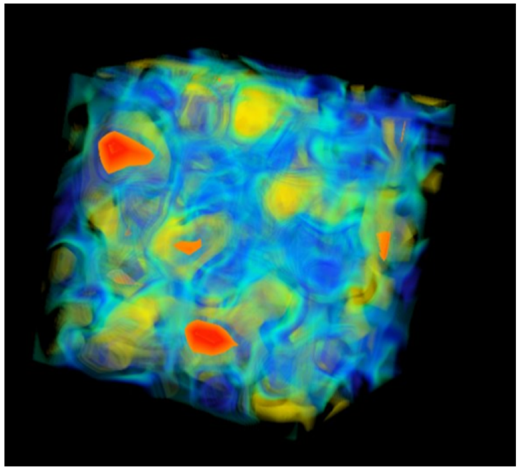
Introduction to Lattice QCD

Taku Izubuchi



BROOKHAVEN NATIONAL LABORATORY

RIKEN BNL Research Center

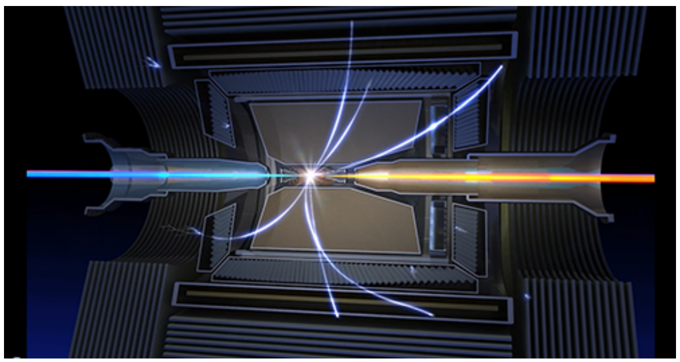
August 1st, 2019 Belle-II school, BNL

Text Book

- Gattringer, Lang, “Quantum Chromodynamics on the Lattice”
- DeGrand, DeTar, “Lattice Methods for Quantum Chromodynamics”
- Montvay, Munster, “Quantum Fields on Lattice”
- Creutz, “Quarks, Gluons and Lattices”

Particle & Nuclear Physics

- [experiment] [QCD, Strong Interaction] [Standard Model theory]



[KEK.JP]

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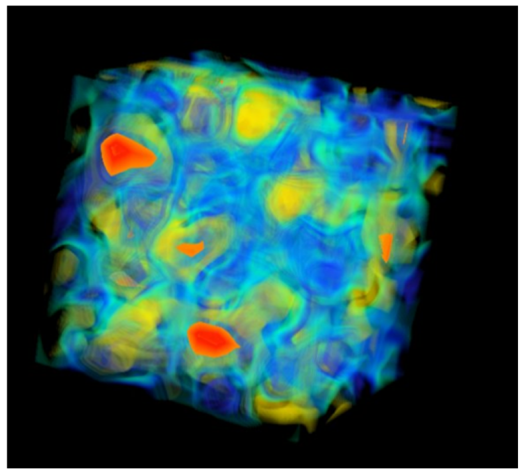

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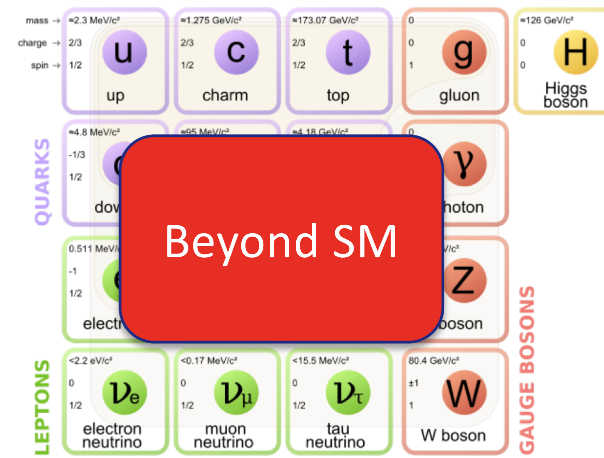
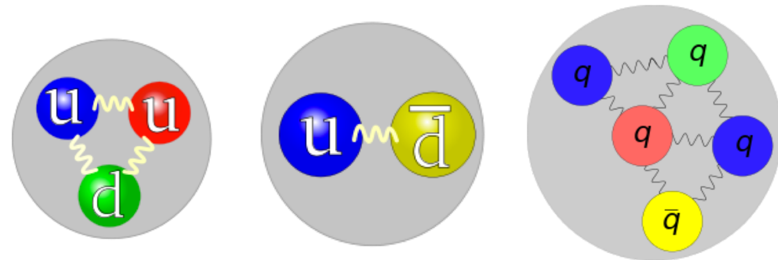


- Goal : understand and check fundamental law of sub-atomic physics, Standard Model, through many interplay between experiments and theory
- To interpret experimental results, one often needs to solve how sub-atomic particles, quarks and gluons interacts each other.

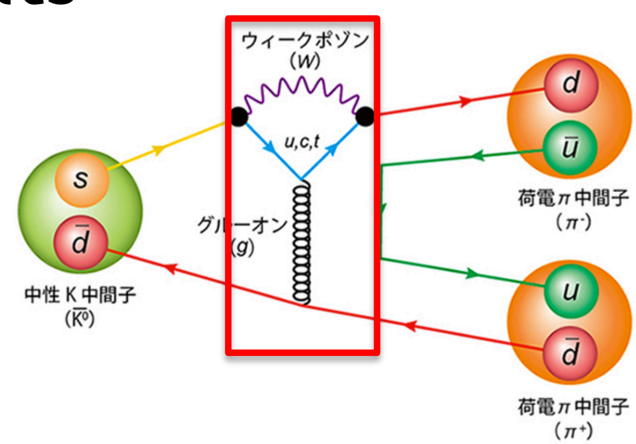
Why study QCD ?

- Strong dynamics



- Make composite "stuff" :
  Baryons, mesons, exotics
  [spectroscopy , energy]

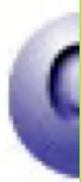
- To interpret experimental results
  [hadronic matrix elements]



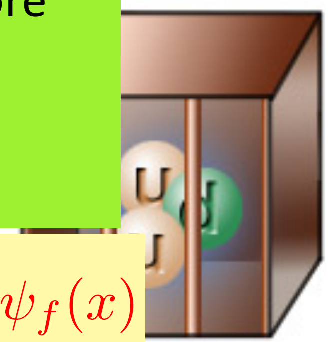
- Precision with reliable error

Quantum Chromo Dynamics （QCD)

- Gluon： 100 times stronger coupling than electromagnetism, strong coupling constant α_s
- Quark： quark mass m_f
- confinement → only color neutral, White, can exist at low energy and low temperature
- Make composite objects and phenomena, large hierarchy of scale
- Inter nuclear force is also from QCD → how nuclei exist



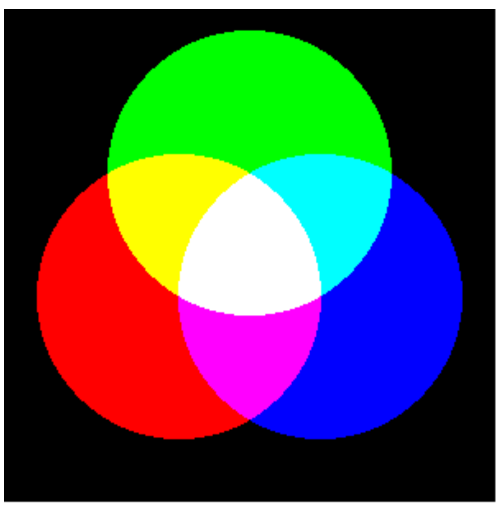
（so far）there is no further sub-structure nor more fundamental law known
If we solve QCD (in principle) we know (almost) everything very precisely

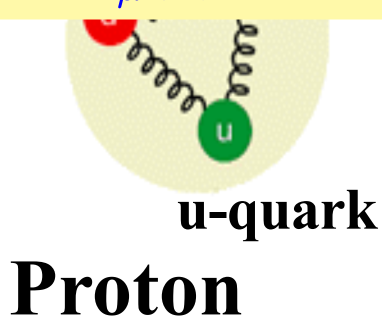


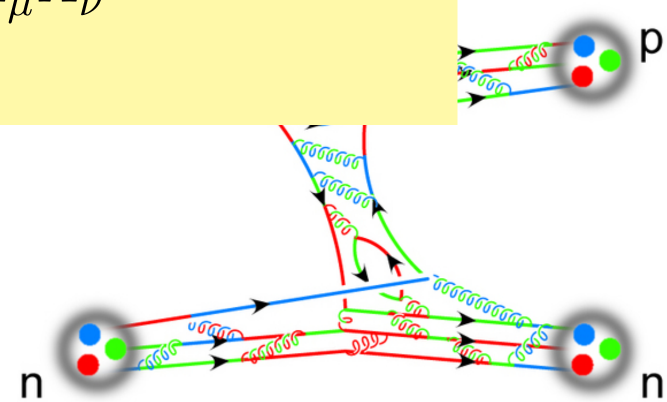
$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4}F^2_{\mu\nu}(x) + \sum_f \bar{\psi}_f(i\gamma^\mu D_\mu - m_f)\psi_f(x)$$

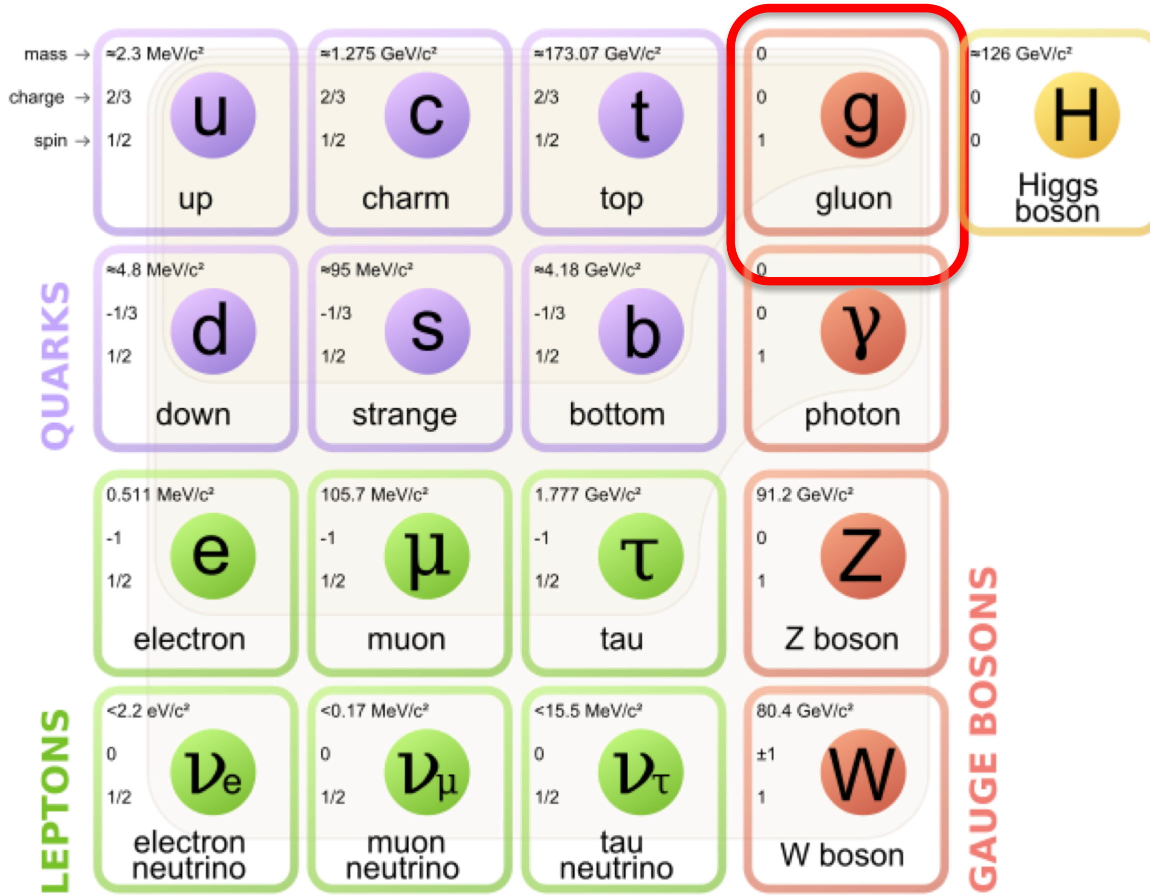
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$D_\mu = \partial_\mu - igT^a A^a_\mu(x)$$








mass →
charge →
spin →
≈2.3 MeV/c²
2/3
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1/2
c
charm
≈173.07 GeV/c²
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g
gluon
≈126 GeV/c²
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Higgs boson
≈4.8 MeV/c²
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QUARKS
0.511 MeV/c²
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91.2 GeV/c²
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Z
Z boson
<2.2 eV/c²
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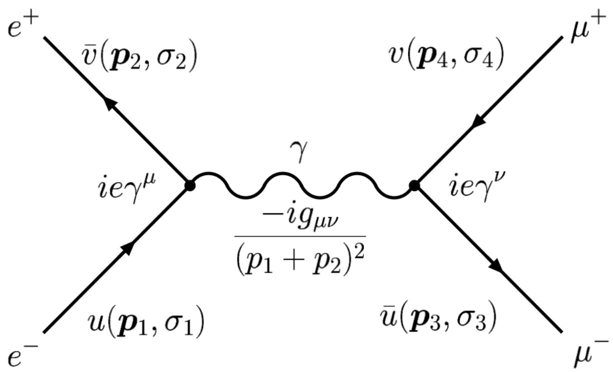
1/2
νe
electron neutrino
<0.17 MeV/c²
0
1/2
νμ
muon neutrino
<15.5 MeV/c²
0
1/2
ντ
tau neutrino
80.4 GeV/c²
±1
1
W
W boson
LEPTONS
GAUGE BOSONS

GLUON

Non perturbative

- Perturbation theory is a powerful tool, it's origin of intuitive understandings.
- Feynman diagram is a perturbative expansion of amplitude
- 

$$\overline{|\mathcal{M}|^2} = \frac{e^4}{4(p_1+p_2)^4}\mathrm{Tr}[(\not{p}_1 + m_1)\gamma^\mu(\not{p}_2 - m_2)\gamma^\nu]\,\mathrm{Tr}[(\not{p}_4 - m_4)\gamma_\mu(\not{p}_3 + m_3)\gamma_\nu]$$

- Convergence of perturbative series ? Precision of calculation ?

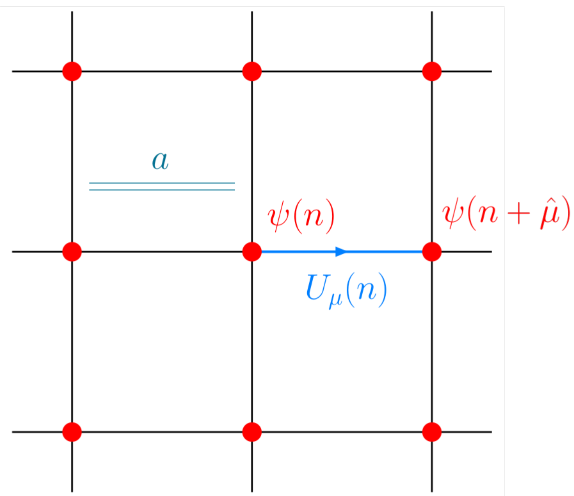
$$\mathcal{M} \propto \exp\left(-\frac{8\pi^2}{g^2}\right) = 0 + 0g^2 + 0g^4 \cdots$$

- Non-perturbative effects are responsible for many important physics

Gluon on Lattice

- Lattice discretization for a non-perturbative analysis of strong dynamics, (K.G. Wilson 1974)

$\Psi(x), A_\mu(x), x \in \mathcal{R}^4$: *continuous infinity*
quantum divergences: *needs regularization and renormalization*



- Discretized Euclidean space-time
- lattice spacing $a \sim 0.1$ fm
  (UV cut-off $|p| \leq \pi/a$)

- $\psi(n)$: Fermion field (Grassmann number)
- $U_\mu(n)$: Gauge field

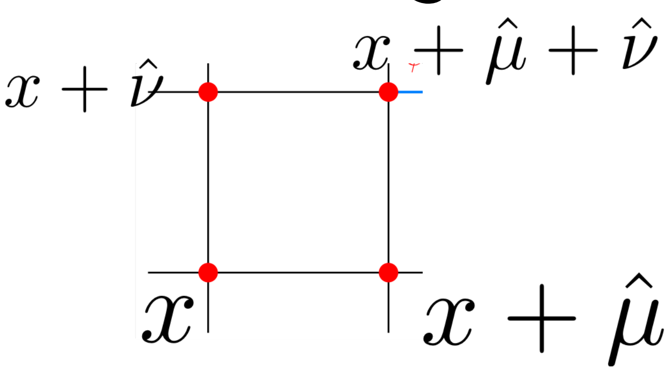
gauge invariance g (x) : SU(3)

$$\psi(x) \to g(x)\psi(x), \bar{\psi}(x) \to \bar{\psi}(x)g^\dagger(x),$$
$$U(x,\mu) \to g(x)U(x,\mu)g^\dagger(x+\mu)$$

- SU(3)-valued gauge filed U(x,μ) “link field”

$$U(x,\mu) = \exp(iaA_\mu(x)) = 1 + iaA_\mu(x) + \cdots$$

- Field strength



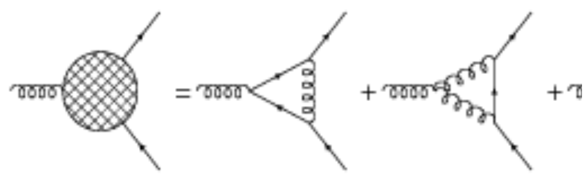
$$\begin{aligned}
&\mathrm{Tr}\left[U(x,\mu)U(x+\hat{\mu},\nu)U^\dagger(x+\hat{\nu},\mu)U^\dagger(x,\nu)\right] \\
&= \mathrm{Tr}\left[e^{igaA_\mu(x)}e^{iga(A_\nu+a\partial_\mu A_\nu)}e^{-iga(A_\mu(x+\partial_\nu A_\mu)}e^{-igaA_\nu}\right] \\
&= \mathrm{Tr}\exp\left[iga^2(\partial_\mu A_\nu - \partial_\nu A_\mu) - g^2a^2[A_\mu, A_\nu]\right] \\
&= 3 - \frac{1}{2}g^2a^4\mathrm{Tr}[F_{\mu\nu}^2]
\end{aligned}$$

