Non-factorizable contribution in $B \to K l^+ l^-$ through the charmonium resonance

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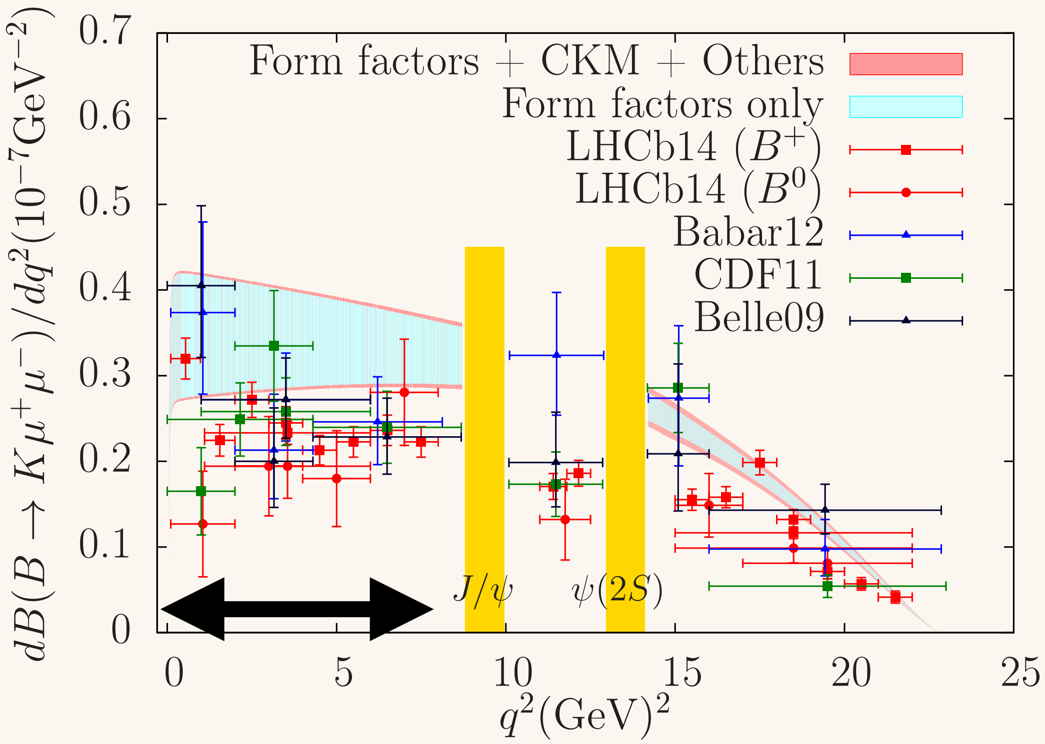
Photo: Granada,, Spain, 2017

◉ Motivation

(1): $B \to K l^+ l^-$ Important as a clean FCNC.
(GIM and loop suppress)

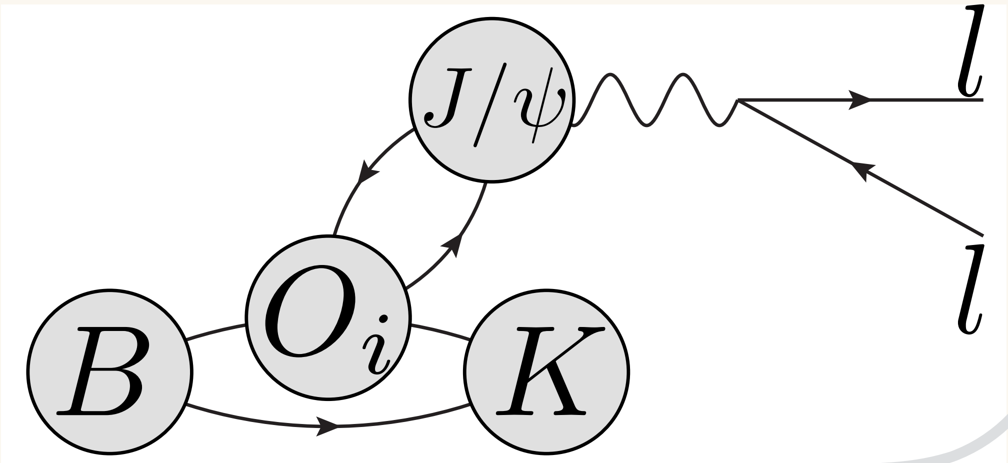


[D. Du et al. (Fermilab, MILC) 1510.02349]

(2): Anomaly in Experiments.

$$q^2 < m_{J/\psi}^2$$

Question: Are the long distance contributions completely taken into account?



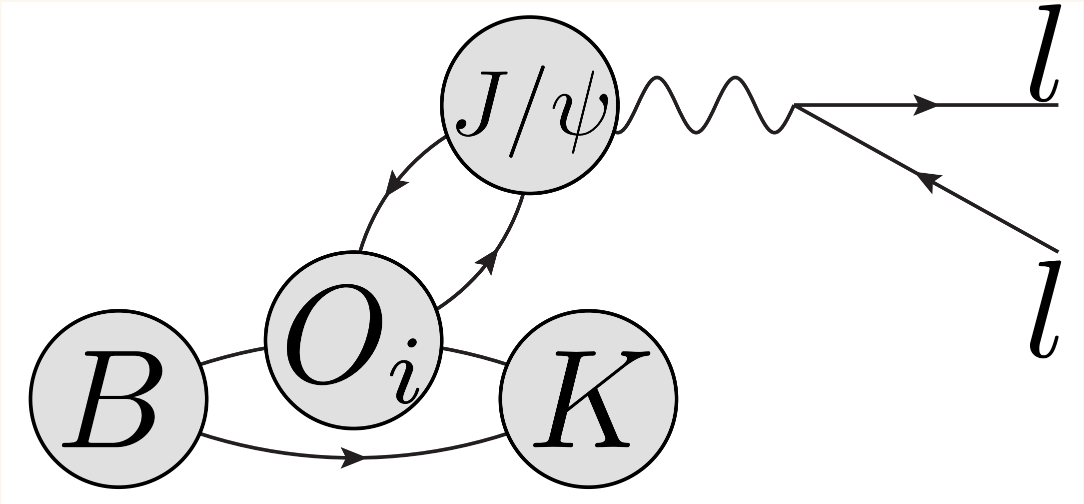
→We calculate the value without factorization scheme.

◉ Charmonium resonance part

◇ We focus on the charmonium part as the diagram which could suffer from non-factrizable contribution.

