Toward a unified description of both low and high p_t particle production in high energy collisions

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OUTLINE

QCD at high transverse momentum:

asymptotic freedom

parton model

collinear factorization (twist expansion)

QCD at high energy (CGC):

breakdown of twist expansion

high gluon density effects

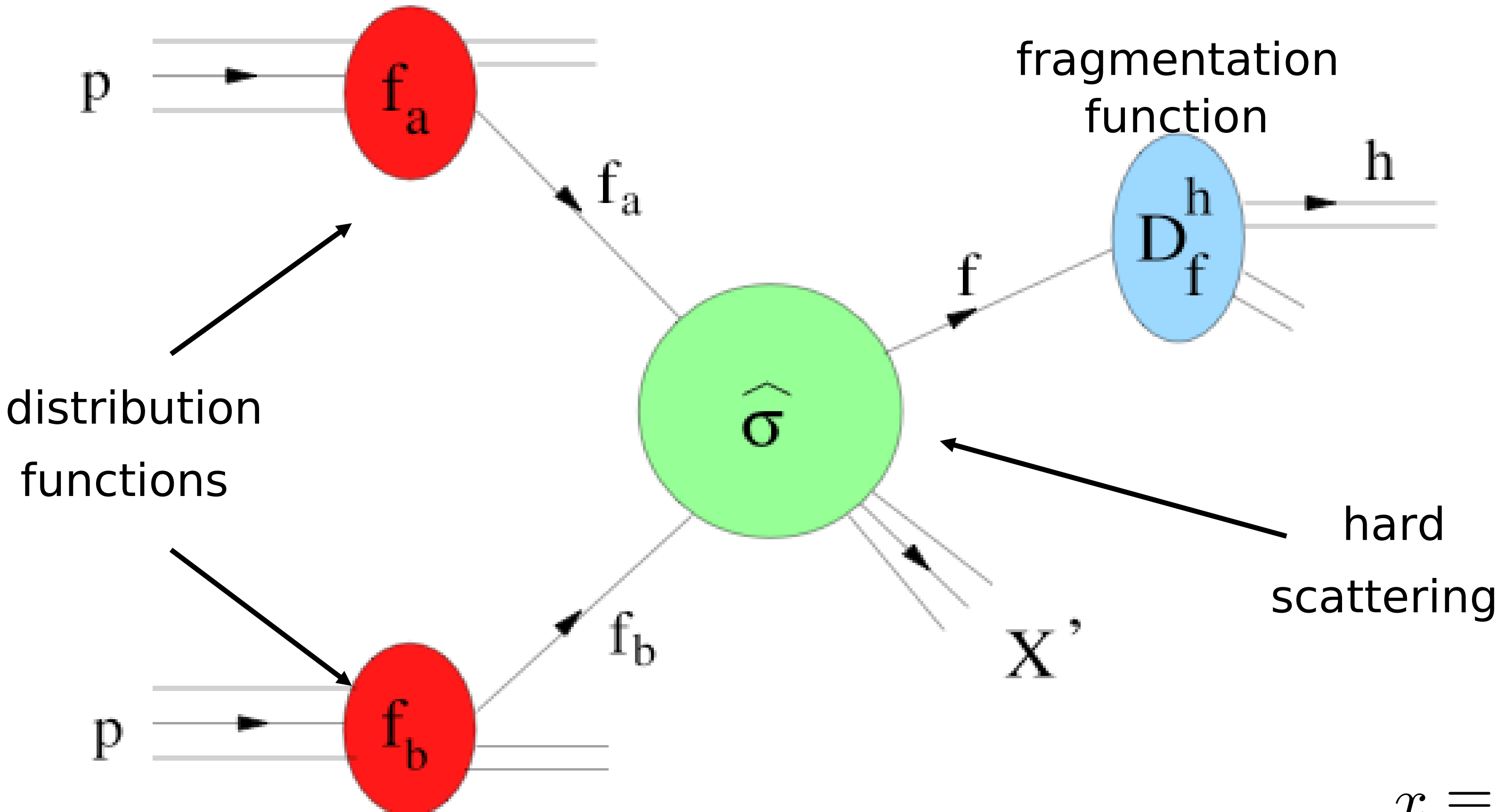
high energy effects

Toward a unified formalism:

beyond eikonal approximation

High p_t particle production: pp collisions

collinear factorization: separation of soft (long distance) and hard (short distance)



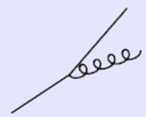
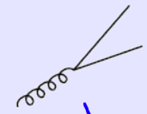
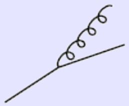
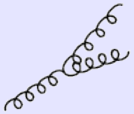
$$x \equiv \frac{p^+}{P^+}$$

$$\frac{d\sigma^{pp\to h\,X}}{d^2p_t\,dy} \sim f_a(x_1) \otimes f_b(x_2) \otimes \hat{\sigma} \otimes D_f^h(z) + \cdots\cdots$$

power corrections

DGLAP evolution equation

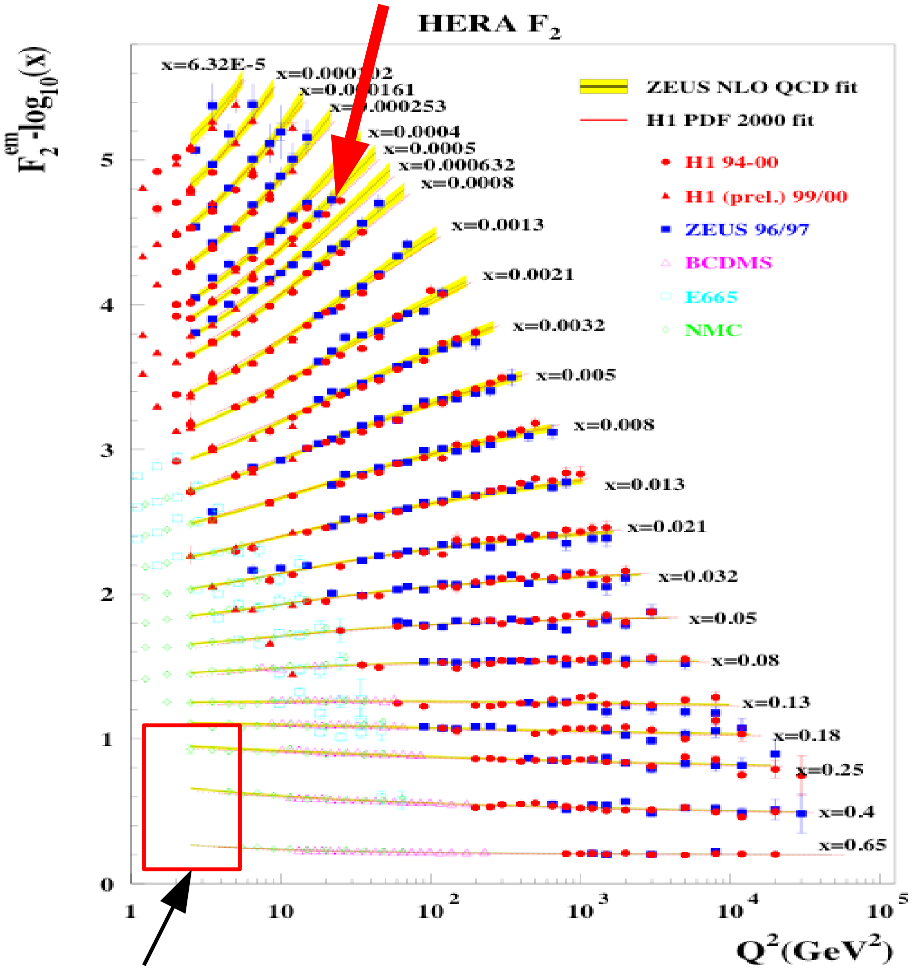
$$\frac{d}{d\ln\mu}\begin{pmatrix} q(x,\mu) \\ g(x,\mu) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix}_{(z,\alpha_s)} \cdot \begin{pmatrix} q(x/z,\mu) \\ g(x/z),\mu) \end{pmatrix}$$

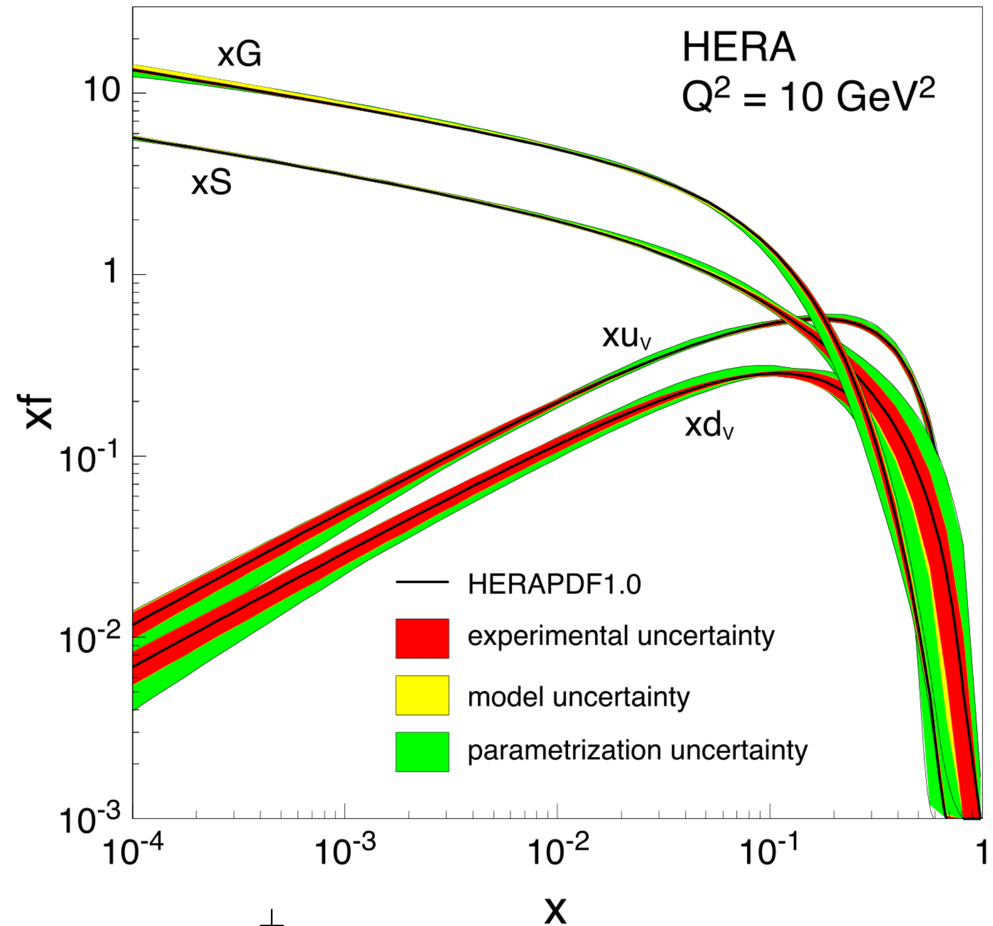
Deep Inelastic Scattering

$$F_2 \equiv \sum_{f=q,\bar{q}} e_f^2 \, x q_f(x, Q^2)$$

QCD: scaling violations



early experiments (SLAC,...):
scale invariance of hadron structure



$$x = \frac{p^+}{P^+}$$

***x** is the fraction of hadron energy carried by a parton*

What drives the growth of parton distributions?