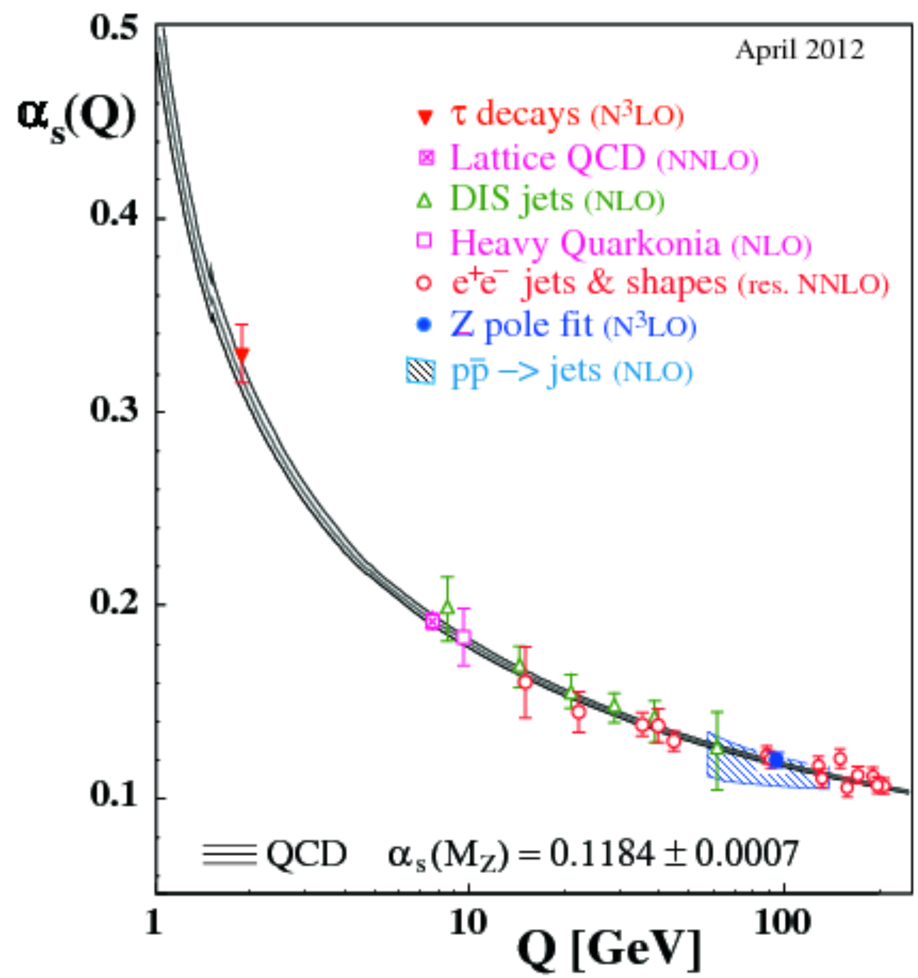
- Wilson’s plaquette action, β= 6 / g^2

$$S_g = \sum_{\square}\beta\left\{1-\frac{1}{3}\mathrm{Retr}(U_\square)\right\} \xrightarrow{a\to 0} \frac{1}{2g^2}a^4\sum_x \mathrm{tr}[F_{\mu\nu}(x)^2] + \mathcal{O}(a^6)$$

Strong coupling constant

- Strong coupling constant $\alpha_s = g^2 / 4\pi$ runs due to the gluon's self coupling





- Asymptotic freedom

- Lattice coupling

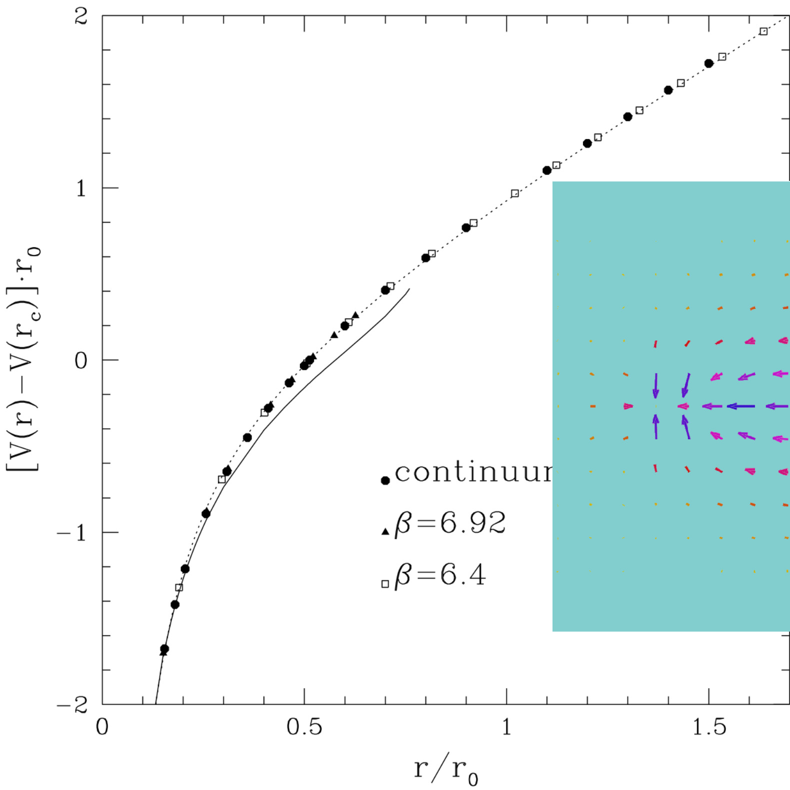
  $\beta = 6 / g^2$

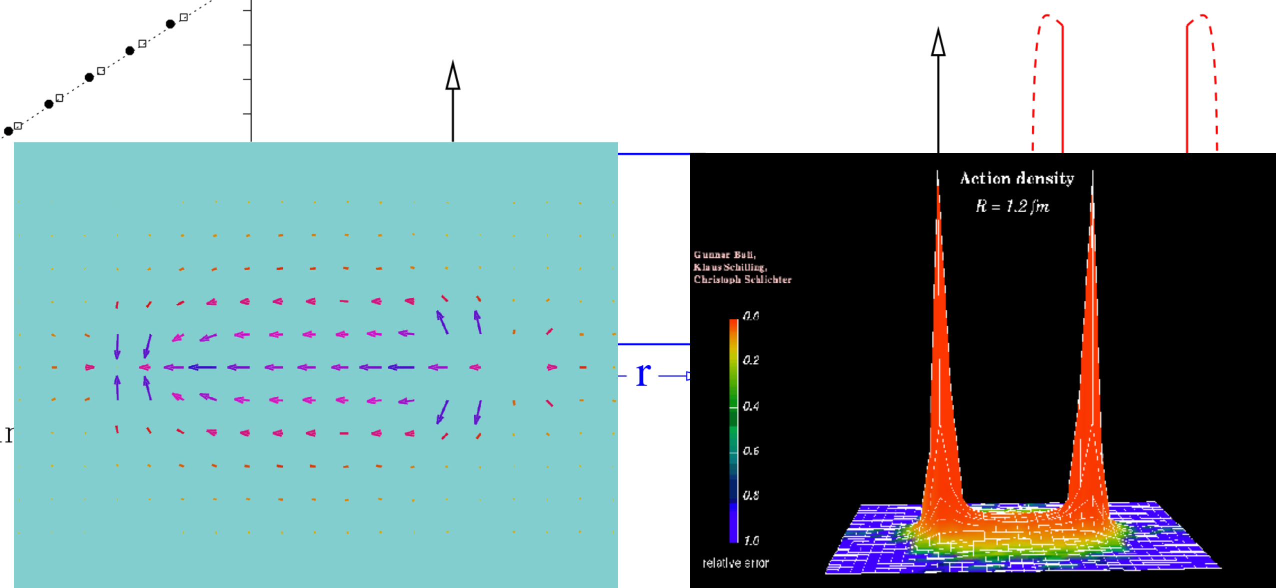
β large <-> small a

Static quark potential

- Infinity heavy quark
- Cornel parametrization

$$V(r) = \sigma r + \frac{\gamma}{r} + \cdots$$



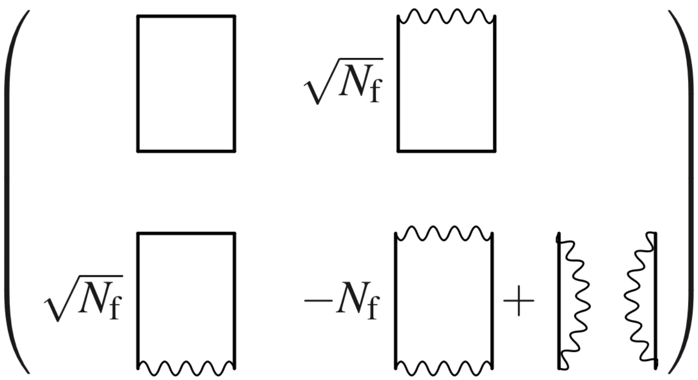


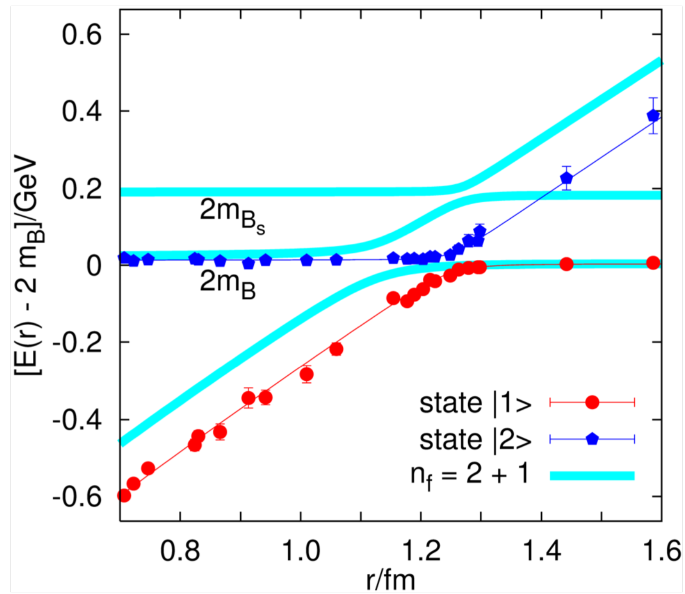
[G. Bali. Home page]

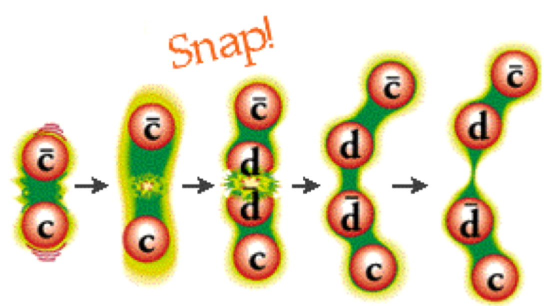
[compilation F. Knechtli, arxiV:1706.00282]

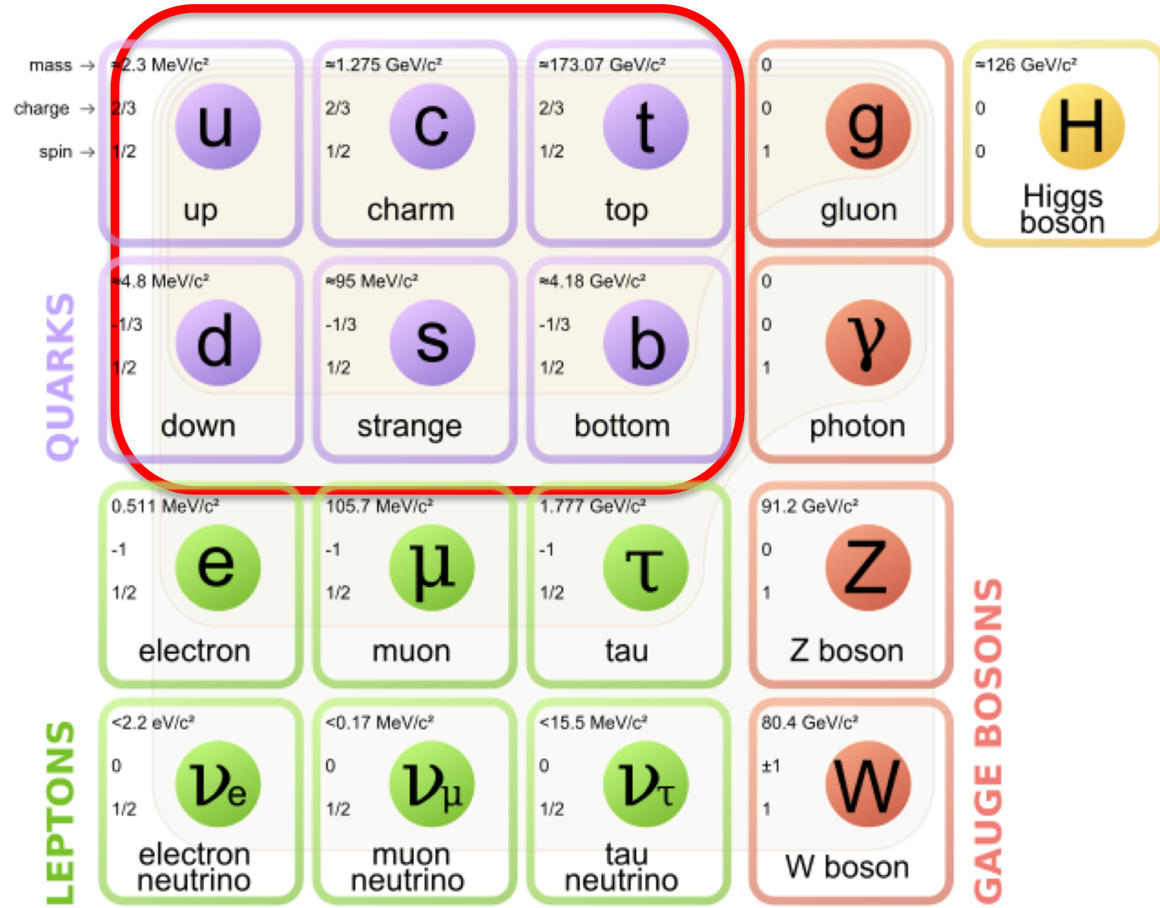
String breaking

- [G. Bali PRD71 (2005) 114513]





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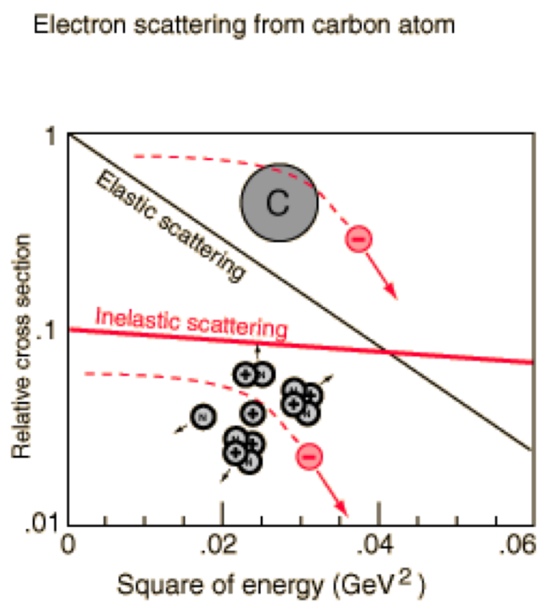
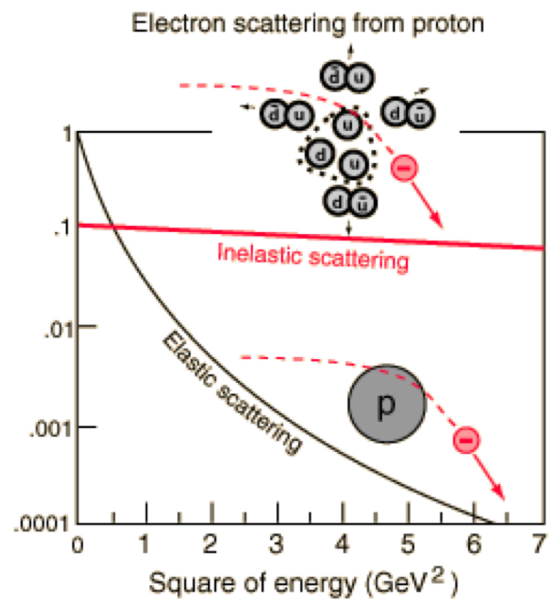
LEPTONS
GAUGE BOSONS

QUARK

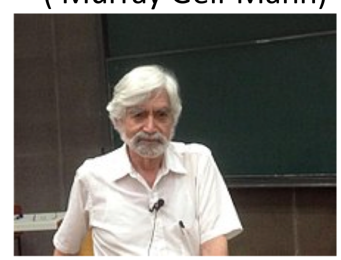
Proton/neutron are made of Quarks

- Electron – nucleon scattering (1968 SLAC)
- Proton has a substructure seen by energetic electrons
- "three quarks for Muster Mark"
  James Joyce *Finnegan's Wake*






(Murray Gell-Mann)


(George Zweig)

[Hyper Physics]

Proton
Charge:
+1

Neutron
Charge:
0

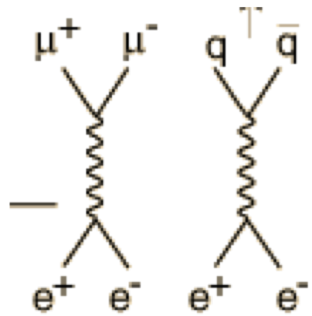
U **mass** 2.3(7)(5) MeV

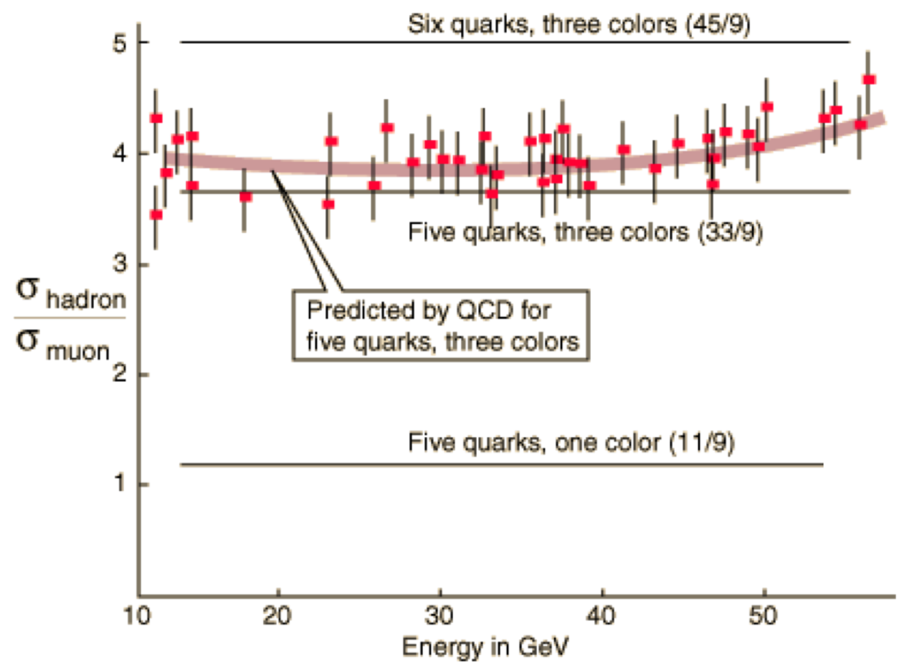
D mass 4.8(5)(3) MeV

U up quark, charge : **2/3**

D down quark, charge: **-1/3**

R-ratio
σ(e+e-→ hadrons) /
σ(e+e-→ μ+μ-)