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left(\sum_{i=1}^{2} (V_{us}^* V_{ub} C_i O_i^u + V_{cs}^* V_{cb} C_i O_i^c) - V_{ts}^* V_{tb} \sum_{i=3}^{10} C_i O_i \right)$$

$$O_1^c = (\bar{s}_i \gamma_\mu P_- c_j)(\bar{c}_j \gamma_\mu P_- b_i)$$
$$O_2^c = (\bar{s}_i \gamma_\mu P_- c_i)(\bar{c}_j \gamma_\mu P_- b_j)$$



→ These 2 kinds operatores are response to long distance…

4 point decay amplitudes

◇ We'd like to calculate B decay amplitudes on the lattice, in a similar way to the K decay amplitudes.

[N.H. Christ et al. (RBC, UKQCD) 1507.03094]

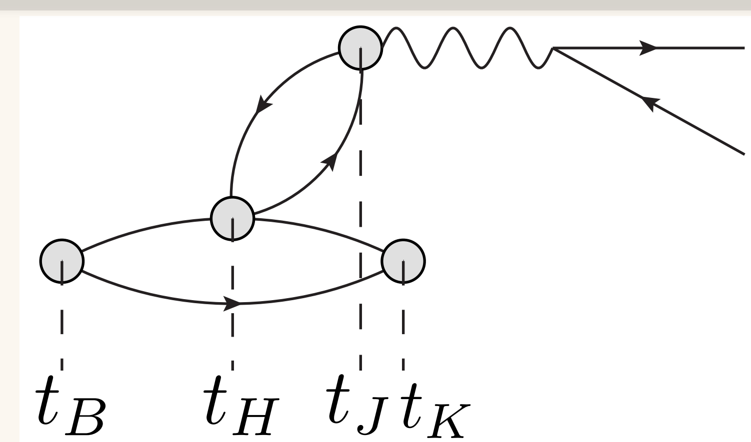
$$\mathcal{A}_\mu\left(q^2\right) = \int \mathrm{d}^4x \, \langle \pi(\mathbf{p}) | \mathrm{T}\left[J_\mu(0) H_{\mathrm{eff}}(x)\right] | K(\mathbf{k}) \rangle$$

↓

$$\mathcal{A}_\mu\left(q^2\right) = \int \mathrm{d}^4x \, \langle K(\mathbf{k}) | \mathrm{T}\left[J_\mu(0) H_{\mathrm{eff}}(x)\right] | B(\mathbf{p}) \rangle$$

◇ The amplitude is calculable from the integration of 4pt.

$$I_\mu\left(T_a, T_b, \mathbf{p}, \mathbf{k}\right) \simeq \int_{t_J - T_a}^{t_J + T_b} \mathrm{d}t_H \left[\quad \right]$$



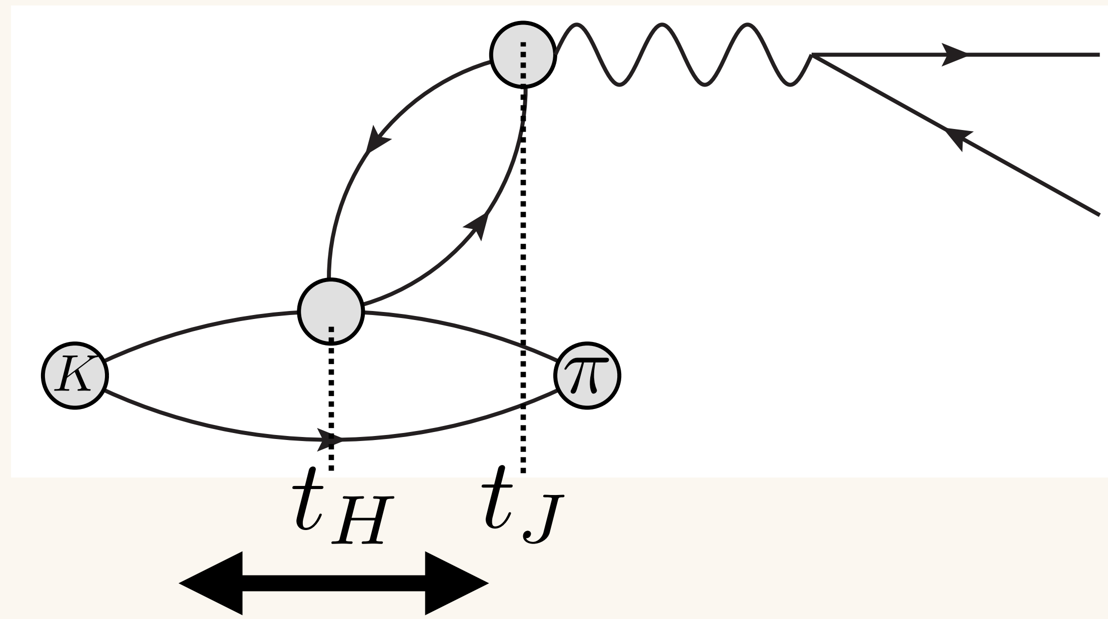
$(0 \ll t_J - T_a \leq t_J + T_b \ll t_K)$

Artificial divergence in amplitude at previous work

◇ $K \to \pi l^+ l^-$ case

[N.H. Christ et al. (RBC, UKQCD) 1507.03094]

$I_\mu =$



$$I_\mu\left(T_a, T_b, \mathbf{p}, \mathbf{k}\right) = -\int_0^\infty \mathrm{d}E \frac{\rho(E)}{2E} \frac{\langle \pi(\mathbf{k}) \left| J_\mu(0) \right| E, \mathbf{p}\rangle \langle E, \mathbf{p} | H_{\mathrm{eff}}(0) | K(\mathbf{p})\rangle}{E_K(\mathbf{p}) - E} \left(1 - e^{[E_K(\mathbf{p}) - E] T_a}\right)$$

