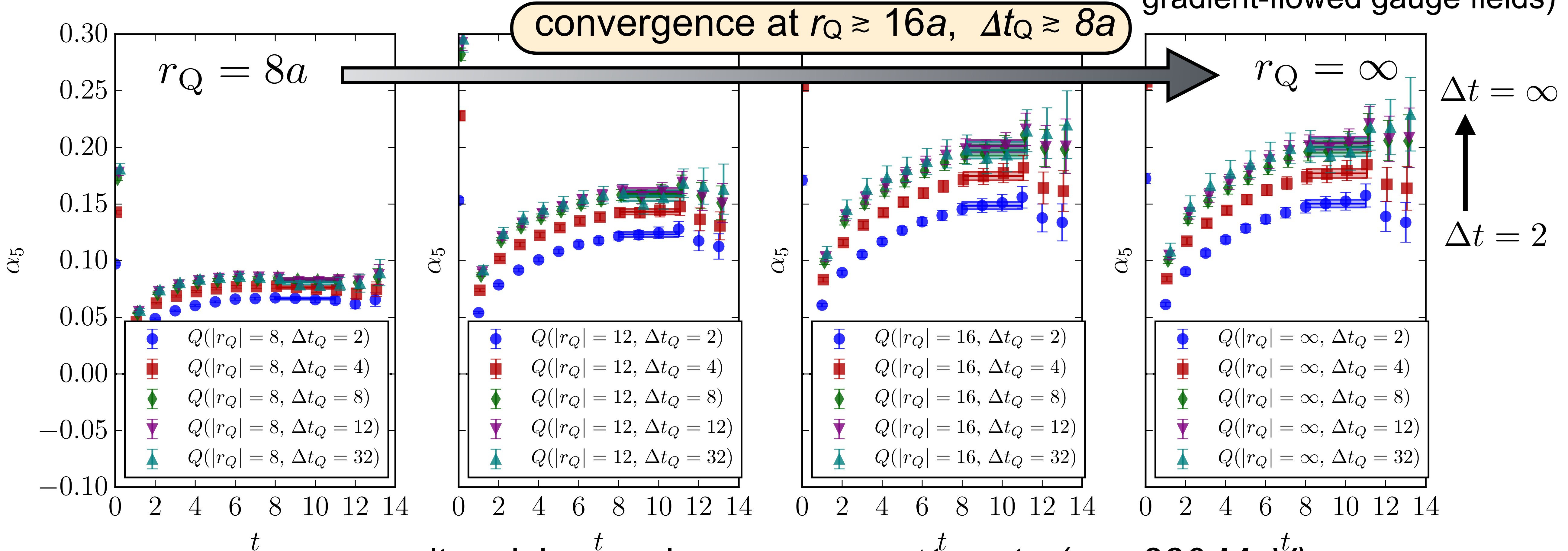
$$|vac\rangle \longrightarrow |vac\rangle_{\cancel{CP}}$$

$\Longrightarrow$ examine parity-mixing angle α_5 as a function of r_Q, Δt_Q :

$$\hat{\alpha}_5^{\rm eff}(t) = -\frac{\mathrm{Tr}\left[\gamma_5\langle N(t)\bar{N}(0)\,\tilde{Q}\rangle\right]}{\mathrm{Tr}\left[\langle N(t)\bar{N}(0)\rangle\right]}$$

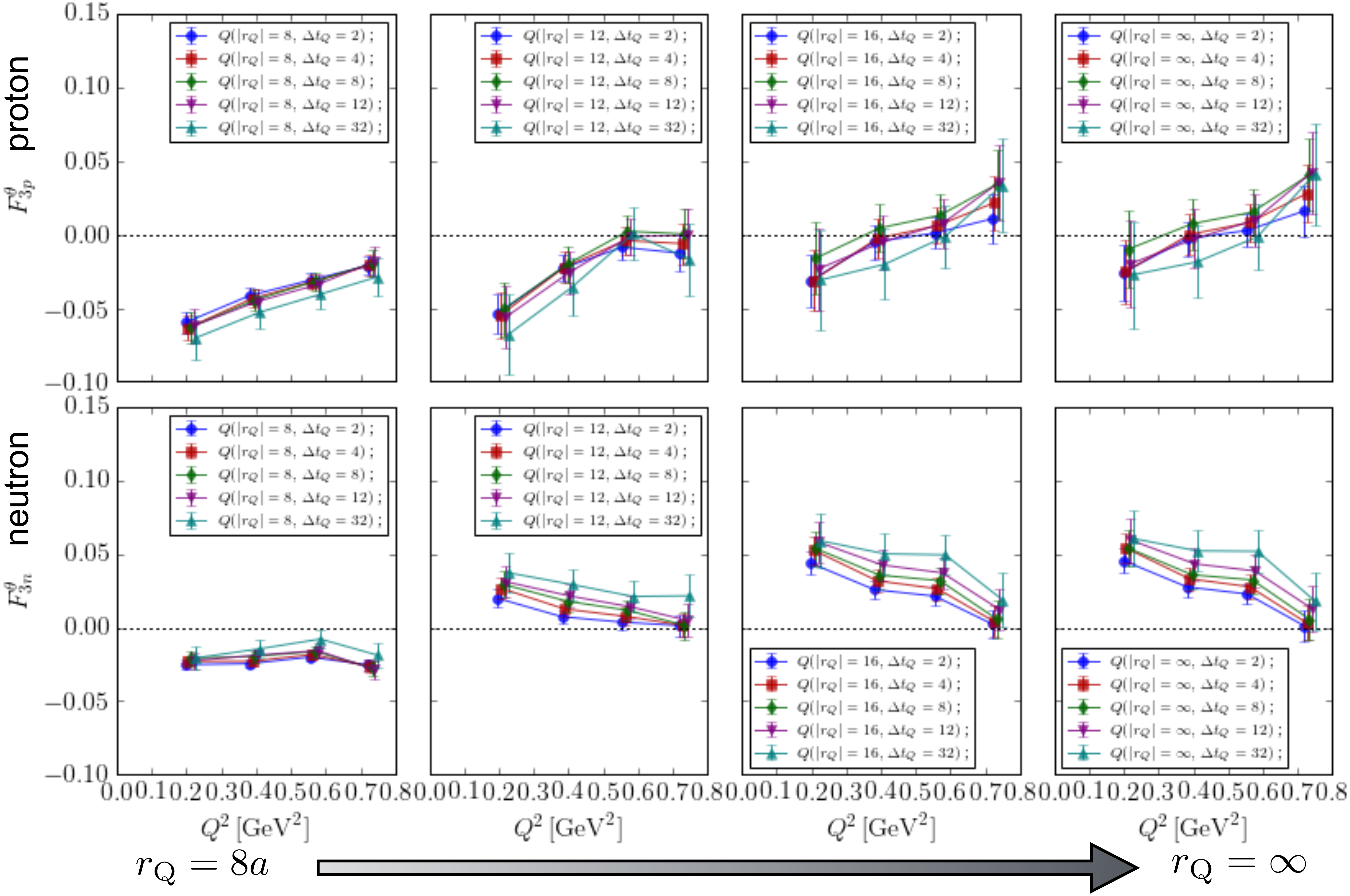


(top.density computed with gradient-flowed gauge fields)



parity mixing angle α_N vs. r_Q, Δt_Q cuts (m_π=330 MeV)

Test at m_π=340 MeV: EDM(Form Factor)



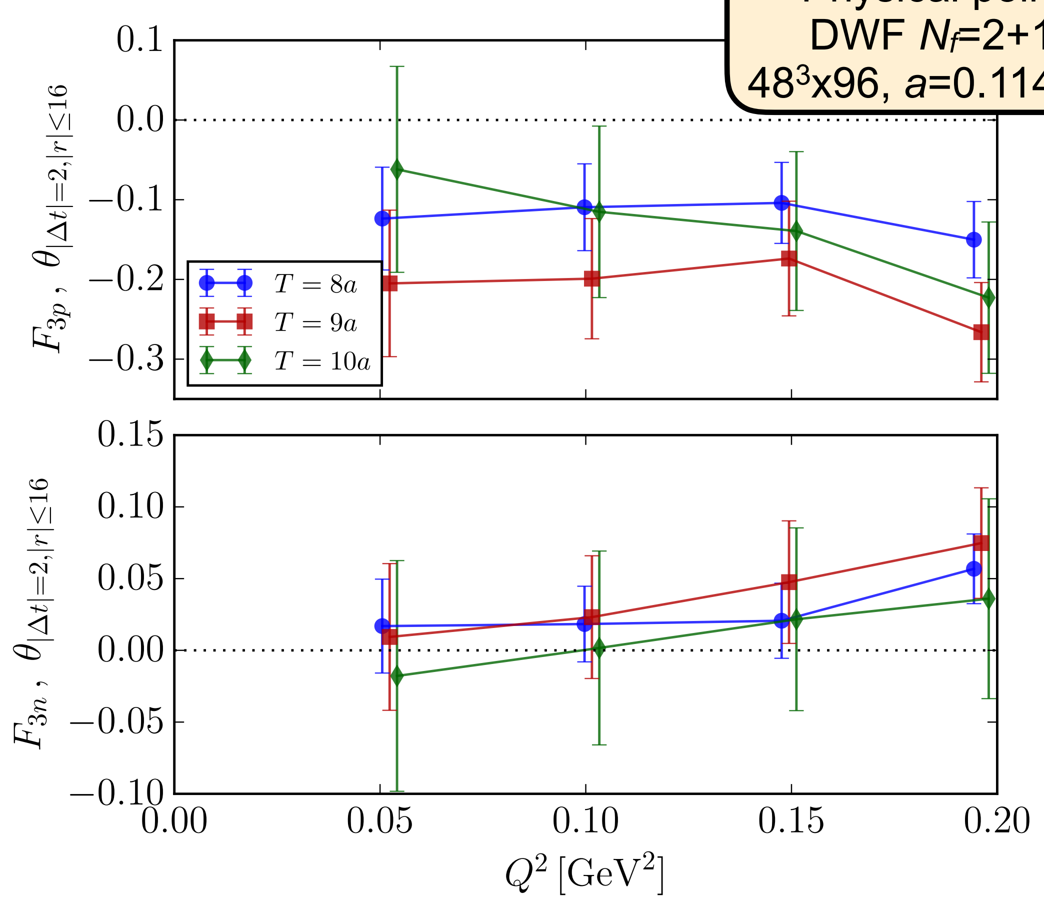
How Hard is θ-nEDM at the Physical Point?