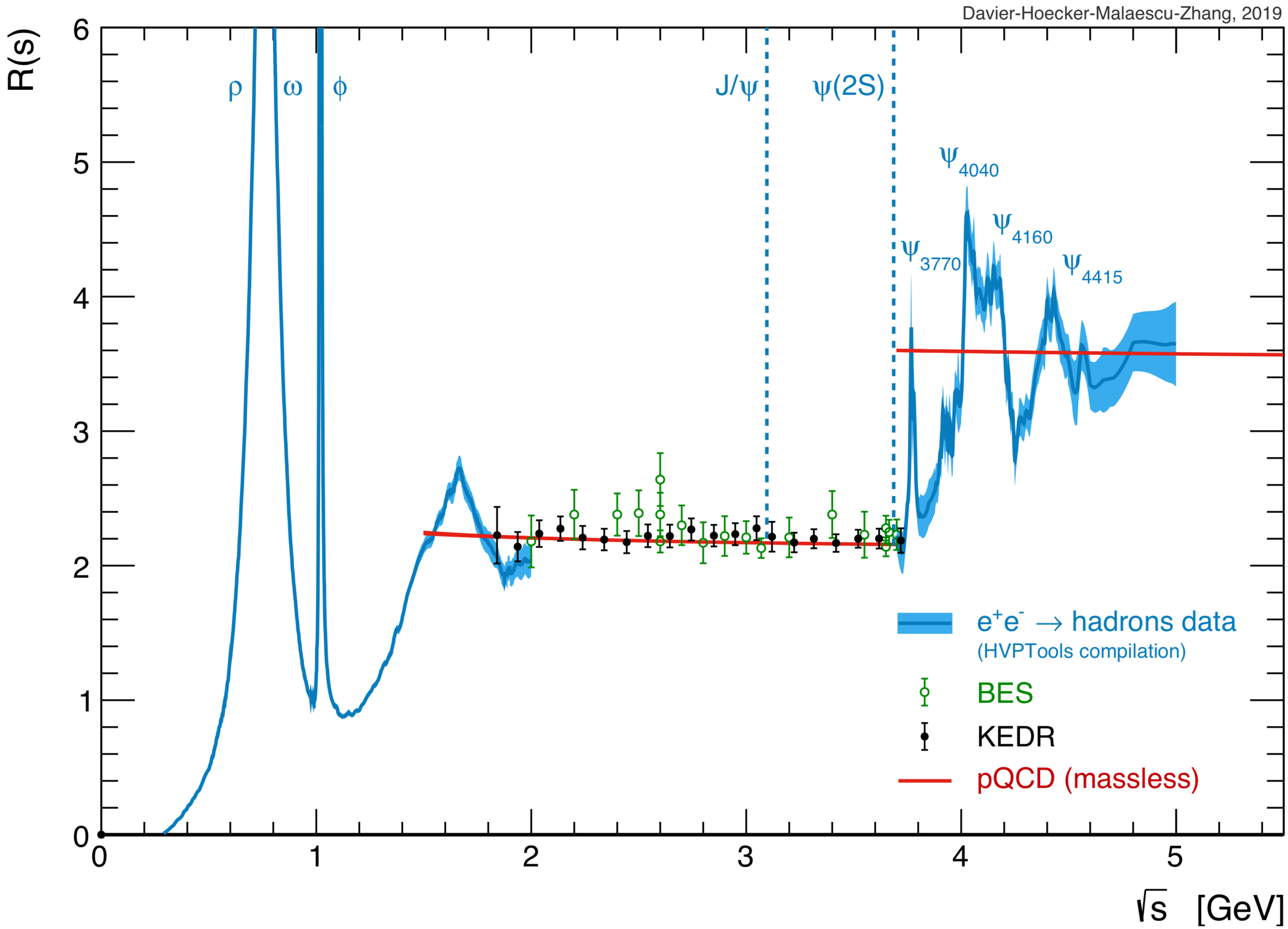


$$\left[\left(\frac{2}{3}\right)^2+\left(-\frac{1}{3}\right)^2+\cdots\right]\times 3=\frac{11}{3}$$

[Hyper Physics]

3 internal d.o.f : color SU(3)

R-ratio

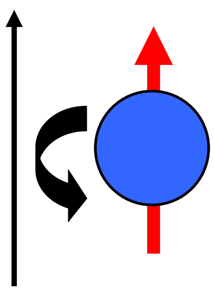


Davier-Hoecker-Malaescu-Zhang, 2019
R(s)
ρ
ω
ϕ
J/ψ
ψ(2S)
ψ 4040
ψ 4160
ψ 3770
ψ 4415
e+e- → hadrons data
(HVPTools compilation)
BES
KEDR
pQCD (massless)
√s [GeV]
0
1
2
3
4
5
6

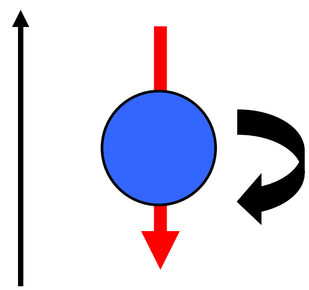
Quark spin & chiral symmetry

- Dirac equation：quark, electron have two degrees of freedom

  SPIN



Right handed



Left handed

- **Massive particles** （slower than speed of light）

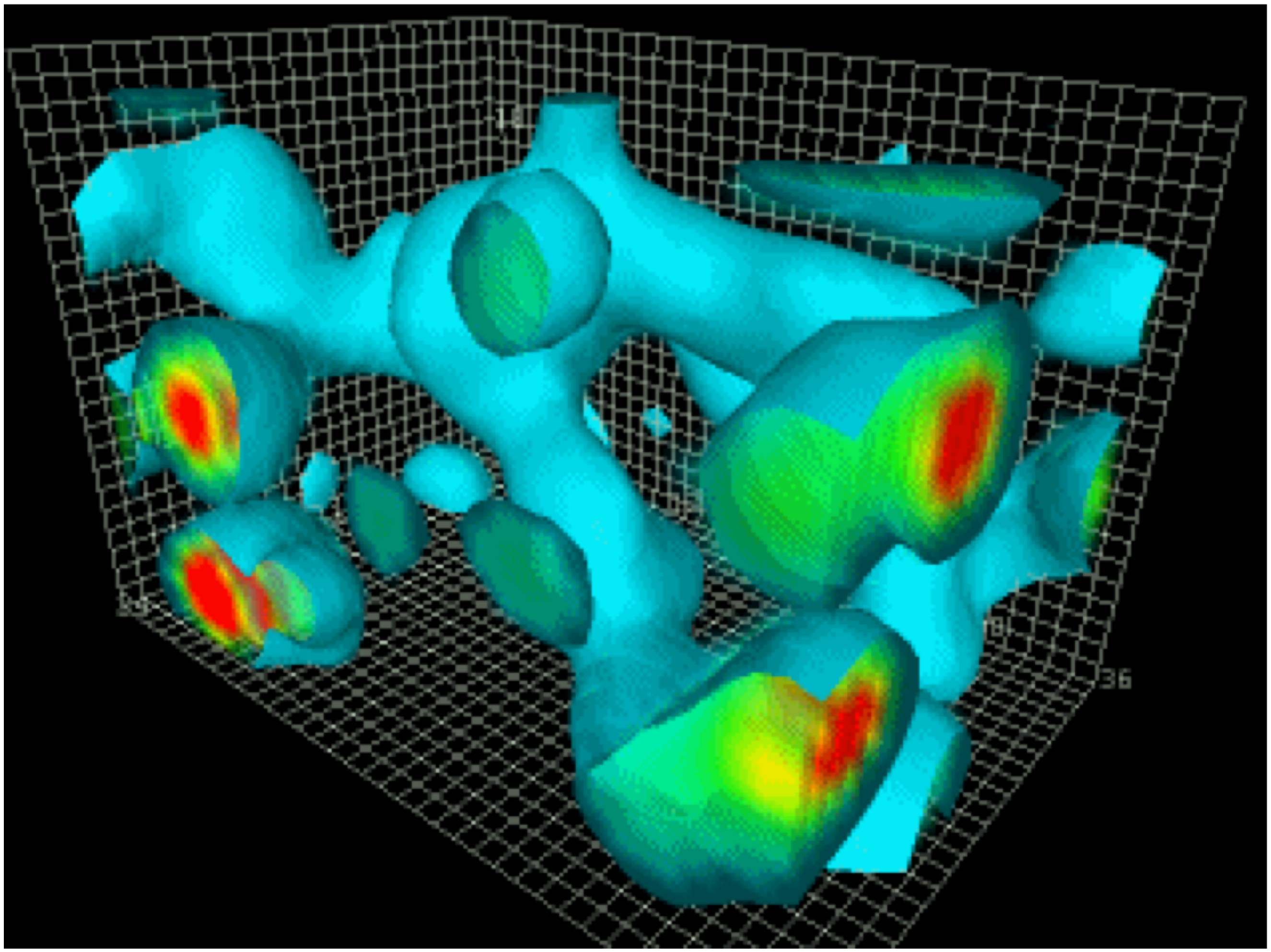
  ⇔ mixing between left and right spins

- **massless particles** （speed of light）

  ⇔ NO mixing between left and right spins

  Chiral symmetry

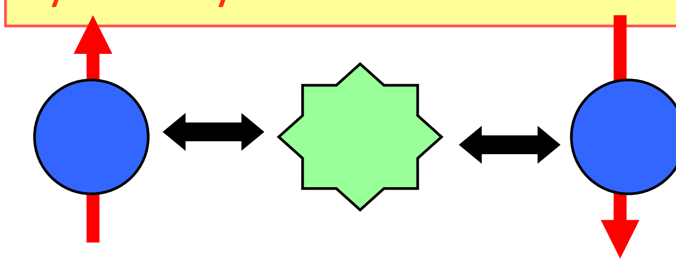
Fluctuation of QCD vacuum and spontaneous breaking of Chiral symmetry



QCD is creating / annihilating instantaneous
" lump" of chiral charges

induce effective mass for quark

quark "picks" this chiral charge, which could be seen as spontaneous breaking of chiral symmetry



(Courtesy of Derek Leinweber, CSSM, University of Adelaide)

Origin of mass

- QCD (T=0): quark and anti-quark creates a pair (chiral condensate) , which is energetically stable than nothing !

$$\langle \bar{q}q \rangle = [242(04)(18) \ \ \mathrm{MeV}]^3$$

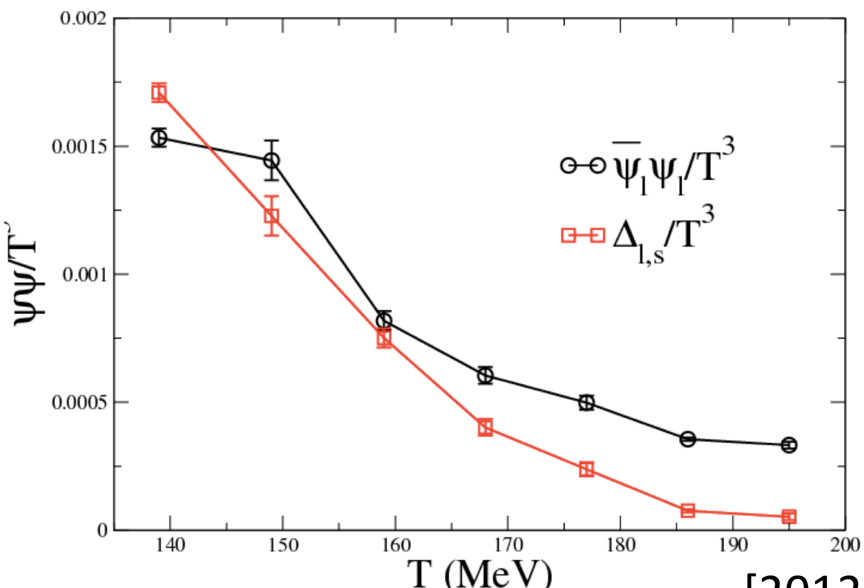
(2010 JLQCD)

- Produced quark an effective mass roughly about a third of proton masses

  3 x 0.25 GeV ~ 1 GeV

  c.f. up down quark mass ~ a few MeV

- At high temperature, chiral condensate decreases (Quark-Gluon Plasma)



[2012 HotQCD]

Y. Nambu

J. Goldstone

[CERN]

cf) Higgs mechanism, or superconductor

fermion action

Lattice fermion and doublers

$$S = \int d^4x \overline{\psi}(x) \not{D} \psi(x)$$

$\Downarrow$ on lattice

$$S = a^4 \sum_n \overline{\psi}_n \gamma_\mu \frac{1}{2a} \left(\psi_{n+\mu} - \psi_{n-\mu} \right)$$

$$S_F = a^4 \sum_x \bar{\psi}(x) \left(\not{D} + m_0 \right) \psi(x)$$

$$\not{D} = \frac{1}{2} \{ \gamma_\mu (\nabla_\mu^* + \nabla_\mu)$$

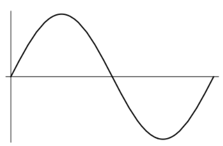
$$\nabla_\mu \psi(x) = \frac{1}{a} \left[U_\mu(x) \psi(x + a\hat{\mu}) - \psi(x) \right]$$

Wilson fermion (breaks chiral sym)

- Doubler problem

(Nielsen-Ninomiya no-go theorem)

$$S = \int_{-\pi}^{\pi} d^4p \overline{\psi}(-p) i\gamma_\mu \sin p_\mu \psi(p)$$



$$\not{D}(p) = i\gamma_\mu \sin p_\mu \rightarrow i\gamma_\mu p_\mu$$

: Particle

$$\not{D}(\pi - p) = i\gamma_\mu \sin(\pi - p_\mu) \rightarrow -i\gamma_\mu p_\mu$$

: Doublers

$\Downarrow$

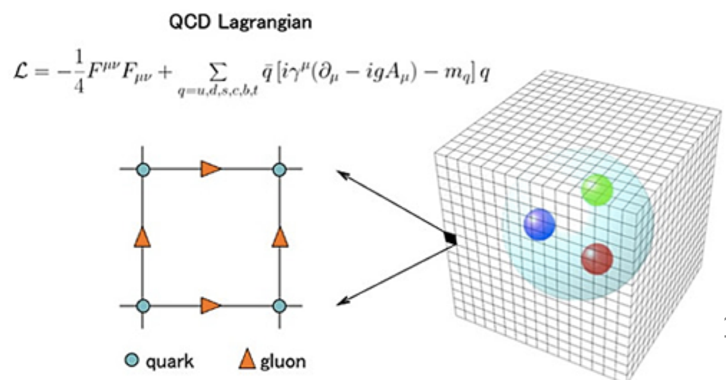
16 fermions. (4 dim.)

$$S = \sum_n \overline{\psi}_n \left[\gamma_\mu D_\mu - \frac{a}{2} D^2 \right] \psi_n,$$

$$S = \int d^4p \overline{\psi} \left[i\gamma_\mu \sin p_\mu + \frac{1}{a} \sum_\mu (1 - \cos p_\mu) \right] \psi.$$

$$\not{D}(p) \rightarrow i\gamma_\mu p_\mu + \mathcal{O}(a),$$

$$\not{D}(\pi - p) \rightarrow -i\gamma_\mu p_\mu + \frac{2}{a} + \mathcal{O}(a).$$



Lattice quark types

- Staggered-types [MILC, HPQCD, BMWc, ….]

Fast, high statistics, Highly Improved Staggered action (HISQ)

4 “taste”, taking [det D]1/4, Non-local action, Unitality ?

- 4D Wilson-types

[Alpha, PACS, BMWc, NPLQCD, PDNME, ….]

No taste problem, exact flavor symmetry, chiral symmetry broken, need to tune to remove O(a) effects (clover term)

Twisted Wilson [ETMC]

Fast, automatic O(a), breaks isospin symmetry

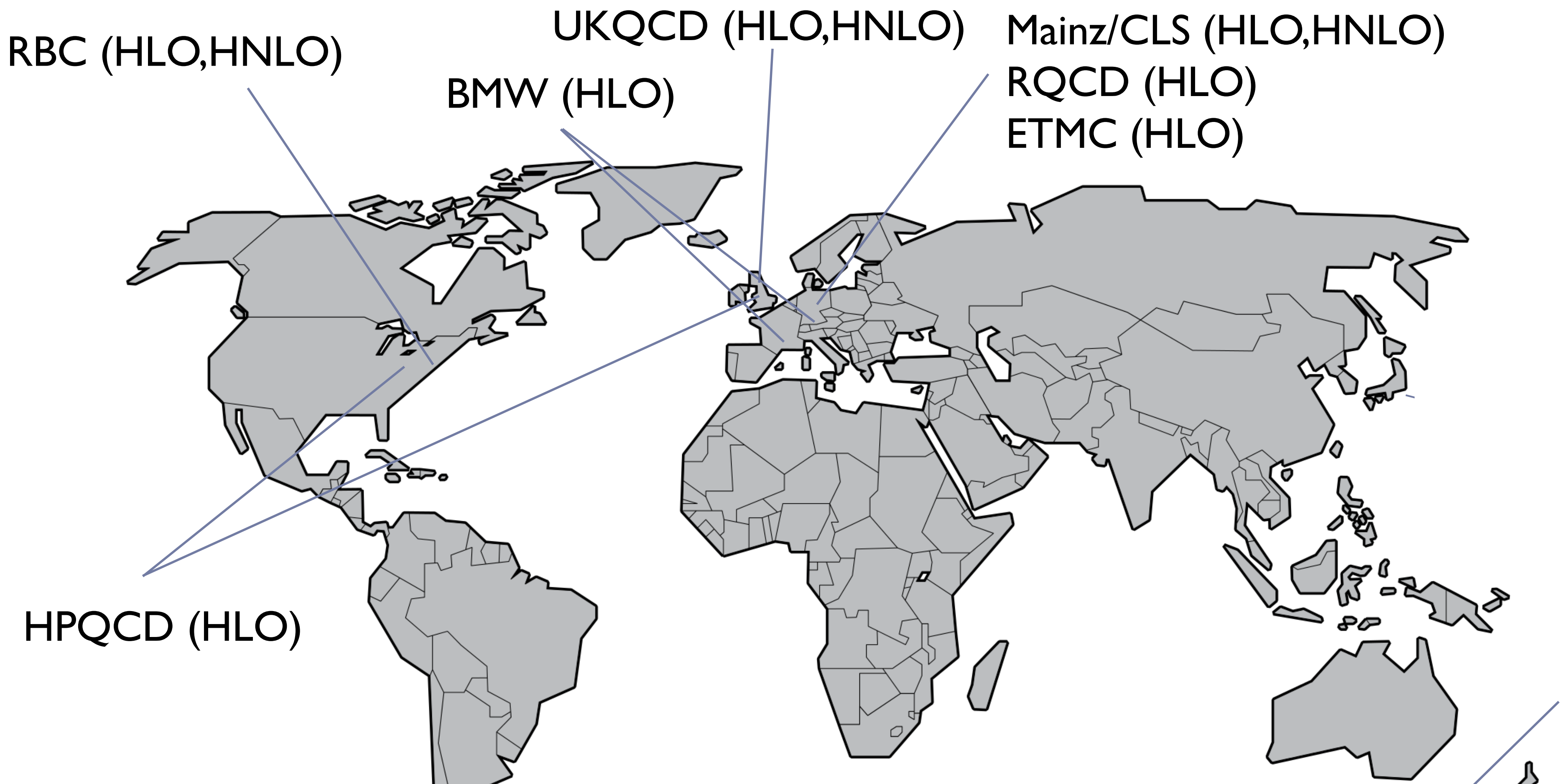
- Domain-Wall Fermion (DWF) / Overlap

[RBC/UKQCD, JLQCD, xQCD….]