Splitting functions at leading order $O(\alpha_S{}^0)$ $(x \neq 1)$

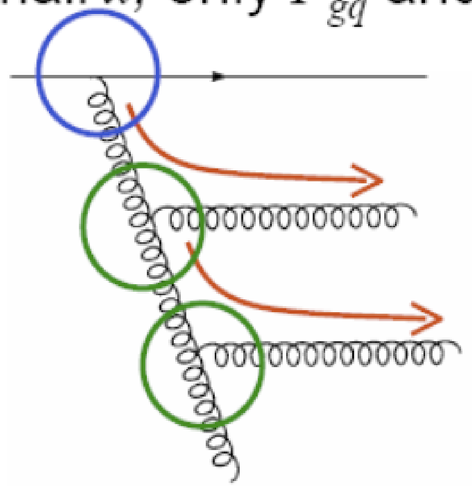
$$P_{qq}^{(0)}(x) = C_F \frac{1+x^2}{1-x}$$

$$P_{qg}^{(0)}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right]$$

$$P_{gq}^{(0)}(x) = C_F \frac{1+(1-x)^2}{x}$$

$$P_{gg}^{(0)}(x) = 2C_A\left[\frac{x}{1-x} + \frac{1-x}{x} + x(1-x)\right]$$

At small x, only P_{gq} and P_{gg} are relevant.



→ Gluon dominant at small x!

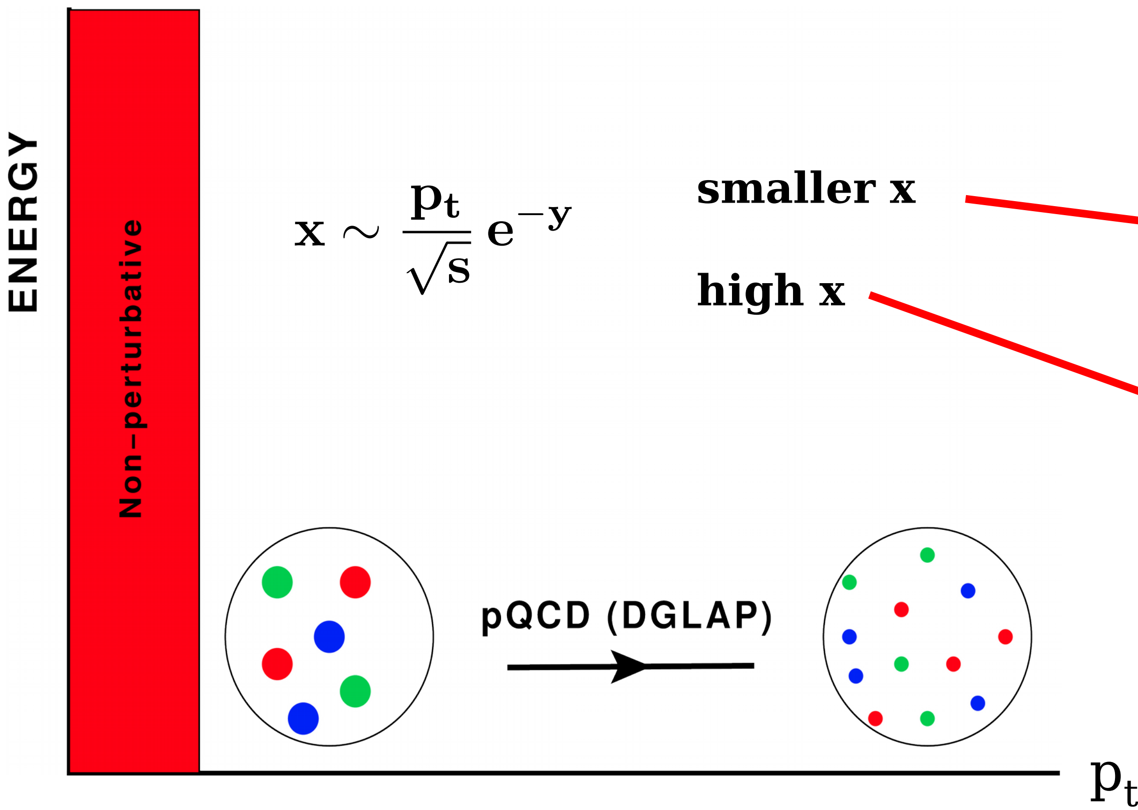
The double log approximation (DLA) of DGLAP is easily solved.

-- increase of gluon distribution at small x

$$xg(x, Q^2) \sim e^{\sqrt{\alpha_s (\log 1/x)(\log Q^2)}}$$

pQCD: the standard paradigm

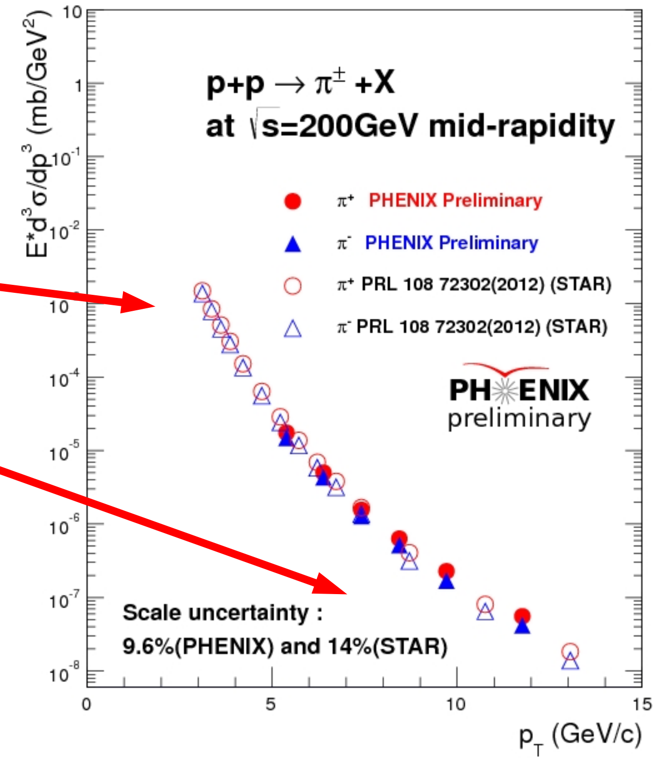
$$E\frac{d\sigma}{d^3p} \sim f_1(x, p_t^2) \otimes f_2(x, p_t^2) \otimes \frac{d\sigma}{dt} \otimes D(z, p_t^2)$$



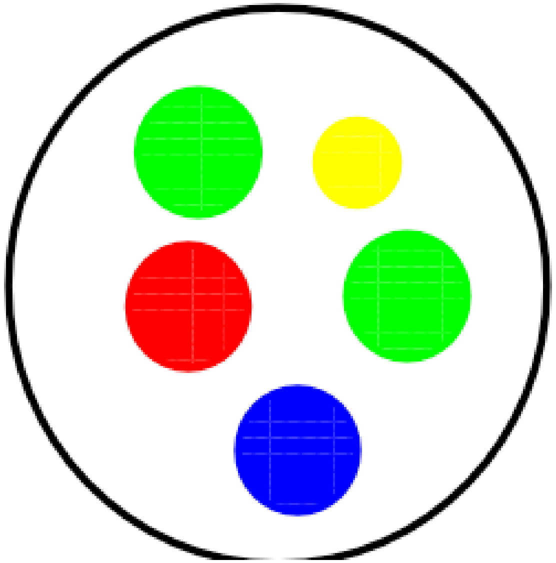
$$x \sim \frac{p_t}{\sqrt{s}}\, e^{-y}$$

smaller x

high x



bulk of QCD phenomena happens at low p_t (small x)