◇ Energy of some intermediate state E are $E_K > E$

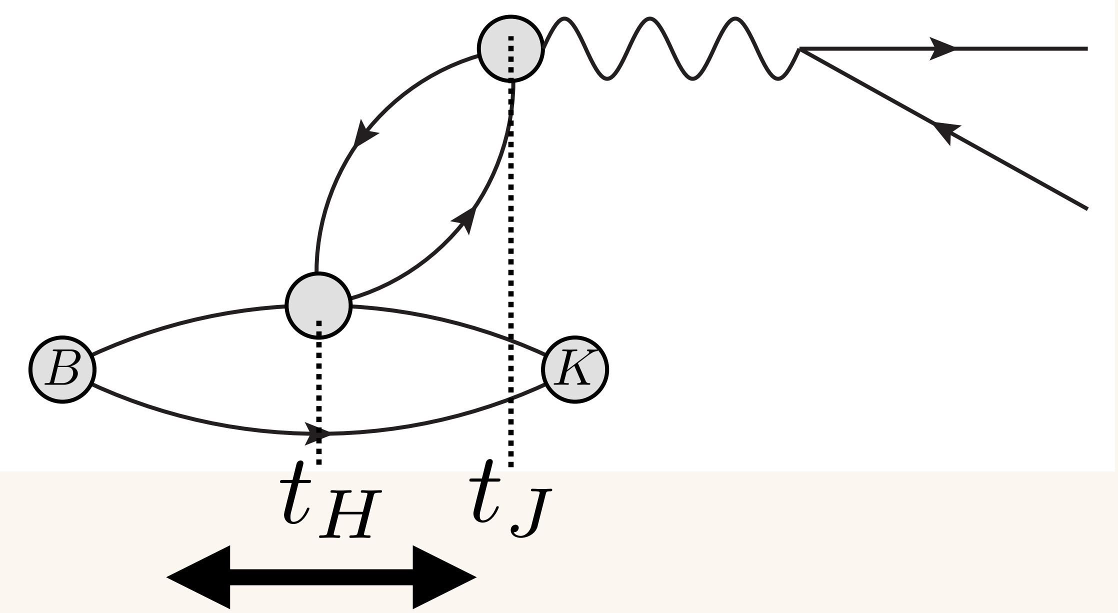
→ Since $T_a \to \infty$,they must be subtracted.

(e.g. $K \to \pi, \pi\pi, \pi\pi\pi$)

Artificial divergence in amplitude

◇ $B \to K l^+ l^-$ case

$I_\mu =$



t_H t_J

$$I_\mu\left(T_a, T_b, \mathbf{p}, \mathbf{k}\right) = -\int_0^\infty \mathrm{d}E \frac{\rho_S(E)}{2E} \frac{\langle K(\mathbf{k})\left|J_\mu(0)\right| E, \mathbf{p}\rangle \langle E, \mathbf{p}|H_{\mathrm{eff}}(0)|B(\mathbf{p})\rangle}{E_B(\mathbf{p}) - E} \left(1 - e^{[E_B(\mathbf{p}) - E]T_a}\right)$$

◇ We take unphysical light bottom and heavy up down quarks.

$$E_B < E_{J/\psi} + E_K$$

→ Artificial divergence does not exist.

◉ Amplitude of $B \to K l^+ l^-$.

◇ From the integration of the 4point correlators, we can extract the amplitude after taking $T_{a,b} \to \infty$ limit.

$$I_\mu\left(T_a, T_b, \mathbf{p}, \mathbf{k}\right) = -\int_0^\infty \mathrm{d}E \frac{\rho_S(E)}{2E} \frac{\langle K(\mathbf{k})\left|J_\mu(0)\right| E, \mathbf{p}\rangle \langle E, \mathbf{p}|H_{\text{eff}}(0)|B(\mathbf{p})\rangle}{E_B(\mathbf{p}) - E}\left(1 - e^{[E_B(\mathbf{p})-E]T_a}\right)$$

$$\mathcal{A}_\mu\left(q^2\right) = \int \mathrm{d}^4 x \left\langle K(\mathbf{k})\left|\mathrm{T}\left[J_\mu(0) H_{\text{eff}}(x)\right]\right| B(\mathbf{p})\right\rangle$$

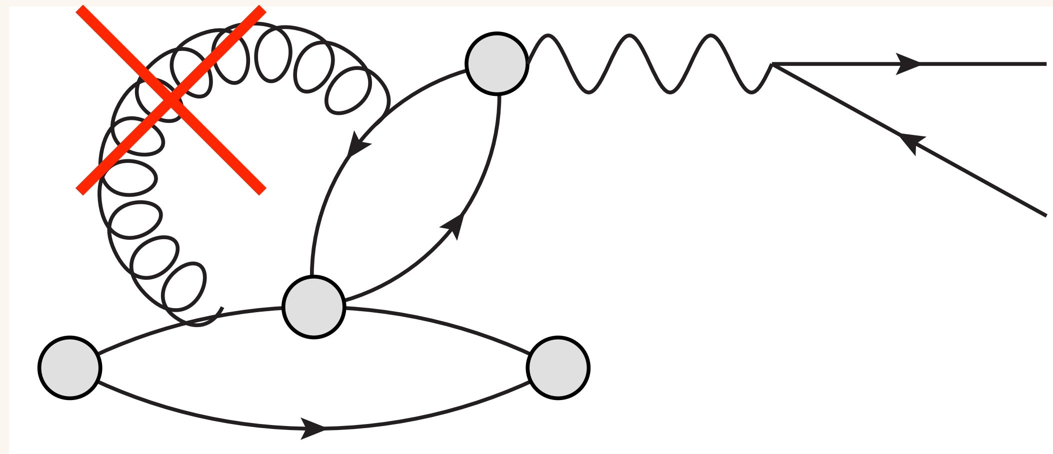
↓

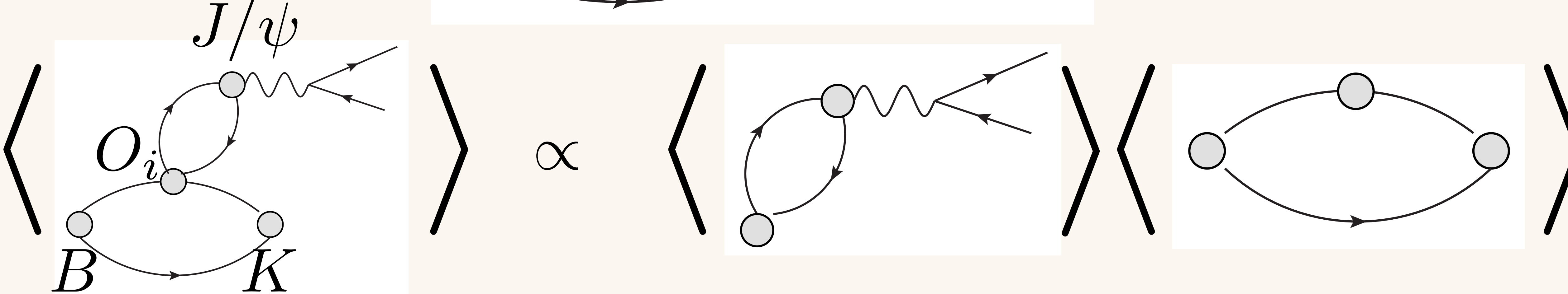
$$A_\mu(q^2) = -i \lim_{T_{a,b}\to\infty} I_\mu(T_a, T_b, \mathbf{k}, \mathbf{p})$$