- m_π = 340 MeV [SS et al] $|F_{3n}(0)| \approx 0.05\cdot\theta$
- m_π = 360 MeV [Guo et al 2015] *(corrected)* $|F_{3n}(0)| \lesssim 0.06\cdot\theta$
- Leading-order ChPT: $|d_n| \sim m_q \sim (m_\pi)^2$: $|F_{3n}(0)| \approx 0.01\cdot\theta$, $|d_n| \approx 0.001\cdot\theta\ e$ fm

$\Longrightarrow \theta_{QCD}$ from ***estimated*** $|F_{3n}(0)| \approx 10^{-2}\cdot\theta$:

- from neutron: $|d_n| \lesssim 2.9\cdot10^{-26}\ e\cdot$cm [Baker et al (2006)] : $|\theta_{QCD}| \lesssim 2.9\cdot10^{-10}$
- from ^{199}Hg: $|d_n| \lesssim 1.6\cdot10^{-26}\ e\cdot$cm [Graner *et al* (2016)] : $|\theta_{QCD}| \lesssim 1.6\cdot10^{-10}$

Physical point
DWF N_f=2+1
48^3x96, a=0.114 fm



- ~ 30 M core-hours on Argonne BlueGene/Q, (connected diagrams only)
$\Longrightarrow$ $|F_{3n}| \leq 0.05$ constraint

Need x30..100 more statistics
to constrain $|F_{3n}| \approx 0.01$:
θ-nEDM remains difficult at the physical point

Path forward: HMC with a dynamical θ_{QCD}-term (exploratory work underway)

Dynamic θ_{QCD} Term in HMC

+ better sampling of Q≠0 sectors (**++** at lighter m_π)
– require new QCD ensembles with ≥2 values of θ

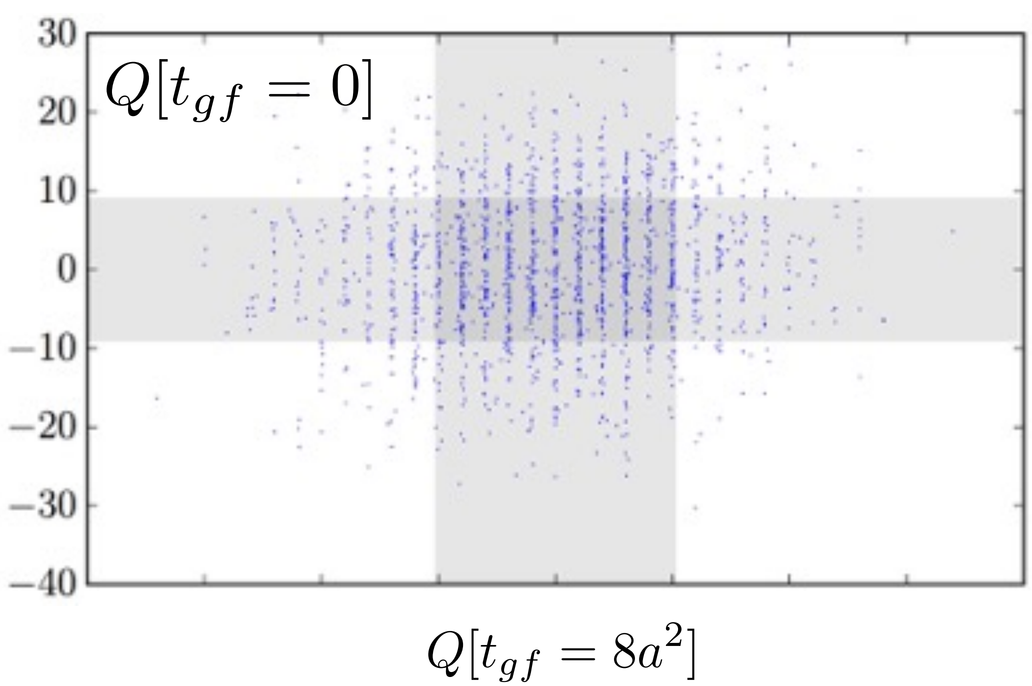
- Analytic continuation $\theta \to i\theta^I$ necessary to make action real and avoid sign problem
  [R.Horsley et al (2008) ; F.K.Guo et al (2015)]
- Adding finite-size θ_{QCD} to HMC: gauge or fermion action

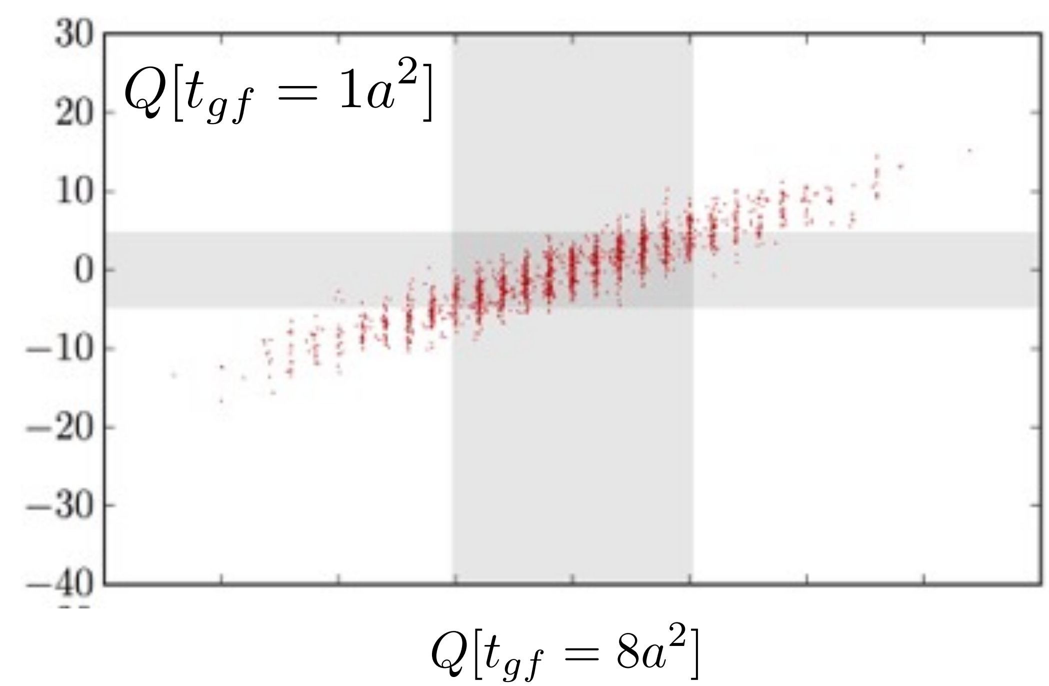
$$\frac{\sum_q \theta^I}{16\pi^2}\underbrace{\mathrm{Tr}[G\widetilde{G}]}_{} \xleftrightarrow{\text{anomaly}} \Pi_q \det\left[\not{D} + \underbrace{e^{\theta^I\gamma_5} m_q}_{}\right]$$

difficult to define locally on a lattice

$m_{qL} \neq m_{qR}$: may be problematic for $\theta^I \gtrsim 1$

Definition of Q needs gradient flow [Luscher'10] (or other link smearing)