(almost) exact chiral symmetry, automatic off-shell O(a) improved, expensive

1. Introduction
Lattice works

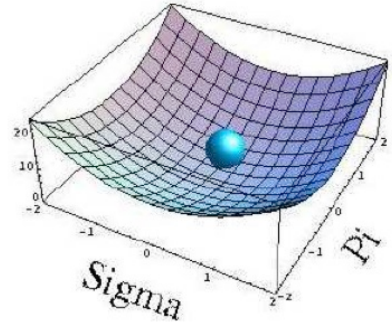
[Slide from Eigo Shintani]



HLO precision: < 5 % (maybe updated)
HNLO: ~ 10 % statistical accuracy

Domain Wall Quarks (for up, down, and strange)

[Kaplan, Shamir, Blum & Soni]

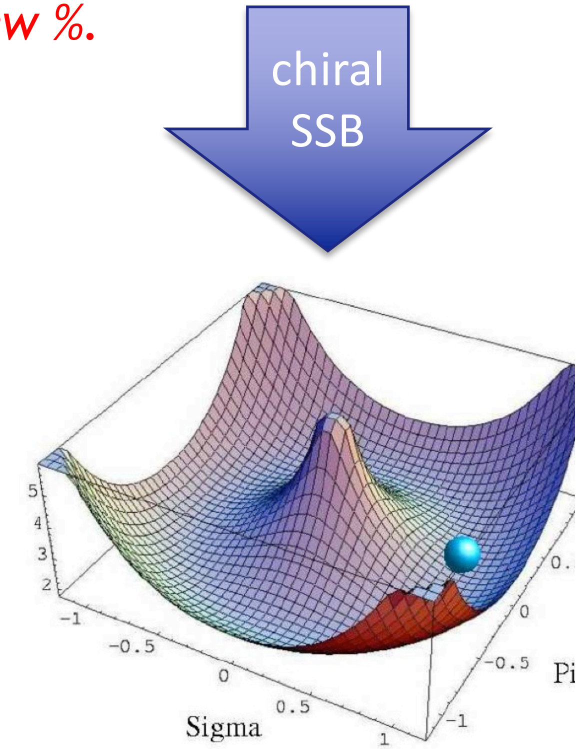


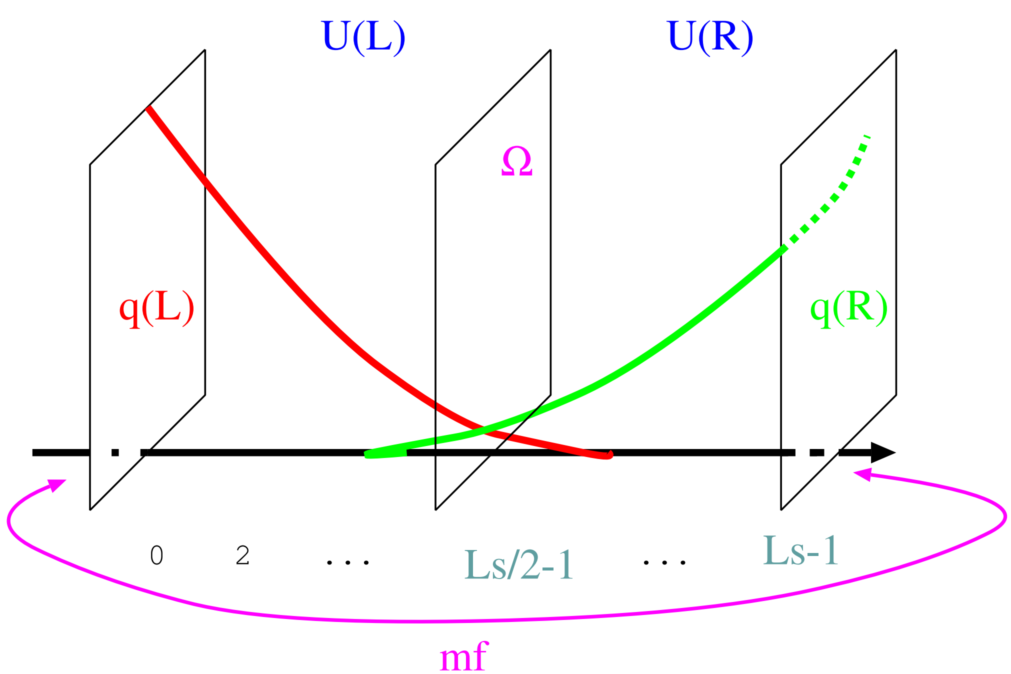
- 4D lattice quark utilizing an ''extra dimension'', L_s. (expensive)
- Almost perfect chiral symmetry

  Small unphysical mixing for the Weak Matrix Elements

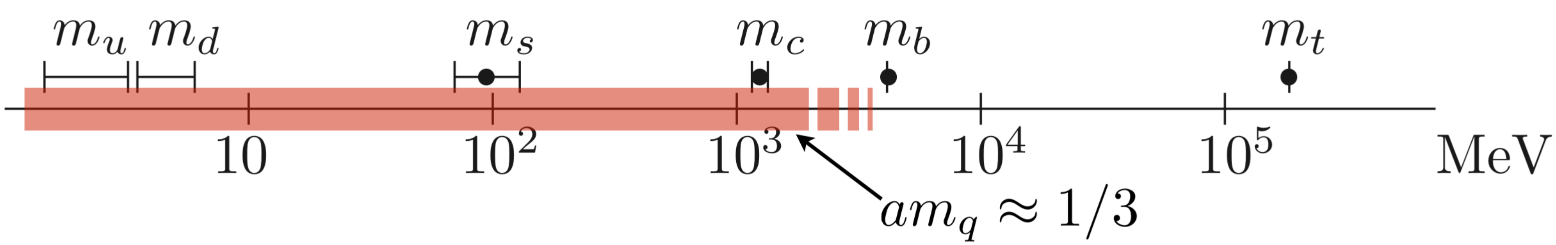
  Error from discretization is small $\mathcal{O}(a^2\Lambda^2_{QCD}) \sim$ *a few %.*

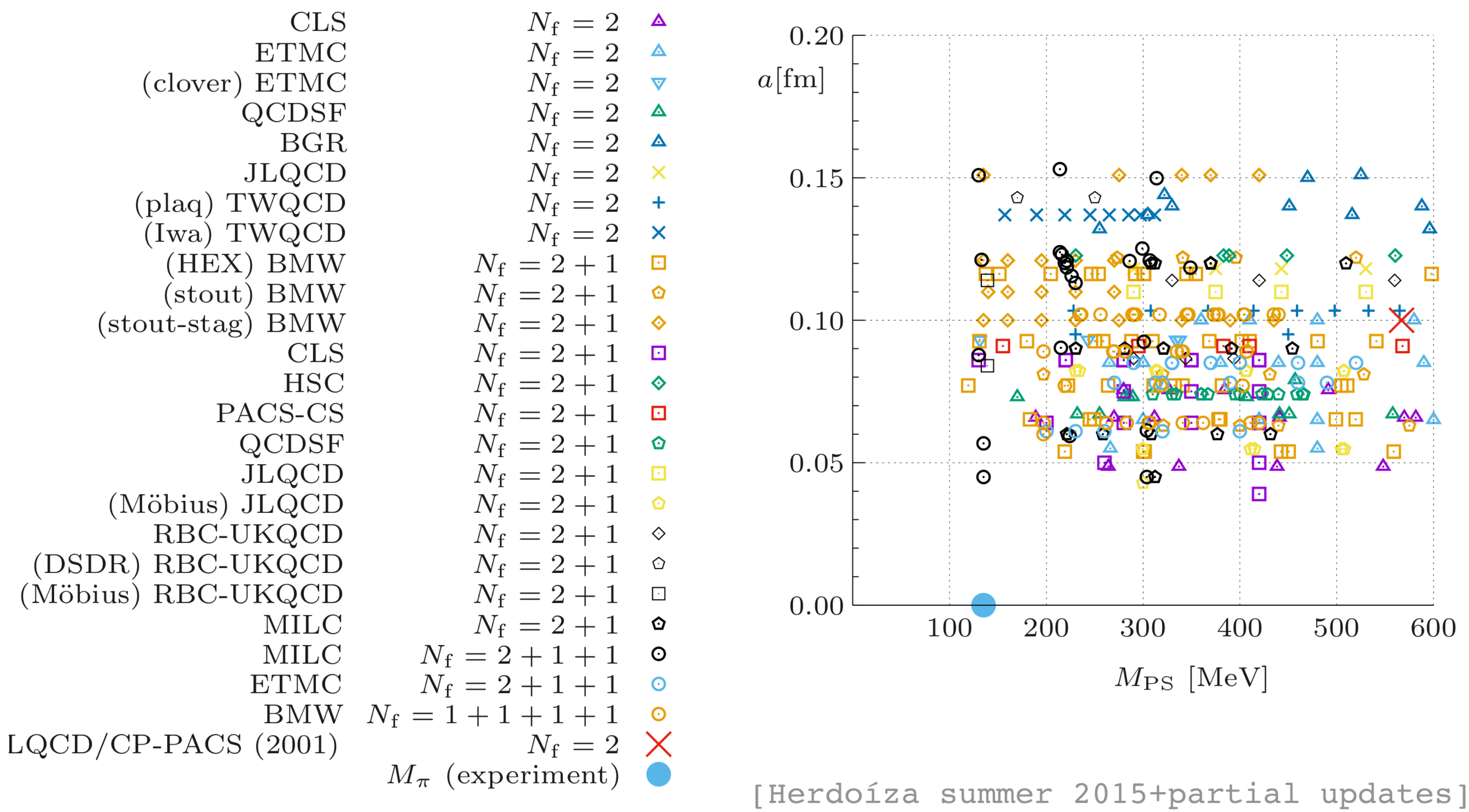
  Chiral extrapolation is simpler, continuum like.
- Unitary theory (at long distance).





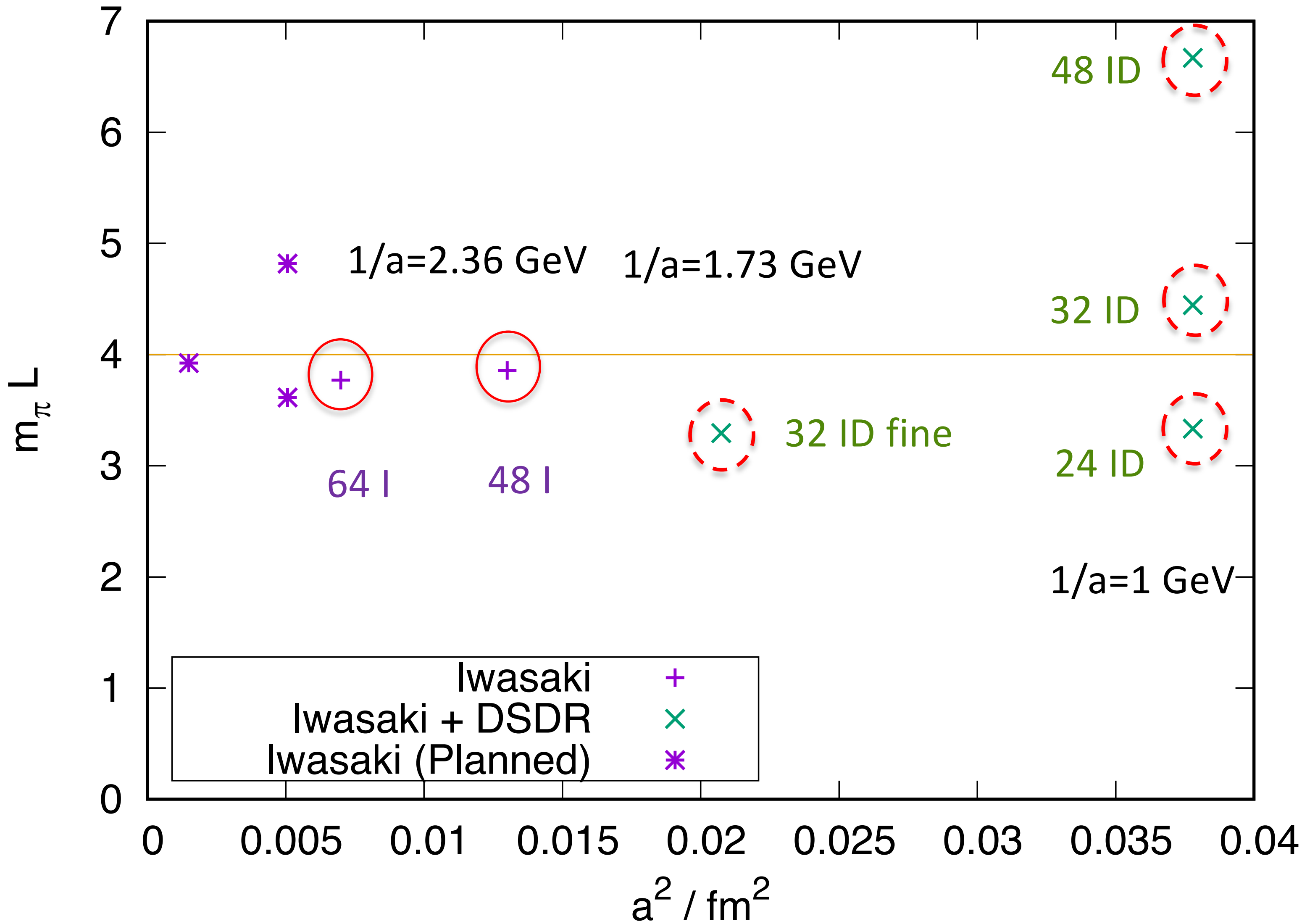
Origin of Proton mass
Emergence of NG boson





[C. Pena, Moriond EW 2018]

Nf=2+1 DWF QCD ensemble at physical quark mass



Lattice QCD calculation

- Solve QCD numerically, and enable direct comparison to experimental results
- Perform Feynman's Path Integral to compute quantum expectation value of an observable O

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}U_\mu(x) \mathcal{D}\psi(x) \mathcal{D}\bar{\psi}(x) e^{-S_G + \bar{\psi}(\not{D}+m)\psi} O[\psi, \bar{\psi}, U_\mu]$$

- To fit protons
  space-time : L ~ 5 fm,

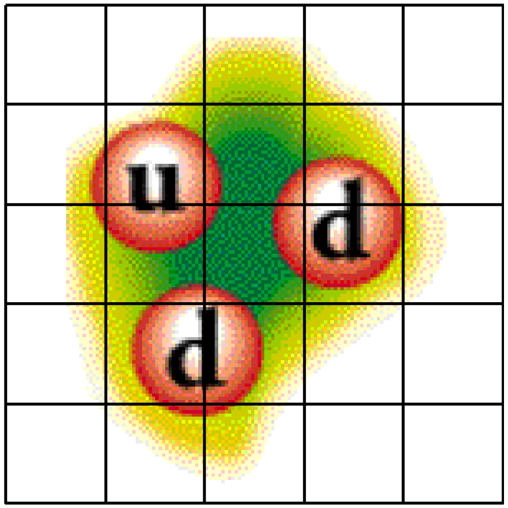
- To avoid large systematic error
  Lattice spacing : a ~ 0.1 fm

  → Integration over ~ $(50)^4$ x 4 x 8 ~ 100 Million variables

- To compute proton mass, typically needs, at least

  ~ 100-1,000 Exa floating point operations
  ~ 10^{20}- 10^{21} floating operations
  ~ 100 days on one rack of Blue Gene / Q
  ~ 100-1000 years on one desktop (if fits)

[© AICS, Y. Kuramashi]

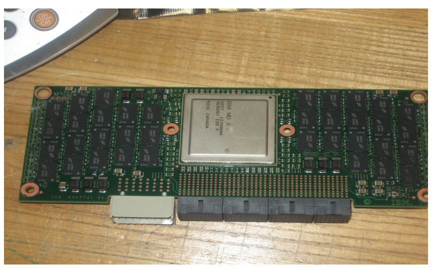


- QCD and SM is the first principle (known to date) : stable problem
- Need precise numerical results to confront big experimental results

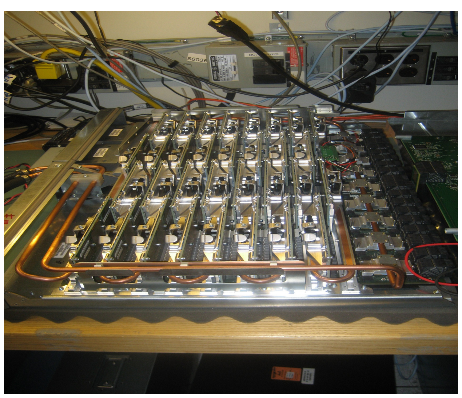
Lattice QCD super computers

- ~ massively parallel computer (1K-10K CPUs)
- ~1 - 10 PFLOPS (1-10 million times faster than PC)
- needs very fast interconnections