Factorization method for $B \to Kl^+l^-$ decay

Factorization

◇ Assume long range gluon exchanging could be ignored





$$\langle P_K | J_\nu^{\bar{c}c}(\bar{c}_i \gamma_\mu P_- c_i)(\bar{s}_j \gamma_\mu P_- b_j) | P_B \rangle = \frac{1}{(\mathrm{Vol.})} \langle 0 | J_\nu^{\bar{c}c} J_\mu^{\bar{c}c} | 0 \rangle \langle P_K | V_\mu | P_B \rangle$$

→ We test this relation and assumption.

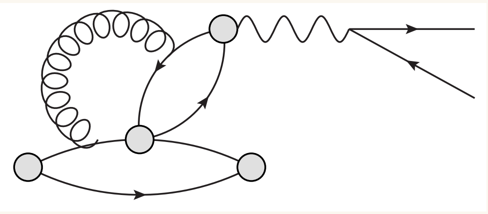
◉ Factorization in previous work at $K \to \pi\pi$

[P.A. Boyle et al. (RBC, UKQCD) 1212.1474]

◇ Factorizable operator O_F and non-factorizable O_{NF}

$$O_F^{(1)} = \left(\bar{l}_i \gamma_\mu P_- l_i\right)\left(\bar{l}_j \gamma_\mu P_- s_j\right)$$

$$O_{NF}^{(8)} = \left(\bar{l}_i \left[T^a\right]_{ij} \gamma_\mu P_- l_j\right)\left(\bar{l}_k \left[T^a\right]_{kl} \gamma_\mu P_- s_l\right)$$



↓ Fierz transformation

$$O_1^l = O_F^{(1)} \qquad O_2^l = \frac{1}{3} O_F^{(1)} + 2 O_{NF}^{(8)}$$

◇ Assume non−factrizable operator $O_{NF}^{(8)}$ could be ignored

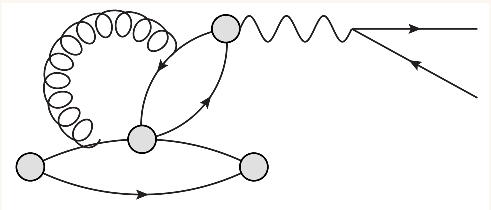
$K \to \pi\pi$, Lattice. $\quad O_2^l \simeq -0.7 O_1^l$

◉ Factorization

◇ Factorizable operator O_F and non-factorizable O_{NF}

$$O_F^{(1)} = (\bar{c}_i \gamma_\mu P_- c_i)\,(\bar{s}_j \gamma_\mu P_- b_j)$$

$$O_{NF}^{(8)} = \left(\bar{c}_i\,[T^a]_{ij}\,\gamma_\mu P_- c_j\right)(\bar{s}_k\,[T^a]_{kl}\,\gamma_\mu P_- b_l)$$



↓ Fierz transformation

$$O_1^c = O_F^{(1)} \qquad O_2^c = \frac{1}{3} O_F^{(1)} + 2 O_{NF}^{(8)}$$

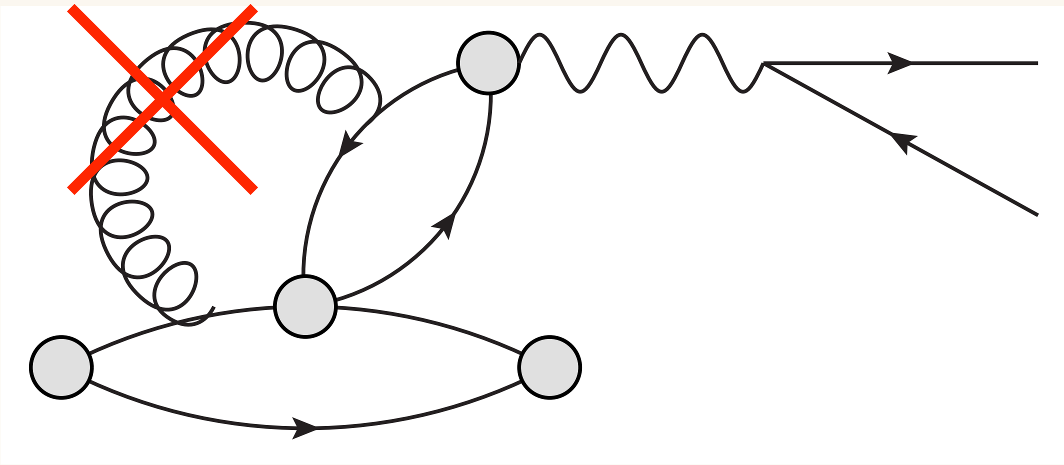
◇ Assume non-factorizable operator $O_{NF}^{(8)}$ could be ignored

→ We test this assumption $O_2^c = \frac{1}{3} O_1^c$.

◉ More on factorization (Perturbation)

→ We test this assumption $O_2^c = \frac{1}{3} O_1^c$.

◇ Note: This relation completely ignore rescattaring.



◇ Rough estimate for this process in perturbation.

$$O_2^c = \left(\frac{1}{3} + \frac{O_{NF}^{(8)}}{O_F^{(1)}} \right) O_F^{(1)}$$

$$3\frac{O_{NF}^{(8)}}{O_F^{(1)}} \simeq 3\frac{\alpha_s(\mu)}{4\pi} \simeq 0.06 \qquad (6\%)$$

Colour suppression

$$3\frac{O_{NF}^{(8)}}{O_F^{(1)}} \simeq 3\frac{\alpha_s(\mu)}{4\pi} \simeq 0.06 \qquad (6\%)$$

◇ Is 6% uncertainty small? (for factorization)

→ Colour suppression in Wilson coefficients.