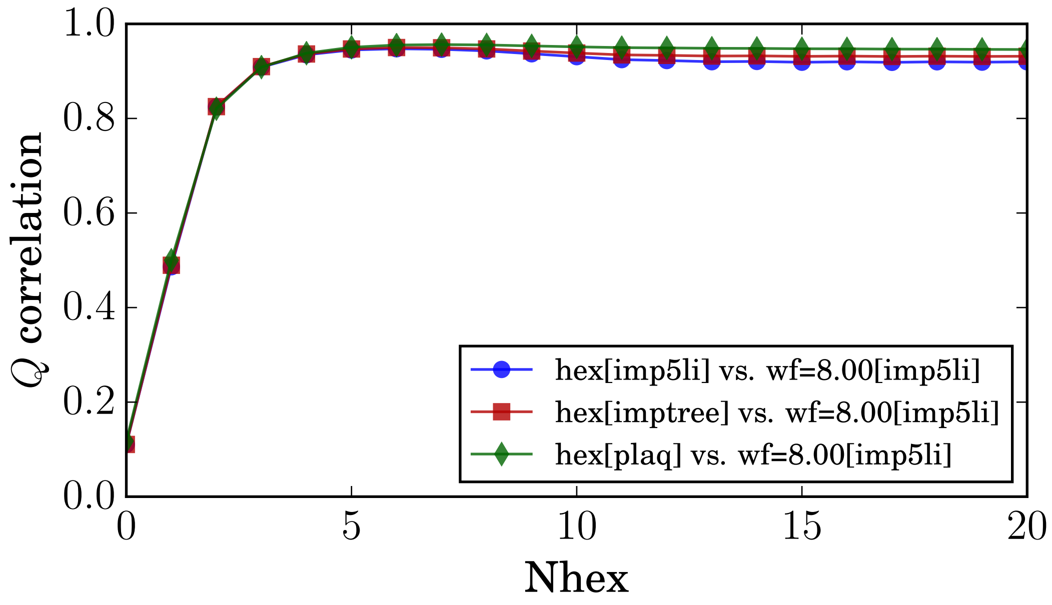


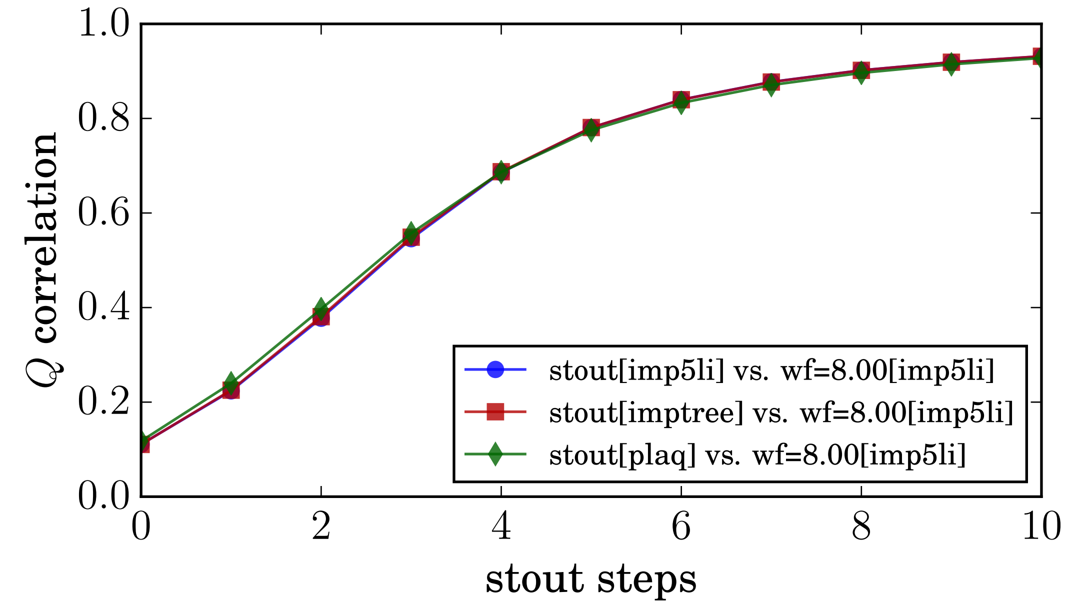


Dynamic θ_{QCD} Term in HMC (2)

HMC requires differentiable smearing

- Both HEX and Stout may work; many steps → more expensive force term

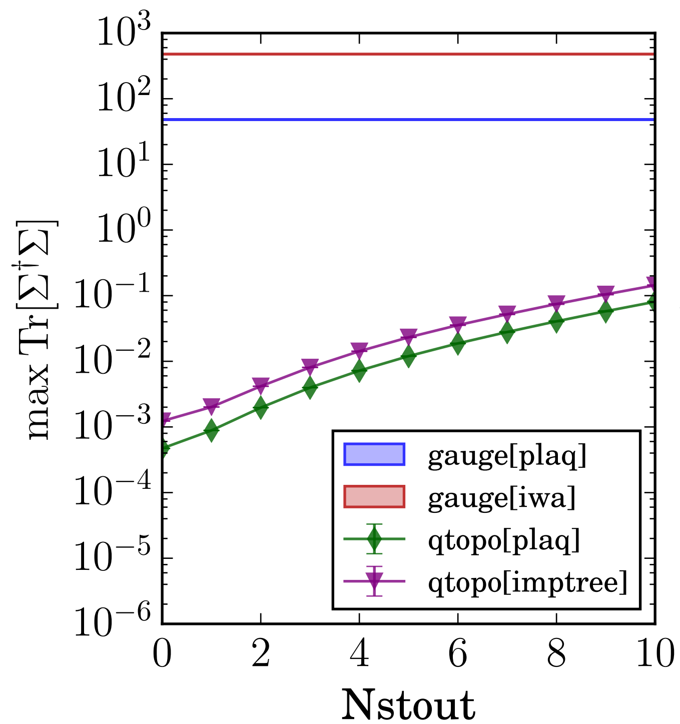


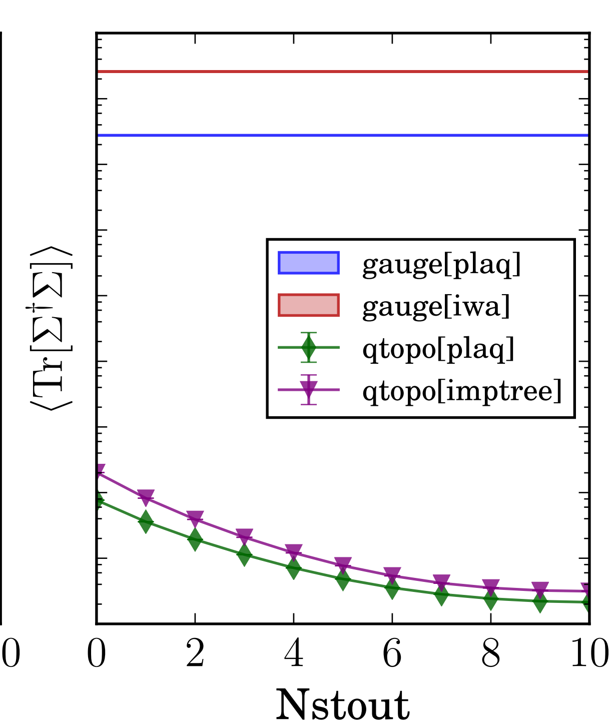


- Will addition of top.density force Σ_x^θ spoil HMC evolution?

$$\frac{dS}{d\tau} = \sum_{x,\mu} \mathrm{ReTr}\left[\Sigma_{x,\mu}\dot{U}_{x,\mu}\right]$$

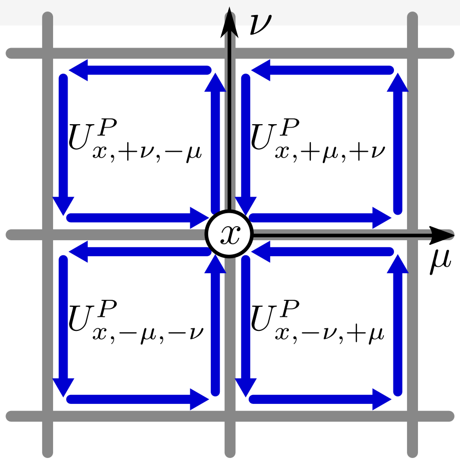
$|\,\Sigma_{x,\mu}\,|_\infty$ is the most important for accept/reject





Evaluated on m_π=340 MeV lattices: $|\Sigma|_\infty$ norm $|\Sigma|_2$ norm

Nucleon EDM Induced by Quark Chromo-EDM



P-,T-odd Dim-5 operator (Dim-6 with Higgs vev)