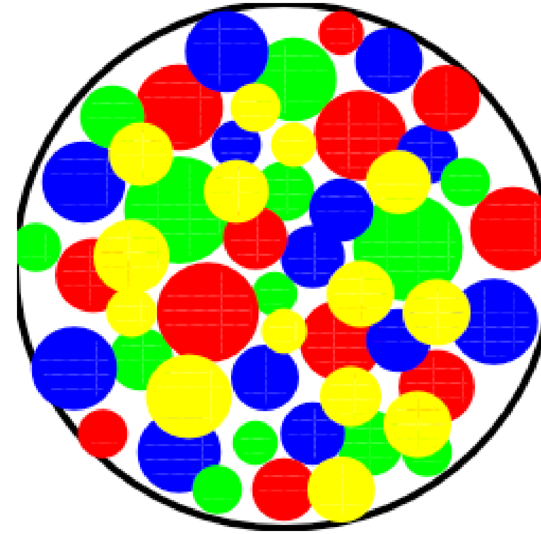


energy ~ 1/x

$$\frac{\alpha_s}{Q^2}\,\frac{xG(x,Q^2)}{\pi r^2} \sim 1$$

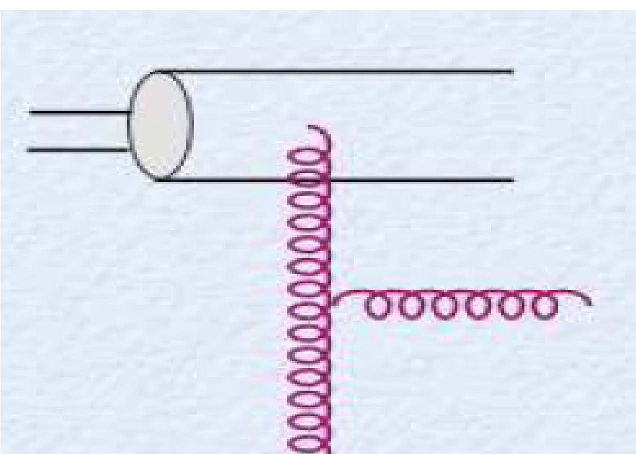
saturation scale

$$Q_s^2(x, b_t, A) \sim A^{1/3}\,(\frac{1}{x})^{0.3}$$



<u>collinear factorization breaks down at small x</u>

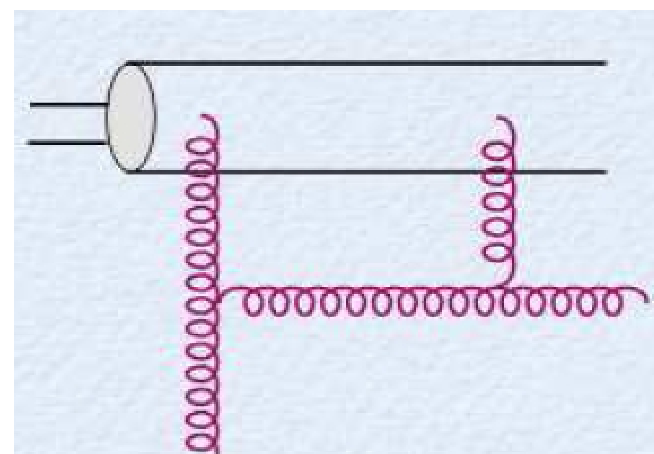
"attractive" bremsstrahlung *vs.* *"repulsive" recombination*



included in pQCD

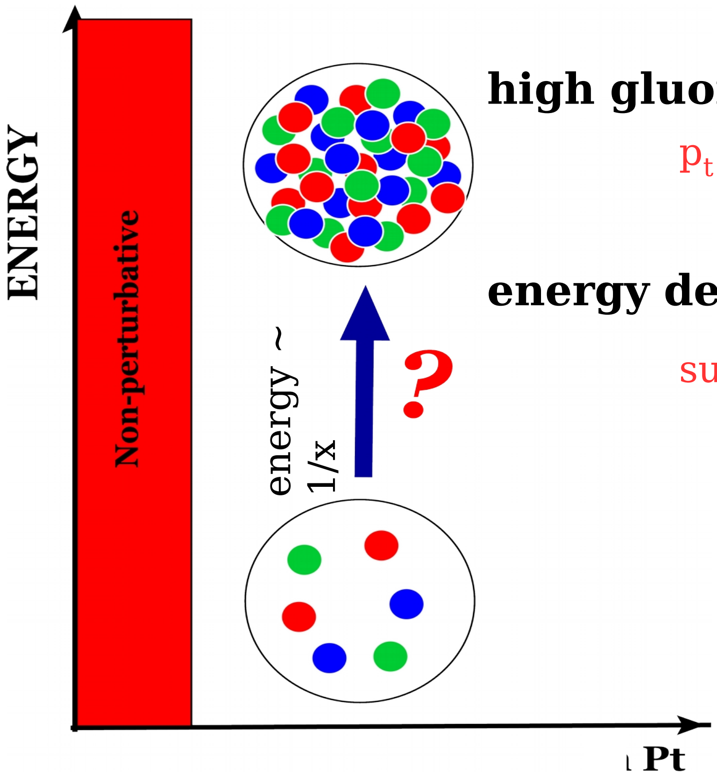
$S \rightarrow \infty,\ \ Q^2\ fixed$

$$x_{Bj} \equiv \frac{Q^2}{S} \rightarrow 0$$



not included in pQCD

A hadron/nucleus at high energy: gluon saturation



high gluon density: multiple scattering via Wilson lines

p_t broadening

energy dependence: x-evolution via JIMWLK

suppression of spectra/away side peaks

$$\mathbf{Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}}$$

$$Q_s^2(x = 3 \times 10^{-4}) \sim 1\, GeV^2$$

for a proton target (quarks)

a framework for multi-particle production in QCD at small x/low p_t

Initial conditions for hydro
Thermalization ?
Long range rapidity correlations
Azimuthal angular correlations
Nuclear modification factor

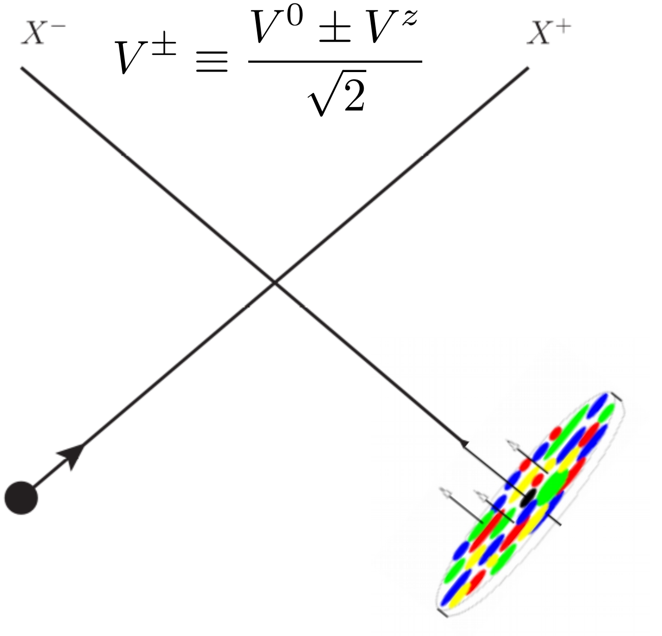
$$\mathbf{x \leq 0.01}$$

eliminate/minimize medium effects (*proton-nucleus*)

Eikonal approximation

$$J^\mu_a \simeq \delta^{\mu -} \rho_a$$

$$V^\pm \equiv \frac{V^0 \pm V^z}{\sqrt{2}}$$



$$D_\mu \, J^\mu \;=\; D_- \, J^- = 0$$

$$\partial_- J^- \;=\; 0 \qquad \text{(in } A^+ = 0 \text{ gauge)}$$

does not depend on x^-

solution to EOM: $$A^-_a(x^+, x_t) \equiv n^- \, S_a(x^+, x_t)$$

with $$n^\mu = (n^+ = 0, n^- = 1, n_\perp = 0)$$
$$n^2 = 2\,n^+ n^- - n_\perp^2 = 0$$

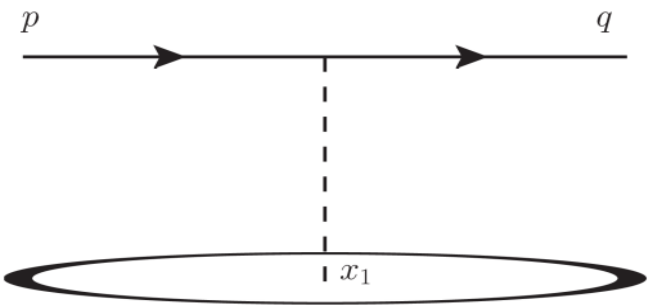
recall (eikonal limit): $$\bar{u}(q)\gamma^\mu u(p) \rightarrow \bar{u}(p)\gamma^\mu \, u(p) \sim p^\mu$$
$$\bar{u}(q)\not{A}\, u(p) \rightarrow p \cdot A \sim p^+ \, A^-$$

multiple scattering of a quark from background color field $A^-_a(x^+, x_t)$

$$i\mathcal{M}_1 = (ig)\int d^4x_1\, e^{i(q-p)x_1}\, \bar{u}(q)\, \left[\not{n}\, S(x_1)\right]\, u(p)$$

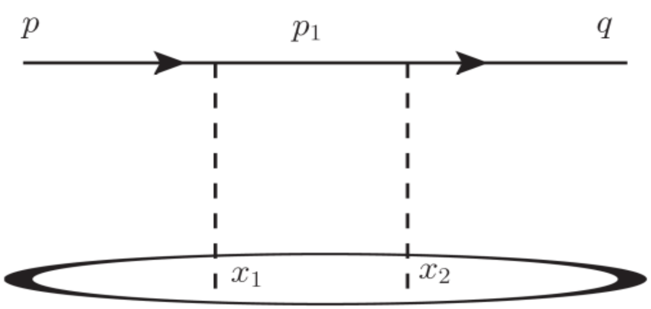
$$= (ig)(2\pi)\delta(p^+ - q^+)\int d^2x_{1t}\, dx_1^+\, e^{i(q^- - p^-)x_1^+}\, e^{-i(q_t - p_t)x_{1t}}$$

$$\bar{u}(q)\, \left[\not{n}\, S(x_1^+, x_{1t})\right]\, u(p)$$



$$i\mathcal{M}_2 = (ig)^2 \int d^4x_1\, d^4x_2 \int \frac{d^4p_1}{(2\pi)^4}\, e^{i(p_1-p)x_1}\, e^{i(q-p_1)x_2}$$

$$\bar{u}(q)\, \left[\not{n}\, S(x_2)\, \frac{i\not{p}_1}{p_1^2 + i\epsilon}\, \not{n}\, S(x_1)\right]\, u(p)$$