(e.g.): NLO in $\overline{\mathrm{MS}}(\mu \simeq 4\ \mathrm{GeV})$

[G. Buchalla et al. hep-ph/9512380]

$$C_1^c O_1^c = C_1^c O_F^{(1)} \qquad C_2^c O_2^c = C_2^c \left(\frac{1}{3} O_F^{(1)} + 2 O_{NF}^{(8)} \right)$$

$$C_1^c + \frac{1}{3} C_2^c \simeq 0.2 \quad C_2^c \simeq 1$$

$$\frac{2 C_2^c O_{NF}^{(8)}}{\left(C_1^c + \frac{1}{3} C_2^c\right) O_F^{(1)}} \simeq 10 \frac{O_{NF}^{(8)}}{O_F^{(1)}} \simeq 0.2 \qquad (20\%)$$

Another estimate

$$\frac{2C_2^c O_{NF}^{(8)}}{\left(C_1^c + \frac{1}{3}C_2^c\right) O_F^{(1)}} \simeq 10 \frac{O_{NF}^{(8)}}{O_F^{(1)}} \simeq 0.2 \qquad (20\%)$$

◇ Another estimate using Breigt-Wigner approximation,

[J. Lyon and R. Zwicky 1406.0566]

$$\frac{2C_2^c O_{NF}^{(8)}}{\left(C_1^c + \frac{1}{3}C_2^c\right) O_F^{(1)}} \simeq -0.5 \qquad (50\%)$$

→ ~50% contribution.

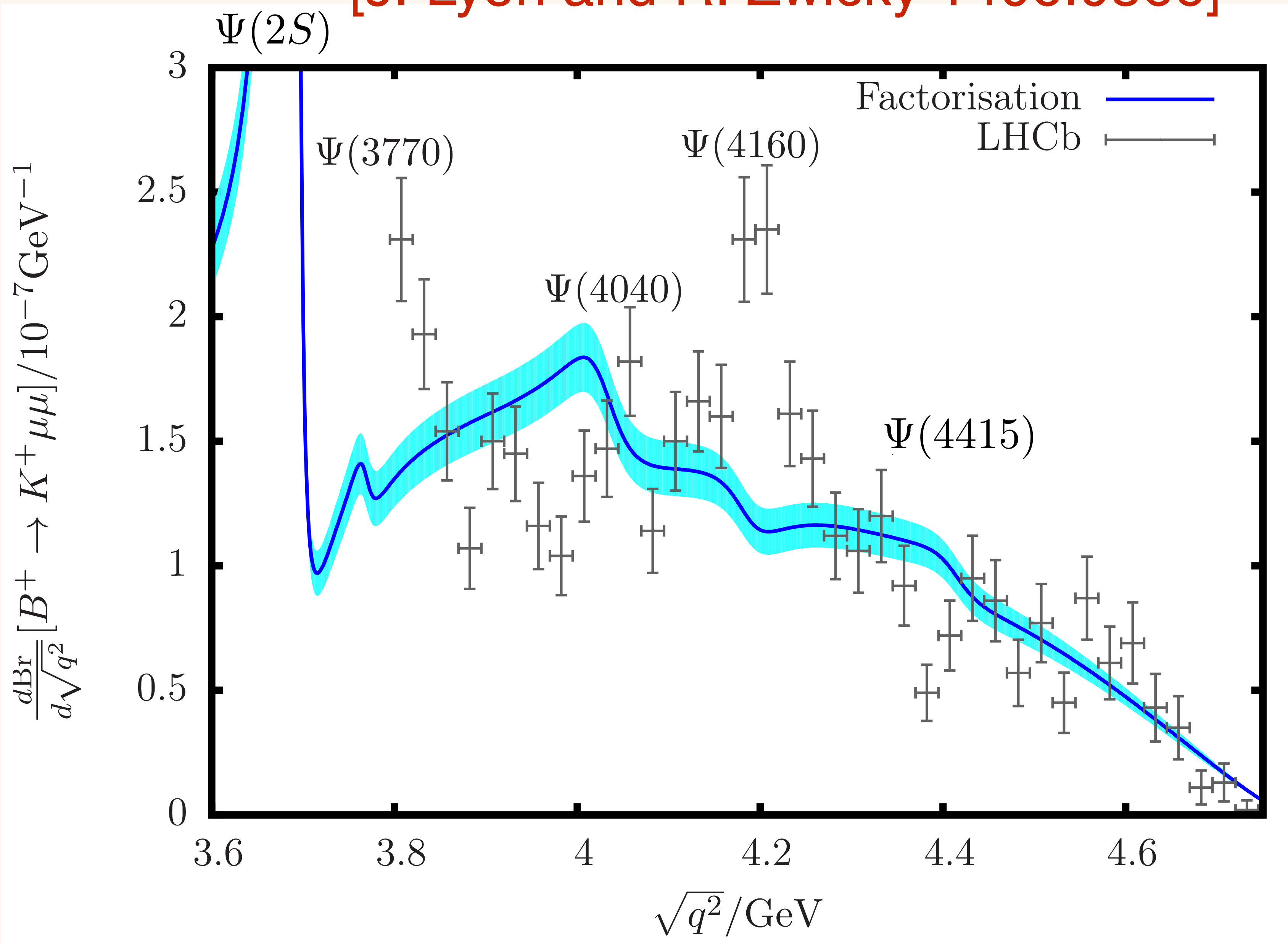
◉ Comparison to experiments

◇ Does factorization represent the experiment?

[J. Lyon and R. Zwicky 1406.0566]

→ No.
(In perturbative study, without rescattering)

→ ~50% contribution?
(From rescattering)

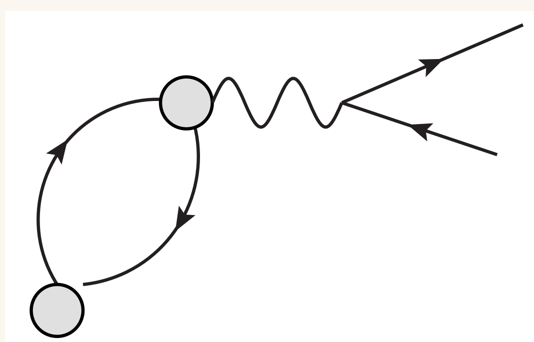
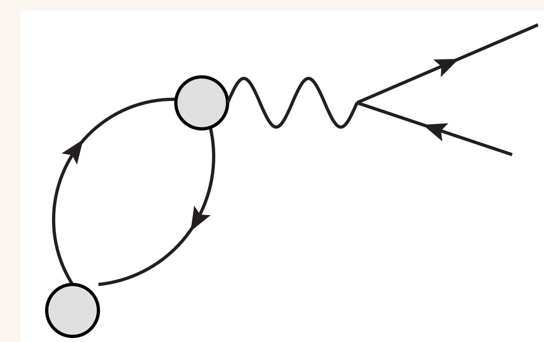


● Estimate of non-factorizable contributions

[J. Lyon and R. Zwicky 1406.0566]

◇ Assuming non-factorizable contributions come from charmonium resonance.

