Introduction: QCD (and electrons)

with short reviews of QFT and pQCD

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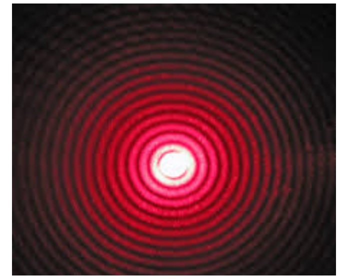
1. **Light and photons: the hidden universe**
2. **The game and stories of quantum fields**
3. **What makes QCD "different"**
4. **e-hadron experiments and the "dark world" (briefly)**
5. **Exploring QCD: short and endless stories in the proton**

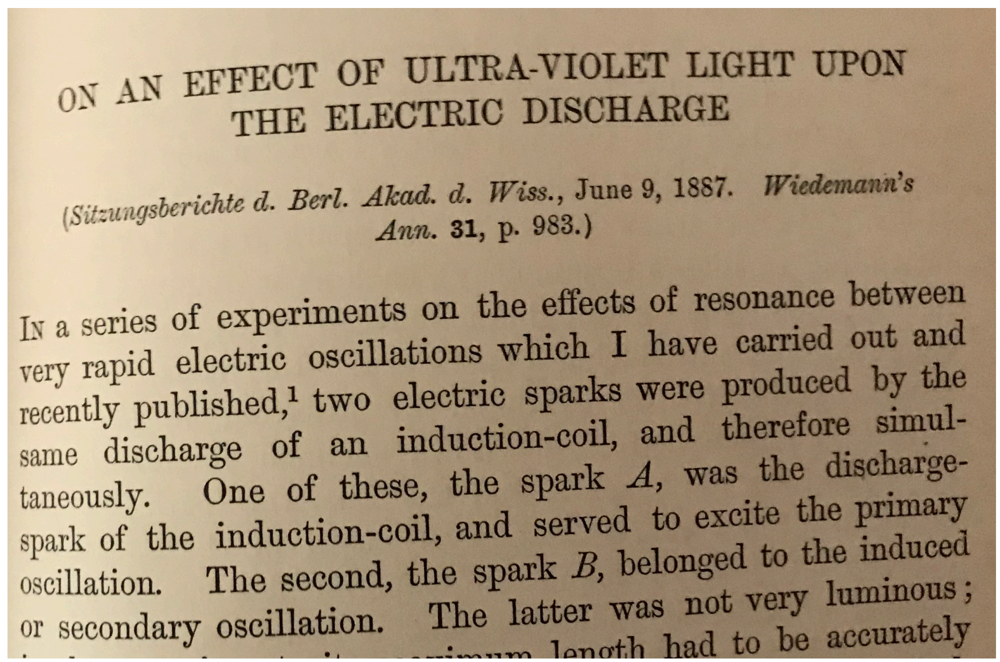
1. Light and photons

"The wave theory of light, which operates with continuous functions has proved itself splendidly . . . (Einstein, 1905)



One should keep in mind, however, that optical observations apply to time averages only . . . the energy of light [is] localized at points in space . . . and can be absorbed or generated only as a whole."

The photoelectric effect (Hertz,1887)



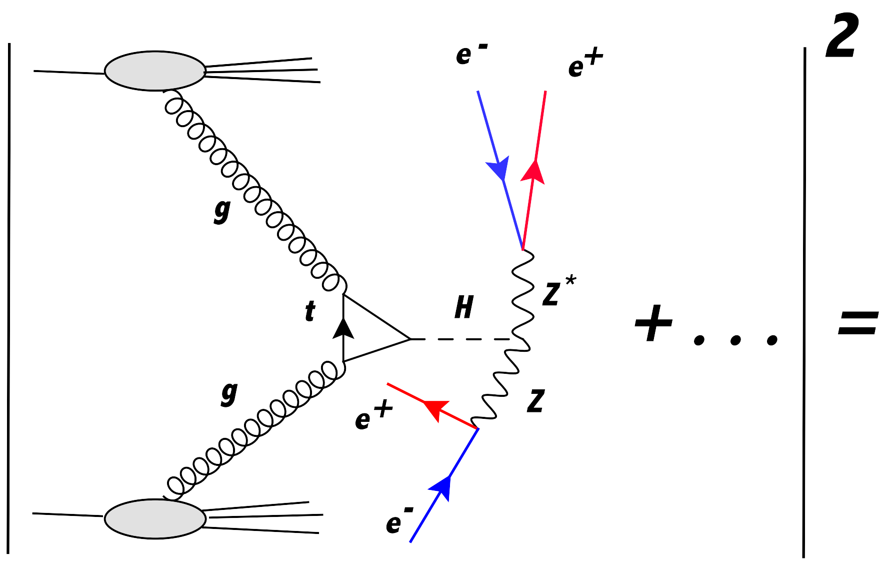
ON AN EFFECT OF ULTRA-VIOLET LIGHT UPON THE ELECTRIC DISCHARGE

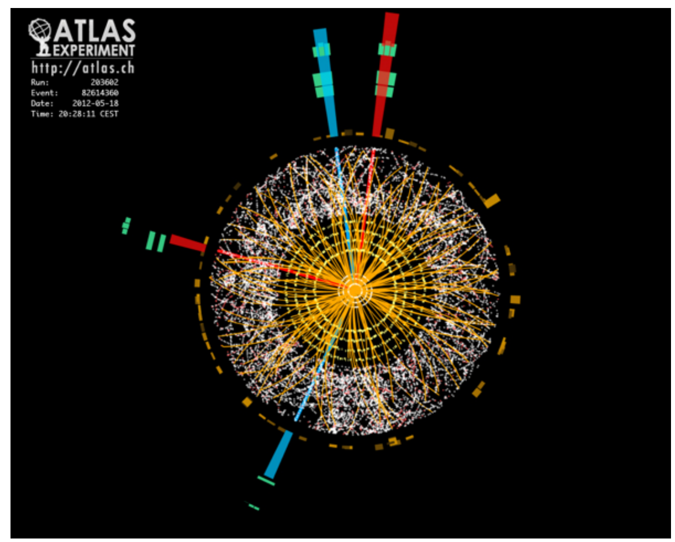
(*Sitzungsberichte d. Berl. Akad. d. Wiss.*, June 9, 1887. *Wiedemann's Ann.* **31**, p. 983.)

In a series of experiments on the effects of resonance between very rapid electric oscillations which I have carried out and recently published,[1] two electric sparks were produced by the same discharge of an induction-coil, and therefore simultaneously. One of these, the spark *A*, was the discharge-spark of the induction-coil, and served to excite the primary oscillation. The second, the spark *B*, belonged to the induced or secondary oscillation. The latter was not very luminous; ... maximum length had to be accurately

Density of photons is proportional to the square of the wave amplitude.

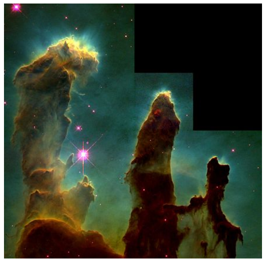
This set the stage for many great discoveries of the twentieth century: all matter (like photons) is described in terms of particles that are created or absorbed at infinitesimal points. At longer scales, these particles combine into "emergent" systems (like light waves).





Emergent "degrees of freedom" are a universal feature of classical and quantum laws of nature

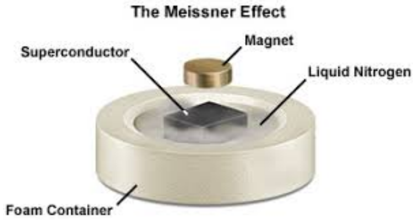
At scales we can see:



From hydrogen gas to galaxies and stars

From atomic magnetic moments to (anti)ferromagnetism

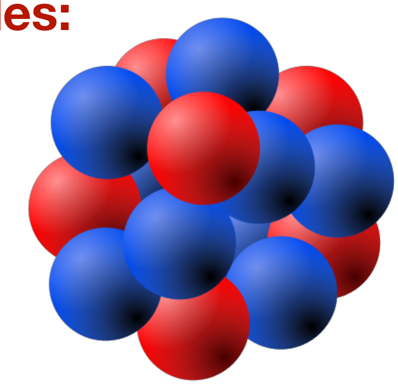
From molecules to phases of matter



From electrons and metals to superconductors

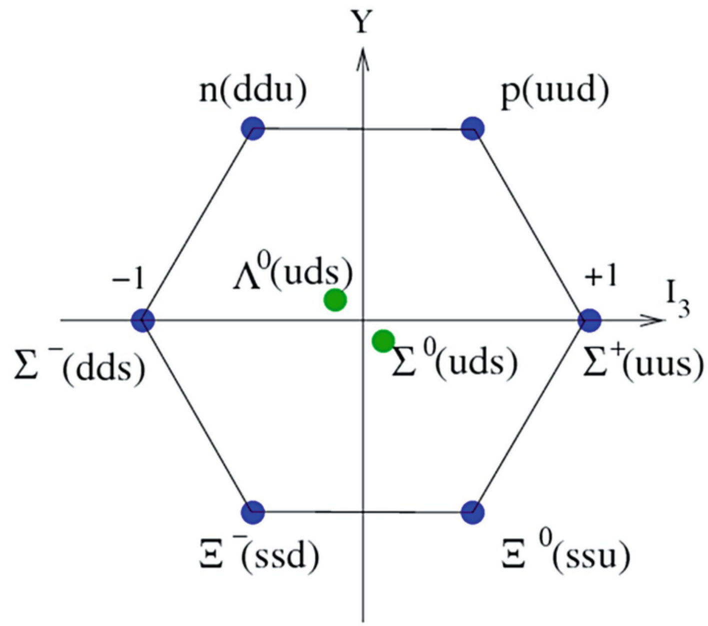
From chemistry to life itself

And at the micro scales:



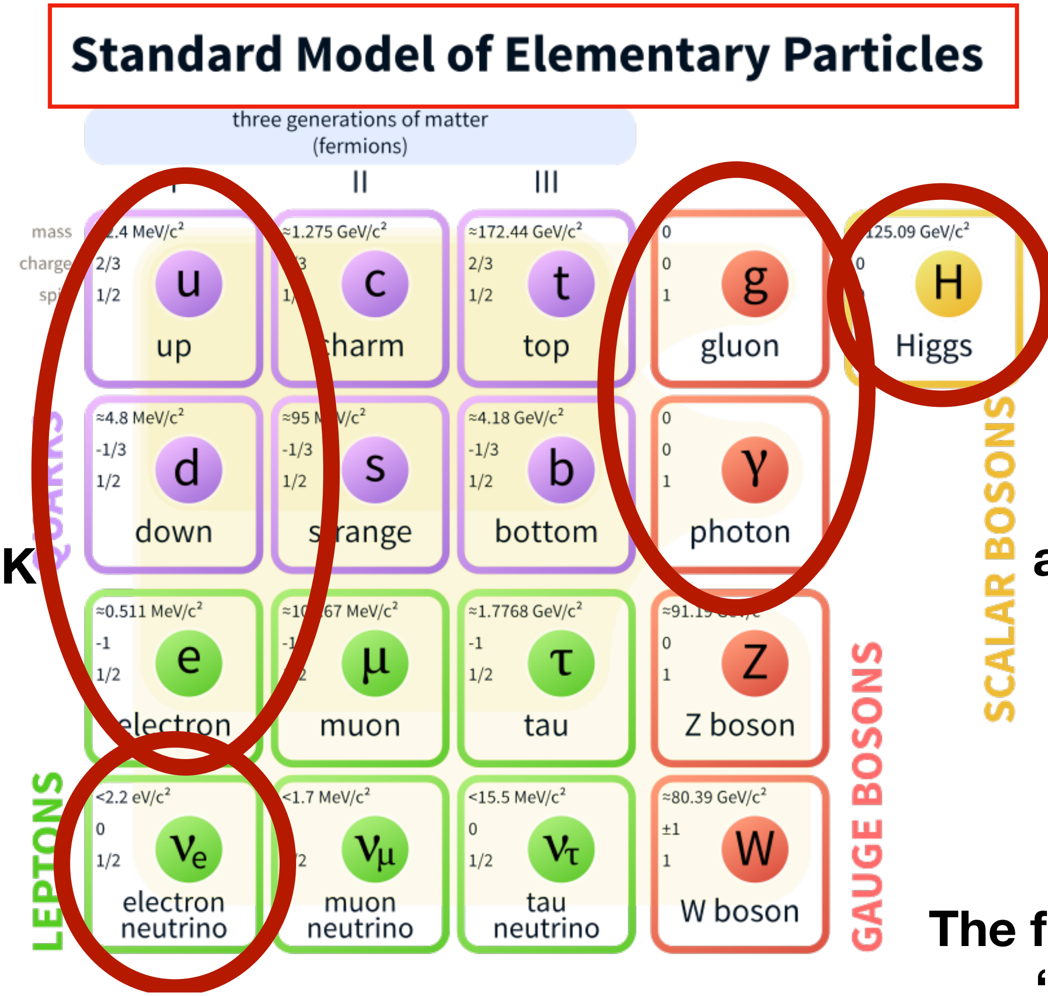
From protons and neutrons to nuclei

From quarks to protons, neutrons and other baryons



At each step, the compound system has features qualitatively different than those of its constituents, yet following from them.

Yet everything we can “see” is made up of the matter and force particles that are players in the game of the



BARYONS:
THREE QUARKS

MESONS:
QUARK/ANTIQUARK

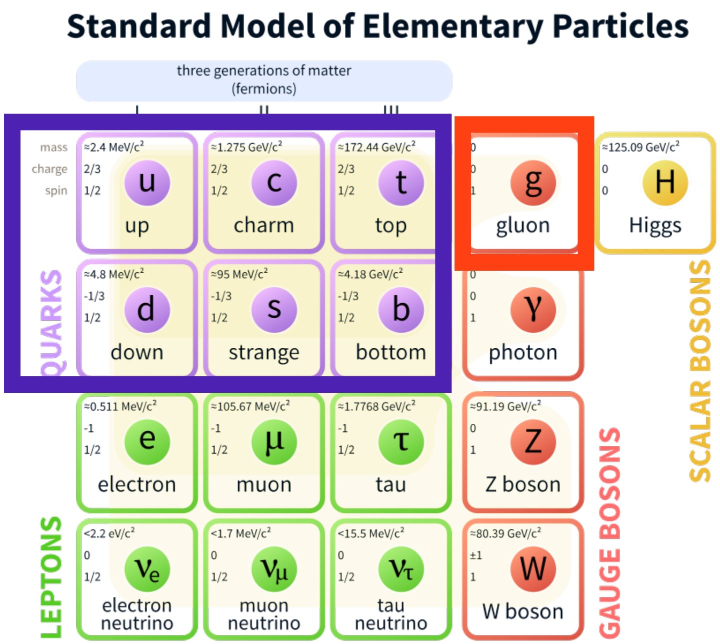
Supply the forces that hold nuclei and atoms together

Don’t usually see them but they’re all over the place

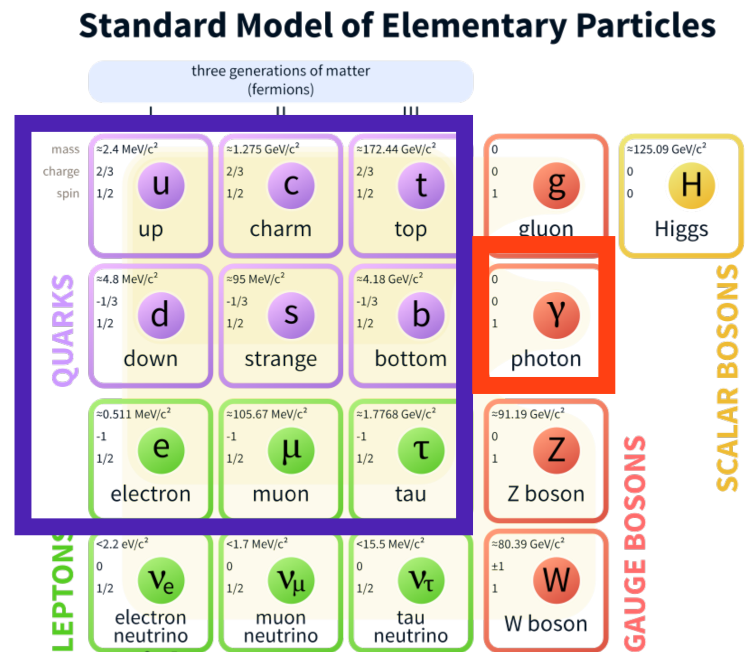
The fermions carry “spin 1/2”
The basic quantum of angular momentum and force particles “spin 1”.
Their spin directions can change, but never, ever slow down.