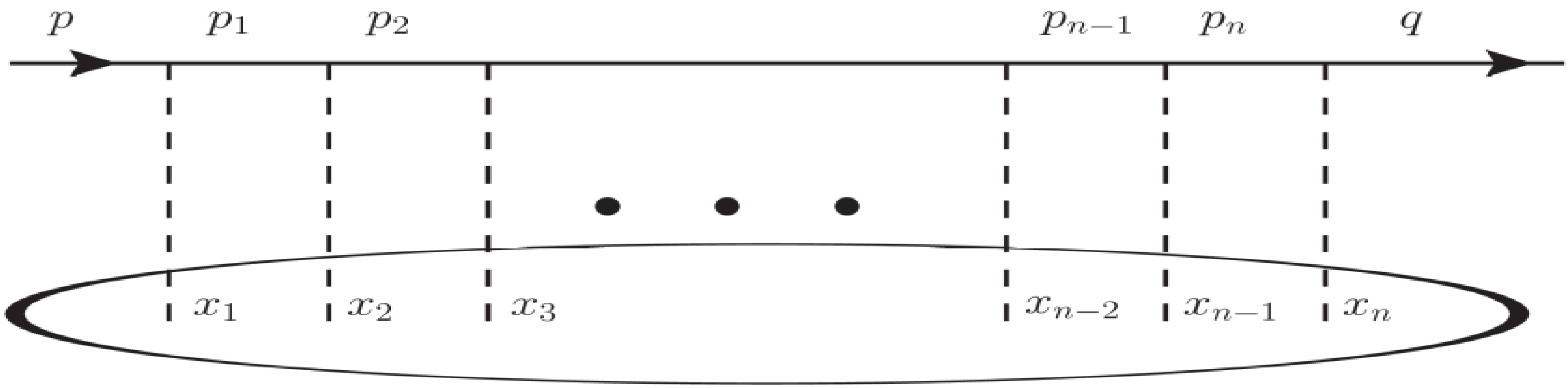
$$\int \frac{dp_1^-}{(2\pi)} \frac{e^{ip_1^-(x_1^+ - x_2^+)}}{2p^+\left[p_1^- - \frac{p_{1t}^2 - i\epsilon}{2p^+}\right]} = \frac{-i}{2p^+}\, \theta(x_2^+ - x_1^+)\, e^{i\frac{p_{1t}^2}{2p^+}(x_1^+ - x_2^+)}$$

contour integration over the pole leads to path ordering of scattering

ignore all terms: $O(\frac{p_t}{p^+}, \frac{q_t}{q^+})$ and use $\not{n}\, \frac{\not{p}_1}{2n\cdot p}\, \not{n} = \not{n}$

$$i\mathcal{M}_2 = (ig)^2\, (-i)(i)\, 2\pi\delta(p^+ - q^+) \int dx_1^+\, dx_2^+\, \theta(x_2^+ - x_1^+) \int d^2x_{1t}\, e^{-i(q_t - p_t)\cdot x_{1t}}$$

$$\bar{u}(q)\, \left[S(x_2^+, x_{1t})\, \not{n}\, S(x_1^+, x_{1t})\right]\, u(p)$$

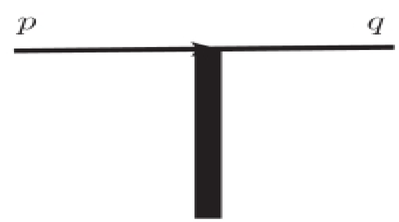


$$
\begin{aligned}
i\mathcal{M}_n \;=\;& 2\pi\delta(p^+ - q^+)\, \bar{u}(q)\, \not{n} \int d^2x_t\, e^{-i(q_t - p_t)\cdot x_t} \\
& \Big\{ (ig)^n\, (-i)^n (i)^n \int dx_1^+\, dx_2^+ \cdots dx_n^+\, \theta(x_n^+ - x_{n-1}^+) \cdots \theta(x_2^+ - x_1^+) \\
& \left[S(x_n^+, x_t)\, S(x_{n-1}^+, x_t) \cdots S(x_2^+, x_t) S(x_1^+, x_t) \right] \Big\}\, u(p)
\end{aligned}
$$

sum over all scatterings $\quad i\mathcal{M} = \sum_n i\,\mathcal{M}_n$

$$
i\mathcal{M}(p,q) = 2\pi\delta(p^+ - q^+)\, \bar{u}(q)\, \not{n} \int d^2x_t\, e^{-i(q_t - p_t)\cdot x_t}\, \left[V(x_t) - 1 \right]\, u(p)
$$

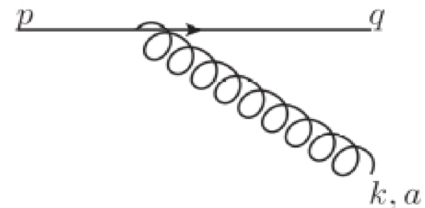
with $V(x_t) \equiv \hat{P}\, \exp\left\{ ig \int_{-\infty}^{+\infty} dx^+\, n^-\, S_a(x^+, x_t)\, t_a \right\}$



$$
\frac{d\,\sigma^{q\,T \to q\,X}}{d^2p_t\, dy} \sim |i\mathcal{M}|^2 \sim\; F.T.\; < Tr\, V(x_t)\, V^\dagger(y_t) >
$$

1-loop correction: energy dependence

basic ingredient: soft radiation vertex (LC gauge)



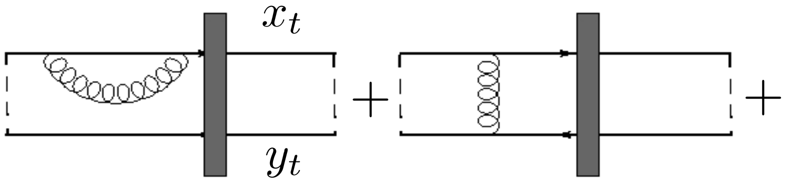
$$g\,\bar{u}(q)\,t^a\,\gamma_\mu\,u(p)\,\epsilon^{\mu}_{(\lambda)}(k) \longrightarrow 2\,g\,t^a\,\frac{\epsilon_{(\lambda)}\cdot k_t}{k_t^2}$$

coordinate space:

$$\int \frac{d^2k_t}{(2\pi)^2} e^{ik_t\cdot(x_t-z_t)} 2\,g\,t^a\,\frac{\epsilon_{(\lambda)}\cdot k_t}{k_t^2} = \frac{2\,i\,g}{2\pi}\,t^a\,\frac{\epsilon_{(\lambda)}\cdot(x_t-z_t)}{(x_t-z_t)^2}$$

x_t, z_t are transverse coordinates of the quark and gluon

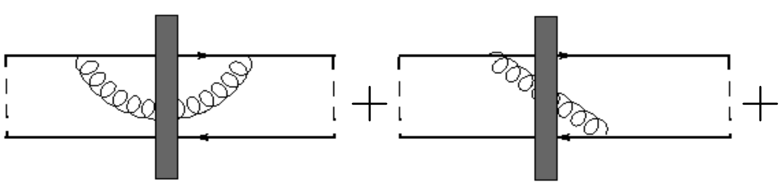
virtual corrections:



$$\longrightarrow Tr\,V(x_t)\,V^\dagger(y_t)$$

a dipole

real corrections:



$$\longrightarrow Tr\,V(x_t)\,V^\dagger(z_t)\,Tr\,V(z_t)\,V^\dagger(y_t)$$

$$\frac{1}{(x_t-z_t)^2} \qquad \frac{(x_t-z_t)\cdot(y_t-z_t)}{(x_t-z_t)^2(y_t-z_t)^2}$$

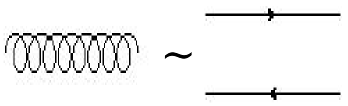
the S matrix

$$S(x_t,y_t) \equiv \frac{1}{N_c} Tr\,V(x_t)\,V^\dagger(y_t)$$

1-loop correction: BK eq.

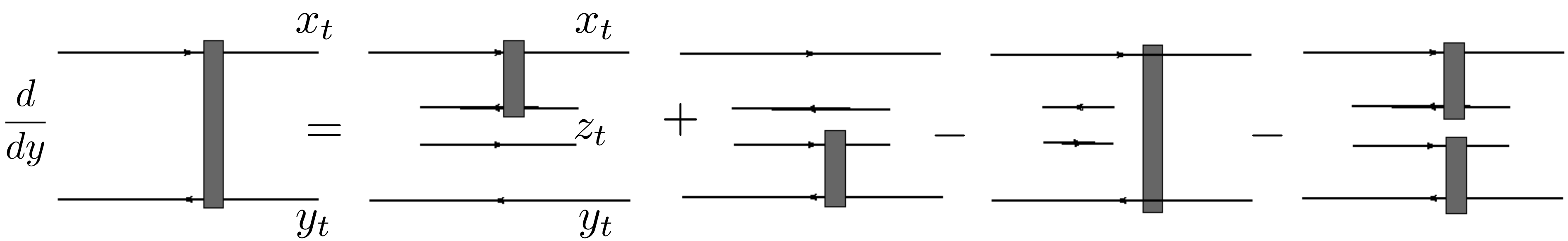
at large N_c

$3 \otimes \bar{3} = 8 \oplus 1 \simeq 8$



$$\frac{d}{dy}T(x_t, y_t) = \frac{N_c \alpha_s}{2\pi^2} \int d^2 z_t \frac{(x_t - y_t)^2}{(x_t - z_t)^2 (y_t - z_t)^2} \left[T(x_t, z_t) + T(z_t, y_t) - T(x_t, y_t) - T(x_t, z_t)T(z_t, y_t)\right]$$

$$T \equiv 1 - S$$



$$\tilde{T}(p_t) \sim \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right] \qquad Q_s^2 \ll p_t^2$$

$$\tilde{T}(p_t) \sim \log\left[\frac{Q_s^2}{p_t^2}\right] \qquad Q_s^2 \gg p_t^2$$

$$\tilde{T}(p_t) \sim \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right]^\gamma \qquad Q_s^2 < p_t^2$$

nuclear modification factor

$$R_{pA} \equiv \frac{\frac{d\sigma^{pA}}{d^2 p_t dy}}{A^{1/3}\frac{d\sigma^{pp}}{d^2 p_t dy}}$$

suppression of p_t spectra

nuclear shadowing

....