QCDCQ at BNL (pre-commercial Blue Gene / Q) 3.5 racks 0.7 Pflops total, 2012-

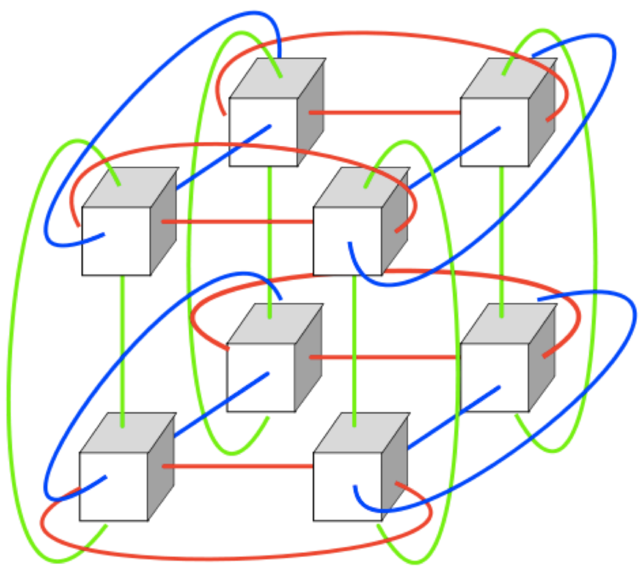


Compute card (16 cores + 16GBmemory



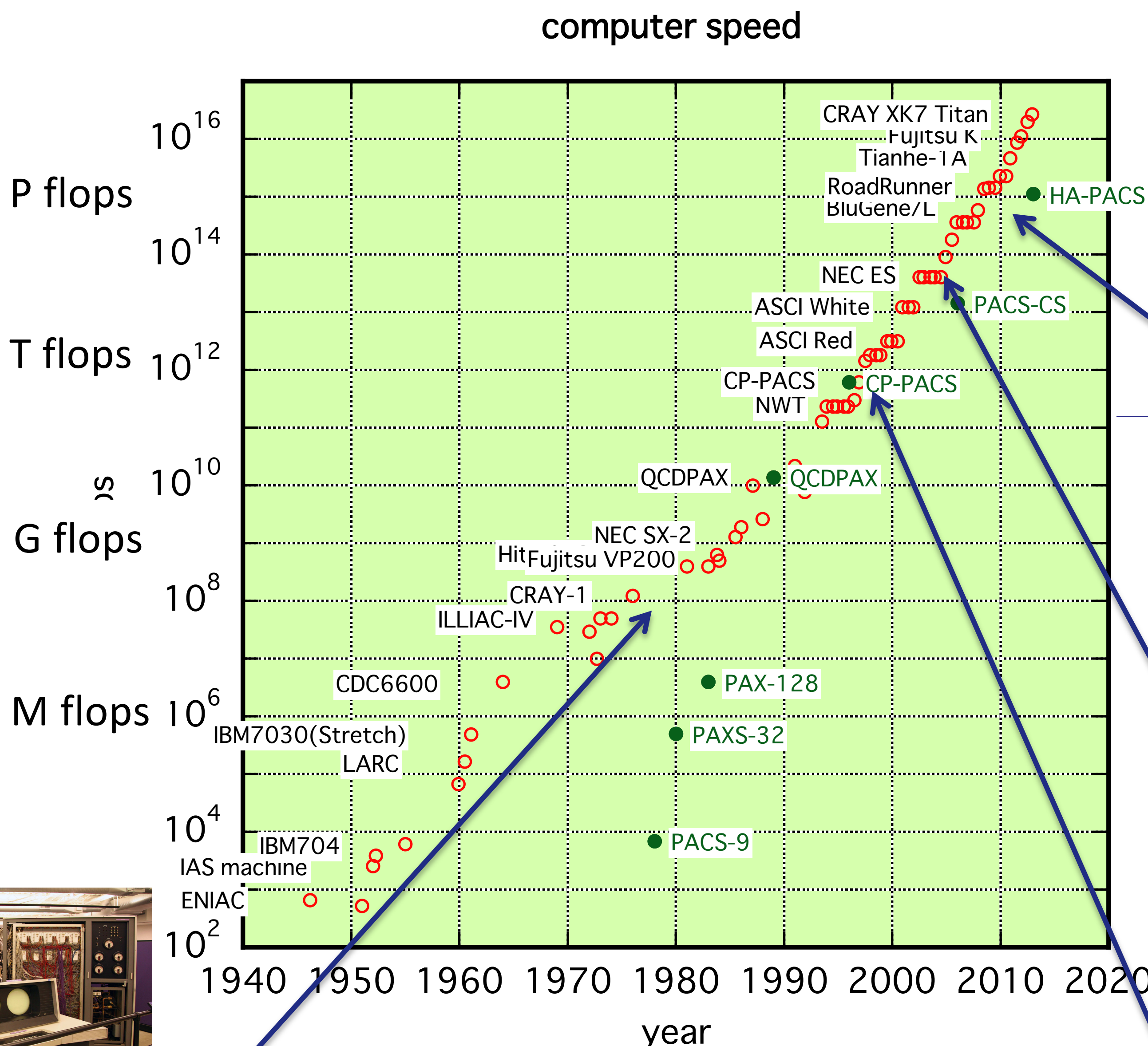
Node Card (32 compute cards, 5D optical link)

BNL, Columbia, RIKEN, U of Edinburgh
+ IBM Watson Lab.



Rack
(32 Node cards + I/O drawers)

pictures : BNL machines



QCDCQ 0.7 P flops 2012

QCDOC 20 T flops 2005

12 racks (1024 DB per rack) ~ 600GFlops @ BNL

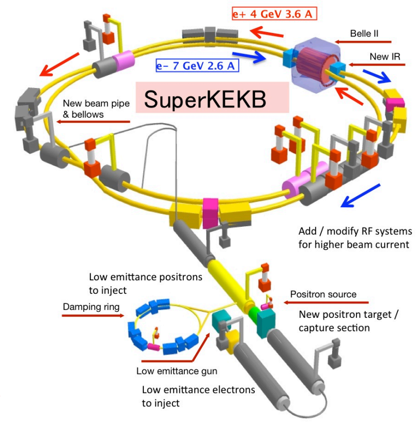
QCDSP 0.6T flops 1998

CDC 6600/7600 40 M flops 1970s

[A. Ukawa]

Lattice Gauge Theory Receipt

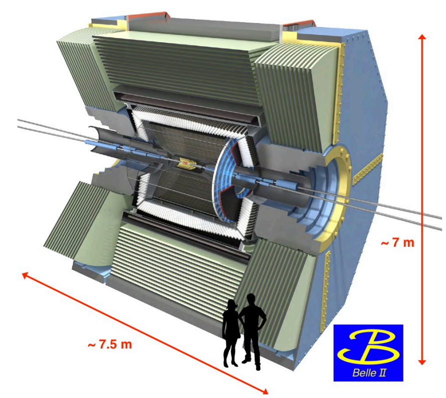
- QCD vacuum ensemble generation $\sim$ Accelerator
  - choice of gauge / sea quark actions
  - Algorithms / Machines **capability machines**
  - for each parameters $(a^{-1}, V, m^{\text{sea}})$



- Physical observable measurements $\sim$ Detector
  - valence quark propagators (low eigenvectors), m_f
  - Hadron n-point green's functions, matrix elements

$$\langle \mathcal{O} \rangle = \int \mathcal{D}U_\mu \mathcal{D}\bar{q}_i \mathcal{D}q_i \mathcal{O} e^{-S_{\text{LGT}}} / \langle 1 \rangle$$

  - Renormalize and Chiral/Volume/continuum extrapolations
  - Algorithms / Machines **capacity machines**



$\Longrightarrow$ The final answers

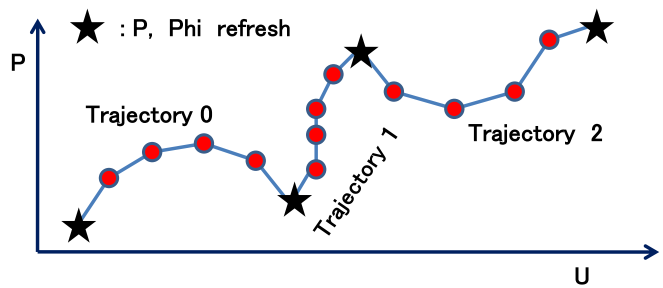
LGT ensemble generation

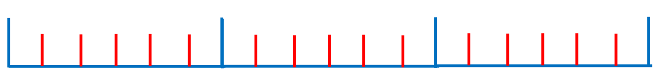
(<-> Ising : spin configuration generation)

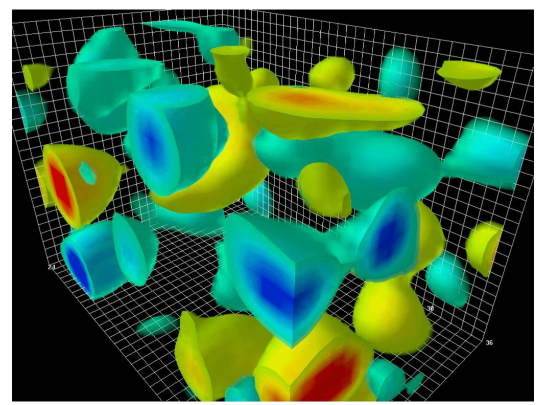
- Monte Carlo to Sample Important configurations of QCD action $e^{-S_{\text{QCD}}}$
- Accumulate samples of QCD vacuum, typically $\mathcal{O}(100) \sim \mathcal{O}(1,000)$ files of gauge configuration $U_\mu(n)$ on disk (1 $\sim$ 10 GB/conf).
- By solving a classical QCD, with an occasional stochastic ''hit'': exactly $\propto e^{-S_{\text{QCD}}}$
- Must generate sequentially $\{U_\mu^{(0)} \to U_\mu^{(1)} \to \cdots\}$, which needs capable machines.

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}U_\mu(x) \prod_{f=1}^{N_f} \det(D + m_f) e^{-S_G} O$$

$$\text{Prob}(U_\mu) \propto \det D_{u,d,s}[U] e^{-S_g}$$







[D. Leinweber]

- RHMC for odd flavor [Clark Kennedy]
- Solve short (long) modes more (less) frequently [Hasenbush's trick]

LGT observable measurement

(<->detector)

- Measurements physical observables on the vacuum ensemble.

$$\langle \mathcal{O} \rangle = \int \underline{\mathcal{D}U_\mu \, \text{Prob}[U_\mu]} \times \mathcal{O}[U_\mu]$$

- Could do Analysis on many configurations independently (trivial parallel jobs) $\longrightarrow$ could also use PC Clusters

- We made hadron operator (EW operators) from quark, and let the quark propagates on each of the generated QCD configuration (by solving the Dirac Eq)

- Obtain hadron mass or QCD matrix elements of operators

Euclidean 2pt Green function

O: hadron operator of target quantum #

$$C_{XY}(t;\vec{p}) = \sum_{\vec{x}} \langle O_X(t,\vec{x}) O_Y^\dagger(0) \rangle e^{-i\vec{p}\cdot\vec{x}}.$$

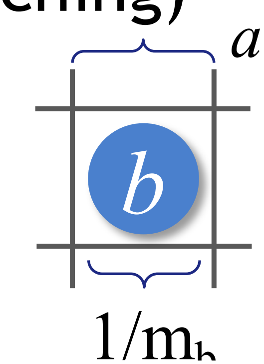
$$C_{XY}(t) \overset{t\to\infty}{\longrightarrow} \frac{1}{2E_0(\vec{p})} \langle 0|O_X(0)|H_0(\vec{p})\rangle \langle H_0(\vec{p})|O_Y^\dagger(0)|0\rangle e^{-E_0(\vec{p})t}$$

$$C_{XY}(t;\vec{p}) = \sum_{i=0}^{\infty} \frac{1}{2E_i(\vec{p})} \langle 0|O_X(0)|H_i(\vec{p})\rangle \langle H_i(\vec{p})|O_Y^\dagger(0)|0\rangle e^{-E_i(\vec{p})t}$$

Input parameters

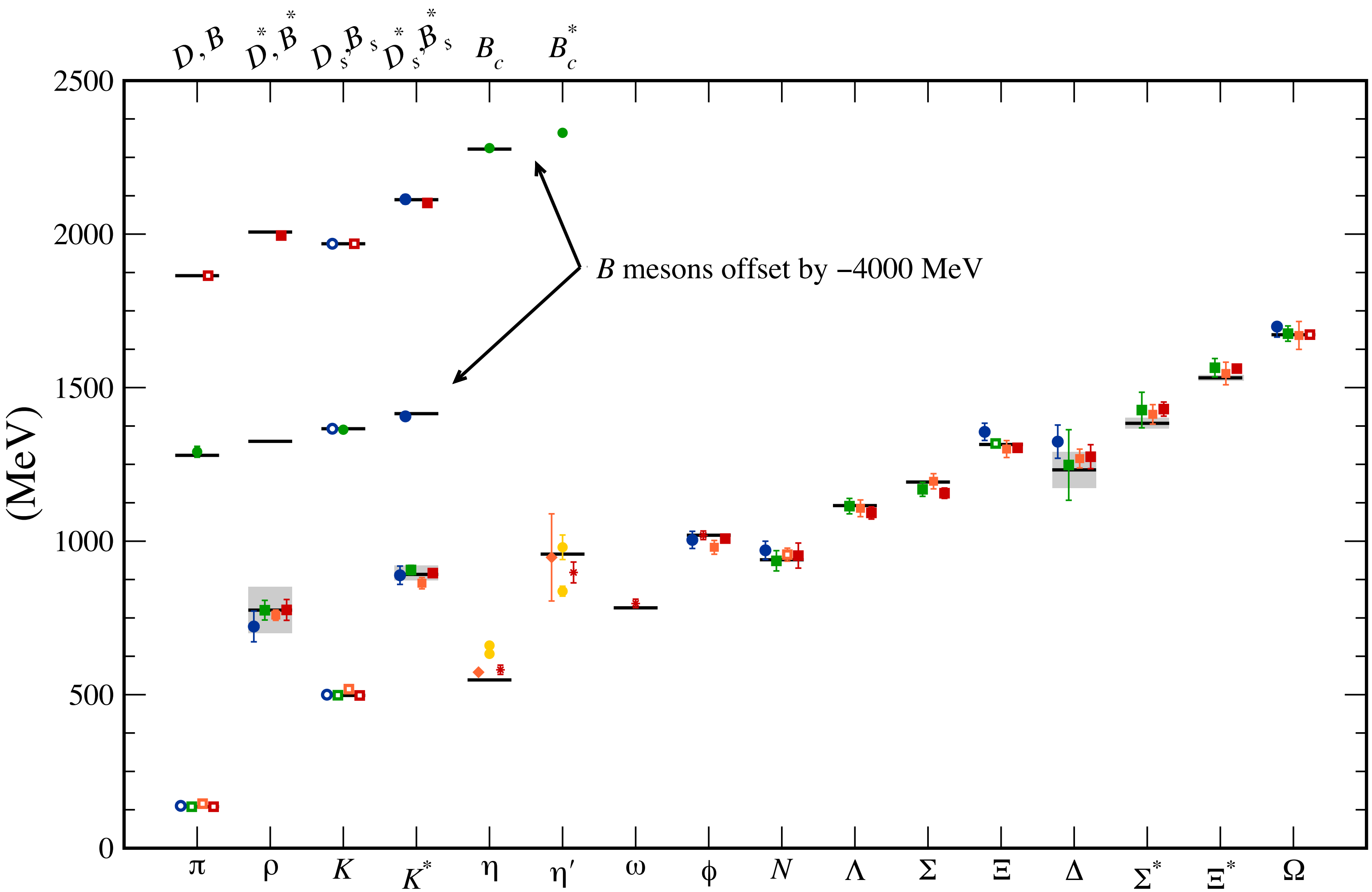
- Same as the continuum theory (coupling, quark mass)
- coupling (lattice spacing a): for given ß , measure one hadron mass (e.g. Omega baryon) (a mΩ) = a x 1.672.45 GeV
- quark mass m_f : measure hadron masses (e.g. charged/neutral π and K)

  $a\, M_\pi(\, a\, m_u,\ a\, m_d\ ) = a \times 0.139\,6$ GeV
- Bottom (& charm) quark needs special action for a large $a\, m_b$ $\rightarrow$ effective quark actions HQET, NRQCD, Fermilab action (needs matching)



APPLICATION – SPECTRUM -

Lattice calculation vs experiments : hadron masses



Input from experiment masses of π, K, Ω

to set

1. up/down quark mass
2. strange quark mass
3. coupling strength
   (or lattice spacing)

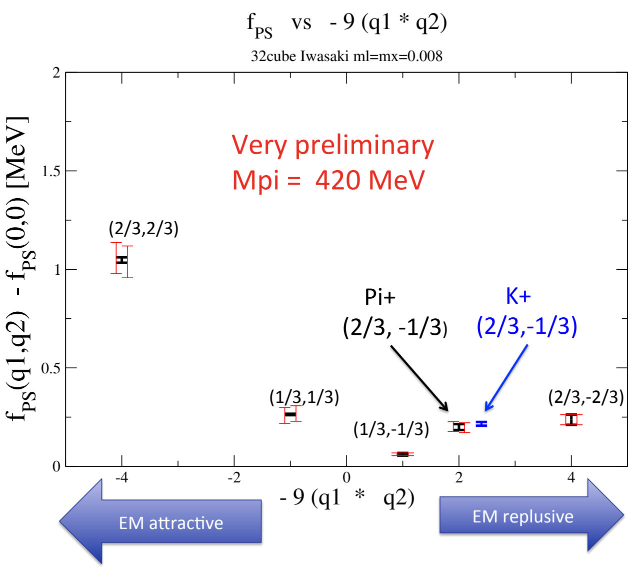
π…Ω: BMW, MILC, PACS-CS, QCDSF; η-η′: RBC, UKQCD, Hadron Spectrum (ω); *D*, *B*: Fermilab, HPQCD, Mohler-Woloshyn

[Kronfeld, 2012]

QCD+QED simulation

[T. Blum et al.]

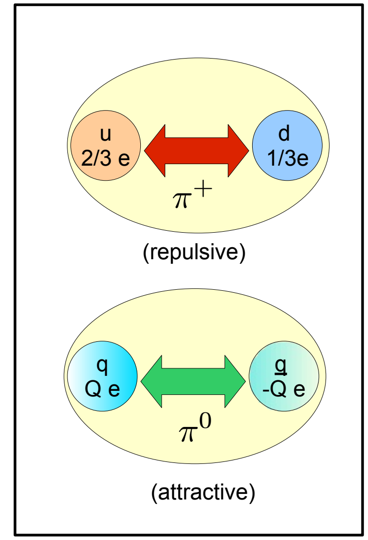
- EM effects on PS decay



- Statistically well resolved by +e/-e averaging.
- c.f. [Bijnens Danielsson 2006]

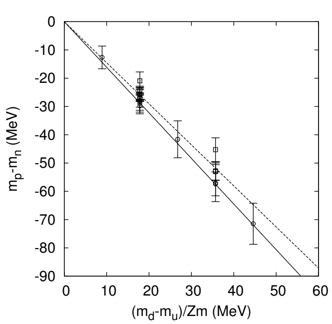
$$f_{\pi^+,\mathrm{NLO}}/F_0 = 0.0039$$
$$f_{K^+,\mathrm{NLO}}/F_0 = 0.0056$$

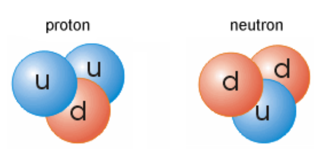
- EM turned on, but mu = md
- Iwasaki-DWF Nf=2+1,
- (2.7 fm)^3, a^{-1} ~ 2.3 GeV