FORCES

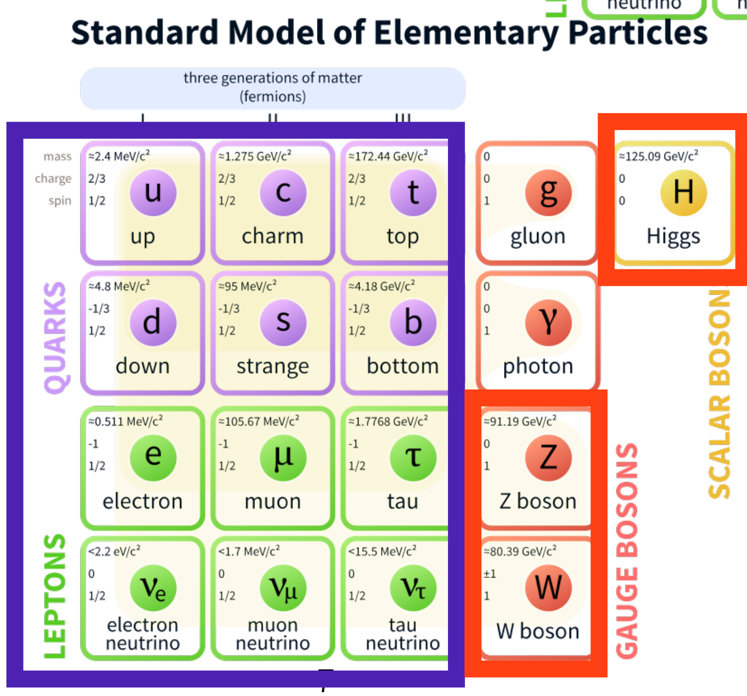
The strongly interacting fermions & their gluon (QCD):



The charged fermions & their photon (QED):



All the fermions experience the weak force:



Where have they been? The three unseen realms

- Right after the Big Bang, all particles mixed freely. Decays were balanced by production, but as the universe expanded and cooled, high energy collisions ceased.

- **Heavy particles decayed and were not replaced, and are now found only in short-lived virtual states, from which they can only emerge with sufficient energy before decaying once again.**

- **Yet other components of the universe appear to be "hidden" from us by not sharing E&M: dark matter.**

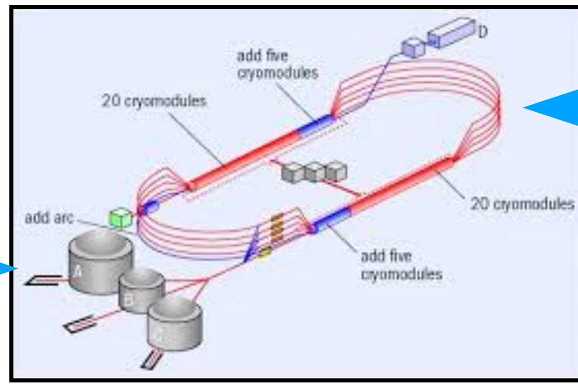
- **Stable quarks and gluons retreated to the tiny volumes of protons and neutrons, surrounded at great relative distances by electrons and photons.**

Experiments and theory at electron hadron colliders seek out matter in each of these realms.

We use the players: quarks, gluons, electrons and photons. . .

Quantum field theory provides the rules. At JLab, for example,

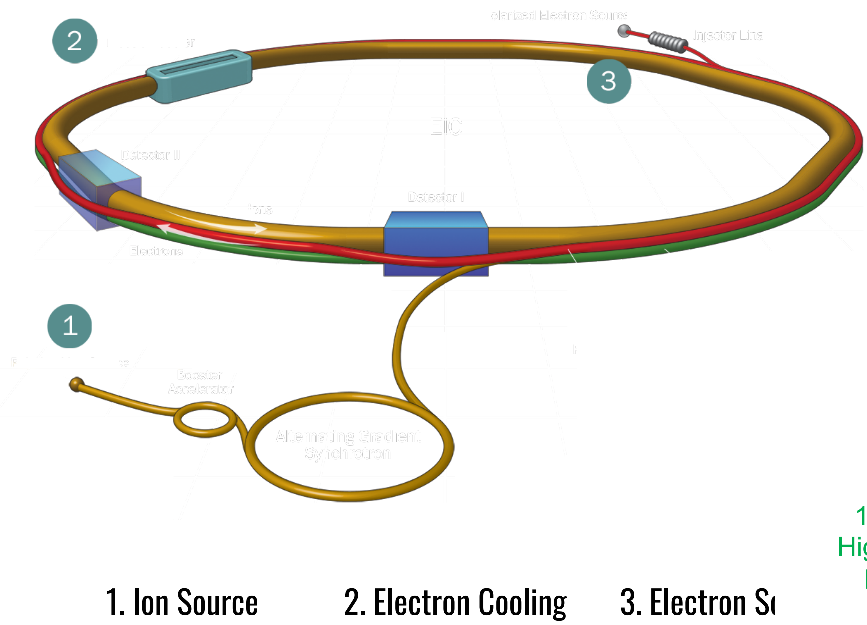
Nucleons await them in the experimental halls.



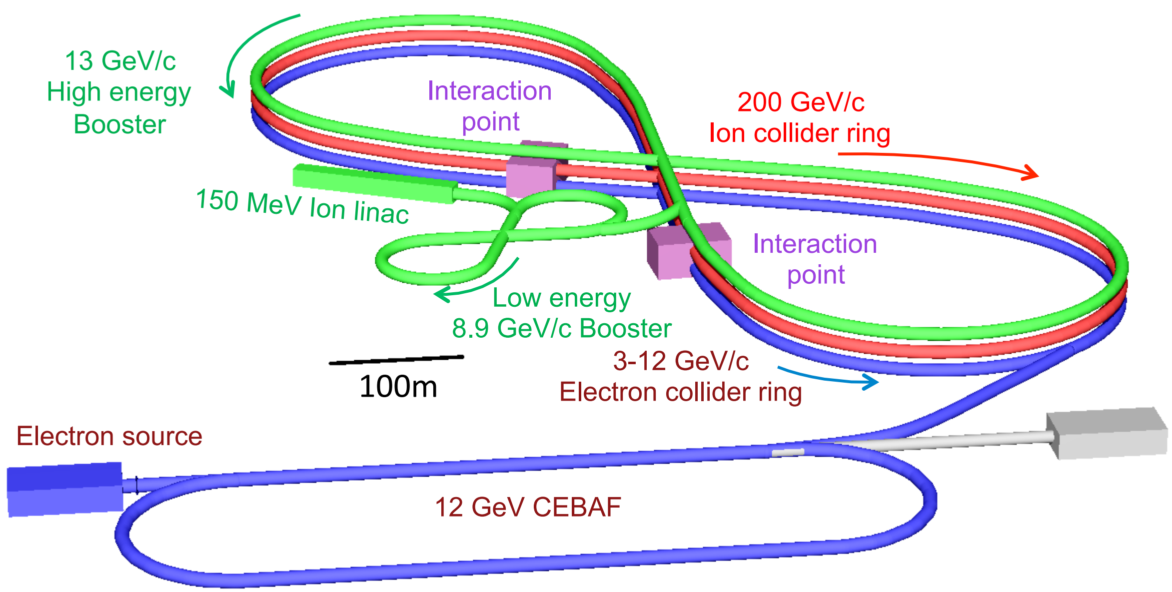
Electrons are accelerated around the loops

Quantum field theory systematizes the "duality" between fundamental and emergent, and we use each to explore the other.

Higher energies will require colliding beams, an EIC:



For example, one of these . . .



2. The Game of Quantum Fields

The Rules: Equations that govern the visible universe.

For ΔT a small time:

("Amplitude" for a new list of particles at time $T + \Delta T$)

=

(ΔT) x [rule for changing the list] x (Lists of particles at time T)

(Schroedinger equation)

Rules act on any list of fermions and bosons

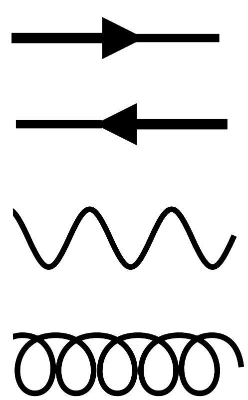
that we represent as

Quark or lepton

Anti-quark or anti-lepton

Photon or weak boson

Gluon

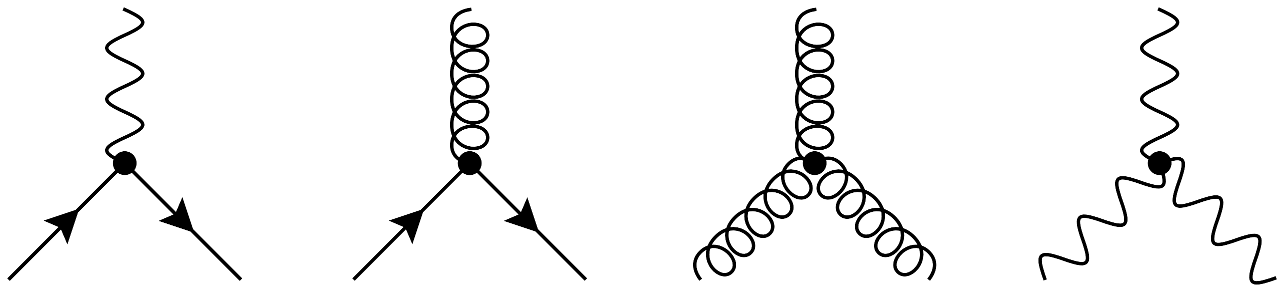


The list of particles is a possible configuration of the field(s) associated with those particles

The Beautiful Theories of the Standard Model

Here are the rules for the electromagnetic, strong and weak forces. They are each proportional to a "charge": electric: e, strong: g_s, or weak: g_W.

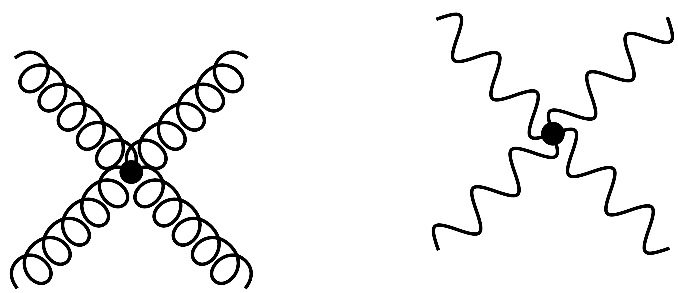
Rules for three particles



charge e **charge g_s** **charge g_s** **charge g_W**

Rules for four particles

Yang-Mills theories (1954) for the strong and weak interactions - charged vector particles.

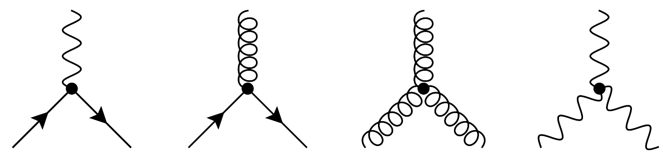


charge $(g_s)^2$ **charge $(g_W)^2$**

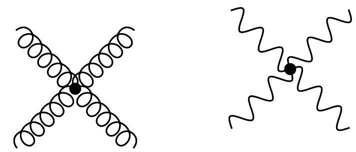
All time directions are allowed! (Dirac, 1928)

So, these:

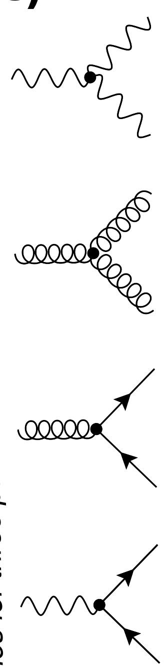
Rules for three particles



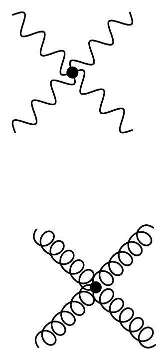
Rules for four particles



Implies these:
(pair production and antiparticles)

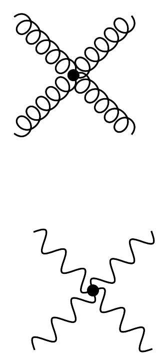


Rules for three particles

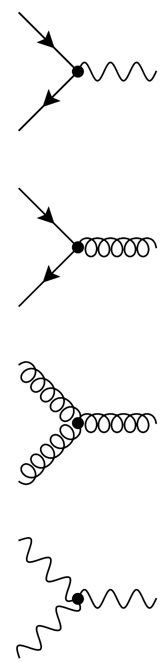


Rules for four particles

and these too:
(pair annihilation)



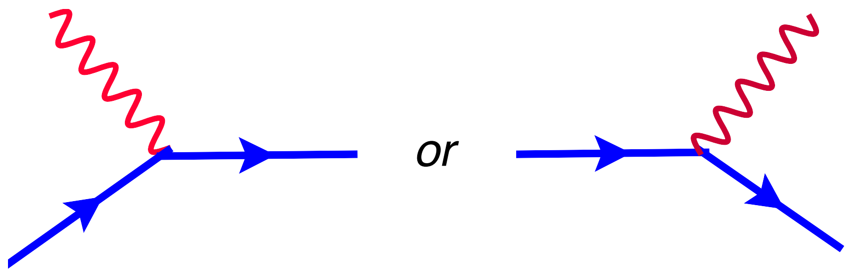
Rules for four particles



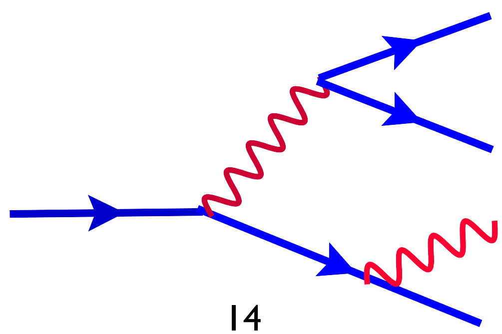
Rules for three particles

Here's how it works: the stories:

There is a kind of restlessness in nature: every single particle is constantly trying to unwind a whole new world out of itself, and at the same time to ravel it back up, always through simple steps that increase or decrease the number of particles by one or two.