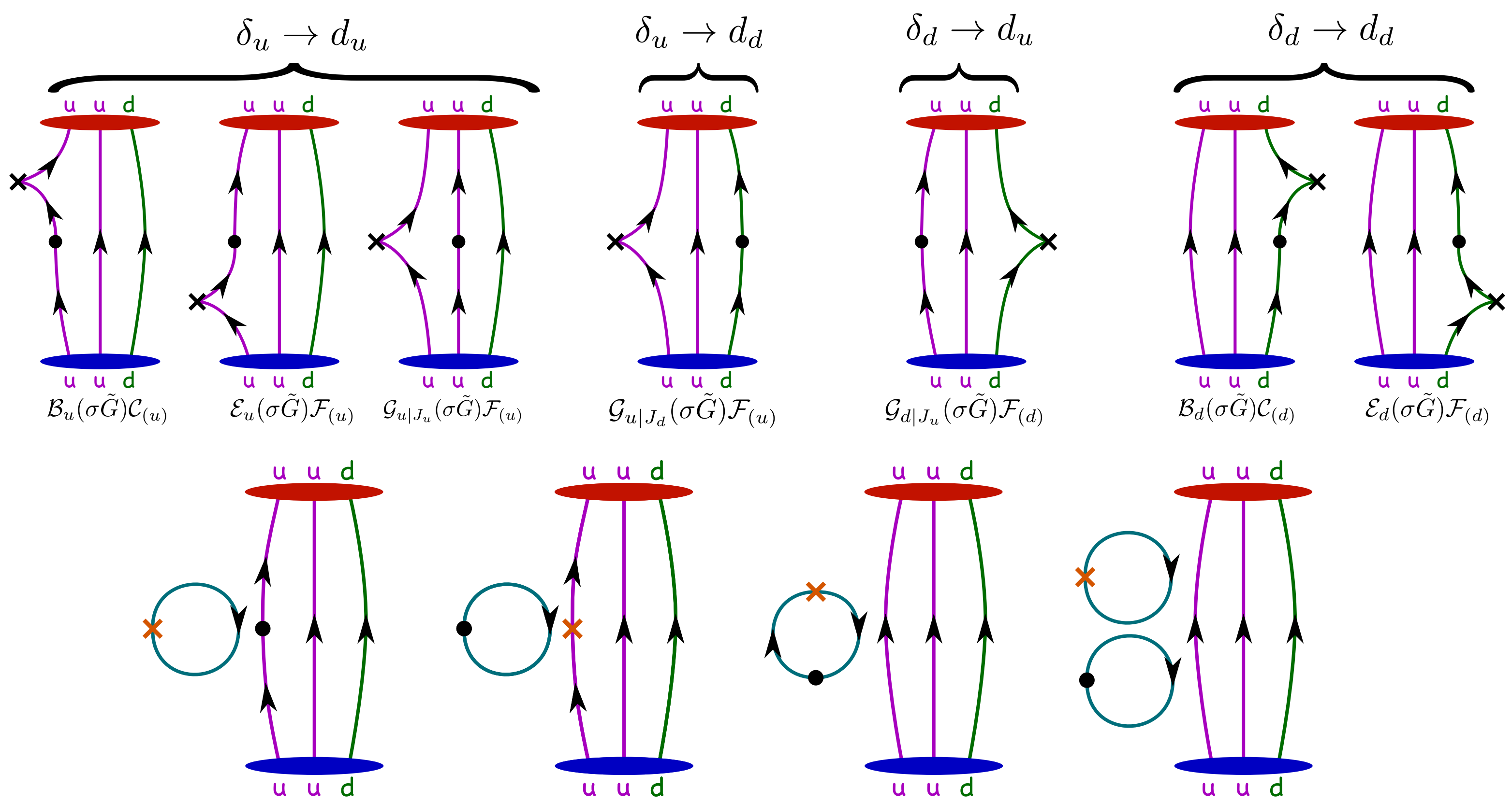
$$\mathcal{L}_{\text{cEDM}} = \sum_{q=u,d} \frac{\tilde{\delta}_q}{2}\, \bar{q}\, [G_{\mu\nu}\sigma^{\mu\nu}\gamma_5]\, q$$

- flavor-dependent CP violation
- predicted by extensions of SM (e.g. 2-Higgs doublet model)
- QCD sum rules, ChiPT have model uncertainty $\Longrightarrow$ need lattice QCD

Lattice diagrams:

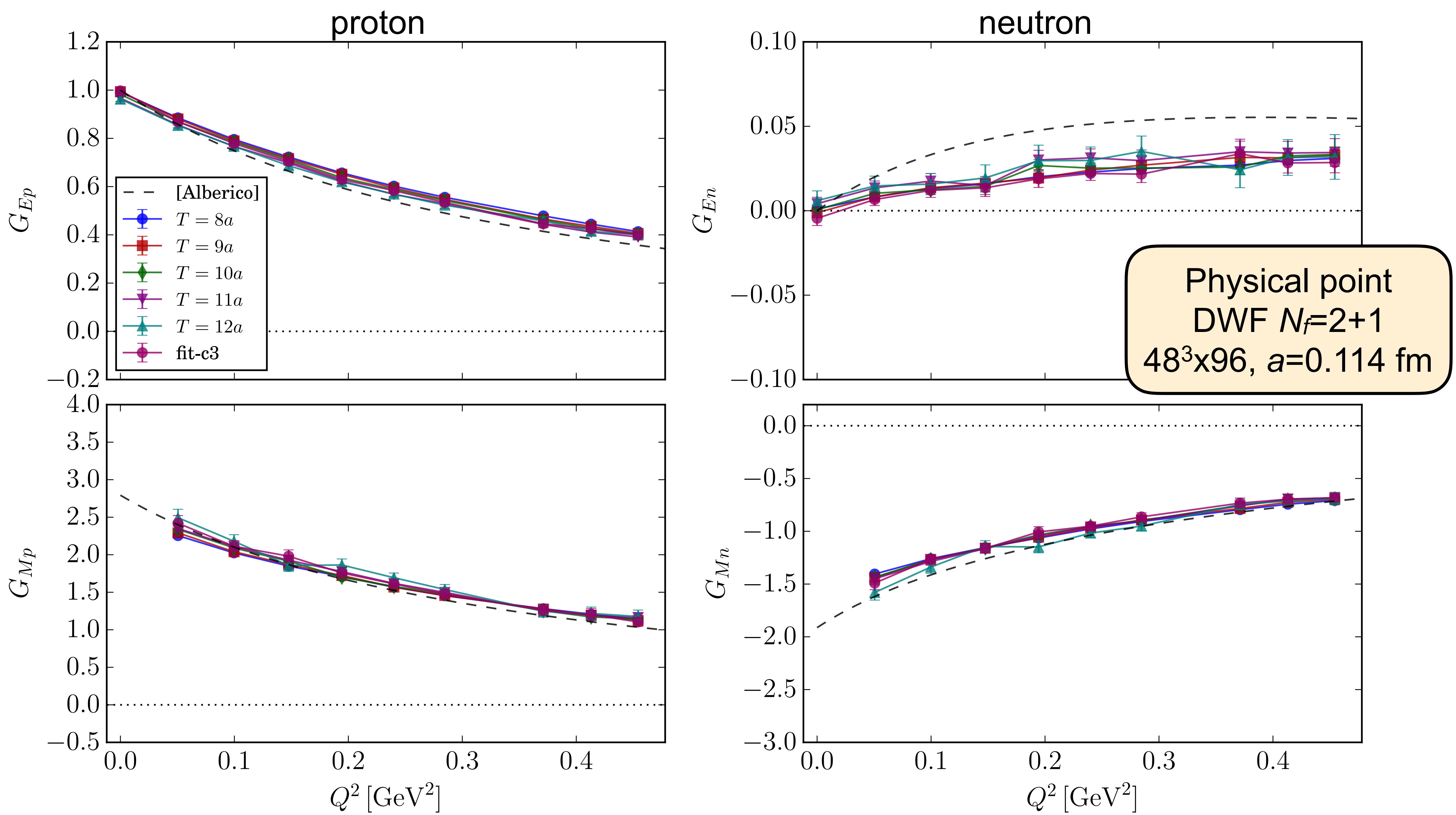
Current work @physical point

Future (additional contributions to isoscalar nEDM/cEDM)



2017, 2018 Advanced Leadership Computing Challenge (135M + 55M core-hours at ANL) for calculations at the physical point