- Proton / Neutron mass difference

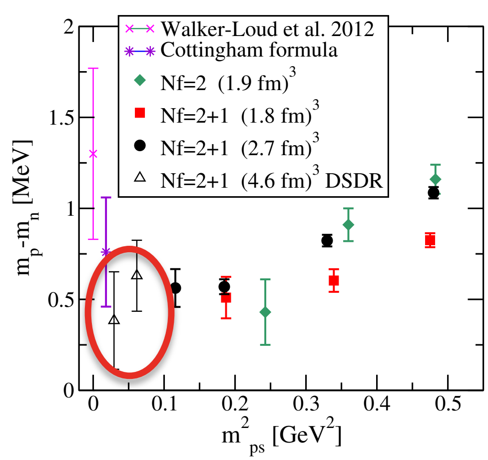
(m_u-m_d) effect





DSDR DWF Nf=2+1
(4.6 fm)^3,
a^{-1} ~ 1.4 GeV

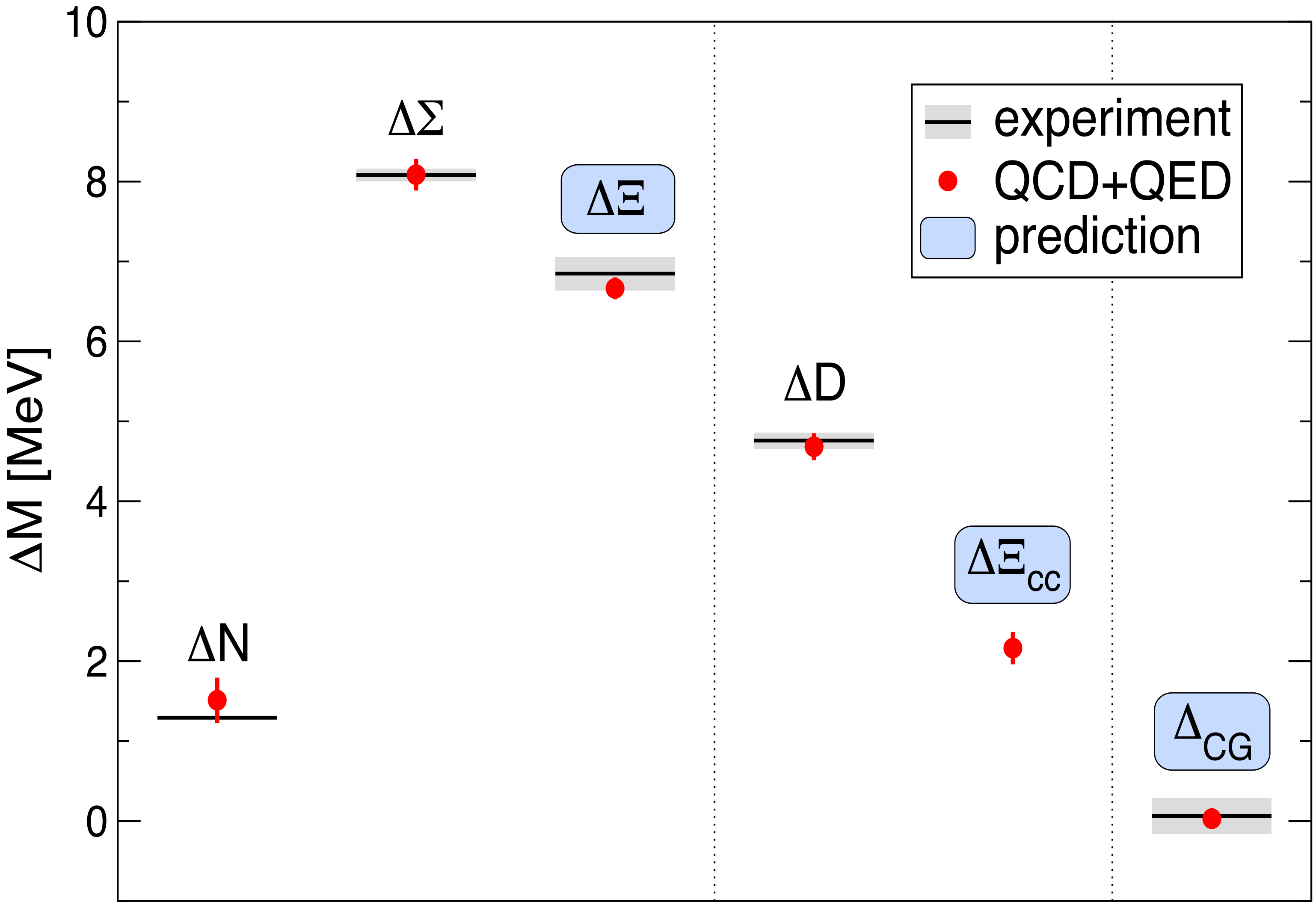
EM effect



	$m_u - m_d$	EM
NPLQCD	2.26(72)	
BLUM	2.51(71)	0.54(24)
RM123	2.80(70)	
QCDSF-UKQCD	3.13(77)	
	2.68(35)	0.54(24)

$$\Longrightarrow M_N - M_p| = 2.14(42) \;\; \mathrm{MeV}$$
$$(\mathrm{experiment:}\;\; 1.2933321(4)\;\mathrm{MeV})$$

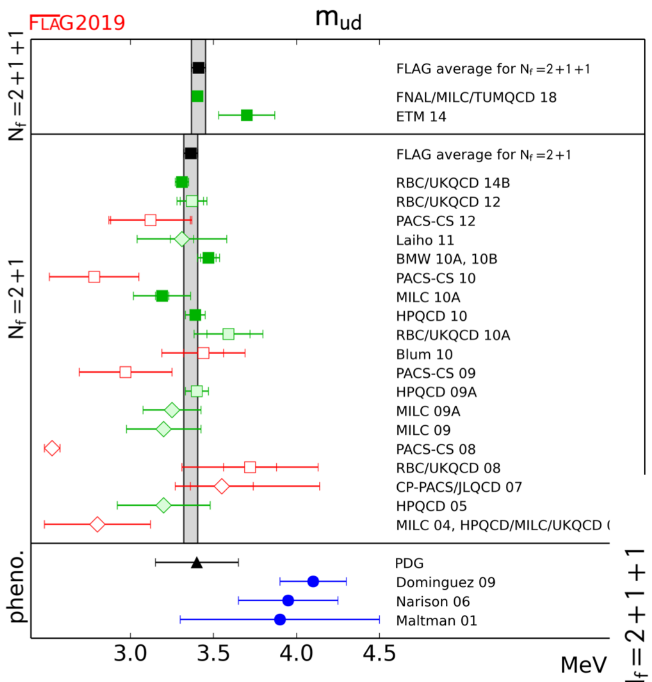
Isospin-breaking spectrum

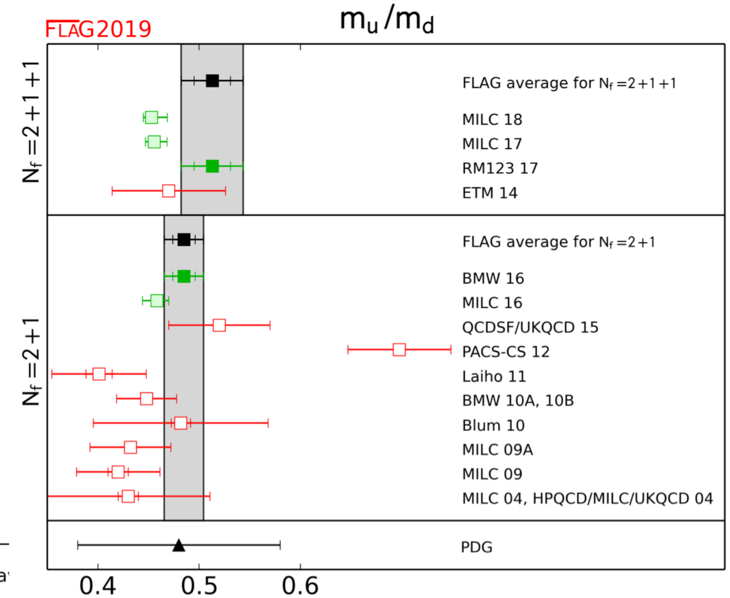


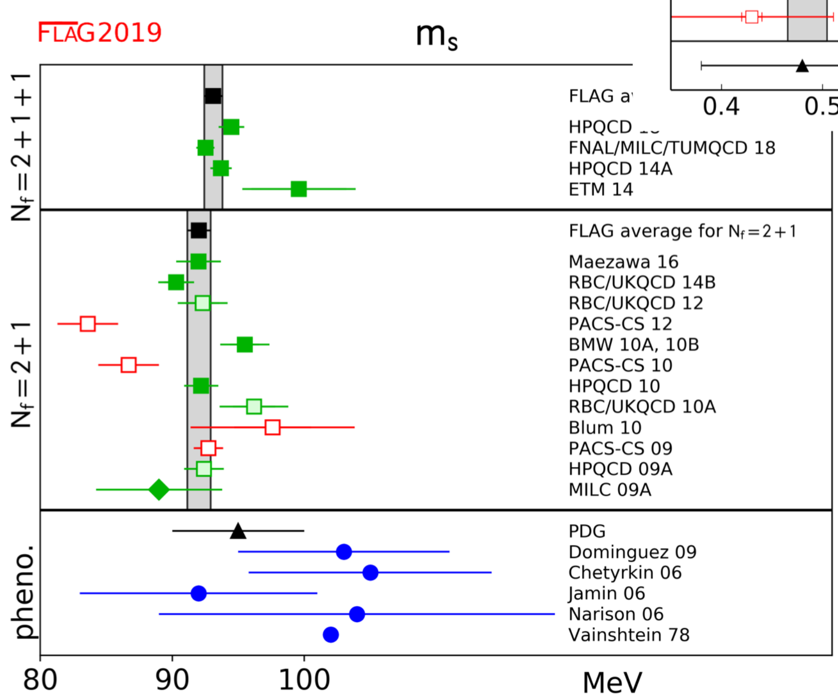
BMWc arXiv:1406.4088

quark mass results

- FLAG (Flavor Lattice Averaging Group)







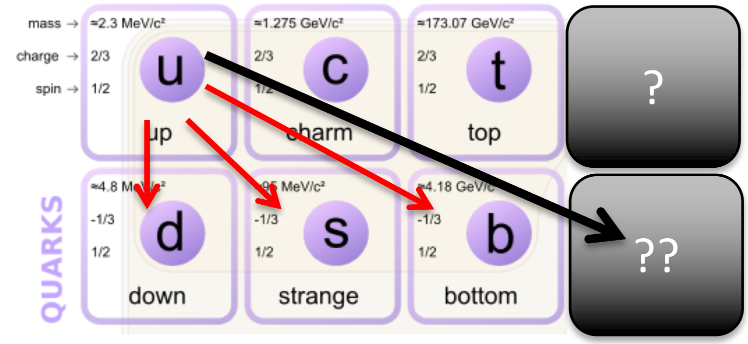
rating for systematic errors

Collaboration	Ref.	publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalization	running	m_{ud}	m_s
Maezawa 16	[33]	A	★	★	★	★	d	–	92.0(1.7)
RBC/UKQCD 14B$^{\ominus}$	[34]	A	★	★	★	★	d	3.31(4)(4)	90.3(0.9)(1.0)
RBC/UKQCD 12$^{\ominus}$	[32]	A	★	○	★	★	d	3.37(9)(7)(1)(2)	92.3(1.9)(0.9)(0.4)(0.8)
PACS-CS 12*	[35]	A	★	■	■	★	b	3.12(24)(8)	83.60(0.58)(2.23)
Laiho 11	[36]	C	○	★	★	○	–	3.31(7)(20)(17)	94.2(1.4)(3.2)(4.7)
BMW 10A, 10B^{+}	[37, 38]	A	★	★	★	★	c	3.469(47)(48)	95.5(1.1)(1.5)
PACS-CS 10	[39]	A	★	■	■	★	b	2.78(27)	86.7(2.3)
MILC 10A	[40]	C	○	★	★	○	–	3.19(4)(5)(16)	–
HPQCD 10**	[41]	A	○	★	★	–	–	3.39(6)	92.2(1.3)
RBC/UKQCD 10A	[42]	A	○	○	★	★	a	3.59(13)(14)(8)	96.2(1.6)(0.2)(2.1)
Blum 10†	[9]	A	○	■	○	★	–	3.44(12)(22)	97.6(2.9)(5.5)
PACS-CS 09	[43]	A	★	■	■	★	b	2.97(28)(3)	92.75(58)(95)
HPQCD 09A$^{\oplus}$	[44]	A	○	★	★	–	–	3.40(7)	92.4(1.5)
MILC 09A	[45]	C	○	★	★	○	–	3.25 (1)(7)(16)(0)	89.0(0.2)(1.6)(4.5)(0.1)
MILC 09	[46]	A	○	★	★	○	–	3.2(0)(1)(2)(0)	88(0)(3)(4)(0)
PACS-CS 08	[47]	A	★	■	■	■	–	2.527(47)	72.72(78)
RBC/UKQCD 08	[48]	A	○	■	★	★	–	3.72(16)(33)(18)	107.3(4.4)(9.7)(4.9)
CP-PACS/ JLQCD 07	[49]	A	■	★	★	■	–	$3.55(19)(^{+56}_{-20})$	$90.1(4.3)(^{+16.7}_{-4.3})$
HPQCD 05	[50]	A	○	○	○	○	–	$3.2(0)(2)(2)(0)^{\ddagger}$	$87(0)(4)(4)(0)^{\ddagger}$
MILC 04, HPQCD/ MILC/UKQCD 04	[51, 52]	A	○	○	○	■	–	2.8(0)(1)(3)(0)	76(0)(3)(7)(0)

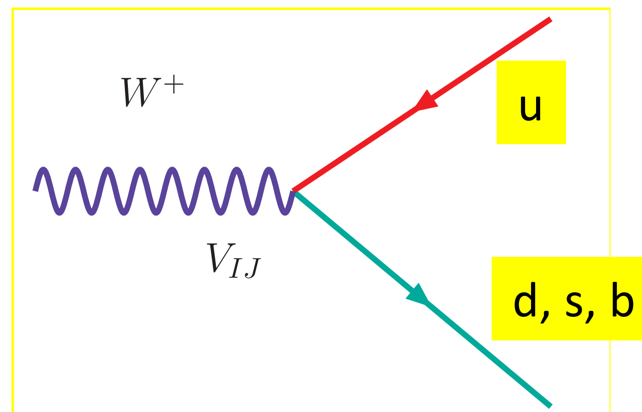
– *For QCD:*

- ★ $[M_{\pi,\mathrm{min}}/M_{\pi,\mathrm{fid}}]^2 \exp\{4 - M_{\pi,\mathrm{min}}[L(M_{\pi,\mathrm{min}})]_{\mathrm{max}}\} < 1$, or at least three volumes
- ○ $[M_{\pi,\mathrm{min}}/M_{\pi,\mathrm{fid}}]^2 \exp\{3 - M_{\pi,\mathrm{min}}[L(M_{\pi,\mathrm{min}})]_{\mathrm{max}}\} < 1$, or at least two volumes
- ■ otherwise

Quark flavor physics



- Cabibbo, Kobayashi and Maskawa (CKM) advocated quark "flavor" could change by emitting W boson (flavor mixing).
- up quark could change into the three down type quarks with mixing amplitude, called CKM matrix element V(ui)
  - down : V(ud) ,
  - strange : V(us),
  - bottom : V(ub)

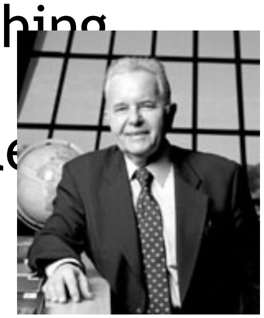


- Due to preservation of total probability,

$$|V(ud)|^2 + |V(us)|^2 + |V(ub)|^2 = 1$$

(CKM unitarity condition)

- If this equation is not fulfilled, something new and unknown, such as 7th quark (or 4th generation) may be responsible



Sub-percent accuracy on Physical point

- charged π and K meson decay into muon and neutrino

$$\Gamma(P \to \ell\nu) = \frac{G_F^2}{8\pi} f_P^2 \, m_\ell^2 M_P \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2 \left|V_{q_1 q_2}\right|^2$$

(experiment)

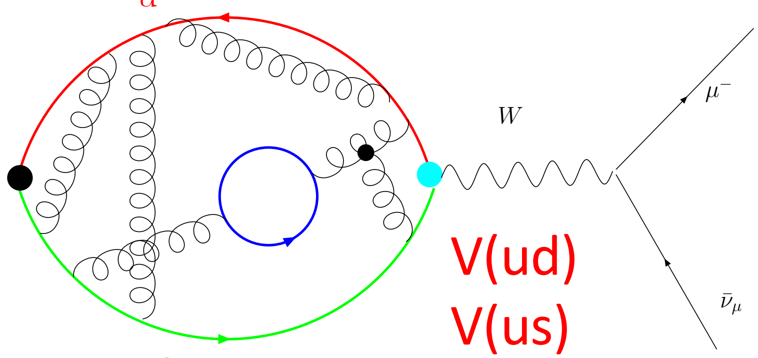
$$\langle 0|\bar{q}_1 \gamma_\mu \gamma_5 q_2|P(p)\rangle = i p_\mu f_P$$

(lattice)

$\pi^+ \to \mu^+ + \nu_\mu$

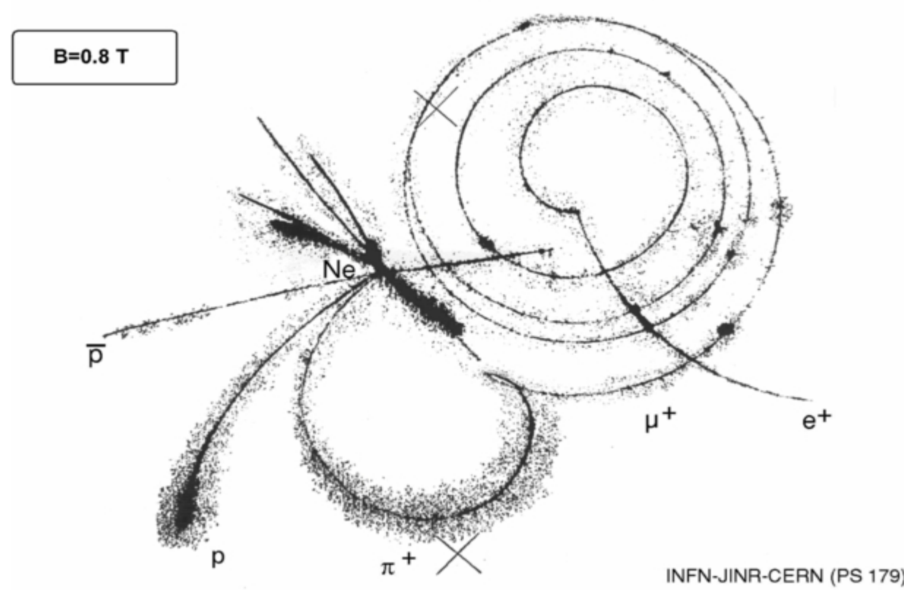
$K^+ \to \mu^+ + \nu_\mu$

u



V(ud)
V(us)