→ Introducing free parameter η_c for fitting

$\langle$  $\rangle$ → $\eta_c \times \langle$  $\rangle$

(e.g.) 50% contribution: $\eta_c = -0.5$

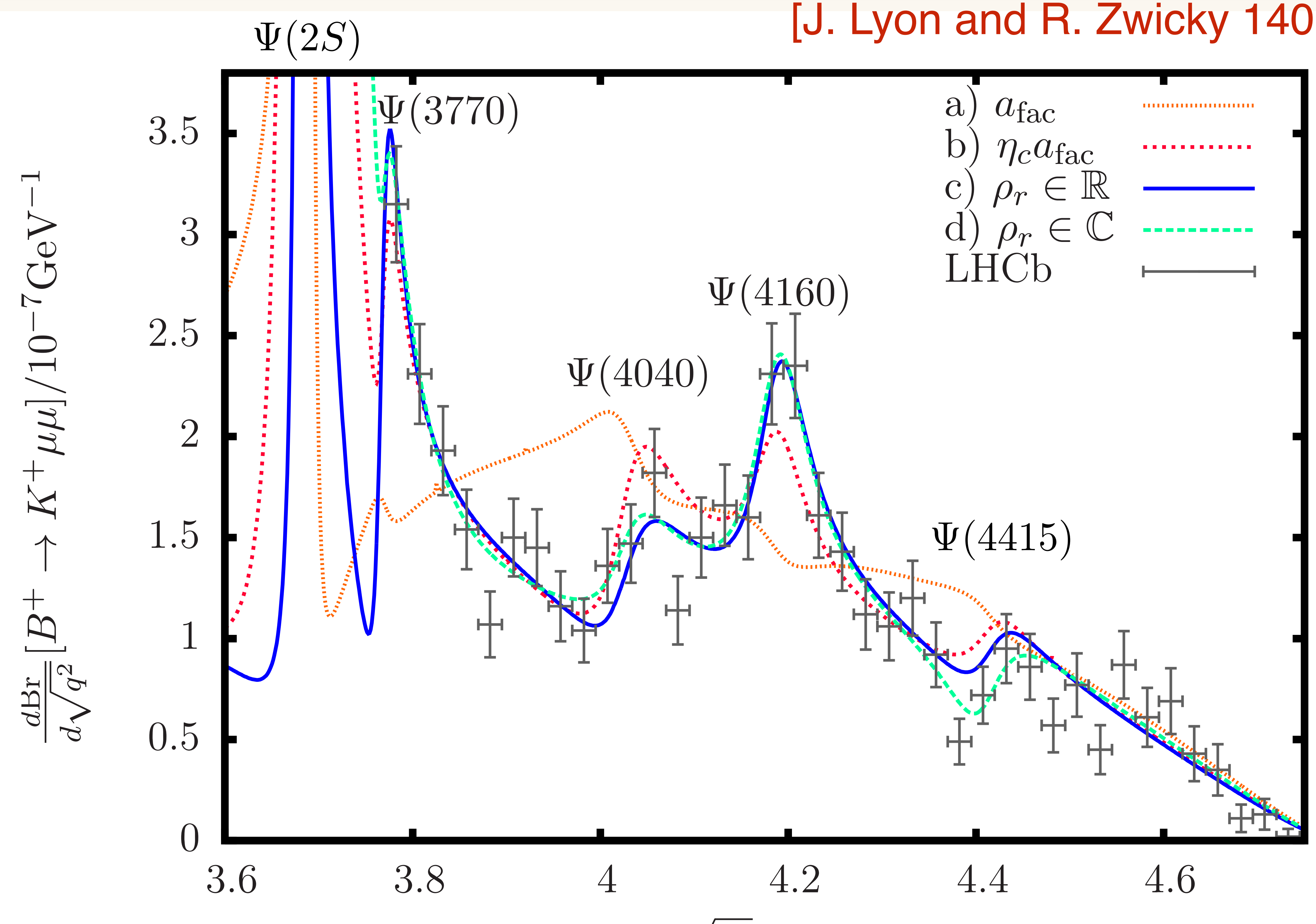
↓

◇ $\eta_c \simeq -2.5$ well represents results from experiments.

◉ Comparison to experiments

◇ $\eta_c \simeq -2.5$ well represents results from experiments.

[J. Lyon and R. Zwicky 1406.0566]



◉ Factorization in perturbation

Perturbation

- ◇ Non-factorizable contribution is sizable.
- ◇ We could estimate the contribution as $\eta_c \simeq -0.5$.

↕

Experiment

- ◇ $\eta_c \simeq -2.5$ well represents results from experiments.

↕

Lattice

→ We test naive factorization $O_2^c = \frac{1}{3} O_1^c$ as a first step.