Physical m_π: Nucleon E&M Form Factors

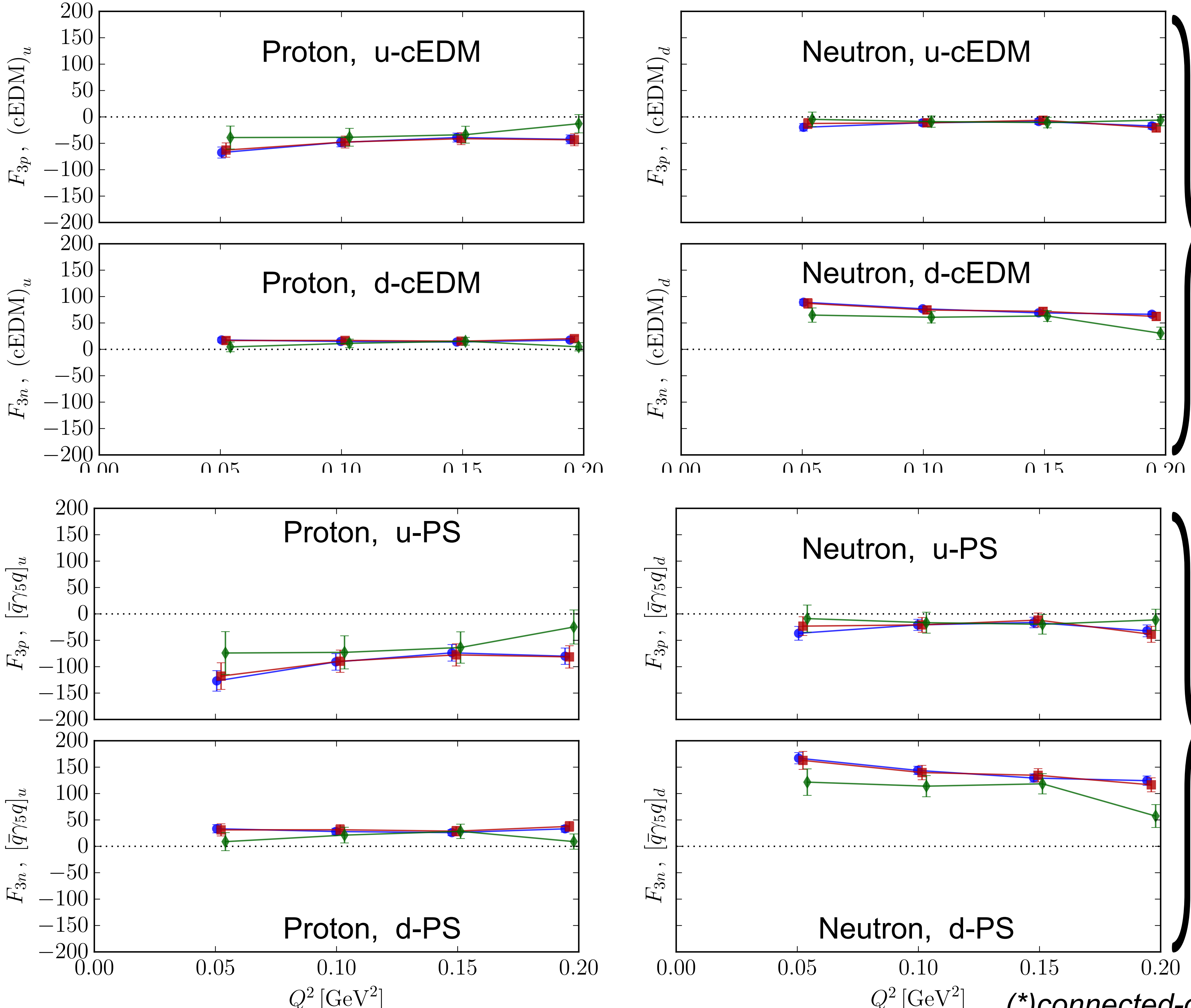


$$G_E = F_1 - \frac{Q^2}{4M^2} F_2$$

$$G_M = F_1 + F_2$$

Proton & Neutron EDM Form Factors (*)

Physical point
DWF N_f=2+1
48^3x96, a=0.114 fm



cEDM-induced electric dipole form factors (EDFF)

pseudoscalar density -induced EDFFs (required for renormalization and mixing subtraction)

(*)connected-only, bare cEDM and PS operators

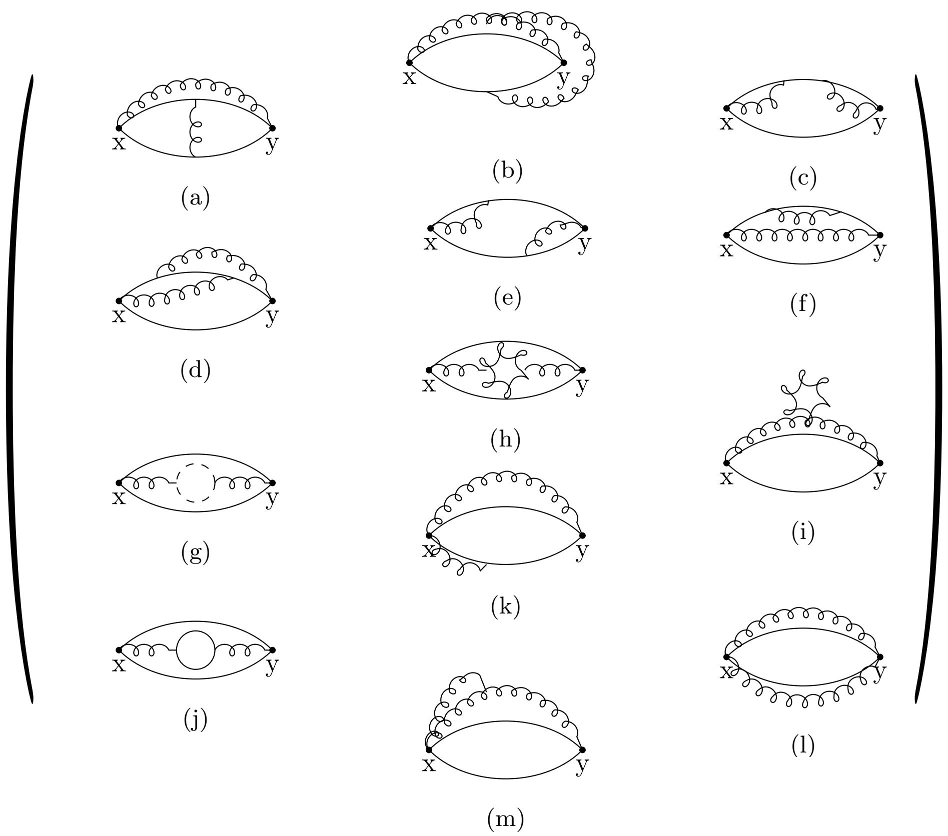
Renormalization of cEDM on a lattice

Need to match CPv operators from lattice QCD to MSbar

- "Momentum-scheme" in Landau gauge [T.Bhattacharya et al , PRD92(2015) 114026]
$\Longrightarrow$mixing with gauge-dependent & EoM-vanishing operators due to gauge fixing

- Alternative: match correlators computed at small distance $X \ll (\Lambda_{\mathrm{QCD}})^{-1}$ (position space method); extend work of [Gimenez et al (2004); Chetyrkin(2010)]

$$\langle \mathcal{C}(X)\mathcal{C}(0)\rangle^{(1)} =$$



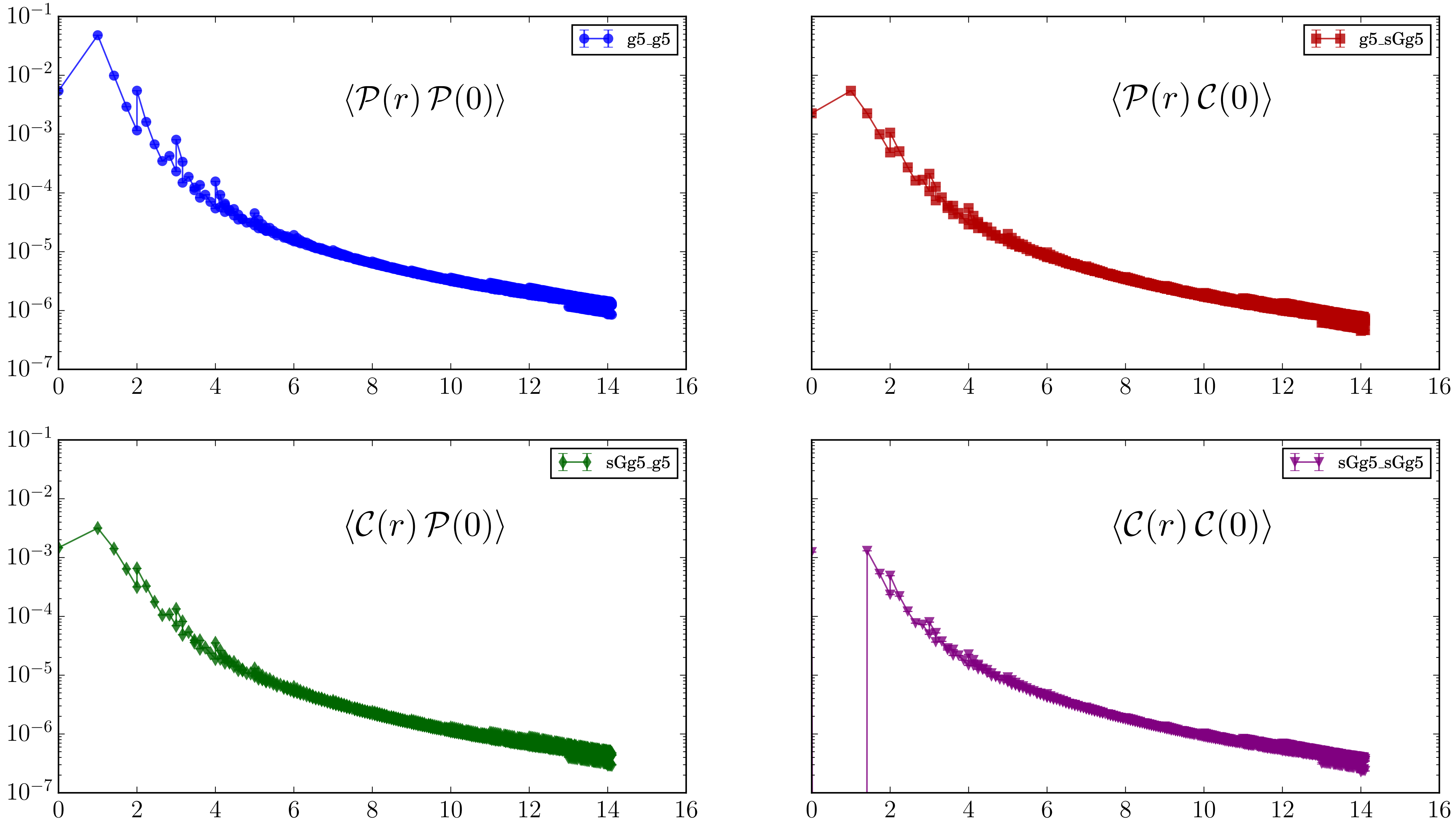
Also $\langle \mathcal{C}(X)\mathcal{P}(0)\rangle$
$\langle \mathcal{P}(X)\mathcal{P}(0)\rangle$