And it keeps going on and on.

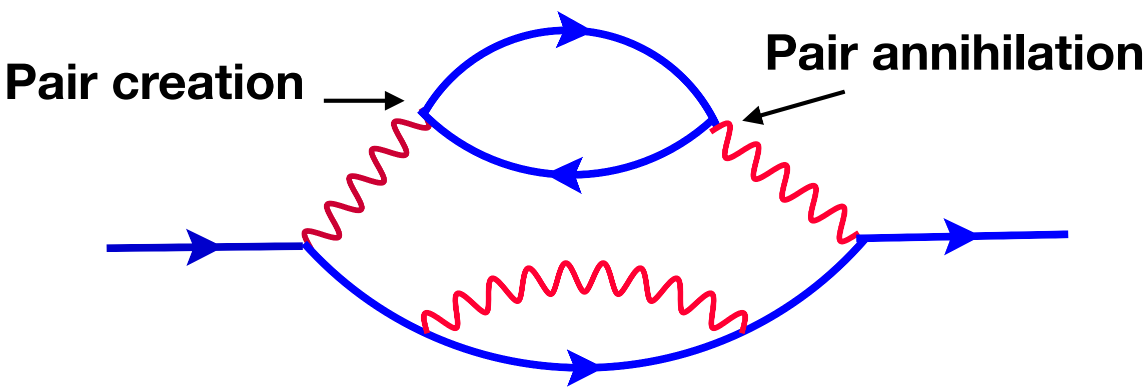


But, sadly, for an isolated electron, all of these states have energy greater than the electron's energy, by some amount, say ΔE.

These are called ``virtual states", and they live only for a time of about

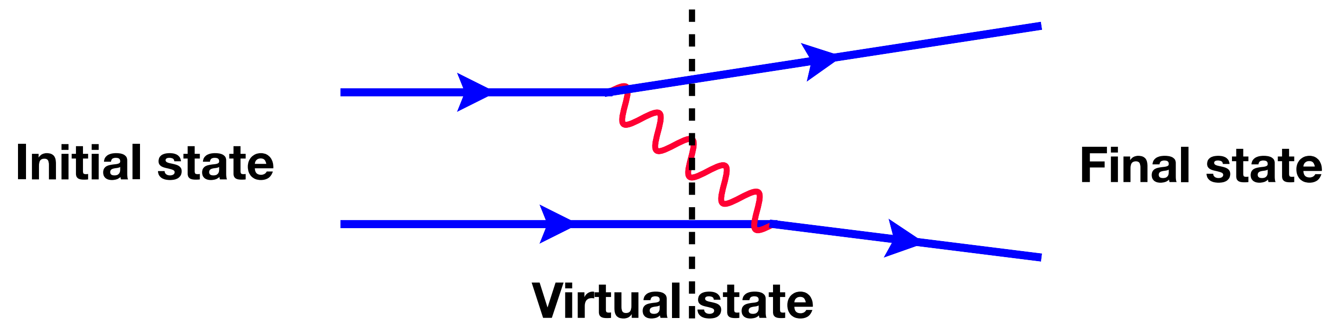
$$T = h/\Delta E$$

before they are wound back in, like

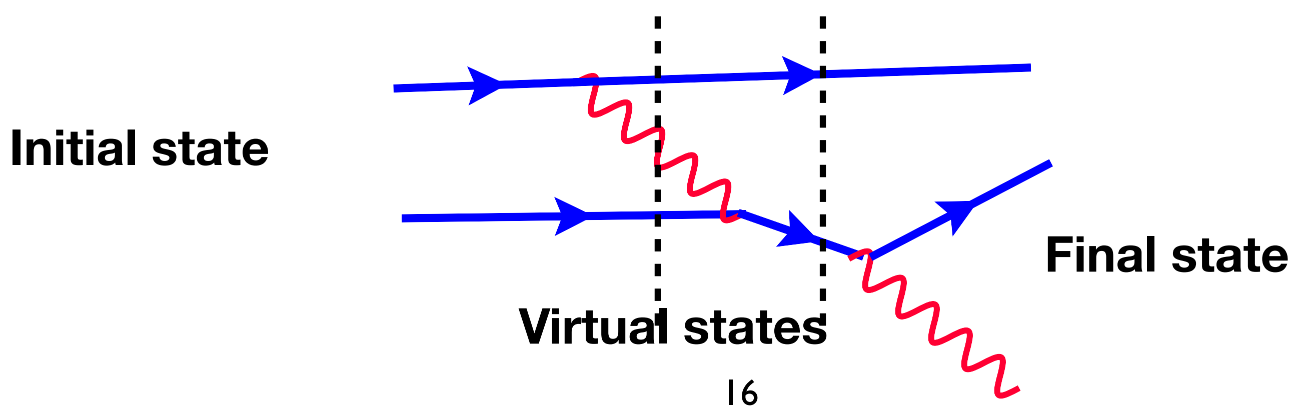


And no matter how hard it tries, each particle in isolation returns always to itself, only to start all over again.

But starting with two particles in a state, together they have enough energy to produce new states by spending short amounts of time in a series of virtual states. “Scattering”.

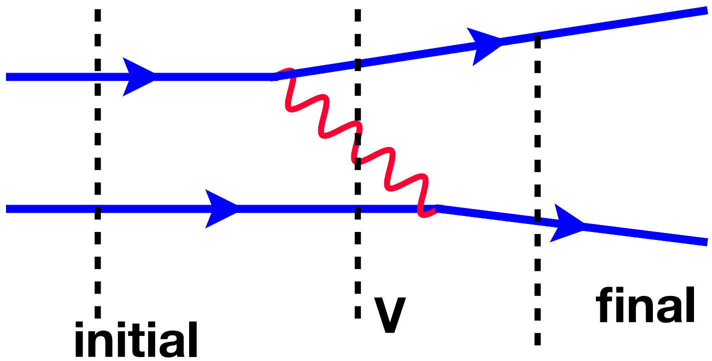


The higher their energy, the more states they can bring about. This is how things happen . . . for example, X-rays are made by adding another virtual state.



From pictures to predictions

For each process given by a picture, there are rules to calculate a wave height, or "amplitude" (Feynman). The amplitude is just a number, found like this:

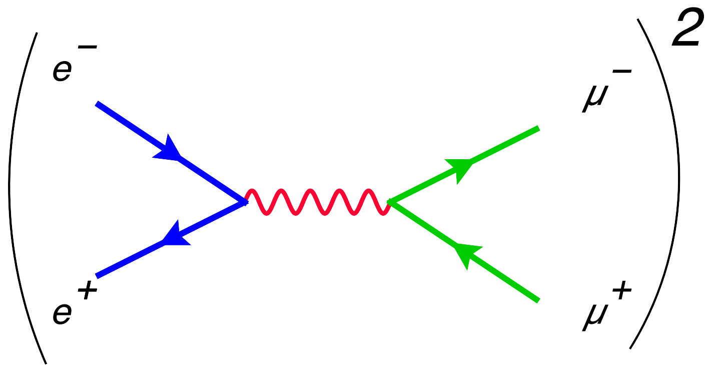


$$= e^2 \left(\frac{\Delta T_V}{h} \right) \times \text{(factors of particle energy)}$$

$$\Delta T_A = \frac{h}{E_V - E_{initial}}$$

The PROBABILITY for any process to happen is proportional to the SQUARE OF THE AMPLITUDE, just as the density of photons in a wave is proportional to the square of the Electric field.

For example, the amplitude that corresponds to the creation of a muon-antimuon pair from an electron-positron pair in a head-on collision, the probability is given by



= $e^4/8\pi$ when the muon is in the same direction as the electron
= $e^4/16\pi$ when the muon is produced at right angles to the electron

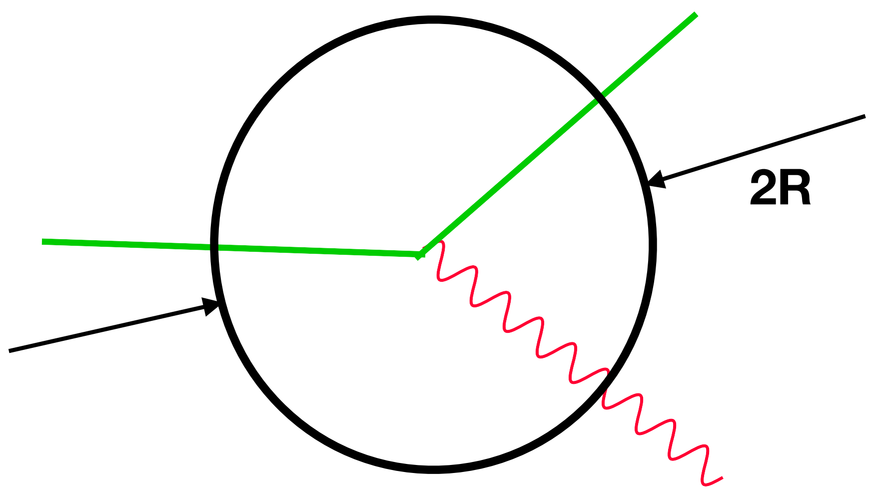
Simple!

But it gets more complicated quickly with more virtual states.
Because e is small, however, this prediction is pretty accurate.

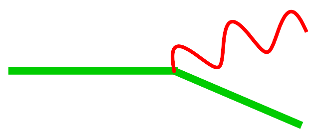
This is quantum field theory.

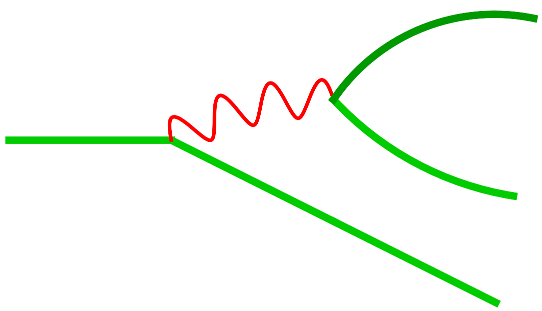
There is no limit to how short a time virtual states might live — no limit to the energy they might have.

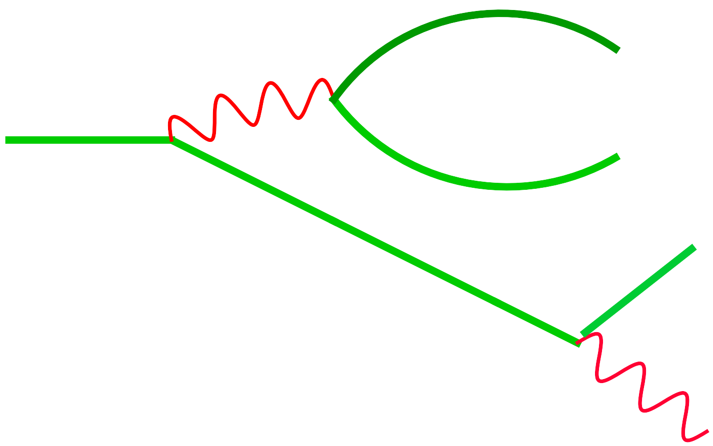
So the closer you look at an interaction, the more you will find. Look inside a radius R, and find not only

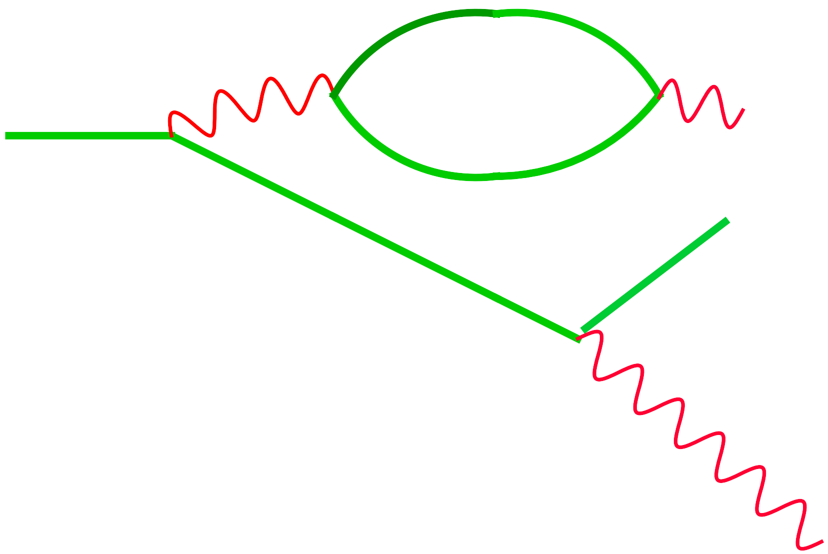


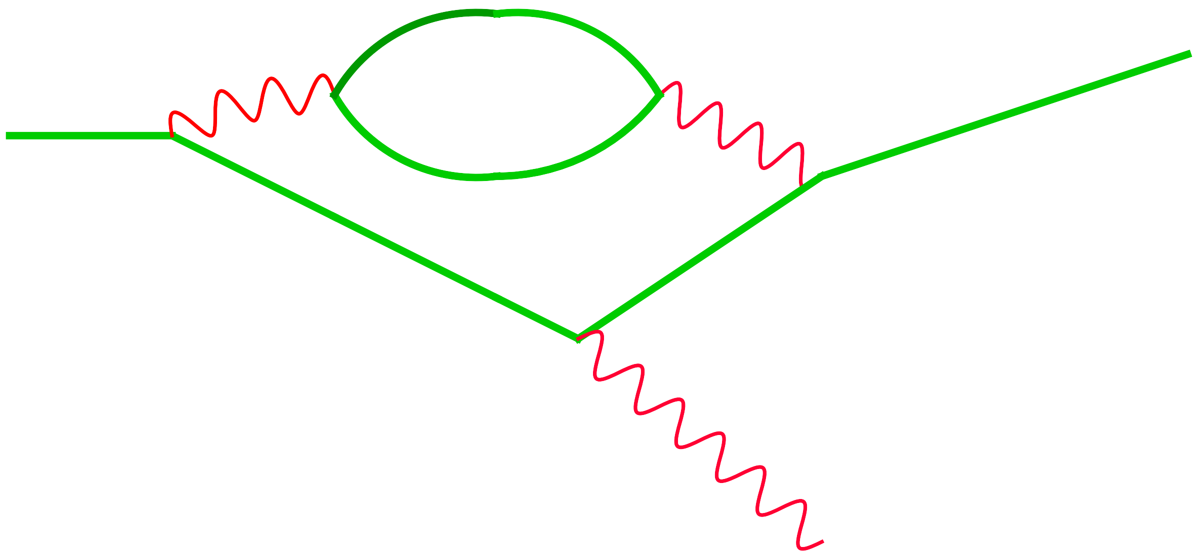
but also . . .