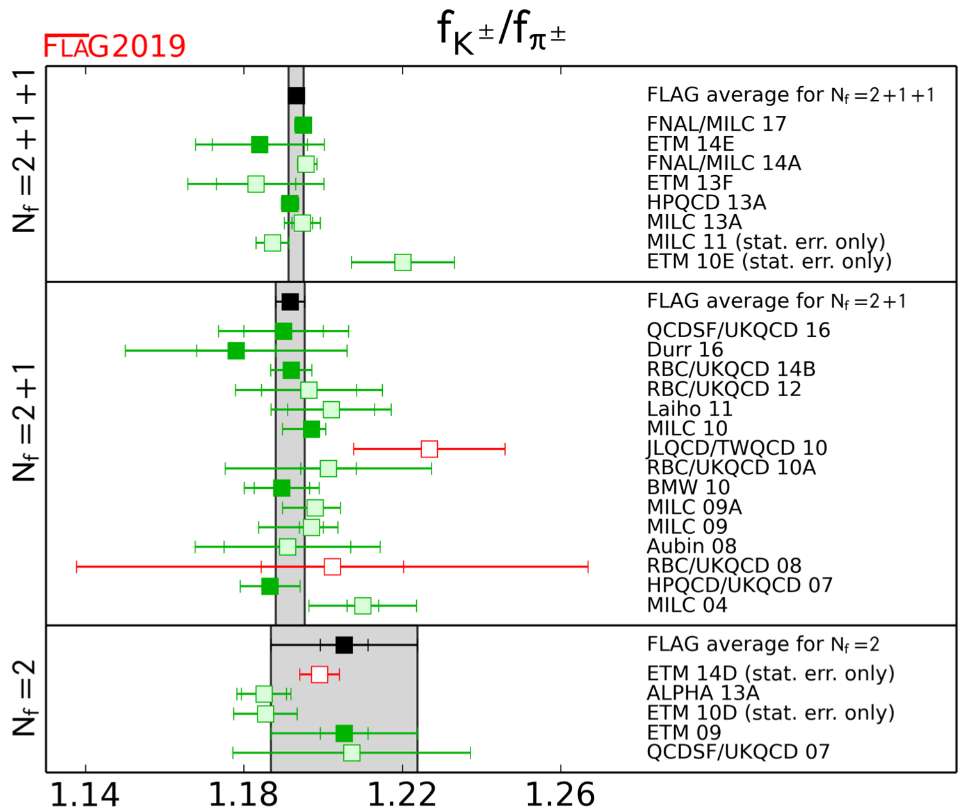
d
s

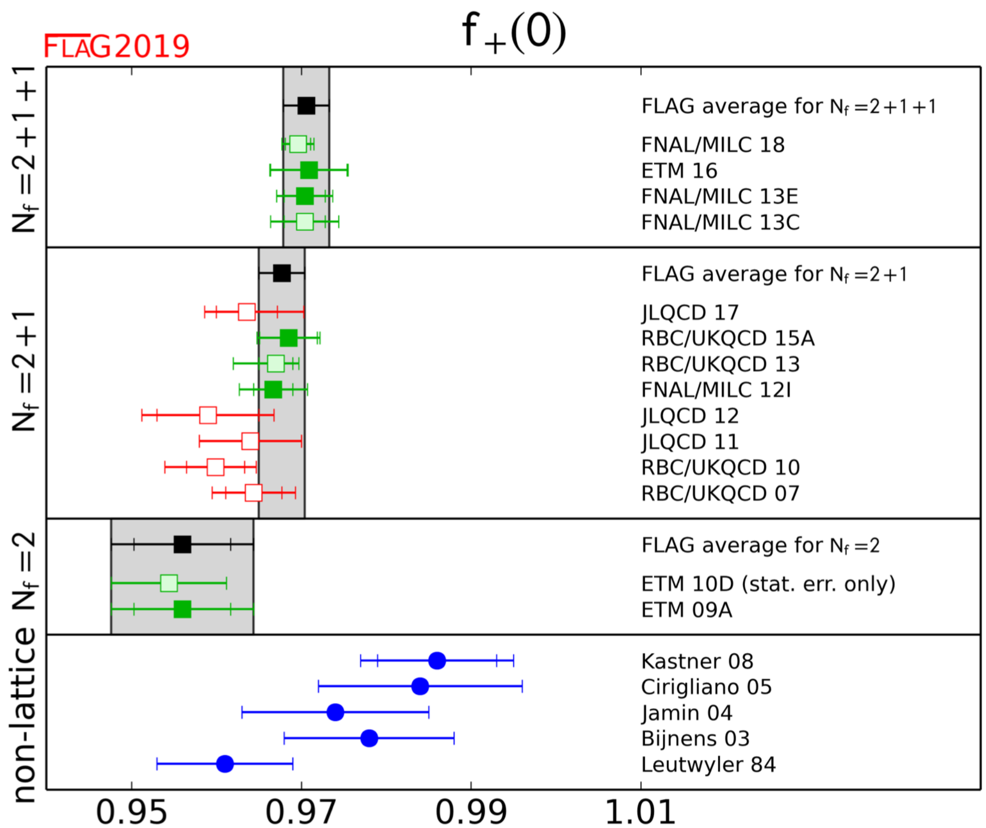


$$\left|\frac{V_{us}}{V_{ud}}\right| \frac{f_K}{f_\pi} = 0.27673\,(29)_{\text{exp}}\,(23)_{\text{th}}$$

Experiment(FLAG2019)

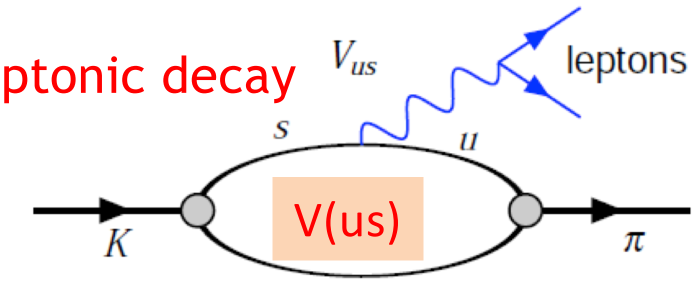
2004 Marciano


FLAG2019
$f_{K^\pm}/f_{\pi^\pm}$
$N_f=2+1+1$
FLAG average for $N_f=2+1+1$
FNAL/MILC 17
ETM 14E
FNAL/MILC 14A
ETM 13F
HPQCD 13A
MILC 13A
MILC 11 (stat. err. only)
ETM 10E (stat. err. only)
$N_f=2+1$
FLAG average for $N_f=2+1$
QCDSF/UKQCD 16
Durr 16
RBC/UKQCD 14B
RBC/UKQCD 12
Laiho 11
MILC 10
JLQCD/TWQCD 10
RBC/UKQCD 10A
BMW 10
MILC 09A
MILC 09
Aubin 08
RBC/UKQCD 08
HPQCD/UKQCD 07
MILC 04
$N_f=2$
FLAG average for $N_f=2$
ETM 14D (stat. err. only)
ALPHA 13A
ETM 10D (stat. err. only)
ETM 09
QCDSF/UKQCD 07
1.14
1.18
1.22
1.26


FLAG2019
$f_+(0)$
$N_f=2+1+1$
FLAG average for $N_f=2+1+1$
FNAL/MILC 18
ETM 16
FNAL/MILC 13E
FNAL/MILC 13C
$N_f=2+1$
FLAG average for $N_f=2+1$
JLQCD 17
RBC/UKQCD 15A
RBC/UKQCD 13
FNAL/MILC 12I
JLQCD 12
JLQCD 11
RBC/UKQCD 10
RBC/UKQCD 07
$N_f=2$
FLAG average for $N_f=2$
ETM 10D (stat. err. only)
ETM 09A
non-lattice
Kastner 08
Cirigliano 05
Jamin 04
Bijnens 03
Leutwyler 84
0.95
0.97
0.99
1.01

A stringent check of Standard Model

- Together with similar analysis on K meson semileptonic decay



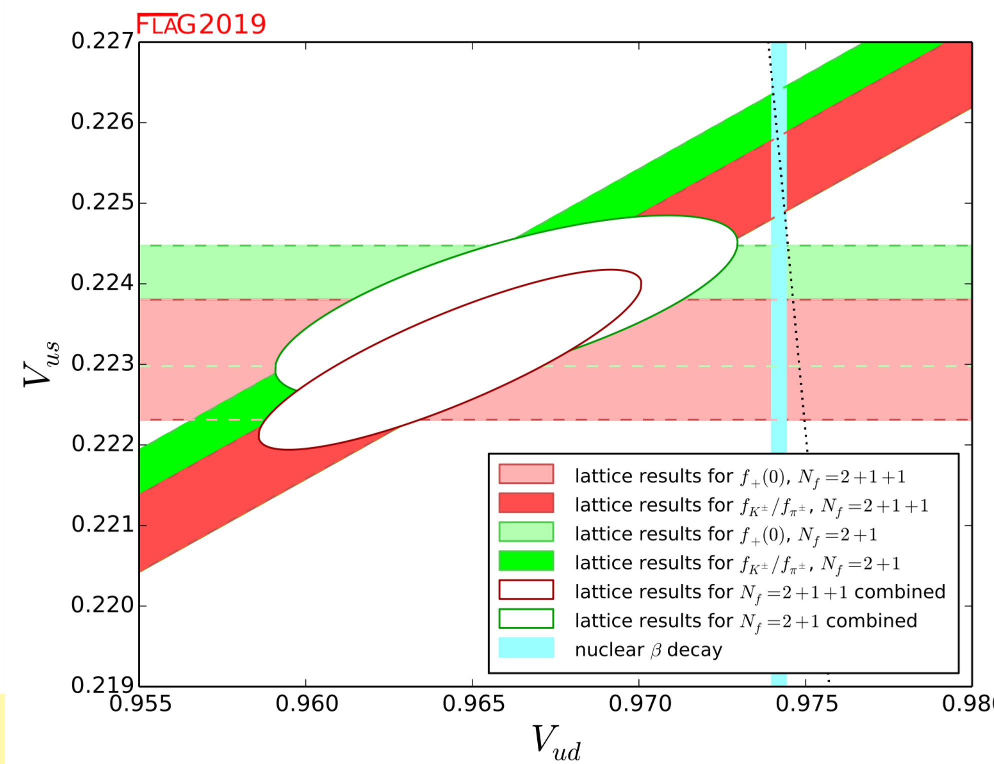
$$K^+ \to \pi^0 + l + \nu_l$$

- Super allowed Beta decay for [Hardy&Towner]

$$|V_{ud}| = 0.97420(21)$$

- V(ub) is negligible

- Very precise check for Standard Model, CKM unitarity [FLAG 2019]



$|V(ud)|^2 + |V(us)|^2 + |V(ub)|^2$

= 0.9797(74) [2.7 σ] (Kl2+Kl3)

= 0.99884(53) [2.2 σ] (Kl3+ β)

= 0.99986(46) [0.3 σ] (Kl2+ β)

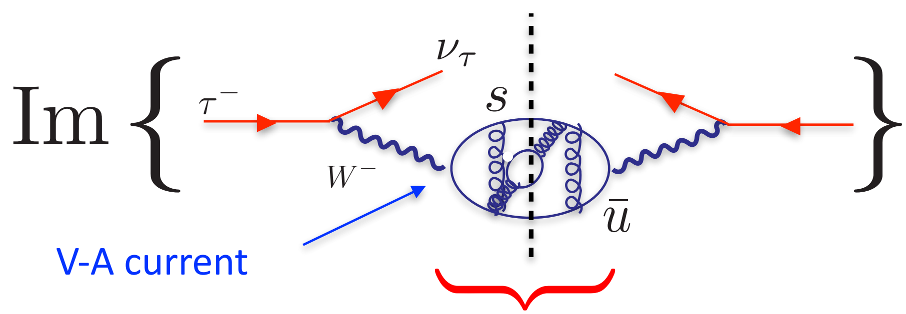
$$C_{KV_\mu D}(t_x,t_y;\vec{p}) = \sum_{i,j} \frac{1}{2m_{D_i} 2E_{K_j}(\vec{p})} e^{-m_{D_i}t_x - E_{K_j}(\vec{p})|t_y|} \times$$

$$\times \langle 0|O_K(t_x,\vec{x})|K_i(\vec{p})\rangle\langle K_i(\vec{p})|V_\mu(0)|D_j(\vec{0})\rangle\langle D_j(\vec{0})|O_D^\dagger(0)|0\rangle.$$

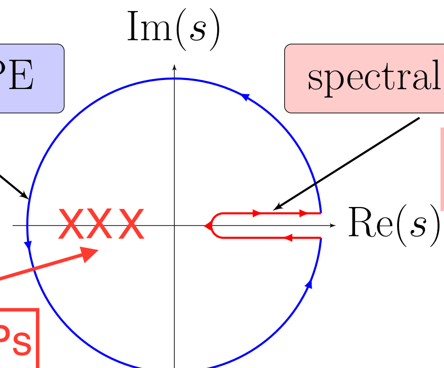
Inclusive tau decay [PRL 18]

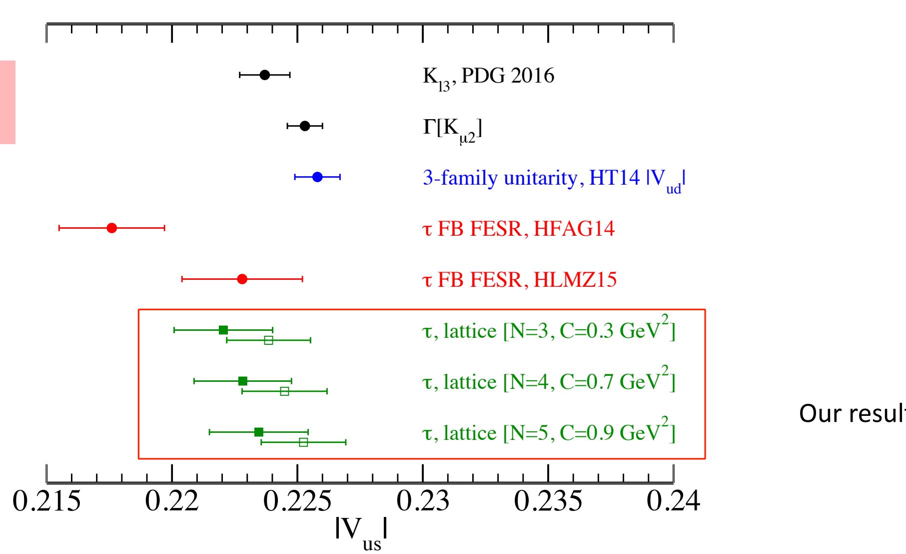
[Hiroshi Ohki]

- A new way to combine
  - τ→ had + ν experimental spectrum
  - Lattice vector/axial vacuum polarization suppressing quark-hadron duality violation



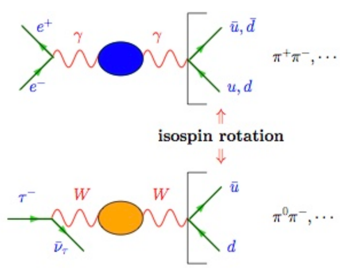
Unitarity and analytic continuation at work !

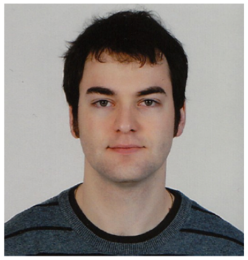




All our results are consistent with each other within 1 σ error, as well as to CKM unitarity.

- Traditional analysis currently gives - 3.4 σ tention in V_{us}
- All our new results are consistent with CKM unitarity and has smaller errors
- Also τ input for muon g-2



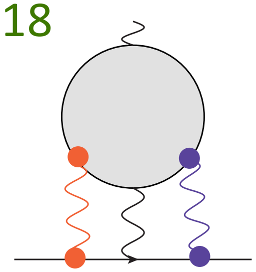


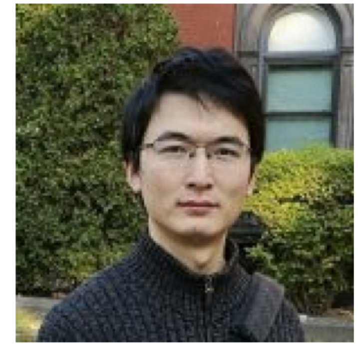
[Lehner, Blum, et al.]

Muon g-2

3 PRLs including an Editors' suggestion
BNL Press Release on 6/13/2018

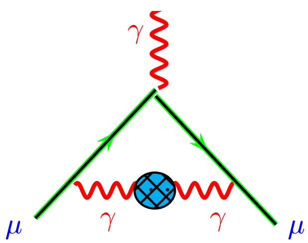
New Muon g-2 @ FNAL, J-PARC
to confirm ~4 σ difference
from SM prediction

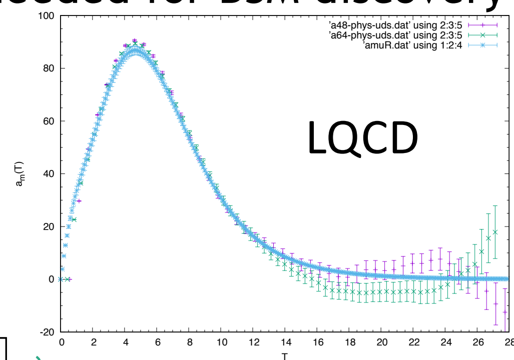


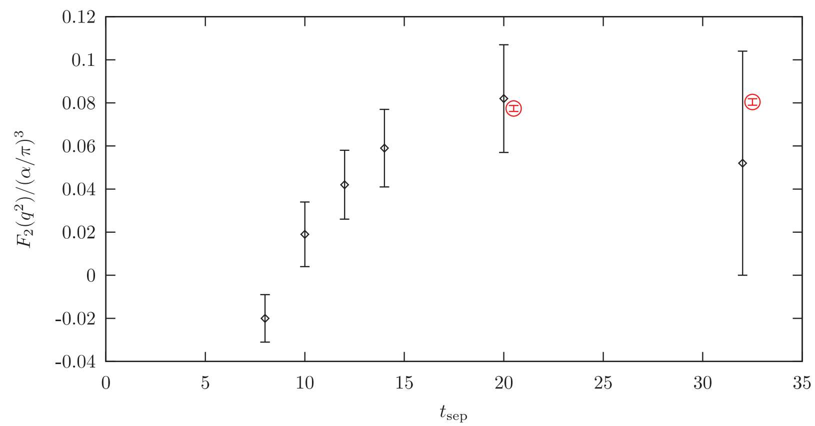


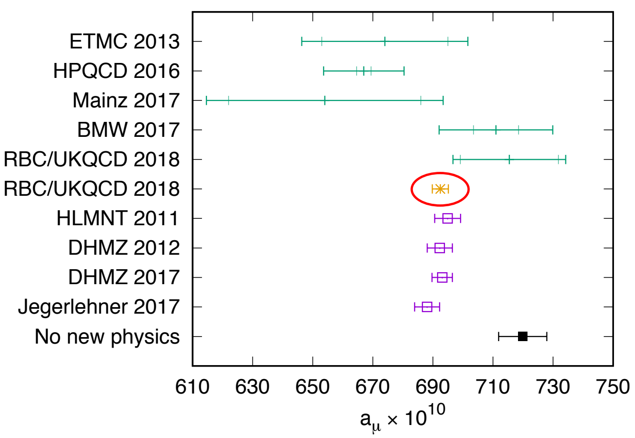
[Luchang's talk]

hadronic contributions needed for BSM discovery

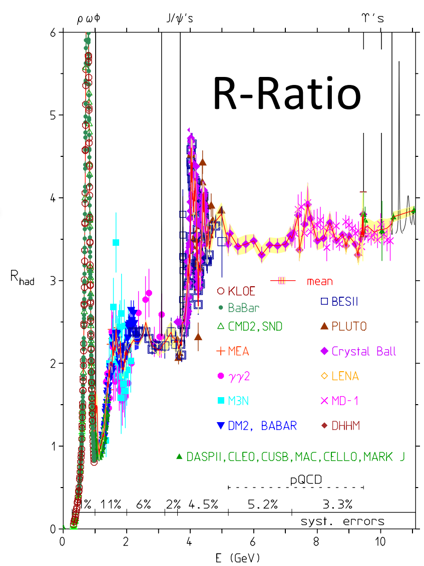






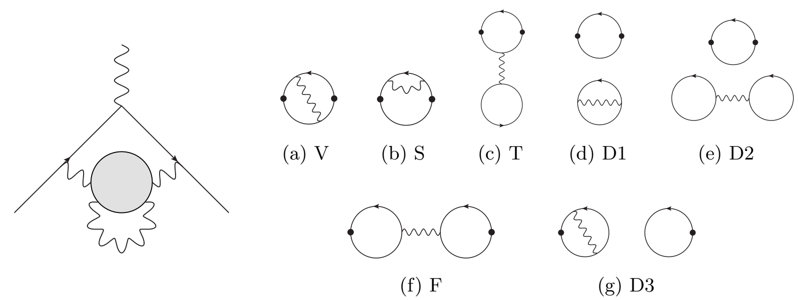


Most precise published result



Cross-check each other
and get the precise and reliable
theoretical guidance to g-2 experiments