$$\mathcal{C} = \bar{q}\big[\frac{1}{2}(\sigma_{\mu\nu}G^{\mu\nu})\gamma_5\big]q$$

$$\mathcal{P} = \bar{q}\gamma_5 q$$

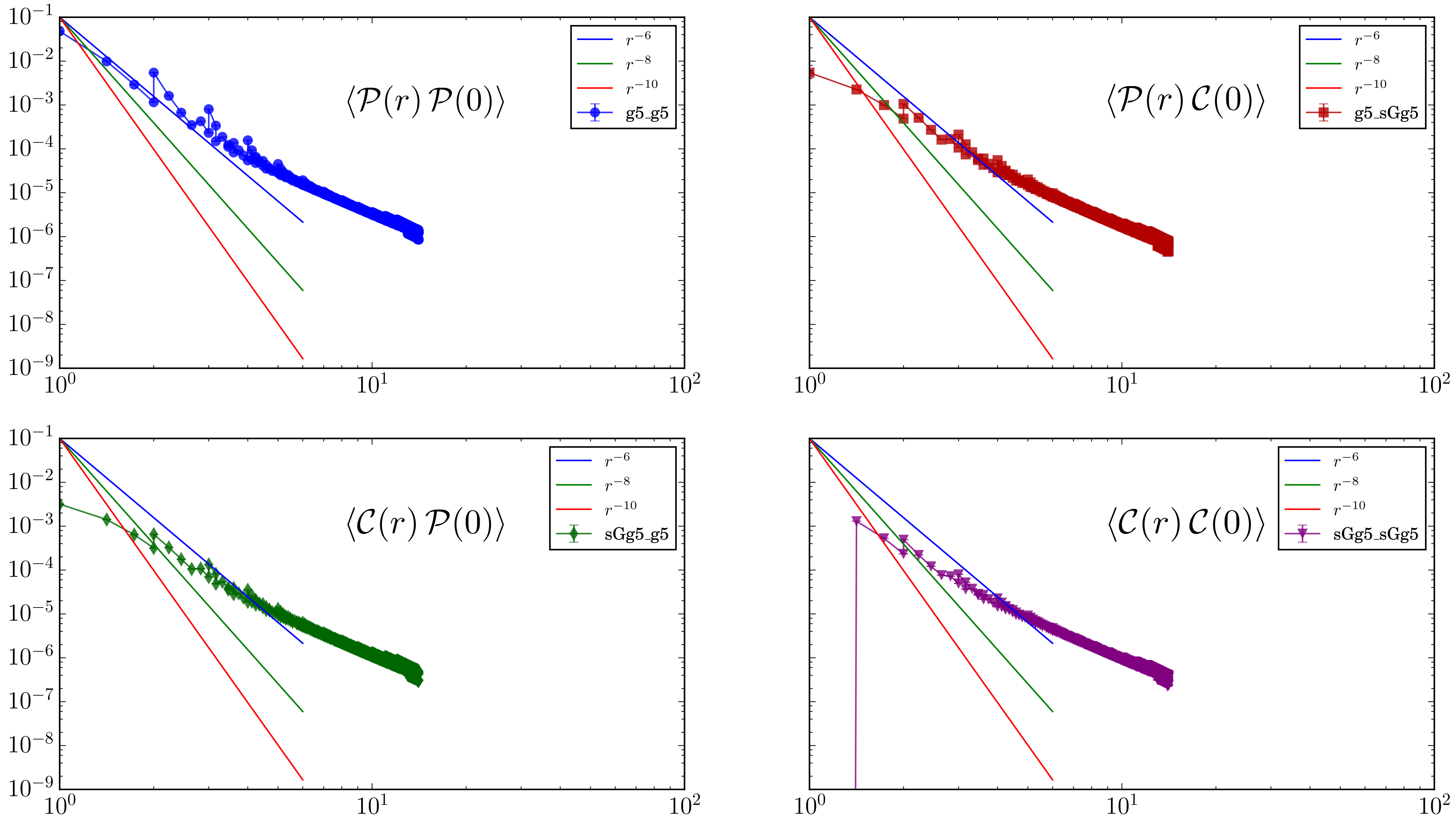
Diagrams computed, lattice data analysis in progress
[work in progress with M.Kellerstein (SBU graduate student)]

Position-space NPR of cEDM operators



no contact terms in <*C(r)C(0)*> : *for r=>(1,1,1,1)*

Position-space NPR of cEDM operators (2)



no contact terms in <*C(r)C(0)*> : *for r=>(1,1,1,1)*

Summary & Outlook

- lattice calculations of Nucleon EDM are necessary to understand bounds (discoveries?) from the next decade of EDM experiments

- Encouraging results for nucleon EDM induced by quark chromo-EDM physical-point
  ~20% stochastic uncertainty for quark cEDM-induced EDM
  Renormalization & mixing subtractions are underway
  Full flavor dependence will require disconnected diagrams & θ_{QCD} -term

- Clear signal for θ_{QCD}-induced nEDM at $m_\pi = 340$ MeV
  Variance-reduction for Q sampling is essential
  Physical $|d_{n,p}| \approx 10^{-3}$ e fm values are in agreement with models&ChPT

- Constraining θ_{QCD}-induced nEDM at the physical point will be challenging
  *O(300-1000) M core*hours may be required even with variance reduction*
  Shall look for alternatives: dynamical θ-therm? coarser lattice spacing?

Stay tuned for the next decade of CPv physics!

BACKUP

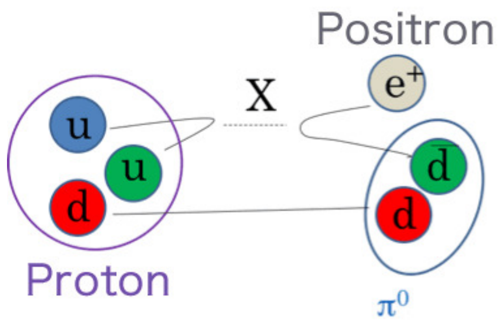
BACKUP

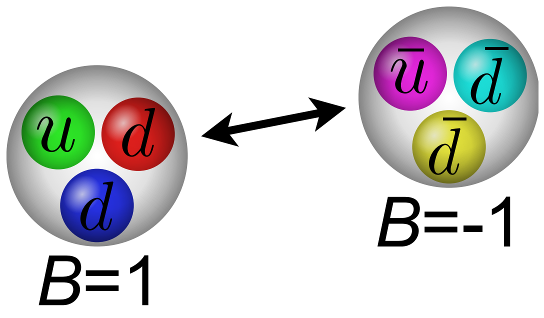
Baryon Number Violation

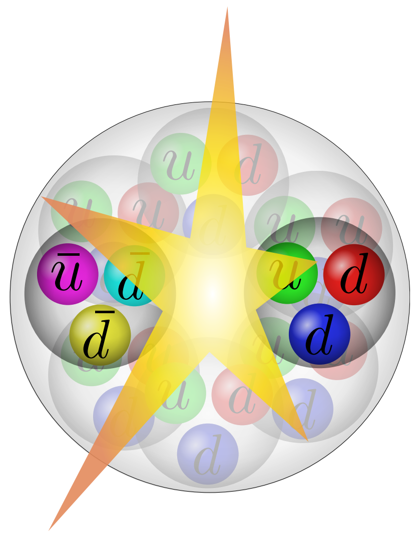
Proton decay and n-n̄ oscillations

Motivation for searches :