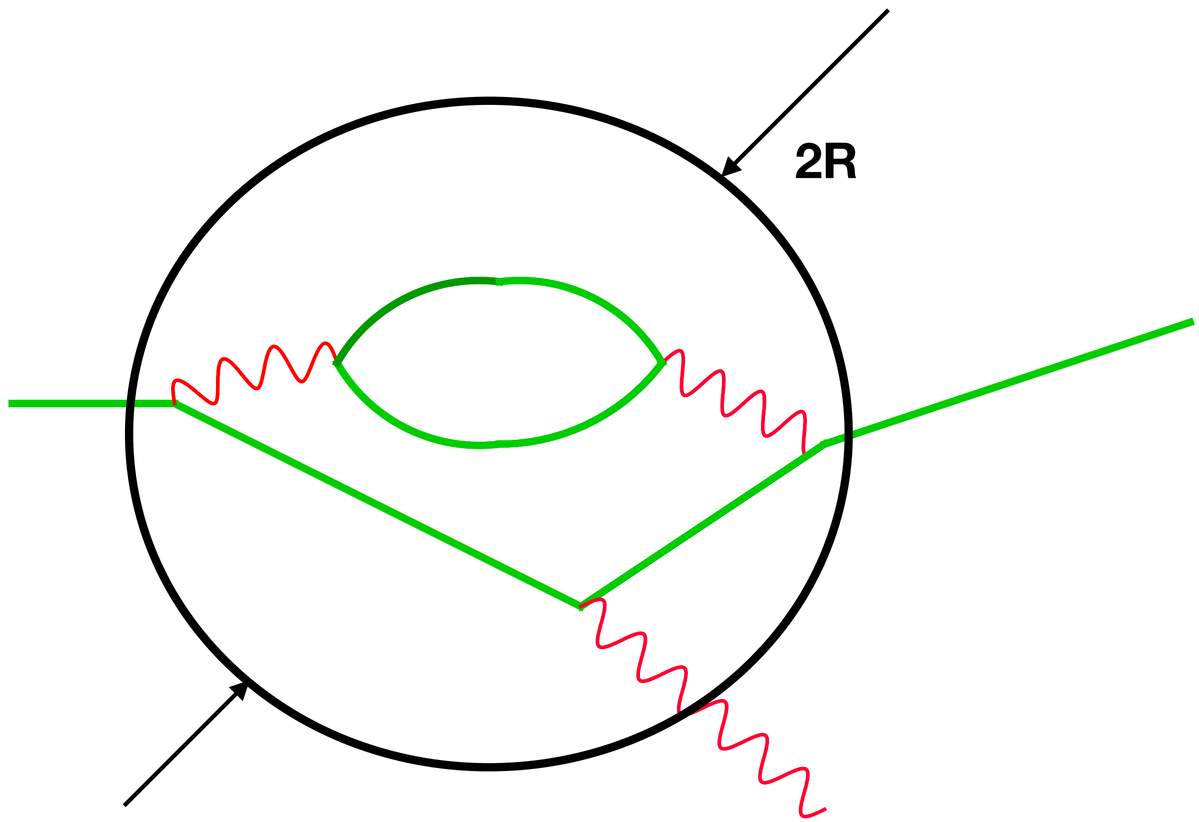






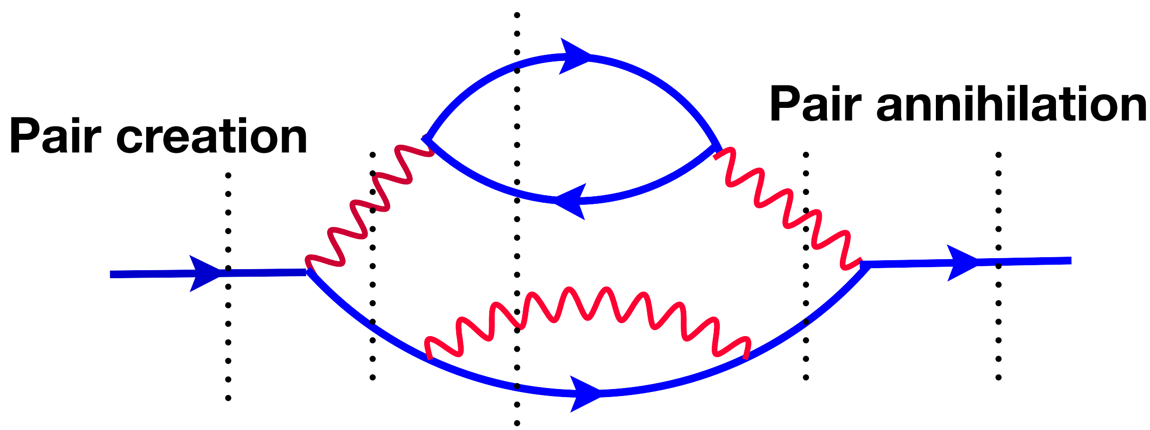


We can actually calculate how all these diagrams depend on R. This is called the "running coupling" and is usually given as a function of the de Broglie wavelength that corresponds to radius R: $\alpha(p=h/\lambda)$ [$\alpha=e^2/4\pi$]. For QED this technique is not such a big effect, but it is very important for the strong interactions.



What do we see? “States” Collections of particles that don’t change in time

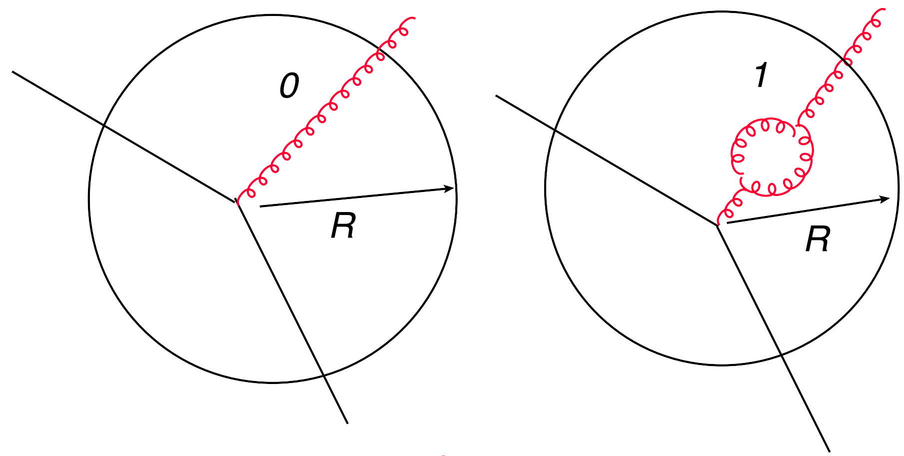
[rule for changing the list] x (Sum of lists of particles) = (Same sum of lists of particles)



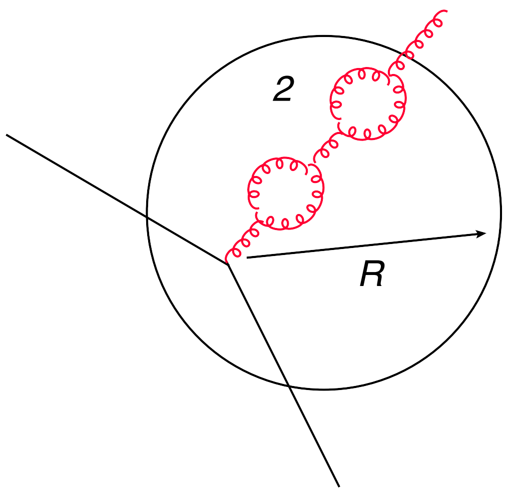
Individual electrons are really collections of these states and more, but its still. “just an electron with some friends”.

3. What makes QCD different

But quarks are very, very different. Why? States with two extra gluons add up to infinity for R about 1 Fermi. There has to be a nearby source to absorb them. Quarks cannot appear alone; this is called “confinement”.



This is not something proven, but demonstrated by “numerical lattice simulations” which provide beautiful agreement in hadronic mass differences



+ . . .

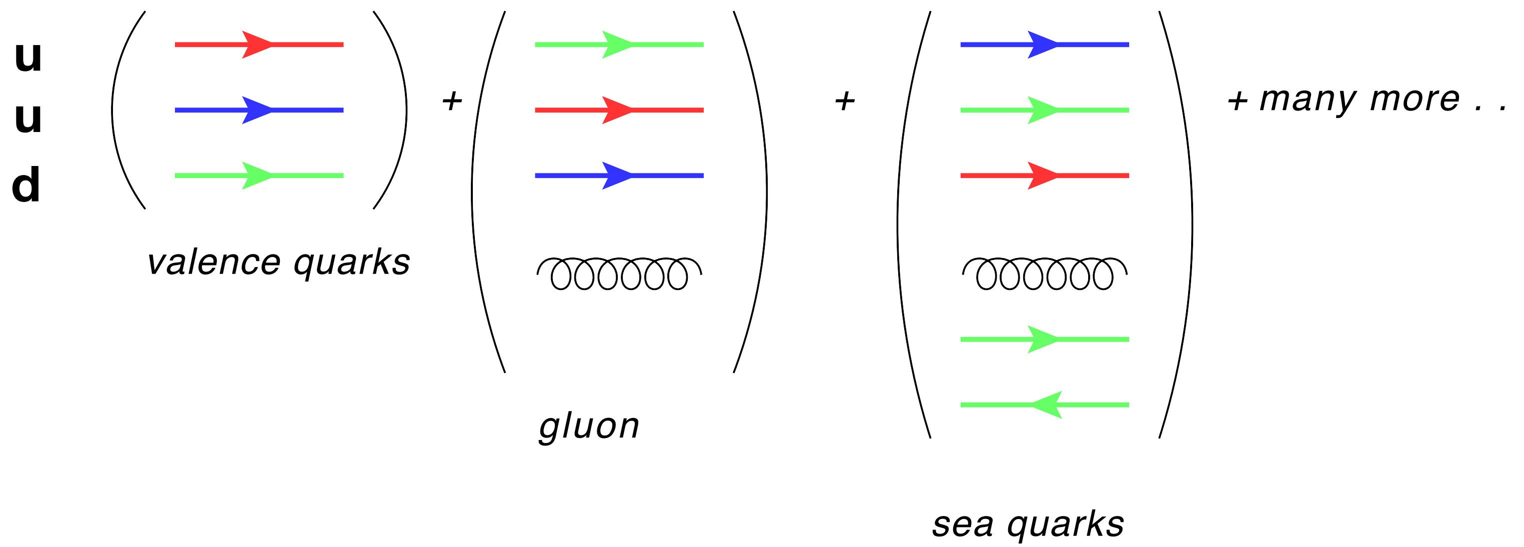
Each increases with R.

For R about 1 fm, they are all equal!

Asymptotic Freedom: Smaller (Larger) R gives weaker (stronger) forces

This means that the "states" of QCD are really different. They are the protons, neutrons and other hadrons, mostly made of three quarks (baryons). and quark-antiquark (mesons).

Our world, of course, is mostly protons, neutrons and the nuclei they can make. In our pictures, they are represented like:



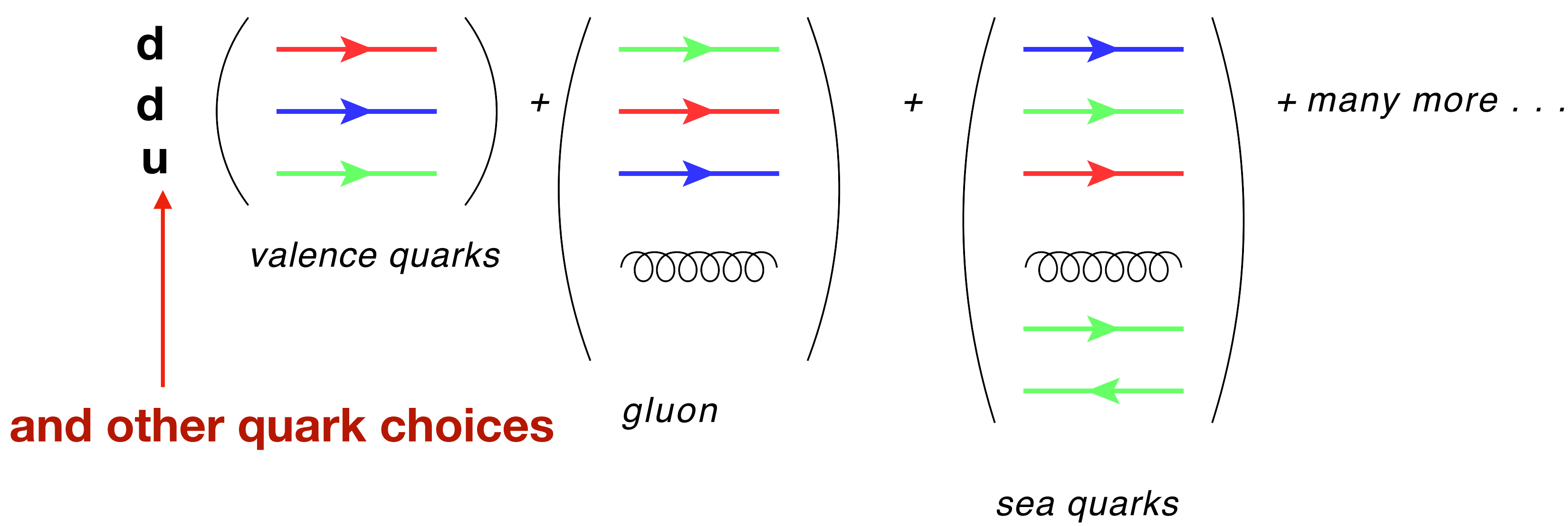
= | proton >

Taken all together, the proton has spin-1/2, the same as an electron or a single quark. It has a definite mass and charge +1. It is extraordinarily stable, and is the ultimate decay product for heavier solutions to the QCD Schrodinger equation.