Preliminary result for the test of factorization

Currenst status

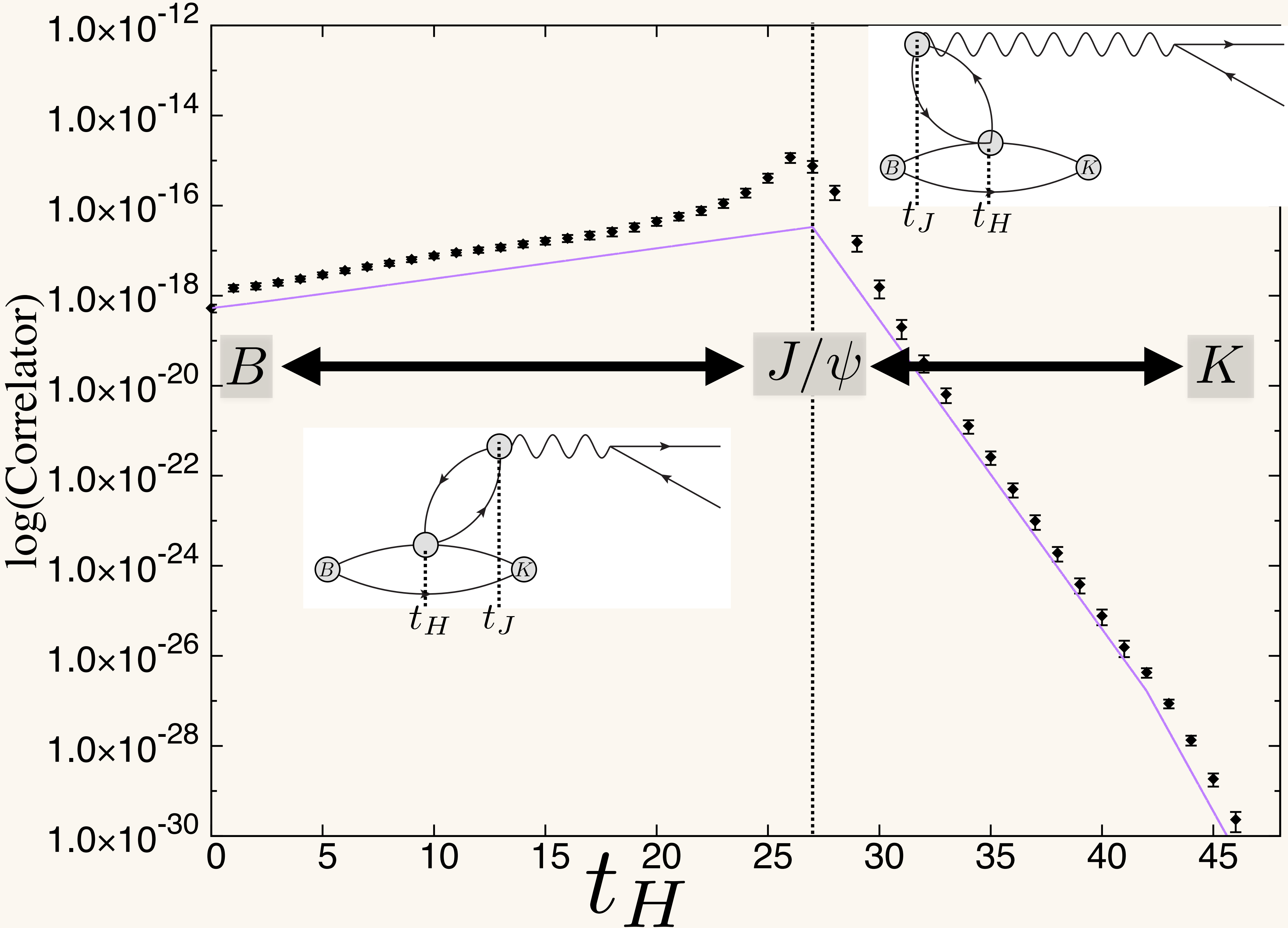
β	a^{-1} [GeV]	$L^3 \times T(\times L_s)$	am_{val}	am_c	am_b
4.35	3.610(9)	$48^3 \times 96(\times 8)$	0.025	0.27287	0.66619

ap	#Conf.	m_π[MeV]	E_K[MeV]	$E_{J/\psi}$[GeV]	m_B[GeV]
$-\frac{2\pi}{L}$ (1,0,0)	377	714(1)	854(3)	3.128(1)	3.44(1)
$-\frac{2\pi}{L}$ (1,1,0)	388	713(1)	962(11)	3.157(1)	3.44(1)

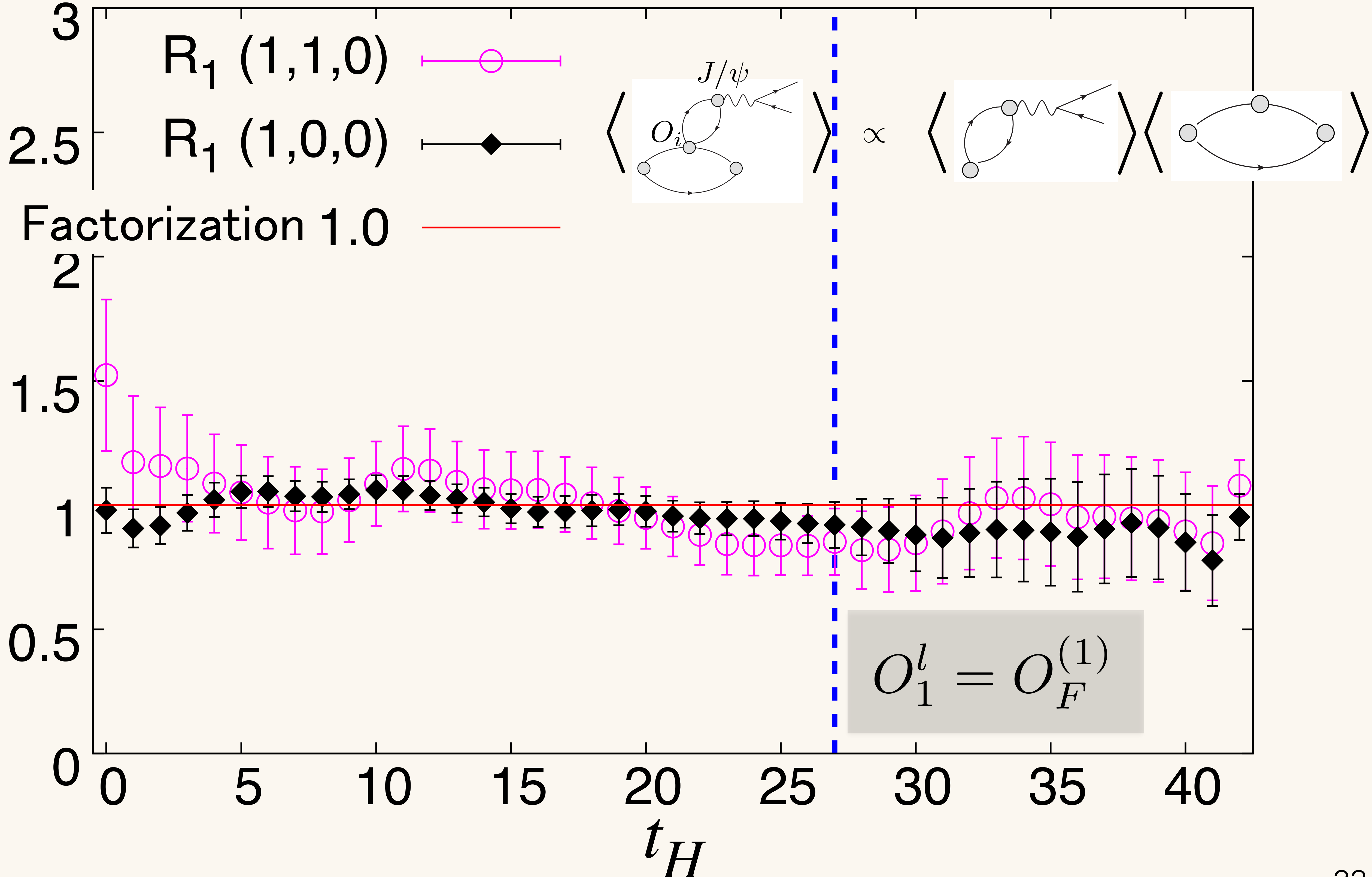
- Heavy up and down mass same with strange.
- Light bottom mass: $m_b = (1.25)^4 m_c$
- Finite momentum at final state $\mathbf{k} = \left(-\frac{2\pi}{L}, 0, 0\right), \left(-\frac{2\pi}{L}, -\frac{2\pi}{L}, 0\right)$