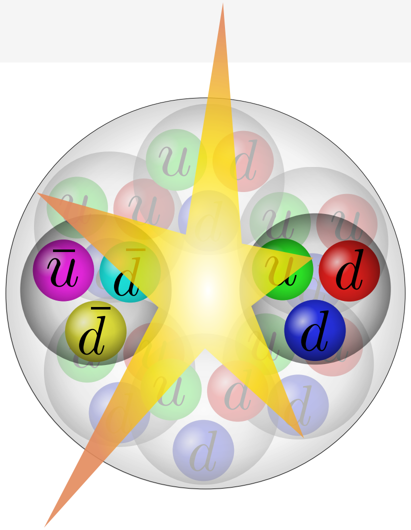
- Baryon number must be violated for baryogenesis (Sakharov's conditions)
  - *N->Nbar transition : ΔB=2*
  - *Proton decay: ΔB=1*
  - *Which one (or both?) realized in nature?*
- Nuclear matter stability
  - *p-decay?*
  - *(nn)-annihilation?*
- Probing BSM physics : *Δ(B–L)=2*
  - *GUT theories constraints from p-decay rate*
  - *Δ(B–L)=2 in (nn̄): connection to lepton number violation ΔL=2 ? neutrino mass mechanism [R.Mohapatra, R.Marshak (1980)]*







Searches for $n \to \bar{n}$ in Nuclei



Nucleus lifetime:

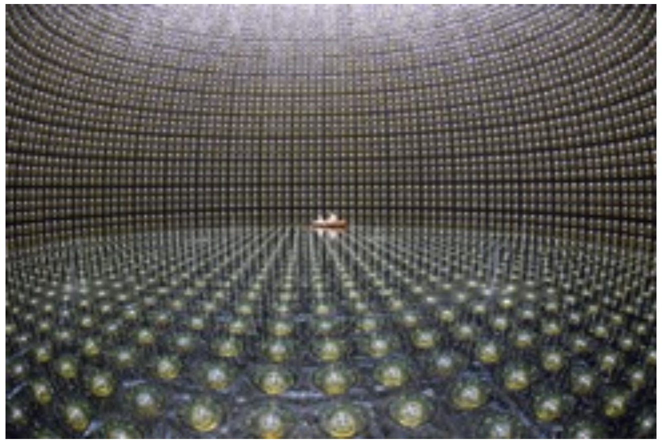
$$T_d = R\tau_{n\bar{n}}^2$$

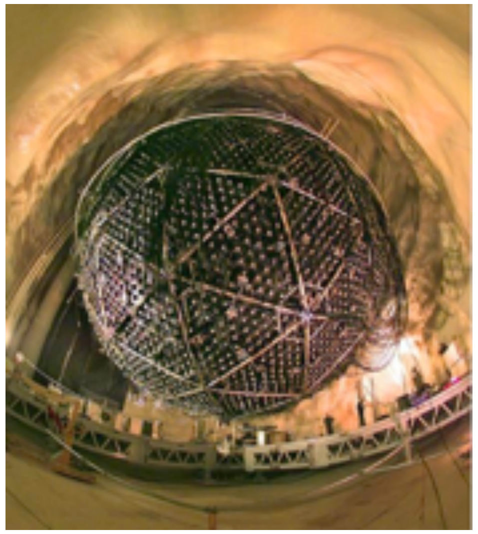
$$R \sim 10^{23}\ s^{-1}$$

Some nuclear model dependence:
e.g. ~ 10-15% for ^{16}O
[E.Friedman, A.Gal (2008)]

Stability of nuclei :

- ^{56}Fe [Soudan 2] $T_d(^{56}Fe) > 0.72 \cdot 10^{32}$ yr $\longrightarrow$ $\boxed{\tau_{n\bar{n}} > 1.4 \cdot 10^8\ \mathrm{s}}$
- ^{16}O [Super-K] $T_d(^{16}O) > 1.77 \cdot 10^{32}$ yr $\longrightarrow$ $\boxed{\tau_{n\bar{n}} > 3.3 \cdot 10^8\ \mathrm{s}}$
- ^{2}H [SNO] $T_d(^{2}H) > 0.54 \cdot 10^{32}$ yr $\longrightarrow$ $\boxed{\tau_{n\bar{n}} > 1.96 \cdot 10^8\ \mathrm{s}}$





Sensitivity is limited by atmospheric neutrinos

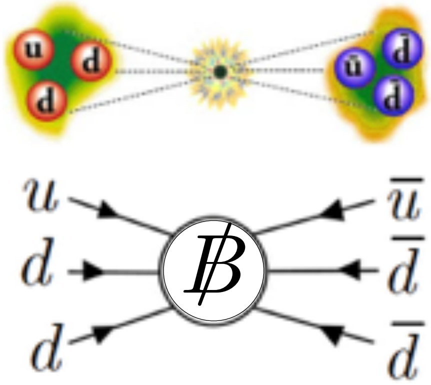
Neutron ↔ Antineutron Transitions and QCD

Effective *ΔB=2* operator: (quark field)^6

From Standard Model extensions:

interaction with a massive Majorana lepton,

unified theories, etc [T.K.Kuo, S.T.Love, PRL45:93 (1980)] [R.N.Mohapatra, R.E.Marshak, PRL44:1316 (1980)]



$$\mathcal{L}_{\rm eff} = \sum_i \left[c_i \mathcal{O}_i^{6q} + \text{h.c.}\right]$$

$$\tau_{n\bar n}^{-1} = \delta m = -\langle \bar n| \int d^4x\, \mathcal{L}_{\rm eff}\, |n\rangle = -\sum_i \frac{c_i}{M_X^5} \boxed{\langle \bar n|\mathcal{O}_i^{6q}|n\rangle}$$

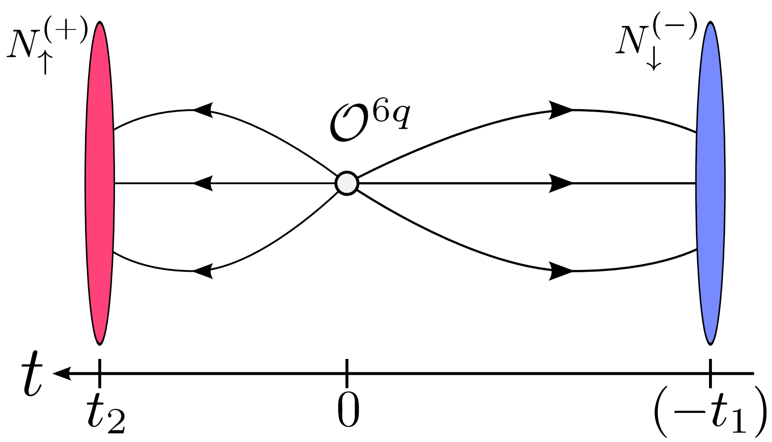
BSM scale suppression of 6-quark Dim-9 operators

nucleon sensitivity to BN-violating eff.interactions

What is the scale for new physics behind n↔$\bar n$?

- Current experimental lower bound on $\tau_{n\text{-}\bar n}$ requires $M_X \gtrsim 10^2$ TeV
- baryon asymmetry puts upper bound on $\tau_{n\text{-}\bar n}$ in models with *ΔB*=2 mechanism (assuming SM-only CPv) e.g. [Babu et al, PRD87:115019(2013)]

Lattice Results & Comparison to Bag Model



$$\langle N_\uparrow^{(+)}(t_2)\,\mathcal{O}^{6\mathrm{q}}(0)\,N_\downarrow^{(-)}(-t_1)\rangle \underset{t_1,\,t_2,\,t_1+t_2\to\infty}{\sim} e^{-M_n(t_2+t_1)}\langle n_\uparrow|\mathcal{O}^{6\mathrm{q}}|\overline{n}_\uparrow\rangle$$

- On a lattice : Calculations with physical chirally symmetric quarks [SS, M.Buchoff, J.Wasem, C.Schroeder (LATTICE 2015)]