Particle production in high energy collisions

pQCD and collinear factorization at high p_t

breaks down at low p_t (small x)

CGC at low p_t

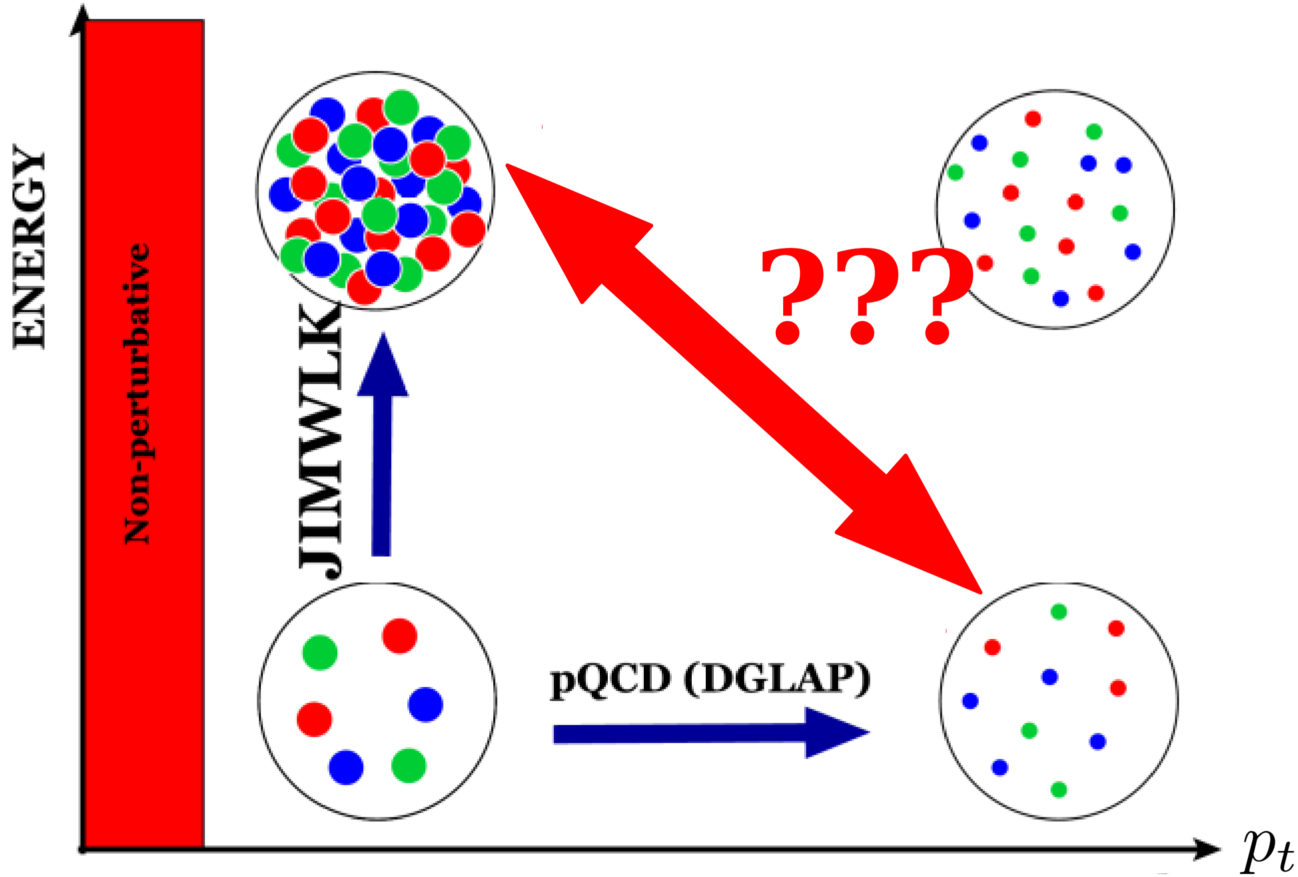
breaks down at large x (high p_t)

need a unified formalism:

CGC at low x (low p_t)

leading twist pQCD (DGLAP) at large x (high p_t)

QCD kinematic phase space



unifying saturation with high p_t (large x) physics?

<u>kinematics of saturation: *where is saturation applicable?*</u>

jet physics, high p_t (polar and azimuthal) angular correlations

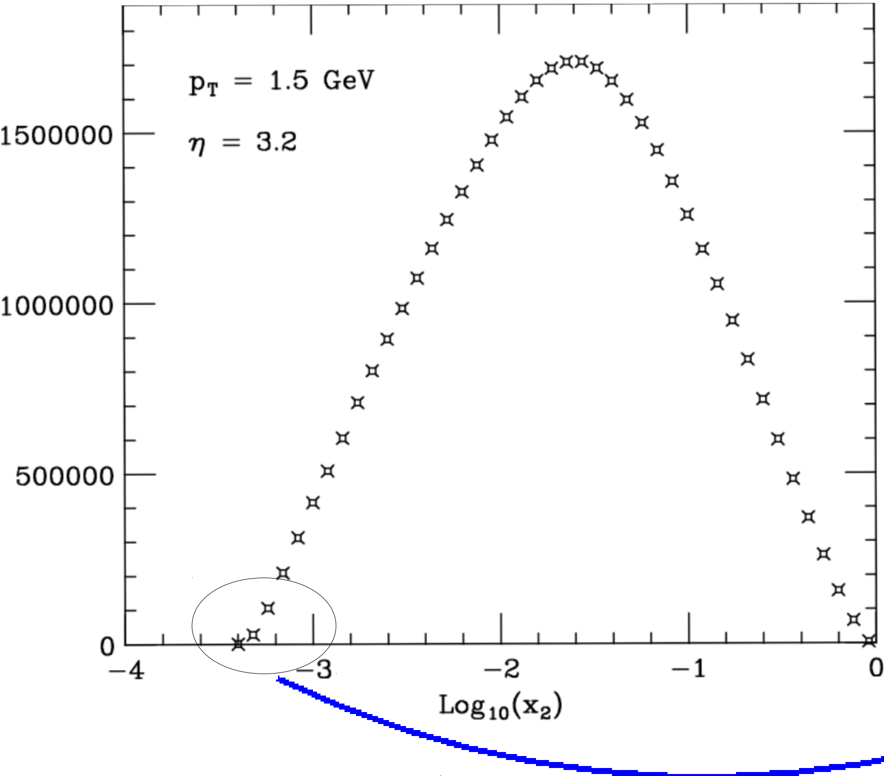
cold matter energy loss, spin physics?, …….

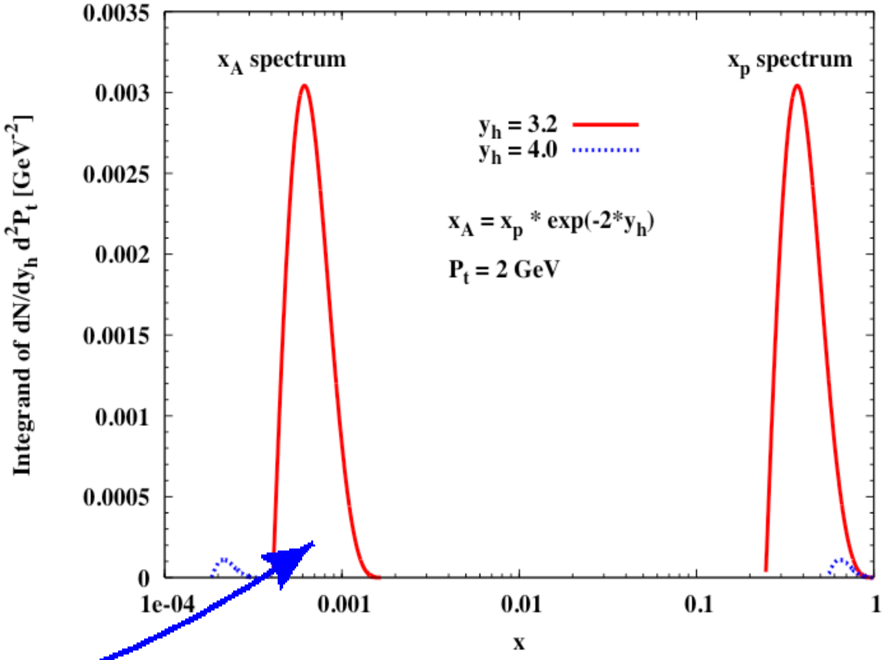
Pion production at RHIC: kinematics

collinear factorization | CGC

GSV, PLB603 (2004) 173-183

DHJ, NPA765 (2006) 57-70





$$\int_{x_{min}}^{1} dx\, xG(x,Q^2)\cdots\cdots \longrightarrow x_{min}G(x_{min},Q^2)\cdots$$

this is an extreme approximation with potentially severe consequences!

how to tackle this problem?

what should be the *starting point/expression/operator*?

pQCD: quark and gluon operators

$$\overline{\Psi}(y^-, 0_t)\gamma^+\Psi(0^-, 0_t)$$

renormalization lead to DGLAP evolution eq.

CGC: correlators of Wilson lines (DIS, Hybrid,....)

$$F_2 \sim Tr\, V(x_t)\, V^\dagger(y_t)$$

renormalization leads to JIMWLK/BK evolution eq.

toward unifying small and large x (multiple scattering)

scattering from small x modes of the target field $A^- \equiv n^- S$ involves only small transverse momenta exchange (small angle deflection)

$$
\begin{aligned}
p^\mu &= (p^+ \sim \sqrt{s},\, p^- = 0, p_t = 0) \\
S &= S(p^+ \sim 0, p^-/P^- \ll 1, p_t)
\end{aligned}
$$

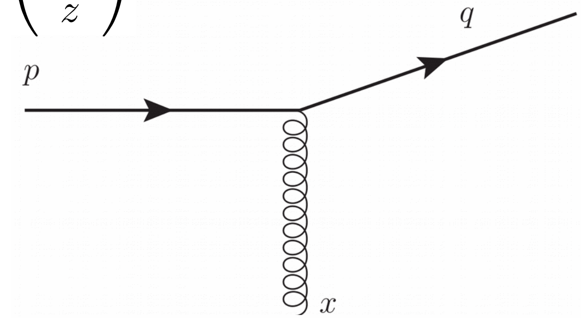
allow hard scattering by including one hard field $A_a^\mu(x^+, x^-, x_t)$ during which there is large momenta exchanged and quark can get deflected by a large angle.

include eikonal multiple scattering before and after (along a different direction) the hard scattering

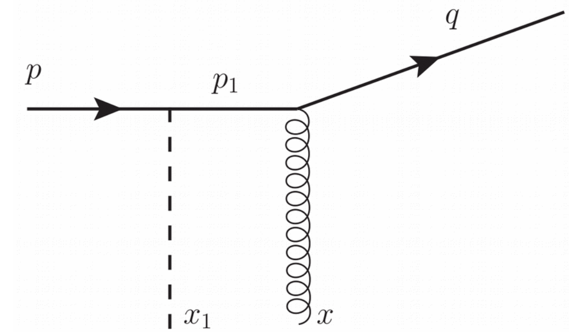
hard scattering: large deflection
scattered quark travels in the new “z” direction: $\bar{z}$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \mathcal{O} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