◉ 4 point functions

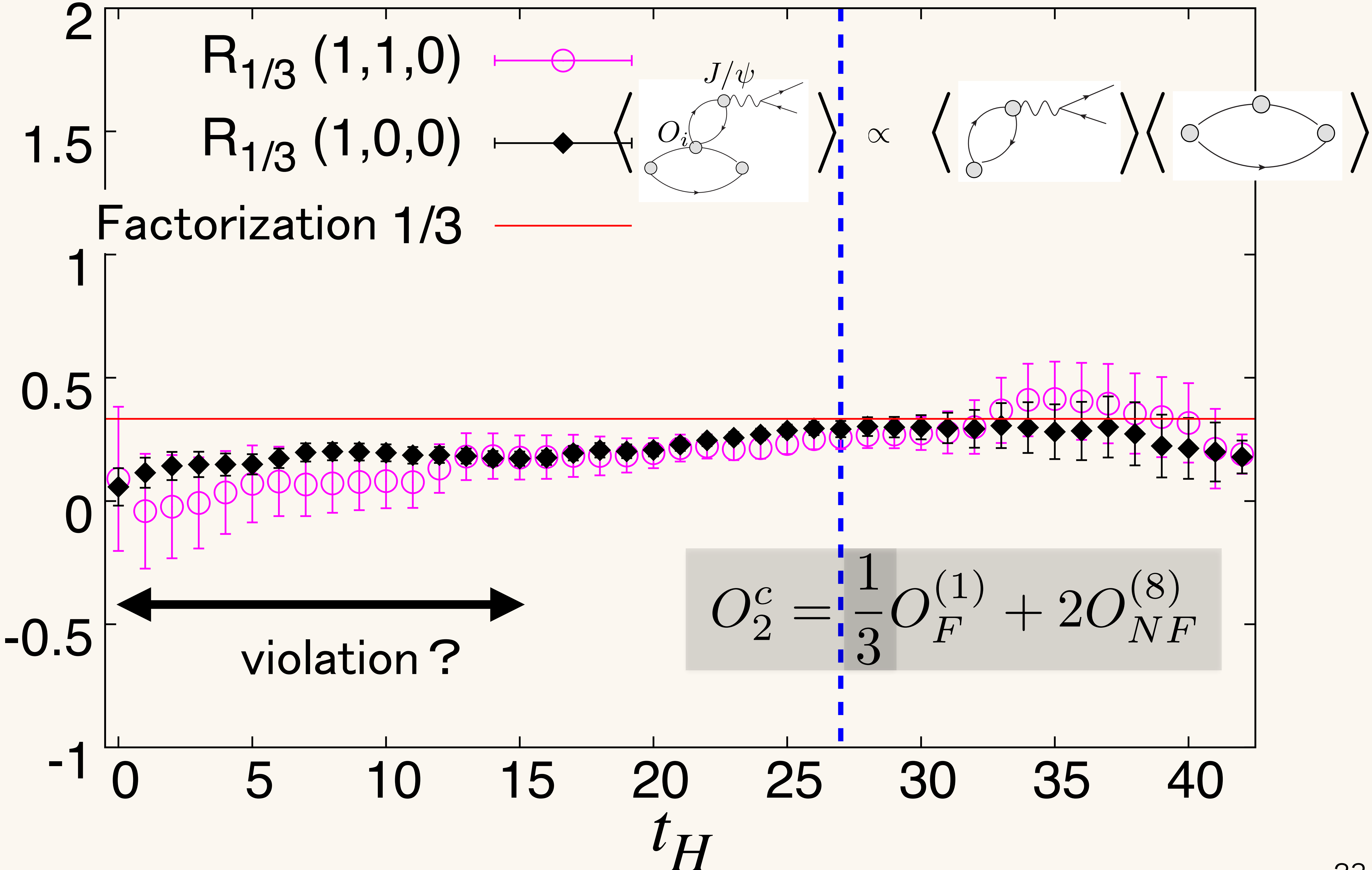
$$\Gamma_{\mu}^{(4)}\left(t_H, t_J, \mathbf{p}, \mathbf{k}\right) = \int \mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{y} e^{-i\mathbf{q}\cdot\mathbf{y}} \left\langle \phi_K\left(t_K, \mathbf{k}\right) \mathrm{T}\left[J_{\mu}\left(t_J, \mathbf{y}\right) H_{\mathrm{eff}}\left(t_H, \mathbf{x}\right)\right] \phi_B^{\dagger}(0, \mathbf{p}) \right\rangle$$



◉ Factorization of 4 point functions



◉ Factorization of 4 point functions



TO DO LIST

We have to…

(1):determine the lattice renormalization constant.

(2):Input more realistic momentum to conserve energy.

$$E_B(0) = E_{J/\psi}(\mathbf{p}) + E_K(\mathbf{p}) \longrightarrow \mathbf{p} \simeq \left(2\left(\frac{2\pi}{L}\right), 2\left(\frac{2\pi}{L}\right), 2\left(\frac{2\pi}{L}\right)\right)$$

(3):Input or extrapolate to physical quark masses.

(4):complete integration and taking limit to extract amplitude.

◉ Summary

◇ We study to extract the charmonium contribution in $B \to K l^+ l^-$ by the lattice calculation.

◇ $B \to K l^+ l^-$ is also calculable similar to $K \to \pi l^+ l^-$ if we use a lighter bottom quark mass to eliminate the artificial divergence.

◇ As a first step, we focus on the validity of the factorization.

◇ So far, sizable non-factorizable contribution might exist in non-perturbative region.