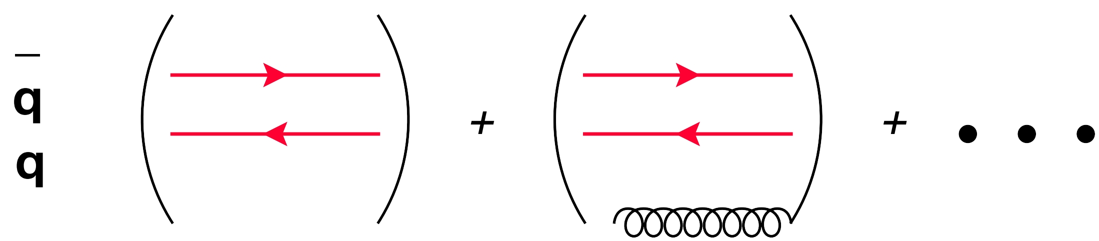
The other "classic" states:

[rule for changing the list] x (Sum of lists of particles)
= (Same sum of lists of particles)

| neutron > =



| meson > =

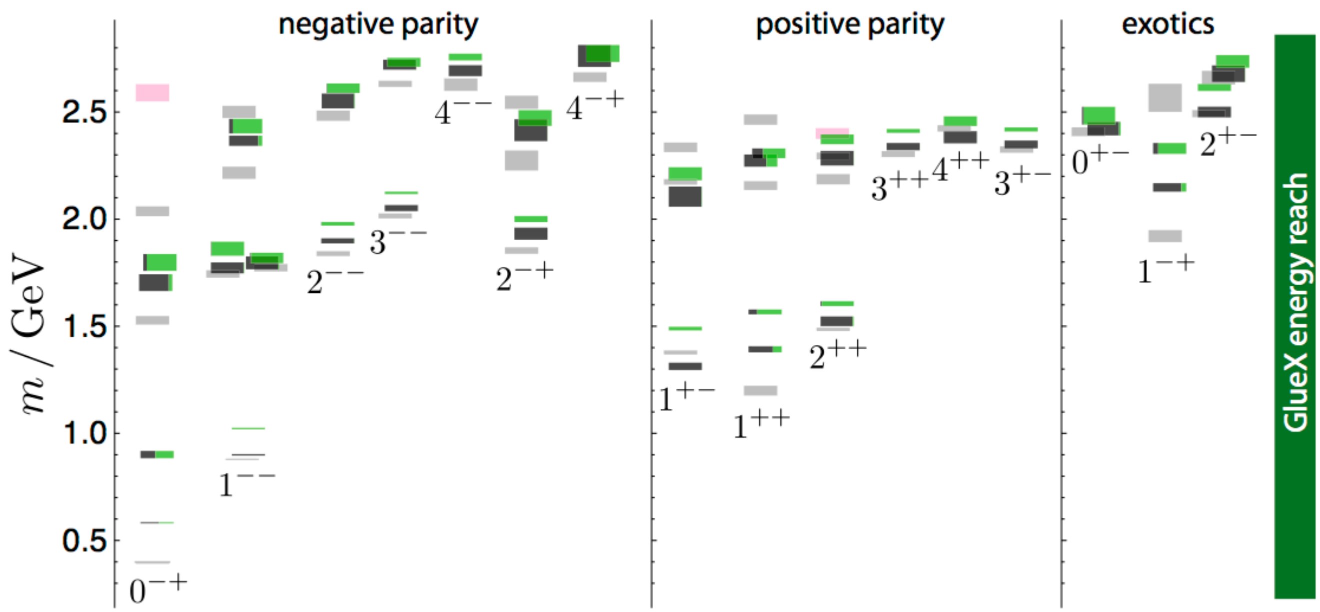


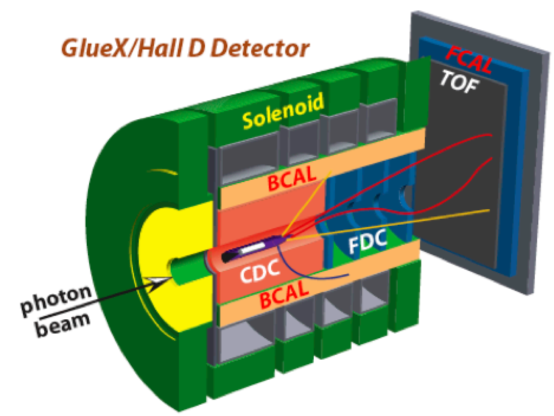
"On the lattice": very roughly — the computer starts with list of just three quarks, or a quark and an antiquark fixed at some position. The state can be given "extra" properties, like spin and left-right symmetry (parity).

Fun part: "uncertainty principles" in QFT mean that states of all energies will emerge.

It then solves the Schoedinger equation (rules for how the list changes in time) and looks for the lowest energy state that is produced.

For example, from the USQCD Collaboration collaboration web site):

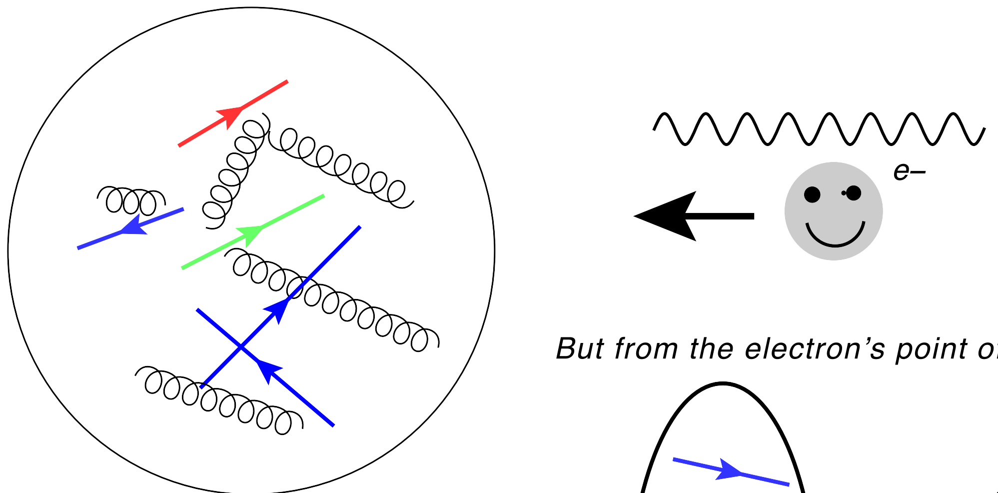




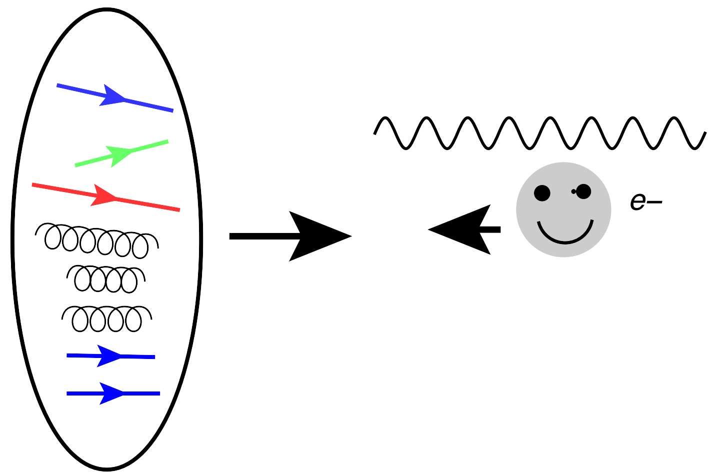
Lattice QCD calculations of the meson spectrum suggest the presence of many exotics.

What a proton looks like, and why you need high energy to see inside:

At rest, a proton looks like this, with partonsgoing every which way.



But from the electron's point of view,they all line up (almost)



To a good approximation, an electron arrives in a virtual state with a single extra photon.   Only that photon interacts directly with quarks in the proton. How much can you get from that?

Quite a lot!   When that photon is absorbed by a quark

The proton may remain "whole", but change direction: elastic scattering.

It may produce an "excited" heavier proton: quasi elastic scattering.

It may break up the proton: inelastic scattering, and produce other particles, anticipated or not in QCD.

If it transfers a lot of energy: "deeply inelastic".

We'll see a little of what we can learn from each of these.

It may also be accompanied not by a photon, but by a short-lived, heavy particle like the Z-boson (a brief detour).

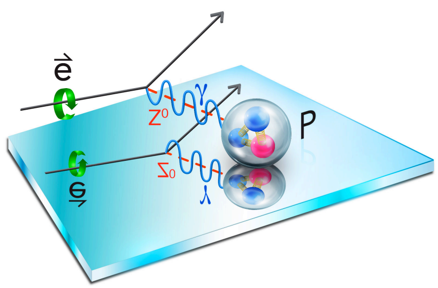
4. Electron-hadron Collisions and Dark World

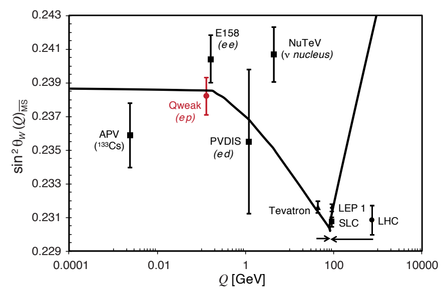
Electron-proton elastic scattering through Z exchange is exquisitely sensitive to the parameters of the Standard Model, through spin-dependent parity violation.

JLab Qweak 2019 (1905.08283)

An EIC will provide a much-needed energy range.

EPJ (2016) 52, 268





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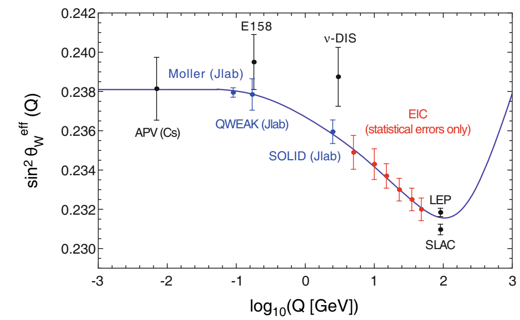
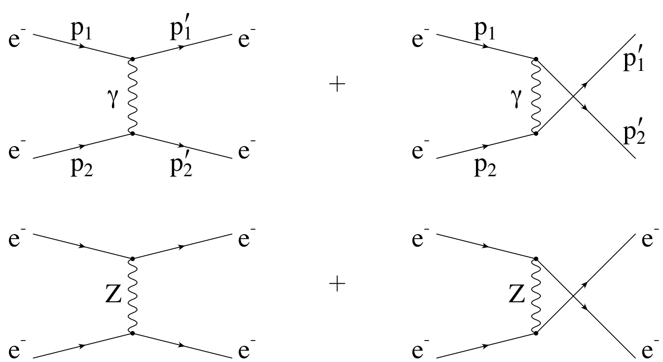
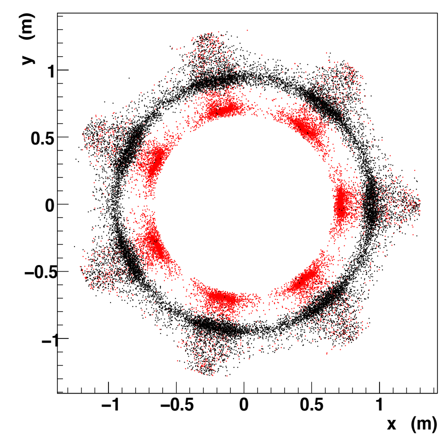


Fig. 74. Projected statistical uncertainties on the $\sin^2\theta_W$ in a series of Q^2 bins ($\sqrt{s} = 140\,\mathrm{GeV}$, $200\,\mathrm{fb}^{-1}$.) The black points are published results while the blue points are projections from the JLab program.

The future JLab "Moeller" experiment will push tests for non-standard forces through measurement of ee elastic scattering, kinematically distinguishable from ep.

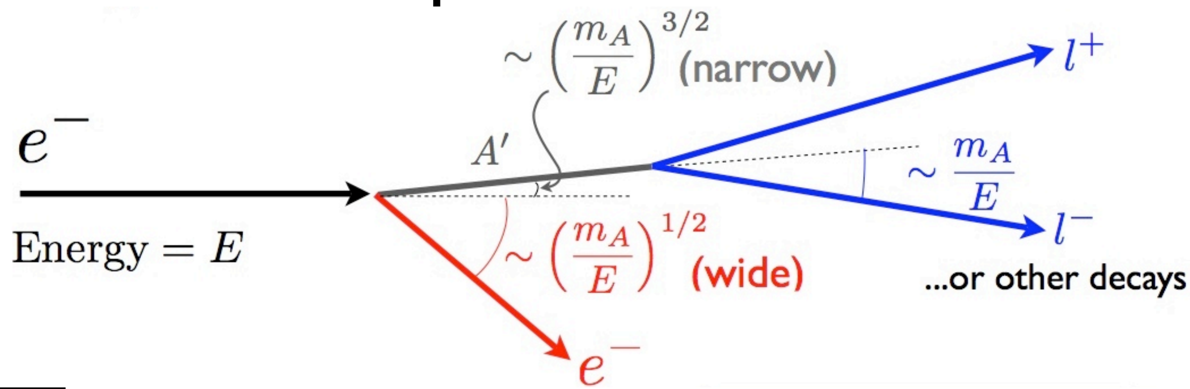




Moeller proposal, 2008

JLab and the Dark World, Cont'd

The electron may emit a new kind of particle: "dark photon", a "portal" to dark matter in many theories, not detectable in previous direct-detection experiments.

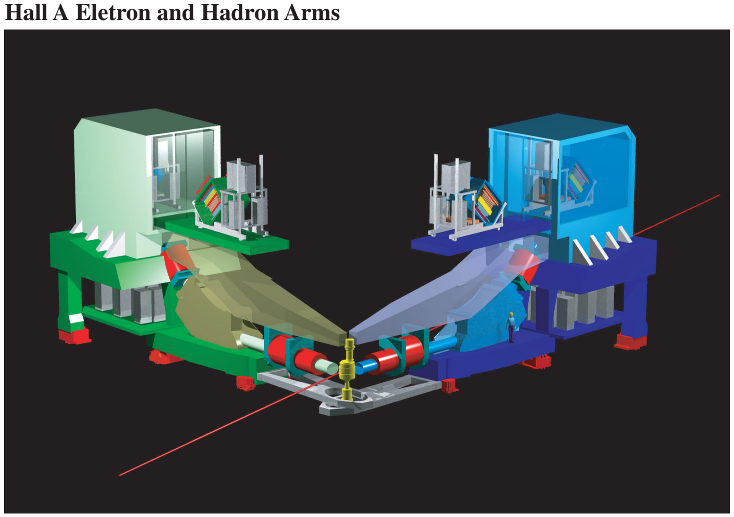


APEX, JLab Hall

Wojtsekhowski, 2018

It can be a portal to a world of leptoquarks

Hall A Eletron and Hadron Arms



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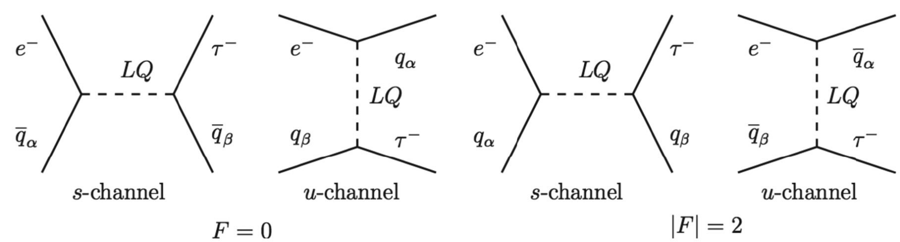


Fig. 73. Feynman diagrams for $e \to \tau$ scattering processes via leptoquarks, which carry fermion number $F = 3B + L$ equal to 0 or ± 2 [311].

EPJ (2016) 52, 268

5. Exploring QCD with electrons

The EIC and the hidden world of the strong interactions.

Through the looking glass into the micro world of the nucleon and nuclei.

The emergent structure that mediates between the point like quarks and the macro world.

What's going on in there?

A selection . . .