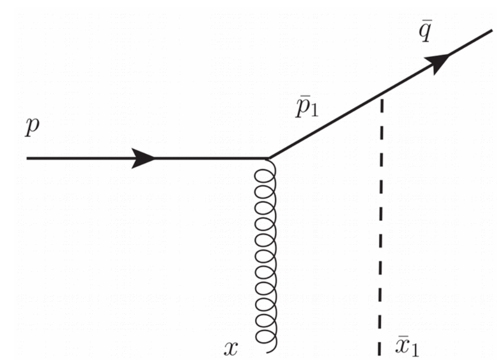
$$i\mathcal{M}_1 = (ig) \int d^4x \, e^{i(\bar{q}-p)x} \, \bar{u}(\bar{q}) \, \left[\not{A}(x) \right] \, u(p)$$



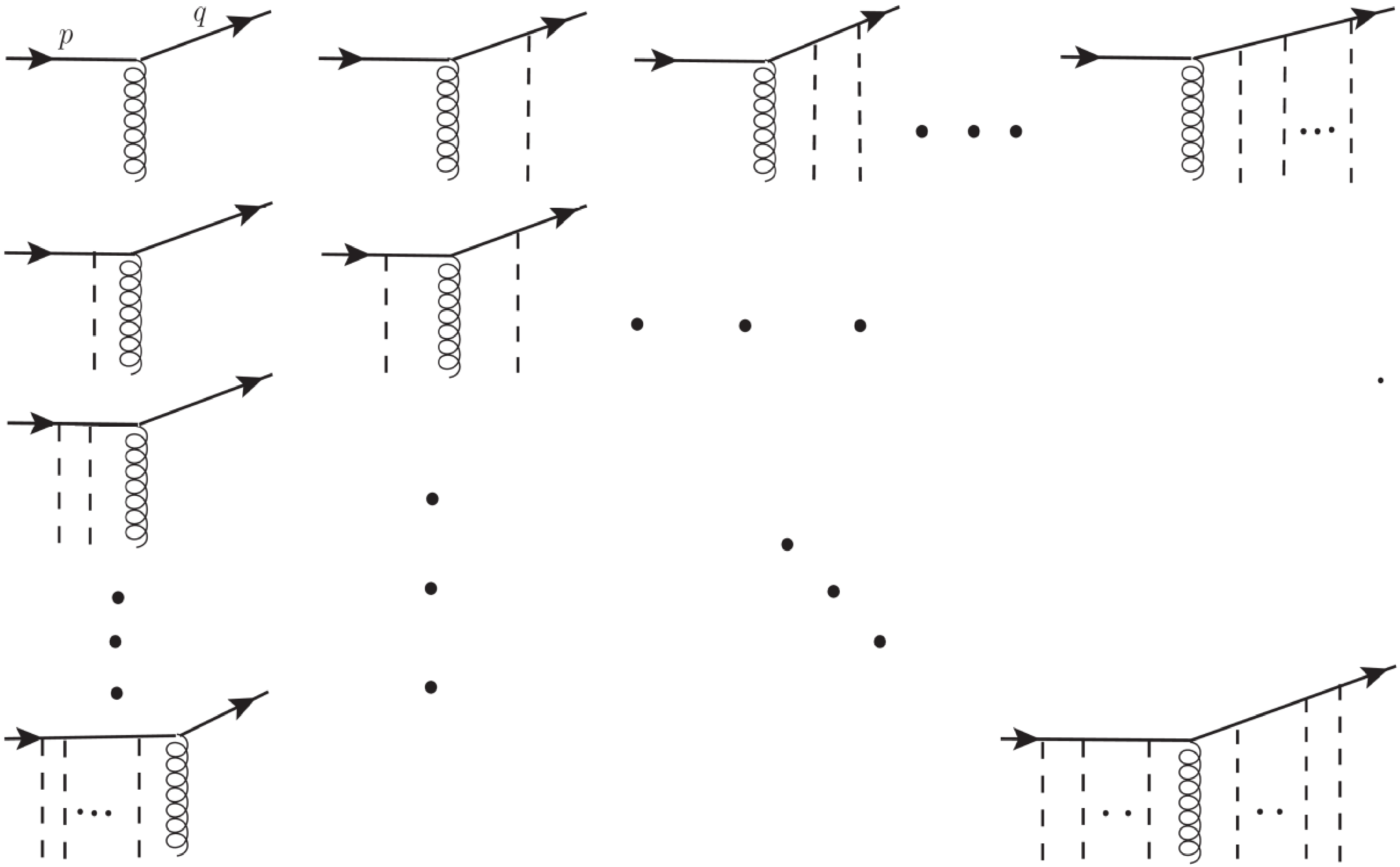
$$i\mathcal{M}_2 = (ig)^2 \int d^4x \, d^4x_1 \int \frac{d^4p_1}{(2\pi)^4} \, e^{i(p_1-p)x_1} \, e^{i(\bar{q}-p_1)x} \, \bar{u}(\bar{q}) \left[\not{A}(x) \, \frac{i\not{p}_1}{p_1^2 + i\epsilon} \, \not{n} \, S(x_1) \right] u(p)$$



$$i\mathcal{M}_2 = (ig)^2 \int d^4x \, d^4\bar{x}_1 \int \frac{d^4\bar{p}_1}{(2\pi)^4} \, e^{i(\bar{p}_1-p)x} \, e^{i(\bar{q}-\bar{p}_1)\bar{x}_1} \, \bar{u}(\bar{q}) \left[\not{\bar{n}} \, \bar{S}(\bar{x}_1) \, \frac{i\not{\bar{p}}_1}{\bar{p}_1^2 + i\epsilon} \not{A}(x) \right] u(p)$$



with $\quad \vec{\bar{v}} = \mathcal{O} \, \vec{v}$

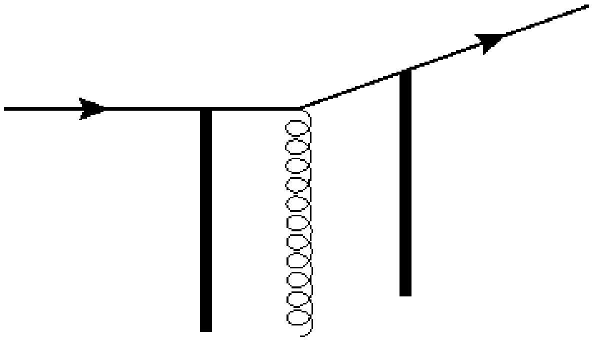

p
q

summing all the terms gives:

$$i\mathcal{M}_1 \quad = \quad \int d^4x\, d^2z_t\, d^2\bar{z}_t \int \frac{d^2k_t}{(2\pi)^2}\, \frac{d^2\bar{k}_t}{(2\pi)^2}\, e^{i(\bar{k}-k)x}\; e^{-i(\bar{q}_t-\bar{k}_t)\cdot\bar{z}_t}\; e^{-i(k_t-p_t)\cdot z_t}$$

$$\bar{u}(\bar{q})\, \left[\overline{V}_{AP}(x^+,\bar{z}_t)\, \not{\bar{n}}\, \frac{\not{\bar{k}}}{2\bar{k}^+}\, \left[ig\not{A}(x)\right]\, \frac{\not{k}}{2k^+}\, \not{n}\, V_{AP}(z_t,x^+)\right]\, u(p)$$

with

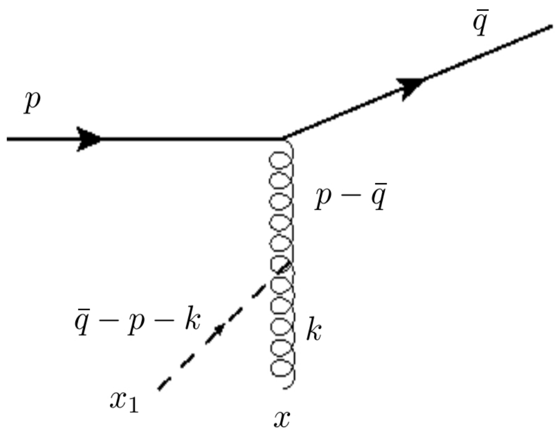


$$\overline{V}_{AP}(x^+,\bar{z}_t) \equiv \hat{P}\, \exp\left\{ig\int_{x^+}^{+\infty} d\bar{z}^+\, \bar{S}_a^-(\bar{z}_t,\bar{z}^+)\, t_a\right\}$$

$$V_{AP}(z_t,x^+) \equiv \hat{P}\, \exp\left\{ig\int_{-\infty}^{x^+} dz^+\, S_a^-(z_t,z^+)\, t_a\right\}$$

can extract the effective quark propagator $\quad i\mathcal{M}(p,\bar{q}) = \bar{u}(\bar{q})\, \tau_F\, u(p)$

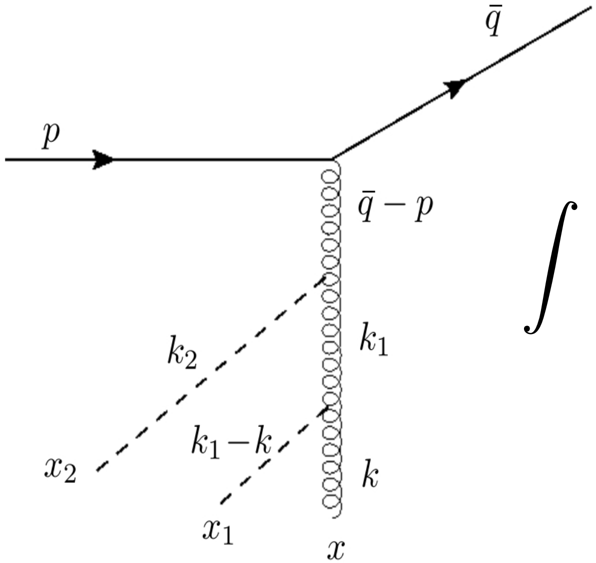
interactions of large and small x modes



$$
\begin{aligned}
i\mathcal{M} \;=\; & f_{acd} \int \frac{d^4k}{(2\pi)^4}\, d^4x\, d^4x_1\, e^{i(\bar{q}-p-k)x_1}\; e^{ikx} \\
& \bar{u}(\bar{q})\,(ig\,\gamma^\mu\, t^a)\, u(p)\, A^c_\lambda(x)\; \left[i\, g\, S^d(x_1)\right] \\
& \frac{1}{(p-\bar{q})^2 + i\epsilon} \left[- g^\mu_\lambda\, n\cdot(p-\bar{q}-k) \right. \\
& \left. + n^\mu \left[p_\lambda - \bar{q}_\lambda\,(1 - \frac{n\cdot k}{n\cdot(p-\bar{q})})\right]\right]
\end{aligned}
$$