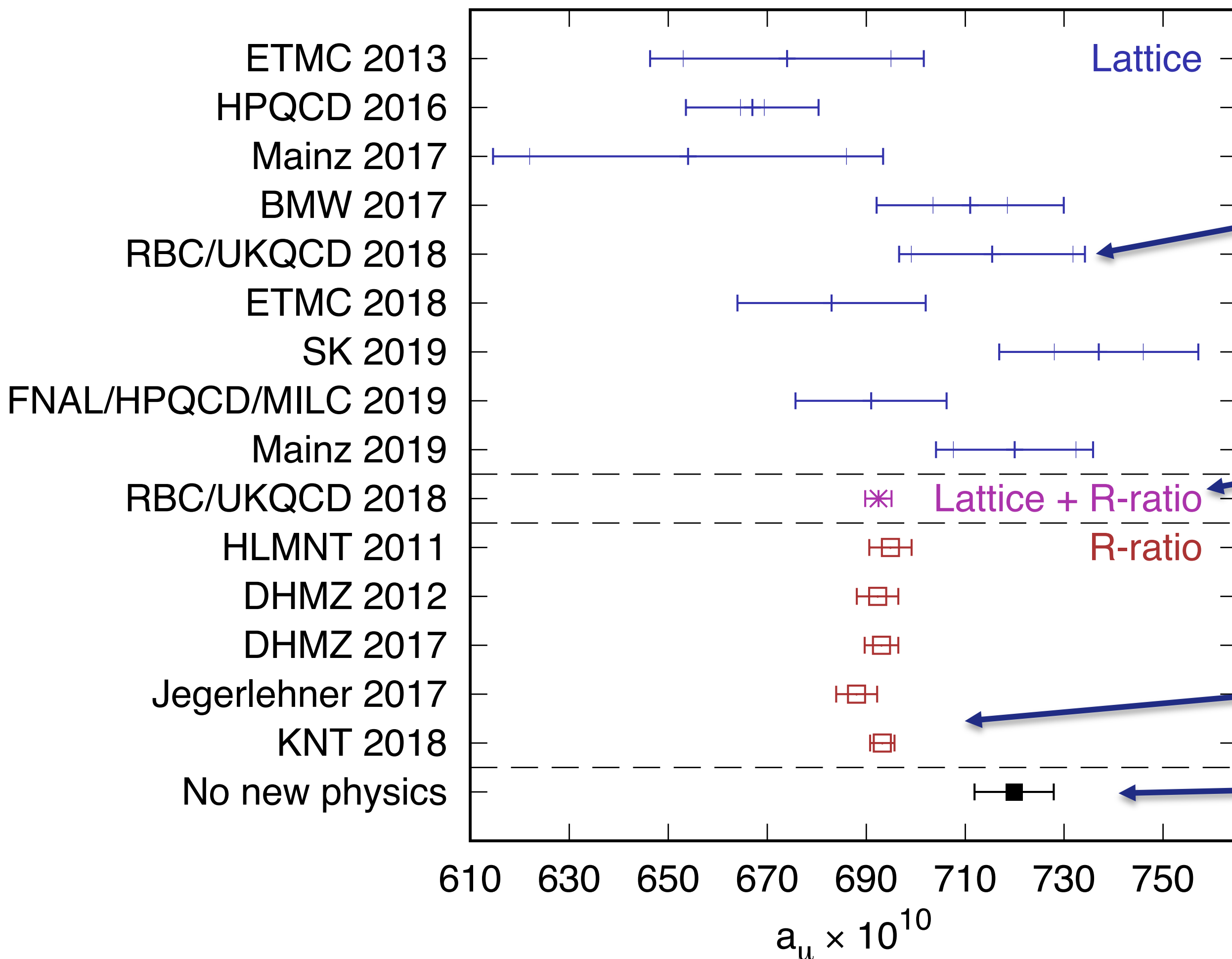
HLbL: x 1000 speed up
at physical quark mass



HVP: disconnected, QED & strong isospin breaking in the first principle calculation

HVP results

[Christoph Lehner Lat19]



Pure Lattice (RBC/UKQCD
715.4 (16.3)(9.2) [2.6%]

Lat+R-ratio Hybrid
692.5 (1.4)(2.3) [0.39%]

KNT18
693.3 (2.5) [0.36%]

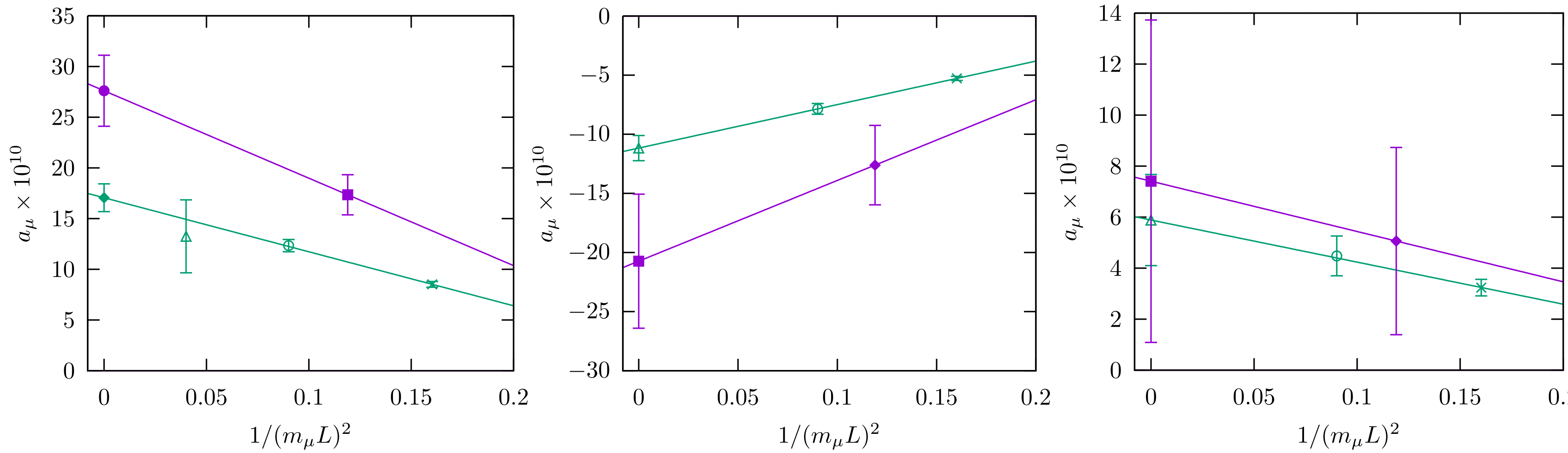
No New Physics

- Significant improvements is in progress for statistical error using 2π and 4π (!) states in addition to EM current (GEVP, GS-parametrization)
- Pure Lattice HVP 5×10^{-10} this year, 1×10^{-10} for long term
- Check BABAR-KLOE tention by windown method, consolidate error at 3×10^{-10}

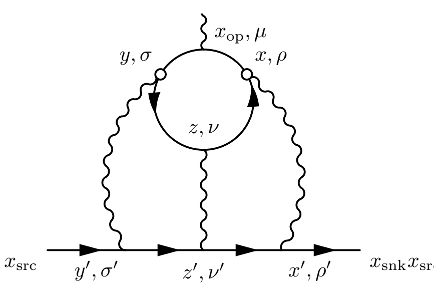
[Blum et al.   Moriond2019]

Using QED_L
[Hayakawa Uno PTP 2008]

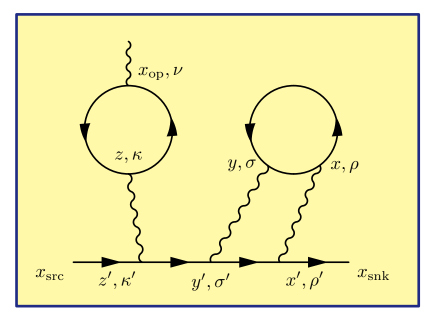
QED_L continuum and infinite volume extrapolation [Blum et al., 2019] (preliminary)



- Iwasaki ensembles: $a \to 0$ ($c_2 = 0$, conn. extrap.: up to 1 fm, 48^3 for $r > 1$ fm)
- I-DSDR ensembles: $L \to \infty$ ($b_2 = 0$)



$$a_\mu^{cHLbL} = (27.61 \pm 3.51_{\rm stat} \pm 0.32_{{\rm sys},a^2}) \times 10^{-10}$$
$$a_\mu^{dHLbL} = -20.20 \pm 5.65_{\rm stat} \times 10^{-10}$$
$$a_\mu^{HLbL} = 7.41 \pm 6.32_{\rm stat} \pm 0.32_{{\rm sys},a^2} \times 10^{-10}$$



$$F_2(a, L) = F_2\left(1 - \frac{c_1}{(m_\mu L)^2}\right)(1 - c_2 a^2)$$

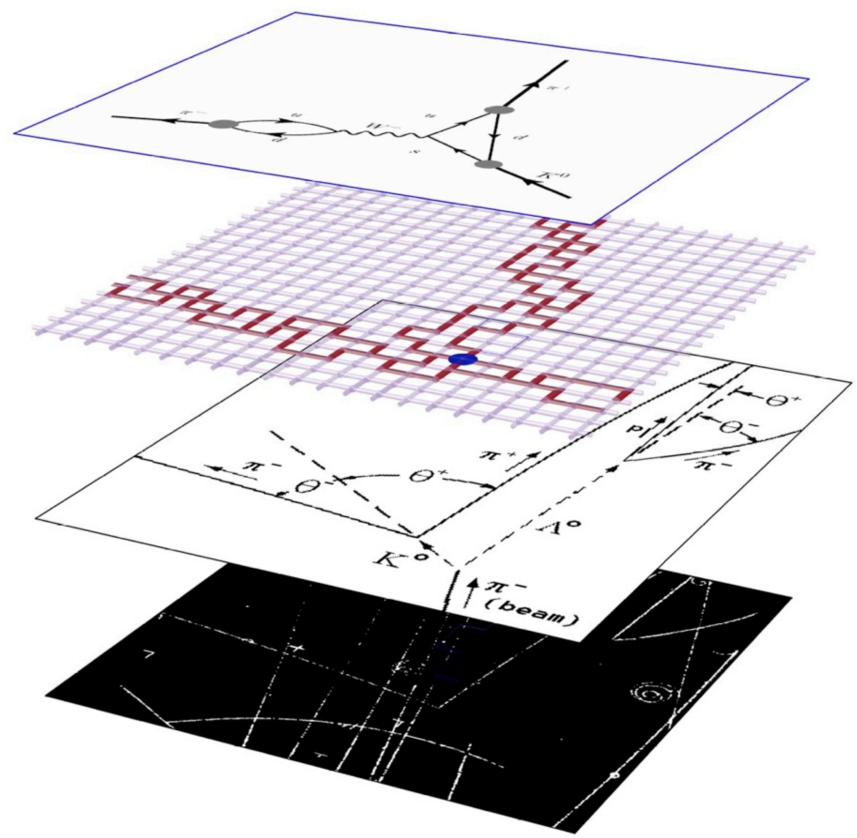
K → π π Decays

- quark flavor mixing (strange quark -> down quark) in
  Cabibbo Kobayashi Maskawa Theory (2008 Nobel prize)
- First direct methods (Lellouch Luscher formula)
  PRL 108, 141601 (2012)
- Finer lattice spacing
  ~15% error -> ~5% error
- Realistic kinematics for
  I=0 by G-parity boundary condition

  40 years old home work : ε'/ε

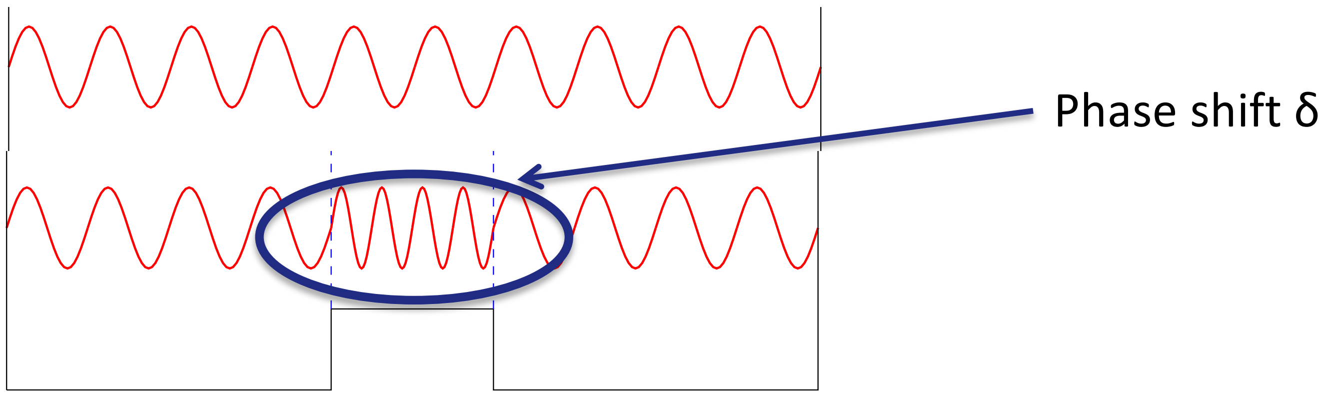
- K_L K_s mass difference rare Kaon decay ?

2012 Ken Wilson Lattice Award



K → ππ decay on lattice

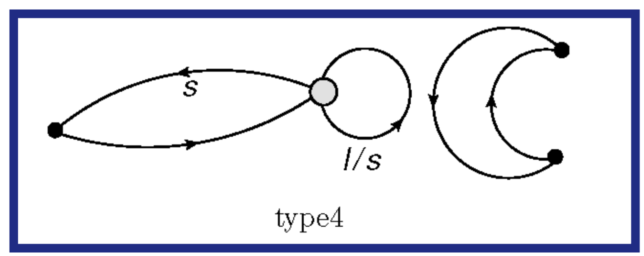
- Relates energy on finite volume $E_{\pi\pi}(V)$ to phase shift δ to obtain complex Amp(K→ππ) = $|A_I|\, e^{i\delta I}$ (Luscher, Lellouch-Luscher)



- Momentum of pions are controlled by boundary condition (anti-periodic or G-parity b.c.)
- Mixing and Renormalization of operator is done using non-perturbative renormalization (NPR)
- Chiral Symmetry is curtail
- I=2 channel is under control, I=0 is still a challenge due to disconnected diagrams. [PRL (2015). update this year]

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right)_{\mathrm{SM}} = 1.38(5.15)(4.43)\times 10^{-4}$$

cf. $$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right)_{\mathrm{exp}} = 16.6(2.3)\times 10^{-4}$$

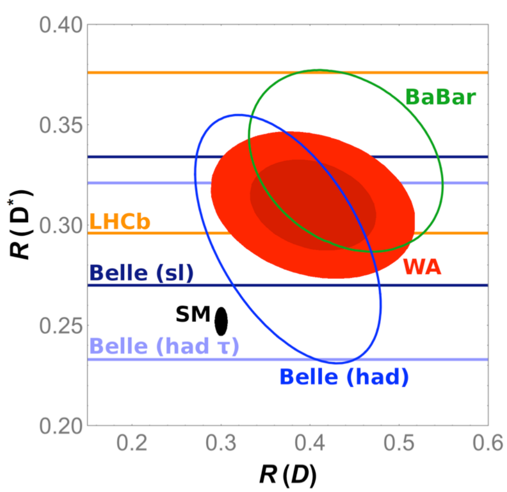


[Soni's lecture , next]

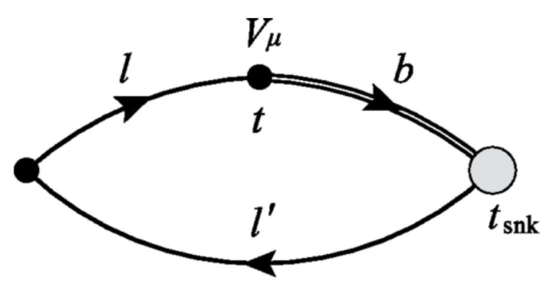
B physics

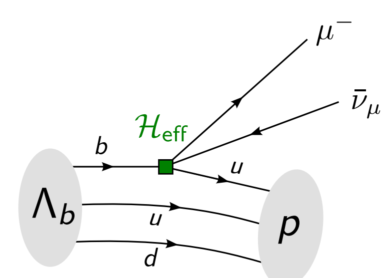
- Provide crucial theoretical guidance for B factories (LHCb and Belle-II), CKM physics
  - Semileptonic form factor B→π l ν
  - 1 → 2 transition (FCNC)
    B → Kπ l^+l^- , ππ $l^+\nu$
  - |Vcs| from $\Lambda_c \to \Lambda$ [Meinel, PRL (2017)]
  - |Vub| from $\Lambda_b \to p$ decay

$\mathbf{V_{ud}}$ $\pi \to l\nu$	$\mathbf{V_{us}}$ $K \to l\nu$ $K \to \pi l\nu$	$\mathbf{V_{ub}}$ $B \to \pi l\nu$
$\mathbf{V_{cd}}$ $D \to l\nu$ $D \to \pi l\nu$	$\mathbf{V_{cs}}$ $D_s \to l\nu$ $D \to Kl\nu$	$\mathbf{V_{cb}}$ $B \to Dl\nu$ $B \to D^*l\nu$
$\mathbf{V_{td}}$ $B_d \leftrightarrow \bar{B}_d$	$\mathbf{V_{ts}}$ $B_s \leftrightarrow \bar{B}_s$	$\mathbf{V_{tb}}$



B→Dτν :





$$\mathscr{R}_D^{SM} = \frac{\mathscr{B}(\bar{B} \to D\tau^-\bar{\nu}_\tau)}{\mathscr{B}(\bar{B} \to De^-\bar{\nu}_e)} = 0.300 \pm 0.008$$

$$\mathscr{R}_{D^*}^{SM} = \frac{\mathscr{B}(\bar{B} \to D^*\tau^-\bar{\nu}_\tau)}{\mathscr{B}(\bar{B} \to D^*e^-\bar{\nu}_e)} = 0.252 \pm 0.003$$

Summary & Conclusions

- LQCD provides non-perturbative ab-initio analysis of QFT. Systematically improvable.

- LQCD now delivers a lot of promised results essential to make experiments meaningful, enables a lot of important physics analysis which has not been possible before.

- Wide spectrum of important studies in Particle and Nuclear Physics. Many basic quantities are determined < 1% ! Precision with reliable error/

- Hot topics : B physics, muon g-2, multi-hadron including $K \to \pi\pi$, EW box, CP violation in Nucleon (EDM), …

Stay tuned ! Thank you !