- **DIS valence distributions and a short review of pQCD.   TMDs.**
- **Excited nucleons and duality**
- **Elastic scattering and the nucleon radius**
- **DVCS, GPDs and nucleon structure**

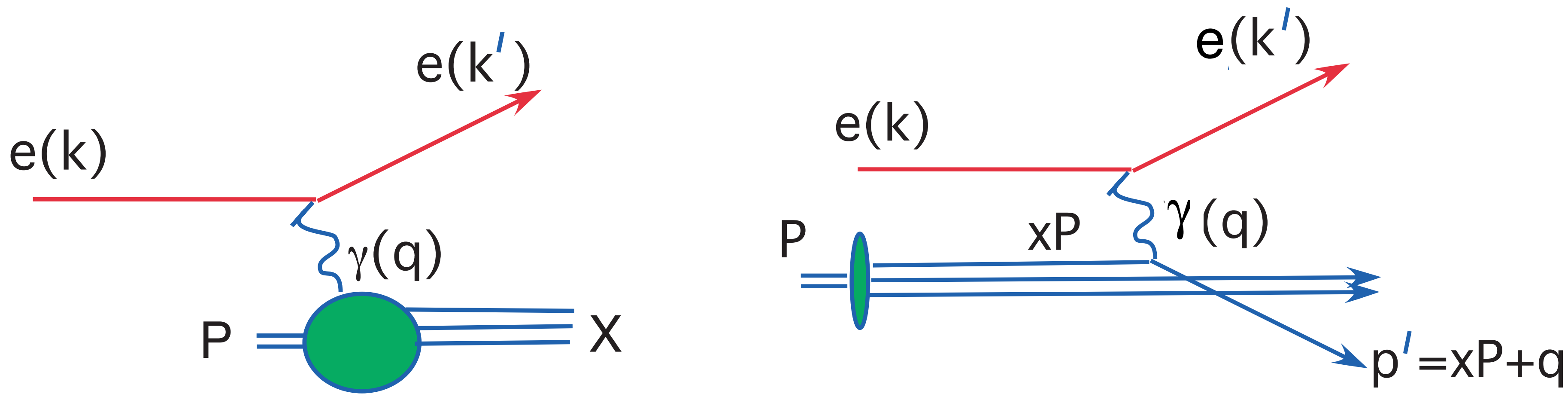
DIS and Transverse Momentum Distributions

- **To make a long story short: Quantum Chromodynamics (QCD) reconciled the irreconcilable. Here was the problem.**

1. **Quarks and gluons explain spectroscopy, but aren't seen directly – confinement.**

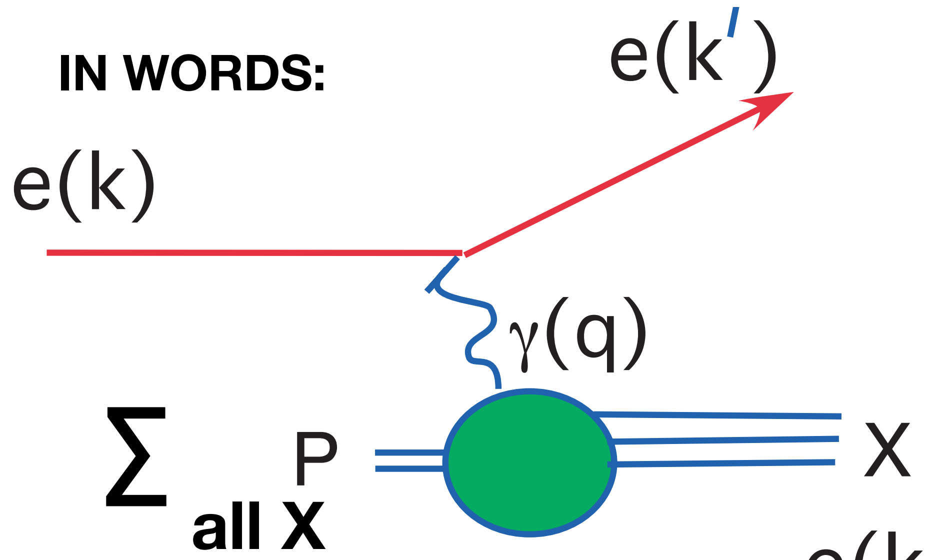
2. **In highly ("deep") inelastic, electron-proton scattering, the inclusive cross section was found to well-approximated by lowest-order elastic scattering of point-like (spin-1/2) particles (="partons" = quarks here) a result called "scaling":**

$$\frac{d\sigma_{e+p}(Q, p\cdot q)}{dQ^2}\Big|_{\text{inclusive}} \quad \propto \quad F\left(x = \frac{Q^2}{2p\cdot q}\right) \frac{d\sigma^{\text{free}}_{e+\text{spin}\,\frac{1}{2}}}{dQ^2}\Big|_{\text{elastic}}$$

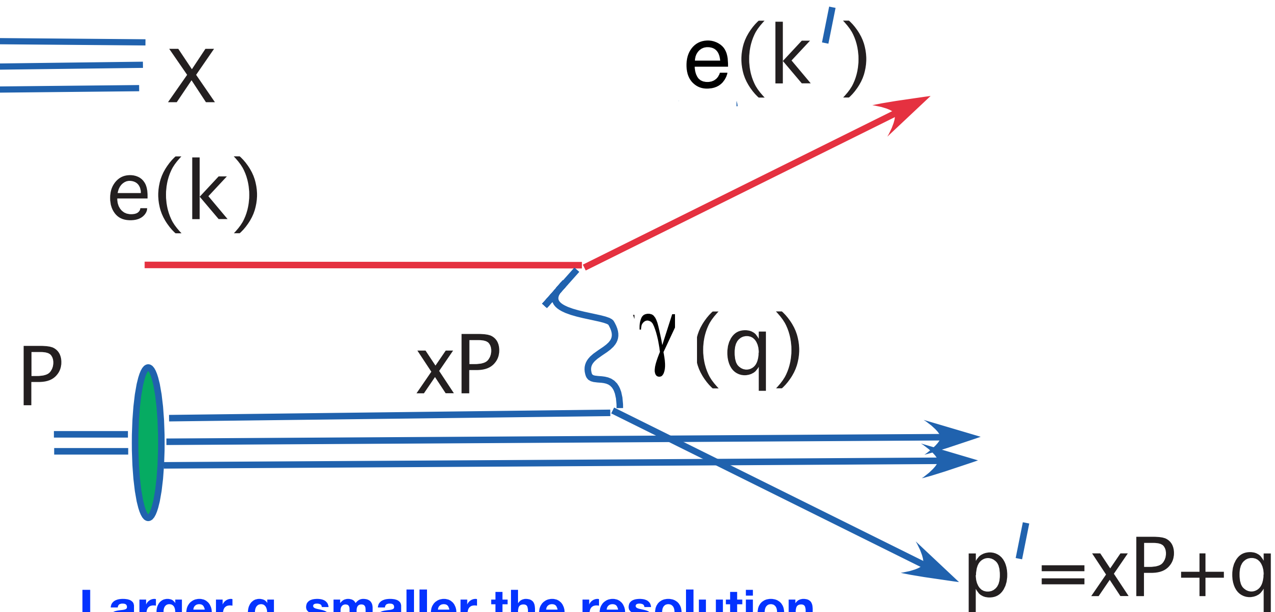


IN WORDS:

The "Inclusive" probability (Cross section) was found to be . . .



proportional to the scattering probability for free charged quarks.

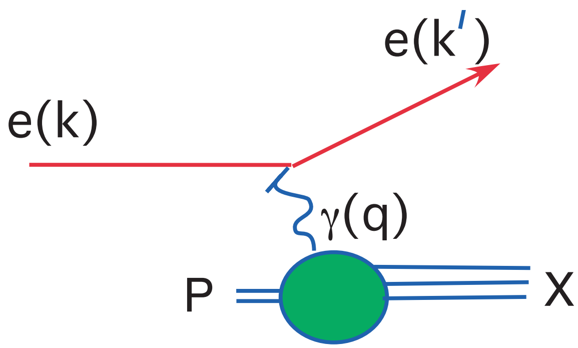


Larger q, smaller the resolution.
x is the fraction of proton's momentum carried by the parton.
DIS measures how partons share the nucleon's momentum. In the short time of the scattering, the quark is effectively fee. Confinement is too late to affect the inclusive cross section.

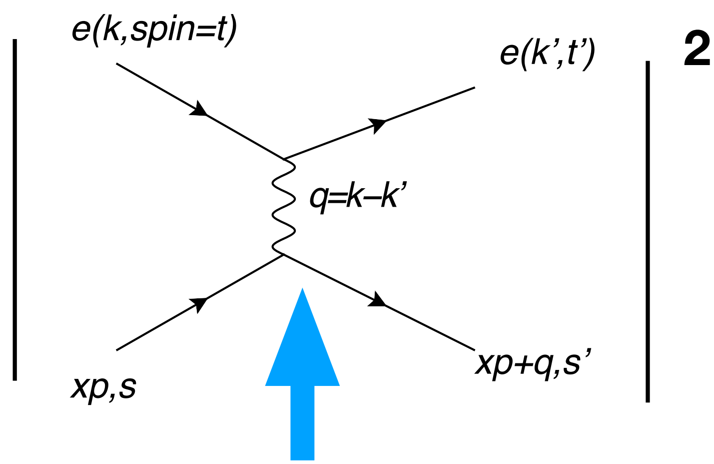
But the free quarks never showed up in experiments.
HOW CAN A CONFINED PARTICLE SCATTER FREELY?
ASYMPTOTIC FREEDOM.

Basic example of a *factorized cross section:*

Product of a "universal" F(x) with a "process dependent" partonic (quark or gluon) cross section that we can compute with perturbative rules. Photon scattering can also depends on the spin of the quark.



= F(x,spin)

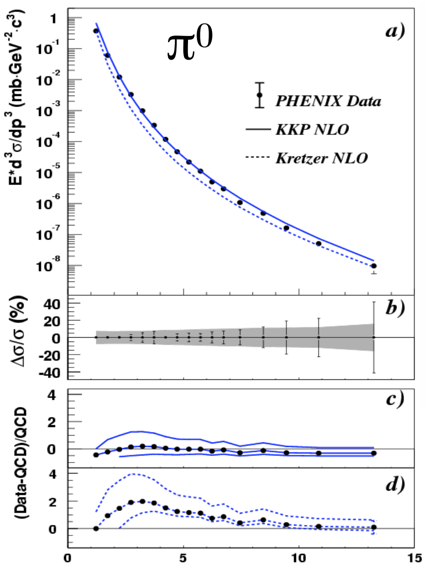


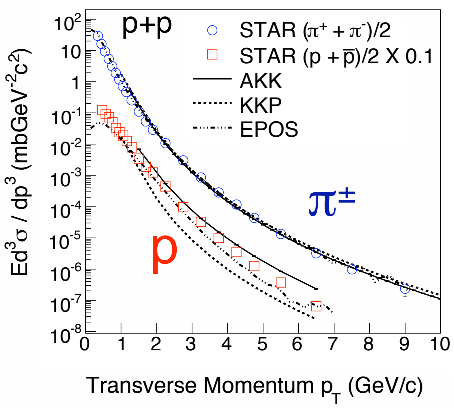
Just a single "story" - all the rest "sums to unity" if we don't ask what happens to the quark!

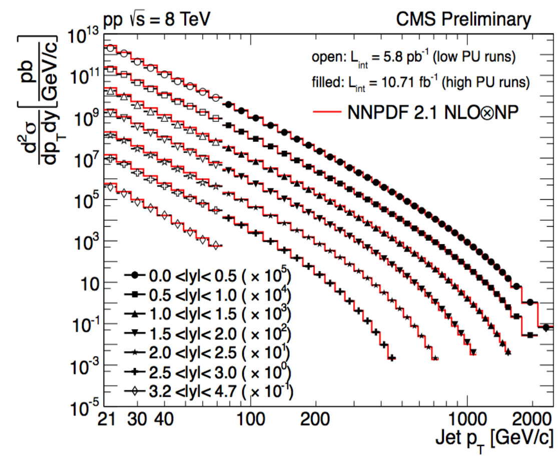
F(x) here is a "quark distribution" — just a function of x and spin

Important "corrections" can be computed in principle.

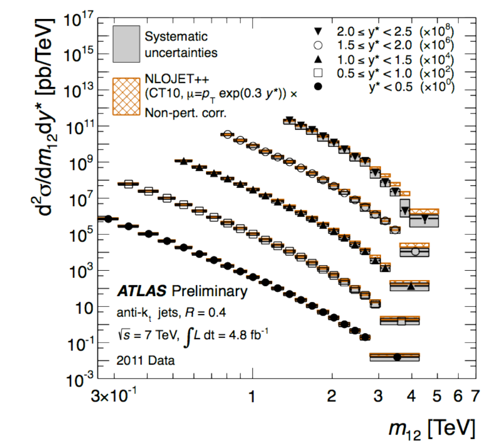
With these methods can describe both particles and jets in pp at 200 GeV . . . at 8 TeV







(CMS-PAS-SMP-12-012)



(ATLAS-CONF-2012-021)

Especially for the single-particle inclusive cross sections at RHIC, the range of agreement was a surprise. A great impetus for polarization, AA, pA *and* eA studies. In ratios, at least we understand the denominator!

Here's a very short course in pQCD terminology and formalism, starting with "collinear" factorization (momentum fraction). The general form . . .

$$Q^2\sigma_{\rm phys}(Q,m,f) \;=\; \omega_{\rm SD}(Q/\mu,\alpha_s(\mu),f) \,\otimes\, f_{\rm LD}(\mu,m) + \mathcal{O}\left(\frac{1}{Q^p}\right)$$

μ = factorization scale;
m= IR scale (*m may* **be perturbative)**

- **"New physics" in $\omega_{\rm SD}$; $f_{\rm LD}$ "universal" – for a given target or observed particle**
- **Almost all collider applications. Enables us to compute the Energy-transfer-dependence in $|\langle Q,\,{\rm out}|A+B,\,{\rm in}\rangle|^2$.**
- **But again, requires a smooth weight for final states!**

- **Whenever there is factorization, there is evolution**

$$0 = \mu \frac{d}{d\mu} \ln \sigma_{\rm phys}(Q, m)$$

$$\mu \frac{d \ln f}{d\mu} = -P(\alpha_s(\mu)) = -\mu \frac{d \ln \omega}{d\mu}$$

- **Wherever there is evolution there is resummation,**

$$\sigma_{\rm phys}(Q, m) = \sigma_{\rm phys}(q, m) \ \exp\left\{ \int_q^Q \frac{d\mu'}{\mu'} P\left(\alpha_s(\mu')\right) \right\}$$

- For example: "collinear" factorization for a (non-singlet) DIS structure function:

$$F_2(x,Q^2) = \int_x^1 d\xi \; C_a\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu)\right) \; f_{a/A}(\xi,\mu)$$

which factorizes into a simple product under moments,

$$\begin{aligned}\tilde{F}_2(N,Q^2) &= \int_0^1 dx x^{N-1}\, F_2(x,Q^2) \\ &= \tilde{C}_a\left(N, \frac{Q^2}{\mu^2}, \alpha_s(\mu)\right) \; \tilde{f}_{a/A}(N,\mu)\end{aligned}$$

- & then we know $\tilde{P}(N,\alpha_s) = \gamma_N = \gamma_N^{(1)}(\alpha_s/\pi) + \ldots$, and we get