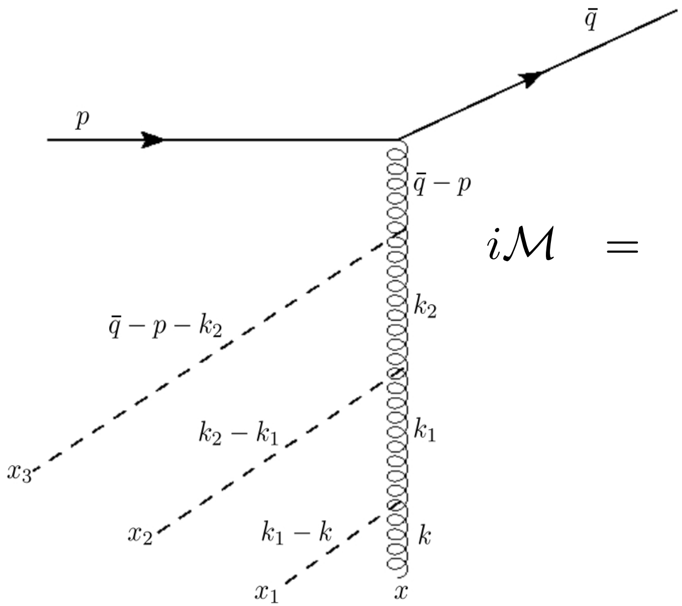
performing k^- integration sets $x_1^+ = x^+$

$$
\begin{aligned}
i\mathcal{M} \;=\; & 2 f_{acd} \int d^4x\, e^{i(\bar{q}-p)x} \\
& \bar{u}(\bar{q})\, \frac{\left[\not{n}\,(p-\bar{q})\cdot A_c(x) - \not{A}_c(x)\, n\cdot(p-\bar{q})\right]}{(p-\bar{q})^2}\, (ig\, t^a)\, u(p)\; \left[i\, g\, S^d(x^+, x_t)\right]
\end{aligned}
$$



$$\int \frac{dk_1^-}{(2\pi)} \frac{e^{ik_1^-(x^+-x_2^+)}}{2(\bar{q}^+-p^+)\left[k_1^- - \frac{k_{1t}^2-i\epsilon}{2(\bar{q}^+-p^+)}\right]} \sim \theta(x^+-x_2^+)$$

$$\begin{aligned} i\mathcal{M} &= 2\, f_{abc}\, f_{cde} \int d^4x\, dx_2^+\, \theta(x^+-x_2^+)\, e^{i(\bar{q}^+-p^+)x^- - i(\bar{q}_t-p_t)\cdot x_t} \\ & \bar{u}(\bar{q})\, \frac{\left[\not{n}\,(p-\bar{q})\cdot A_e(x) - \not{A}_c(x)\, n\cdot(p-\bar{q})\right]}{(p-\bar{q})^2}\, (ig\, t^a)\, u(p) \\ & \left[i\, g\, S_d(x^+, x_t)\right]\, \left[i\, g\, S_b(x_2^+, x_t)\right] \end{aligned}$$



$$i\mathcal{M} \;=\; \frac{2(i)^2}{(\bar{q}-p)^2}\, f^{abc} f^{cde} f^{egf} \int d^4x\, dx_2^+ dx_3^+\, \theta(x^+ - x_2^+)\, \theta(x_2^+ - x_3^+)$$

$$\bar{u}(\bar{q})\,(ig\, t^a)\left[n\cdot(p-\bar{q})\not{A}_f(x) - (p-\bar{q})\cdot A_f(x)\not{n}\right] u(p)$$

$$\left[i\, g\, S_g(x^+, x_t)\right]\; \left[i\, g\, S_d(x_2^+, x_t)\right]\; \left[i\, g\, S_b(x_3^+, x_t)\right]$$

$$e^{i(\bar{q}^+ - p^+)x^- - i(\bar{q}_t - p_t)\cdot x_t}$$

recall

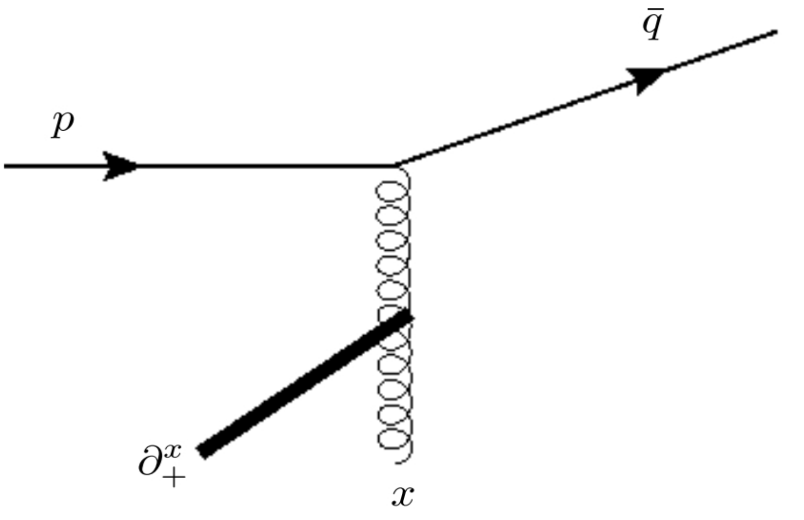
$$\partial_{x^+}\left[U^\dagger_{AP}(x_t, x^+)\right]^{ab} \;=\; (if^{bca})\,[igS_c(x^+, x_t)]$$

$$+\;(if^{bce})\,(if^{eda})\int dx_1^+ \theta(x^+ - x_1^+)[[igS_c(x^+, x_t)]\,[igS_d(x_1^+, x_t)]$$

$$+\;(if^{bch})\,(if^{gdf})\,(if^{fea})\int dx_1^+\, dx_2^+\, \theta(x^+ - x_1^+)\,\theta(x_1^+ - x_2^+)$$

$$[[igS_c(x^+, x_t)]\,[igS_d(x_1^+, x_t)]\,[[igS_c(x_2^+, x_t)] + \cdots\cdots$$

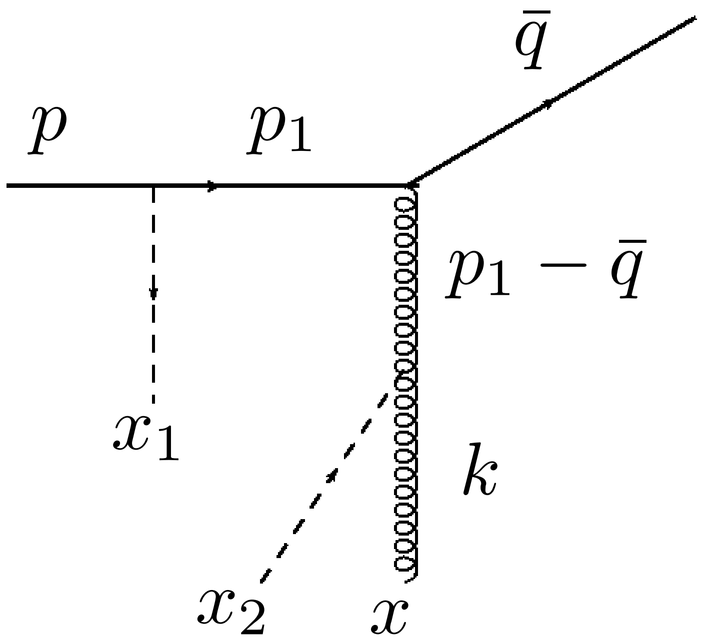
all re-scatterings of hard
Gluon can be re-summed



$$i\mathcal{M}_2 = \frac{2\,i}{(p-\bar{q})^2} \int d^4x\, e^{i(\bar{q}-p)x}\, \bar{u}(\bar{q}) \Big[(ig\, t^a) \left[\partial_{x^+} U^{\dagger}_{AP}(x_t, x^+) \right]^{ab} \left[n\cdot(p-\bar{q}) \not{A}_b(x) - (p-\bar{q})\cdot A_b(x) \not{n} \right] \Big] u(p)$$

with $$U_{AP}(x_t, x^+) \equiv \hat{P}\, exp \left\{ ig \int_{-\infty}^{x^+} dz^+\, S_a^-(z^+, x_t)\, T_a \right\}$$

but there is more!



both initial state quark and hard gluon interacting:

integration over p_1^-

$$\int \frac{dp_1^-}{2\pi} \frac{e^{ip_1^-(x_1^+ - x^+)}}{[p_1^2 + i\epsilon]\,[(p_1 - \bar{q})^2 + i\epsilon]}$$

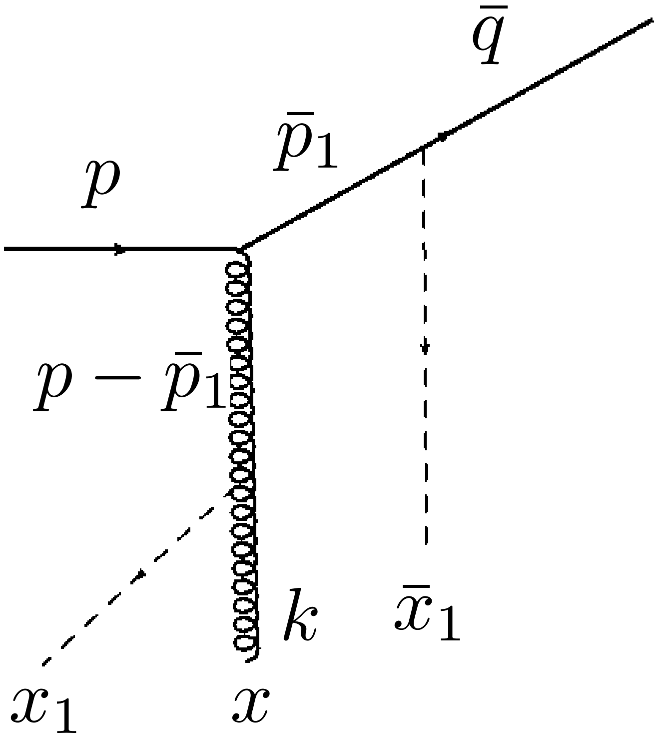
both poles are below the real axis, we get

$$\frac{e^{i\frac{p_{1t}^2}{2p^+}(x_1^+ - x^+)}}{\left[\frac{p_{1t}^2}{2p^+} - \bar{q}^- - \frac{(p_{1t} - \bar{q}_t)^2}{2(p^+ - \bar{q}^+)}\right]} + \frac{e^{i\left[\bar{q}^- + \frac{(p_{1t} - \bar{q}_t)^2}{2(p^+ - \bar{q}^+)}\right](x_1^+ - x^+)}}{\left[\bar{q}^- + \frac{(p_{1t} - \bar{q}_t)^2}{2(p^+ - \bar{q}^+)} - \frac{p_{1t}^2}{2p^+}\right]}$$

ignoring phases we get a cancellation!