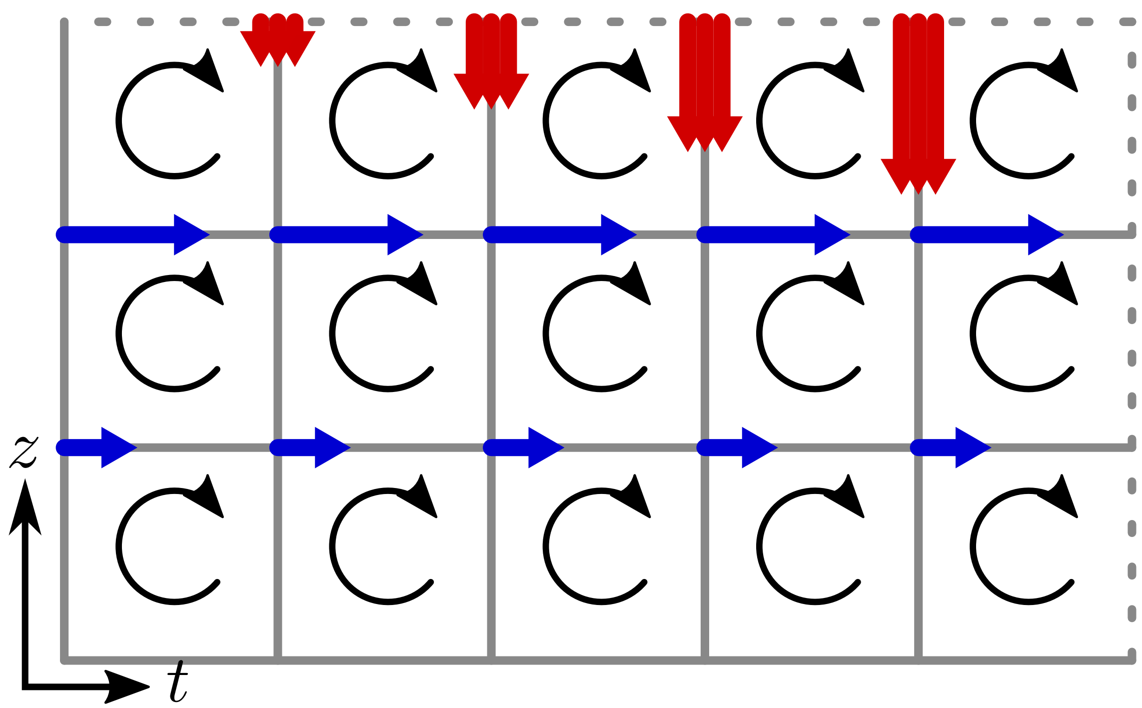
	$\mathcal{O}^{\overline{MS}}(2\text{ GeV})$	Bag "A"	$\frac{\text{LQCD}}{\text{Bag "A"}}$	Bag "B"	$\frac{\text{LQCD}}{\text{Bag "B"}}$	
$[(RRR)_{\mathbf{3}}]$	0	0	–	0	–	
$[(RRR)_{\mathbf{1}}]$	45.4(5.6)	8.190	5.5	6.660	6.8	EW-singlet n-n̄ tree-lev.
$[R_{\mathbf{1}}(LL)_{\mathbf{0}}]$	44.0(4.1)	7.230	6.1	6.090	7.2	
$[(RR)_{\mathbf{1}}L_{\mathbf{0}}]$	-66.6(7.7)	-9.540	7.0	-8.160	8.1	
$[(RR)_{\mathbf{2}}L_{\mathbf{1}}]^{(1)}$	-2.12(26)	1.260	-1.7	-0.666	3.2	EW non-singlet n-n̄ at 1 loop
$[(RR)_{\mathbf{2}}L_{\mathbf{1}}]^{(2)}$	0.531(64)	-0.314	-1.7	0.167	3.2	
$[(RR)_{\mathbf{2}}L_{\mathbf{1}}]^{(3)}$	-1.06(13)	0.630	-1.7	-0.330	3.2	
	$[10^{-5}\text{ GeV}^{-6}]$	$[10^{-5}\text{ GeV}^{-6}]$		$[10^{-5}\text{ GeV}^{-6}]$		

Rate of ($n\overline{n}$)-osc. in terms of BSM couplings [arXiv:1809.00246, submitted to PRL]

$$\tau_{n\text{-}\bar{n}}^{-1} = \frac{10^{-10}\text{ s}^{-1}}{(100\text{ TeV})^{-5}}\Big|6.8(2.9)C_1^{\overline{\text{MS}}}(\mu) - 21.2(6.0)C_2^{\overline{\text{MS}}}(\mu)$$
$$+12.0(3.5)C_3^{\overline{\text{MS}}}(\mu) + 0.217(98)C_5^{\overline{\text{MS}}}(\mu)\Big|_{\mu=2\text{ GeV}}.$$

Background Electric Field

Accessing magnetic and electric moments at Q^2=0
Imag.Minkowski/Real Euc. electric field on a lattice
[W.Detmold et al (2009)] : calculation of hadron polarizabilities



Full flux through the "side" of the periodic box

$$= q\Phi = 2\pi \cdot n$$

Constant Electric field has to be quantized,

$$\mathcal{E}_{\min} = \frac{1}{|q_d|}\frac{2\pi}{L_x L_t}$$

$$U_\mu \to e^{iqA_\mu} U_\mu$$

$$A_z(z,t) = n\,\mathcal{E}_{\min}\cdot t$$

$$A_t(z, t = L_t - 1) = -n\,\mathcal{E}_{\min}\cdot L_t z$$

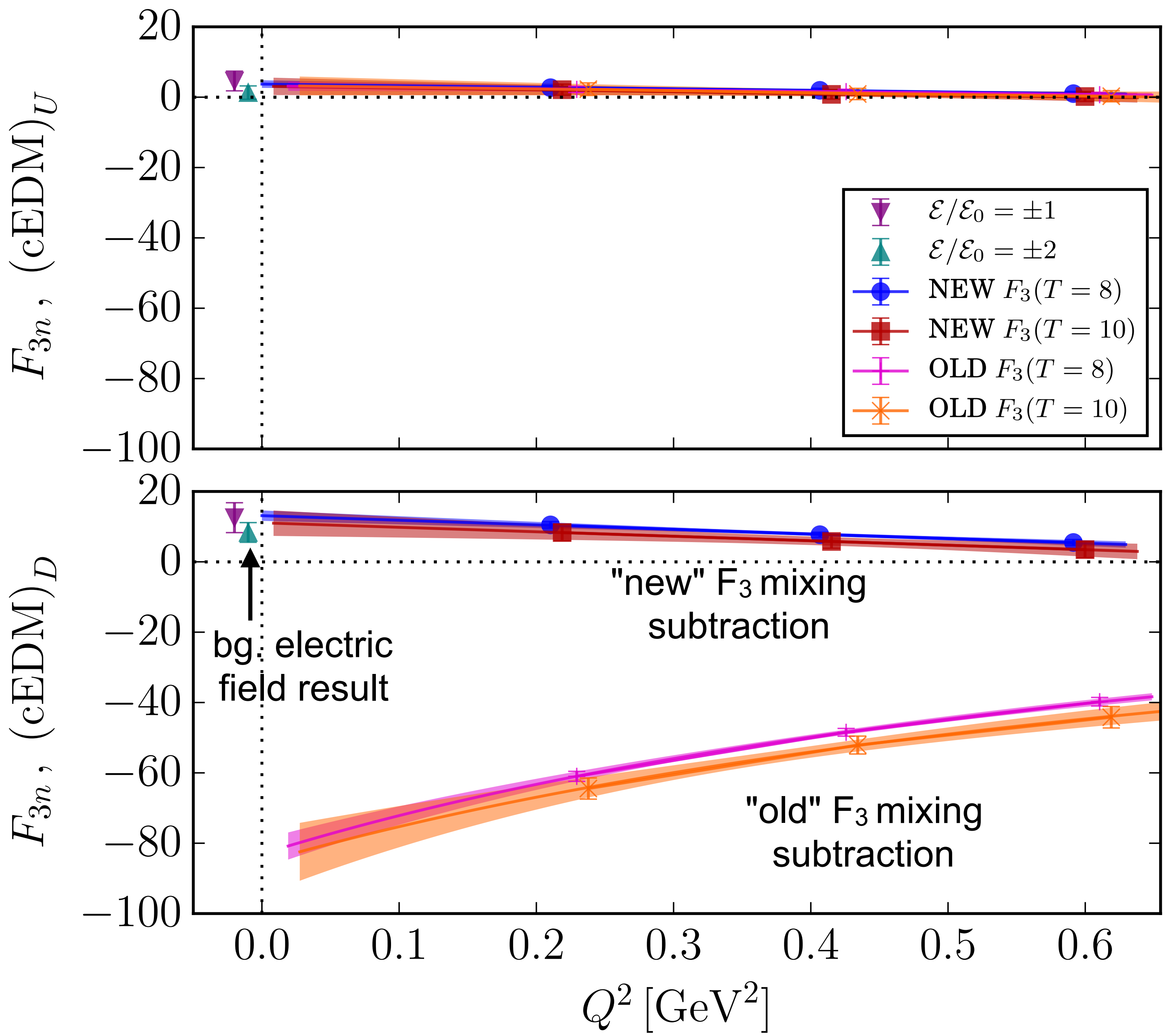
Electric field on a 24^3x64 lattice

$$\mathcal{E} = \frac{6\pi}{L_x L_t} \approx 0.037\ \mathrm{GeV}^2$$

$$\approx 186\ \mathrm{MV/fm}$$

Unambiguous determination of EDM from the energy shift

Energy Shift vs. Form Factors (Neutron)



Mixing $\alpha_U \approx 0$

No F_2 contribution to F_3

$$“F_{3n}^U” \approx [F_{3n}^U]_{\mathrm{true}}$$

Mixing $\alpha_D \approx 30(0.2)$

Large F_2 contribution to "F_3"

$$“F_{3n}^D” = [F_{3n}^D]_{\mathrm{true}} - 2\alpha_D F_{2n}$$

[S.Aoki, SNS, *et al* (2017) arXiv:1701.07792]

*Agreement between the **new** F_3 formula and the energy shift method*