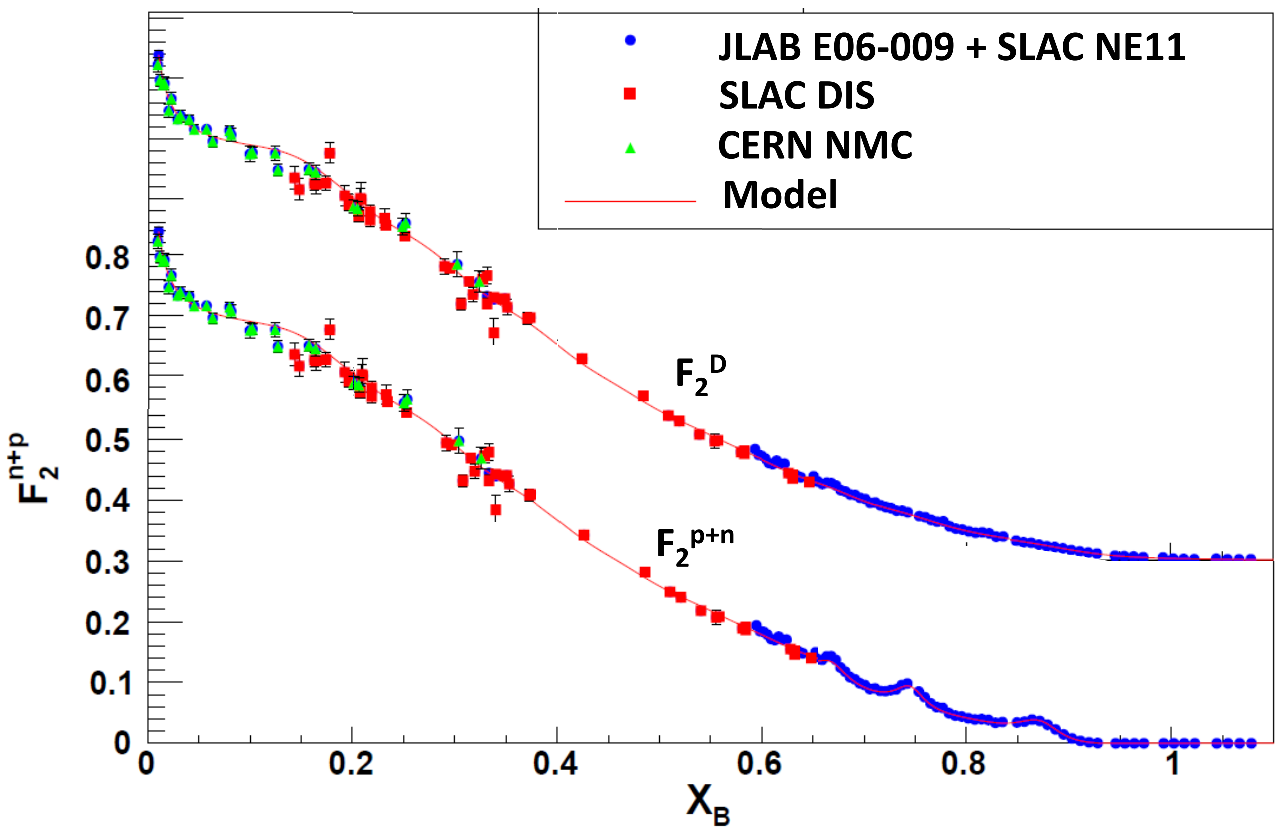
$$\tilde{F}_2(N,\mu) = \tilde{F}_2(N,\mu_0) \; \exp\left[-\frac{1}{2} \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \; \gamma(N,\alpha_s(\mu')) \right]$$

- and with $\alpha_s(\mu) = 4\pi/b_0 \ln(\mu^2/\Lambda^2_{\rm QCD})$, this is

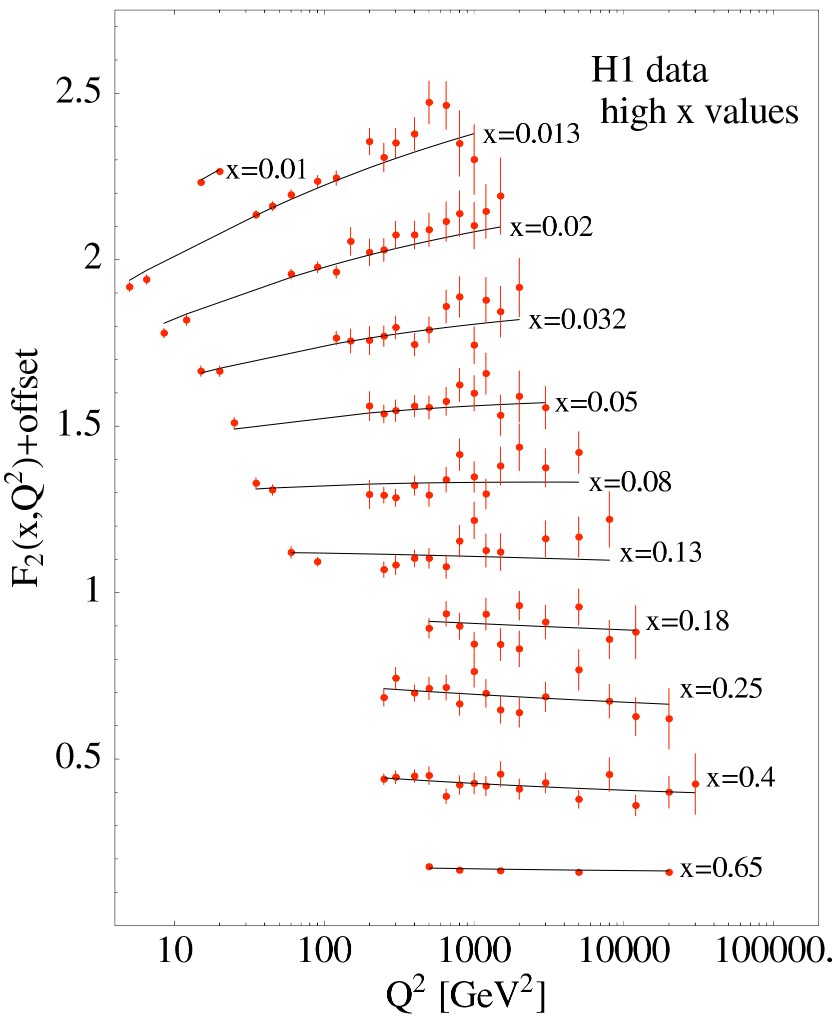
$$\tilde{F}_2(N,Q) \; = \tilde{F}_2 q/H(N,Q_0) \left(\frac{\ln(Q^2/\Lambda^2_{\rm QCD})}{\ln(Q_0^2/\Lambda^2_{\rm QCD})}\right)^{-2\gamma_N^{(1)}/b_0}$$

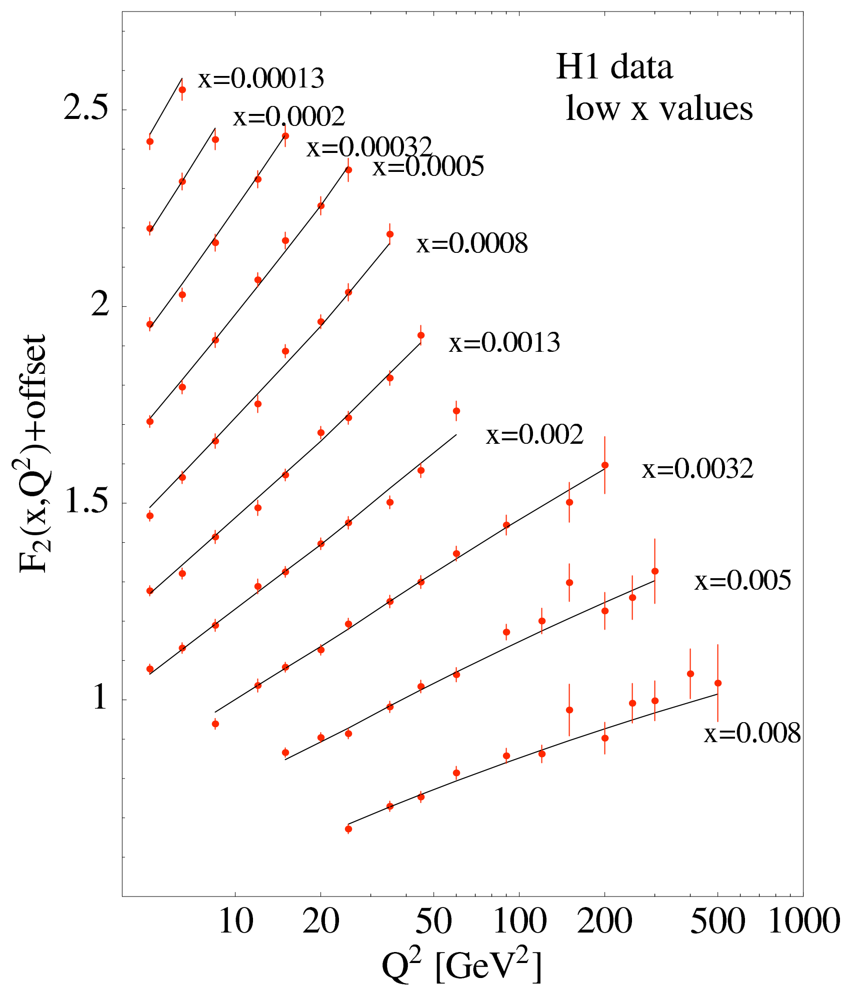
In effect, a beam of electrons is a machine for detecting quarks.

Unprecedented senstivity to large-x quark distributions.
(JLab ED6-009 Hall C 2019)
Of special interest both to new-physics searches at high energy, and to nuclear structure.



- **It works quite well.** *Approximate scaling at moderate* $\boldsymbol{x}$*, pronounced evolution for smaller* $\boldsymbol{x}$*:*





A similar analysis applies to all the structure functions: (here QCD only)

$$W_{\mu\nu} = \frac{1}{2\pi}\sum_X \langle PS|J_\mu(0)|X\rangle\langle X|J_\nu(0)|PS\rangle(2\pi)^4\delta^{(4)}(p_X - P - q)$$

$$= \quad -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)F_1(x,Q^2) + \left(P_\mu - \frac{P\cdot q}{q^2}q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2}q_\nu\right)\frac{2}{P\cdot q}F_2(x,Q^2)$$

$$\epsilon_{\mu\nu\lambda\sigma}q^\lambda\left\{S^\sigma\frac{1}{P\cdot q}g_1(x,Q^2) + (P\cdot qS^\sigma - q\cdot SP^\sigma)\frac{1}{(P\cdot q)^2}g_2(x,Q^2)\right\}.$$

With Spin: require polarized distributions: Δf(x), f = G,u,d,s. . .

There is a lot still to learn, especially regarding polarized distributions: how much spin parsons carry:

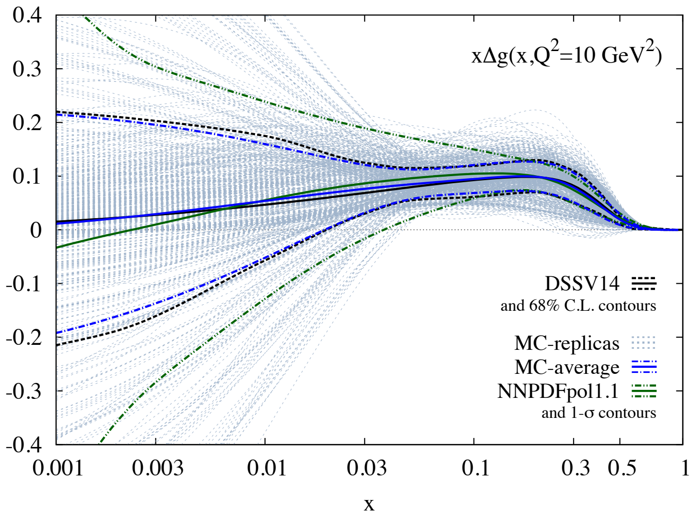


FIG. 1: The ensemble of replicas (dotted blue lines) for the NLO gluon helicity density $\Delta g(x, Q^2)$ at $Q^2 = 10\,\mathrm{GeV}^2$ shown along with its statistical average (solid blue line) and variance (dot-dashed blue lines). The corresponding results from the DSSV14 fit (black lines) and the NNPDFpol1.1 analysis (green lines) are shown for comparison; see text.

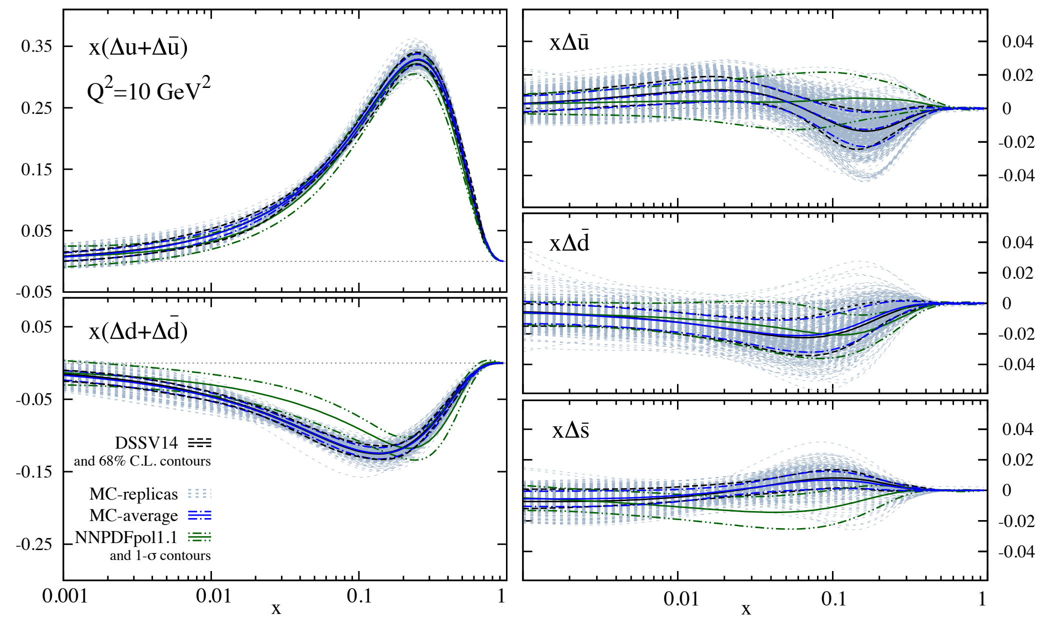


FIG. 2: Same as Fig. 1 but now showing our results for the quark and antiquark helicity PDFs at $Q^2 = 10\,\mathrm{GeV}^2$ in comparison to the analyses of DSSV14 and NNPDFpol1.1.

de Florian, Vogelsang 1902.10548

- **All the foregoing looks "inward" toward the hard scattering. The emphasis is the verification of QCD at the shortest distances and the prediction of new physics signals at high energy.**
- **The path to a fuller understanding of the theory involves looking away from the hard scattering . . .**
- **In two qualitative ways:**
  1. **Far back into the initial state, before the collision: the structure of nuclei in the language of partons.**
  2. **Far forward to the final state, after the collision: hadronization.**
- **The questions we seek to answer will still be phrased in the language of hadronic structure and formation in terms of partonic degrees of freedom.**

To start, the quark doesn't have to be moving exactly in the same direction as the proton!

Other measurements (single-inclusive DIS) are sensitive to this extra motion transverse momentum distributions or TMDs.

These cross sections factorize too, into parton distributions, and "fragmentation functions" that depend on the "extra" transverse motion of the quarks and gluons

Corrections here are even more important, but can still be computed in principle.

With electron scattering we can measure the momentum fractions of quarks, their transverse momentum variations, and how their spin is connected to the spin of the proton and to their transverse momentum

This gives a family of functions to measure — and together they demonstrate the full correlation of single quarks to the momentum and spin of the proton.

QFT expression for such distributions: itself a story.
A quark disappears, and then reappears.

All this is to pick out the wave corresponding to the scattered quark

And the proton here

The quark reappears here!

The photon scatters the quark here. It disappears from the story.