this can be shown to hold to all orders whenever both initial state quark and hard gluon scatter from the soft fields!

how about the final state quark interactions?



integration over $\bar{p}_1^-$

$$\int \frac{d\bar{p}_1^-}{2\pi} \frac{e^{i\bar{p}_1^-(\bar{x}_1^+ - x^+)}}{[\bar{p}_1^2 + i\epsilon]\,[(p_1 - \bar{p}_1)^2 + i\epsilon]}$$

now the poles are on the opposite side of the real axis, we get both ordering

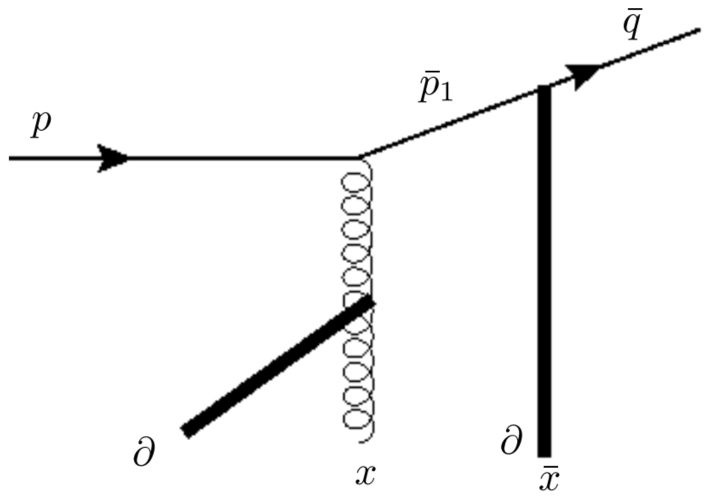
$\theta(x^+ - \bar{x}_1^+)$ and $\theta(\bar{x}_1^+ - x^+)$

ignoring the phases the contribution of the two poles add!

path ordering is lost!

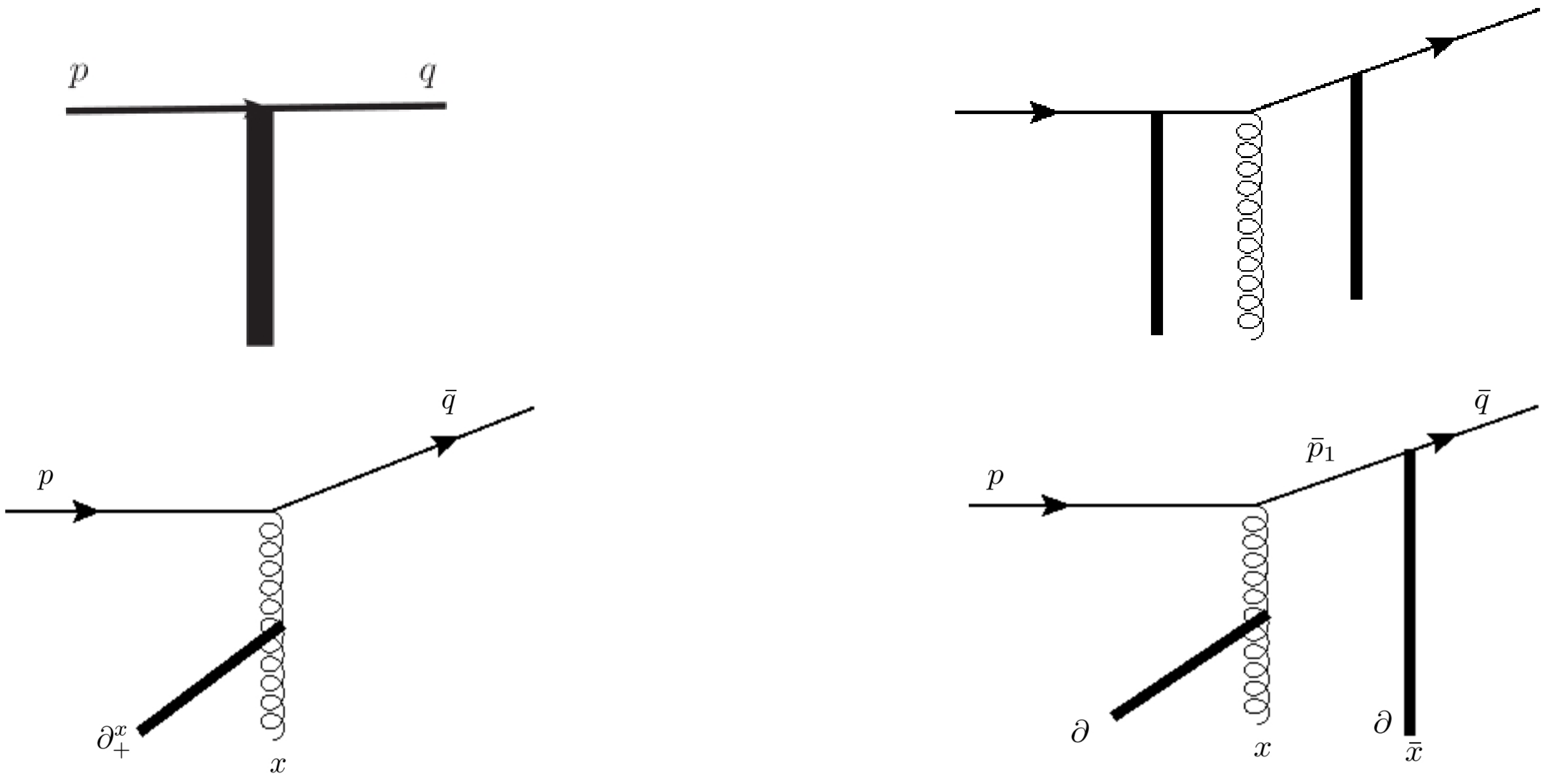
however further rescatterings are still path-ordered before/after $\mathbf{x_1^+}, \bar{\mathbf{x}}_1^+$

these contributions re-sum to



$$
\begin{aligned}
i\mathcal{M}_3 \;=\; & -2\,i \int d^4x\, d^2\bar{x}_t\, d\bar{x}^+ \,\frac{d^2\bar{p}_{1t}}{(2\pi)^2}\, e^{i(\bar{q}^+ - p^+)x^-}\; e^{-i(\bar{p}_{1t}-p_t)\cdot x_t}\; e^{-i(\bar{q}_t - \bar{p}_{1t})\cdot \bar{x}_t} \\
& \bar{u}(\bar{q}) \Bigg[\left[\partial_{\bar{x}^+}\, \overline{V}_{AP}(\bar{x}^+, \bar{x}_t)\right]\, \not{n}\, \not{\bar{p}}_1\, (igt^a)\, \left[\partial_{x^+}\, U^\dagger_{AP}(x_t, x^+)\right]^{ab} \\
& \frac{\left[n\cdot(p-\bar{q})\not{A}^b(x) - (p-\bar{p}_1)\cdot A^b(x)\,\not{n}\right]}{\left[2n\cdot\bar{q}\, 2n\cdot(p-\bar{q})\, p^- - 2n\cdot(p-\bar{q})\,\bar{p}_{1t}^2 - 2n\cdot\bar{q}\,(\bar{p}_{1t}-p_t)^2\right]} \Bigg] u(p)
\end{aligned}
$$

full amplitude: $\mathbf{i}\mathcal{M} = \mathbf{i}\mathcal{M}_{\mathbf{eik}} + \mathbf{i}\mathcal{M}_{\mathbf{1}} + \mathbf{i}\mathcal{M}_{\mathbf{2}} + \mathbf{i}\mathcal{M}_{\mathbf{3}}$



soft (eikonal) limit:

$$\begin{aligned} A^\mu(x) &\rightarrow n^- S(x^+, x_t) \\ n \cdot \bar{q} &\rightarrow n \cdot p \end{aligned}$$

$$i\mathcal{M} \longrightarrow i\mathcal{M}_{eik}$$

cross section: $\quad |\mathbf{i}\mathcal{M}|^2 = |\mathbf{i}\mathcal{M}_{\mathbf{eik}} + \mathbf{i}\mathcal{M}_{\mathbf{1}} + \mathbf{i}\mathcal{M}_{\mathbf{2}} + \mathbf{i}\mathcal{M}_{\mathbf{3}}|^2$

$$|i\mathcal{M}_2|^2 = \frac{8g^2}{(p-\bar{q})^4} \int d^4x\, d^4y\, e^{i(\bar{q}^+ - p^+)(x^- - y^-)}\, e^{-i(\bar{q}_t - p_t)\cdot(x_t - y_t)}$$
$$\left\{ p^+ q^- (p^+ - \bar{q})^2\, A^b_\perp(x) \cdot\, A^c_\perp(y) + 2\,(p^+)^2\, q_\perp \cdot A^b_\perp(x)\, q_\perp \cdot A^c_\perp(y) \right\}$$
$$\left[\partial_{y^+}\, U_{AP}(y_t, y^+)\right]^{ca} \left[\partial_{x^+}\, U^\dagger_{AP}(x_t, x^+)\right]^{ab}$$

other terms are more complicated: *spinor helicity formalism* for Dirac Algebra

DIS: structure functions, di-jet production

PA: single inclusive particle production

Need to clarify, work in progress

The limit of intermediate/large x (high p_t) ?

set S = 0, target gluon distribution function (gauge invariance?)

Matching between small and large x ?

brute force?

strength of gluon field?

Gluon scattering and (photon) radiation

backward-forward asymmetry

One-loop correction: cross sections for both low and high p_t

SUMMARY

CGC is a systematic approach to high energy collisions

CGC breaks down at large x (high p_t)

Toward a unified formalism:

quark scattering from small and large x fields

gluon radiation, 1-loop corrections

particle production in pp, pA in both small and large p_t regions

effective action approach for AA?

jet energy loss from early times