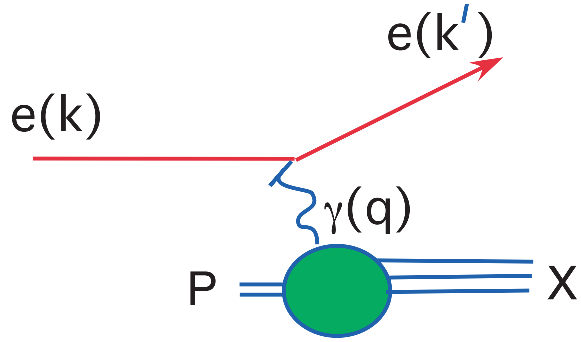
Start with a proton

$$\Phi^q = \frac{1}{2}\int \frac{d\xi^-}{2\pi}\frac{d^2\vec{\xi}_T}{(2\pi)^2}\, e^{ik\cdot\xi}\,\langle P;S|\,\bar{\psi}^q(0)\,\gamma^+\,\mathcal{W}_{TMD}\,\psi^q(\xi)\,|P;S\rangle\Big|_{\xi^+=0}$$

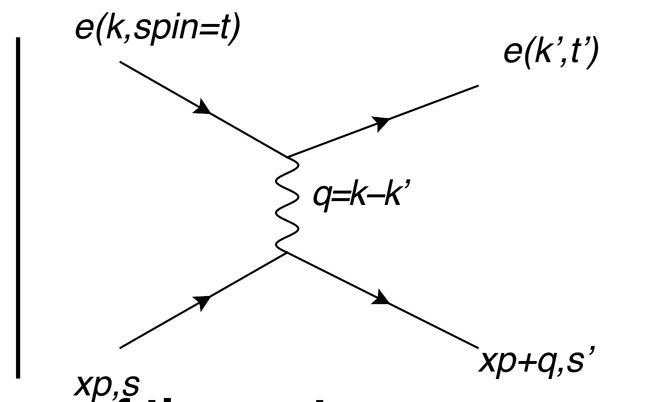
$$= f_1^q(x,\vec{k}_T^{\,2}) + \frac{(\vec{S}_T\times\vec{k}_T)\cdot\hat{P}}{M}\, f_{1T}^{\perp q}(x,\vec{k}_T^{\,2})$$

Electron-positron scattering to excited nucleons

For intermediate momentum transfer compare

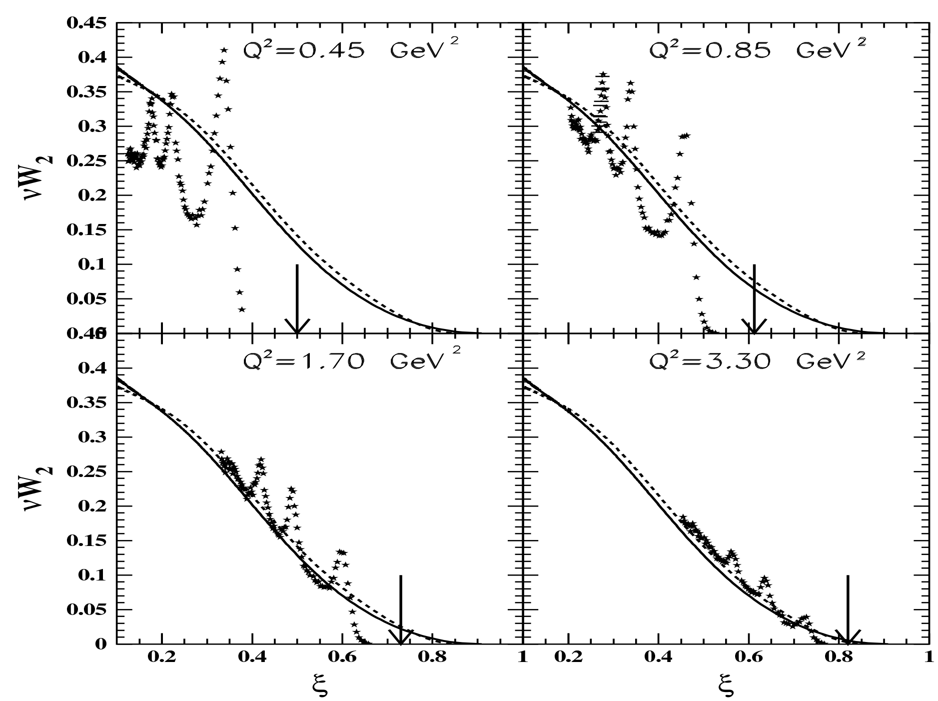


= F(x,spin)



To the production of heavier, unstable versions of the proton. Not just one, but all the quarks are scattered together, and yet the two probabilities are closely related:

Quark-hadron duality:



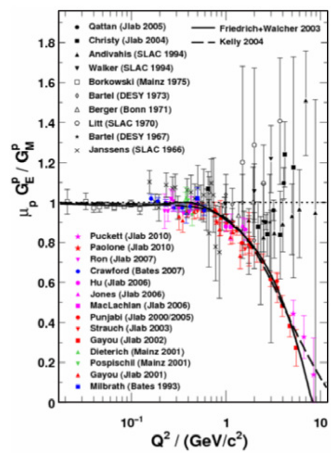
Melnitchouk, Keppel, Ent (2005)

Elastic Scattering and the Nucleon Radius

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left(\frac{E'}{E}\right) \frac{1}{1+\tau} \left(G_E^{p\,2}(Q^2) + \frac{\tau}{\varepsilon} G_M^{p\,2}(Q^2)\right)$$

Still lots to do at large and small momentum transfer

E. Cisbani, 2014

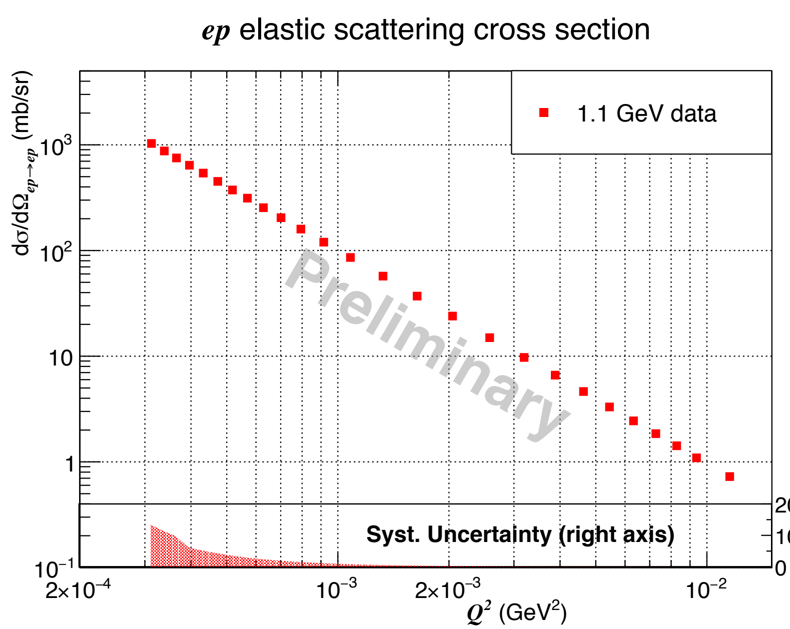


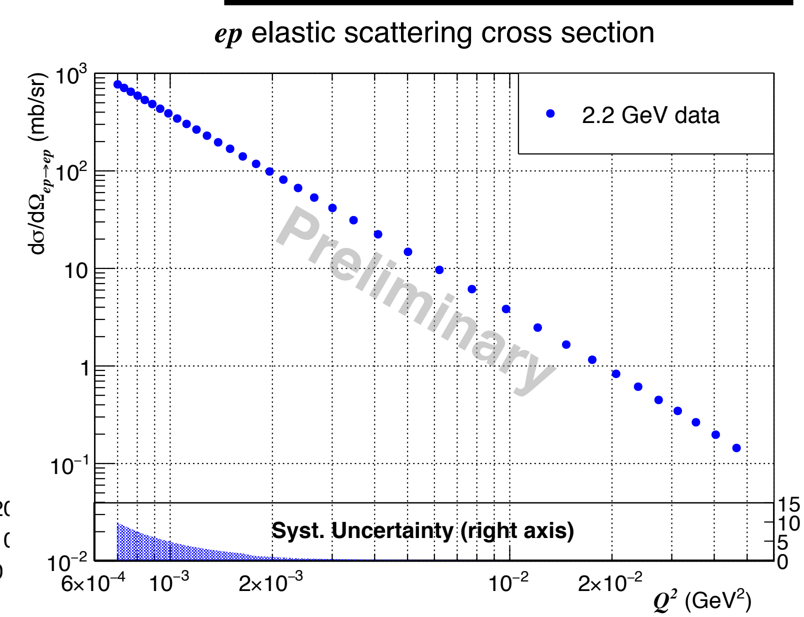
Special interest in low Q²: radius!

pRad:

$$G_E^p(Q^2) = 1 - \frac{Q^2}{6}\langle r^2\rangle + \frac{Q^4}{120}\langle r^4\rangle + \ldots$$

T. Gasparian 2018 ECT



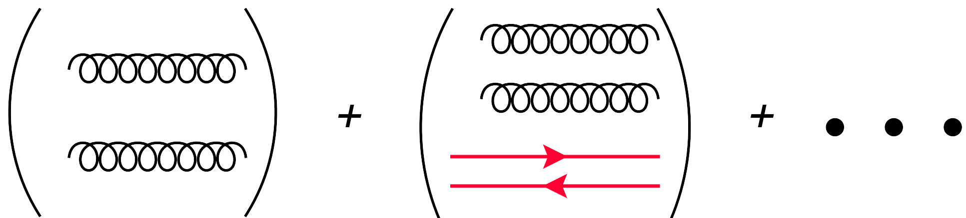


Exotic States

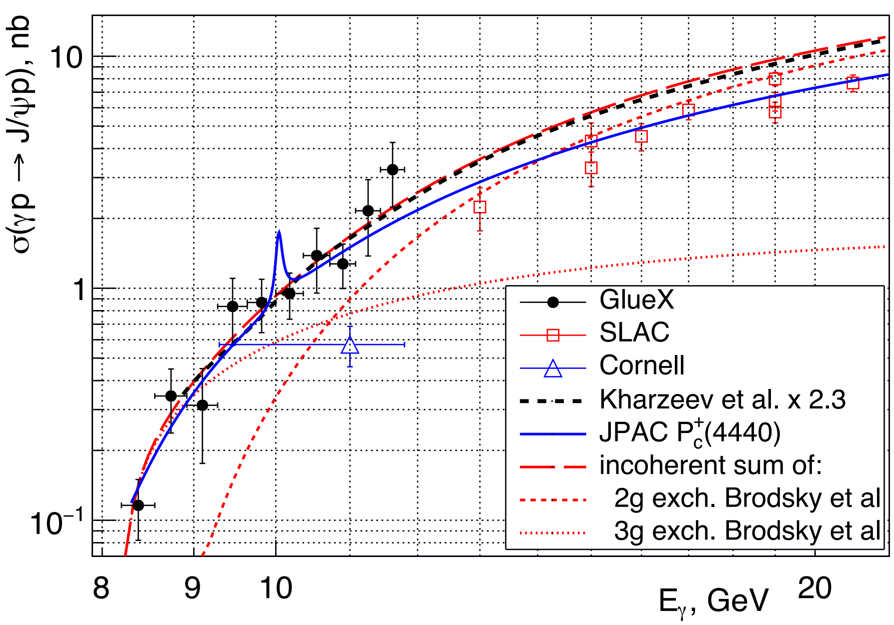
Photons evolve into quark pairs that can pick up gluons in the proton . . .

Computer simulations provide other possible QCD states not found in the classic quark model: glueballs, “multi-quarks”

| glue ball > =



GlueX: using “real” photons to produce exotic and mixed states.
Here, early results on J/Psi (ccbar):



GlueX May 2019
12 GeV

DVCS and Generalized Parton Distributions

Single-quark distributions from DIS and SIDS tell a great story.
But there is much more. Two-photon processes, like

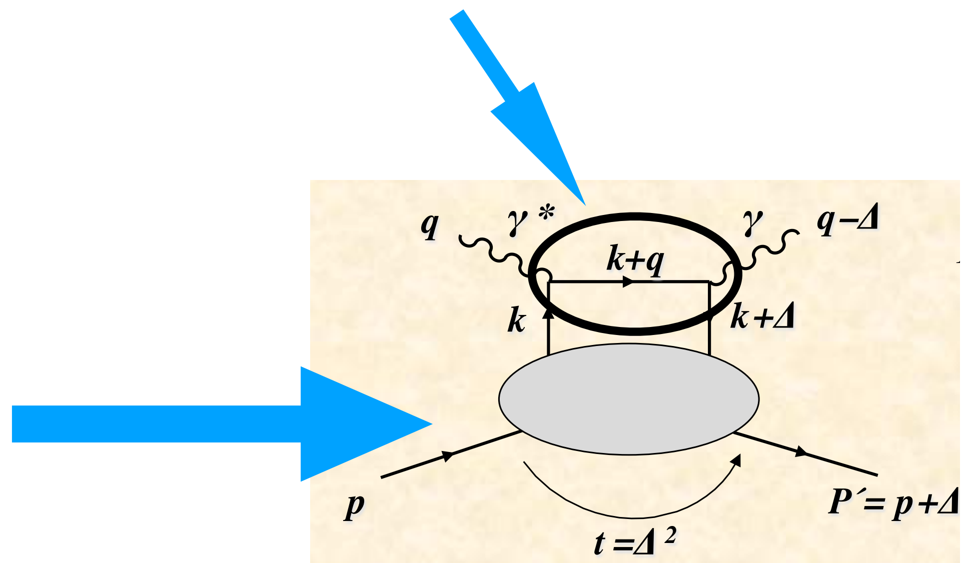
Deeply Virtual Compton Scattering (DVCS)

reveal correlations between positions and momenta within the proton within information on angular momenta and other extended structure:

“tomography”.

By making one photon state very virtual the process factorizes into the story of the proton and the story of the scattering itself.

We can calculate the latter.
We measure the former.



credit: S. Kumano

Δ = q - q’

- **The factorization of collinear distributions (and TMDs) depends on the localization of partons by hard scattering, and the absence of final-state interactions. Such factorizations are sensitive to local properties of hadrons and nuclei, not so much to their coherent structure. This is fine for parton polarizations; not so much for orbital angular momenta. These require generalized parton distributions.** (Ji, Radyushkin)

- **Comparing standard ("diagonal") and generalized (skewed) distributions:**

  1. **(After using the optical theorem) the standard distribution in terms of creation/absorption operators looks like**

$$q(x) = \int d\ell \ \langle p| \ b_q^\dagger(xp+\ell) \ b_q(xp+\ell) \ |p\rangle$$

  2. **While the generalized parton distribution (GPD) is**

$$Q(x,x') = \int d\ell \ \langle (1-\delta)p| \ b^\dagger\left((x-\delta)p+\ell\right) \ b(xp+\ell) \ |p\rangle \, .$$

  3. **Quark collinear momentum fractions**

$$p_0 \int_0^1 x \, q(x)$$

- **Generalized distributions were originally designed to isolate:**
- **Quark and gluon** *total angular* **momenta**

$$\mathbf{J}_q = \int d^3x \left[\psi^\dagger \frac{\boldsymbol{\Sigma}}{2} \psi + \psi^\dagger \mathbf{x} \times (-i\mathbf{D})\psi \right]$$
$$\mathbf{J}_g = \int d^3x \, \mathbf{x} \times (\mathbf{E} \times \mathbf{B})$$

- **The off-diagonal connection ($p' = p + \Delta$)**

$$J_{q,g} = \frac{1}{2} \left[A_{q,g}(0) + B_{q,g}(0) \right]$$

$$\langle p' | \, T^{\mu\nu}_{q,g} \, | p \rangle = \bar{u}(p + \Delta) [\, A_{q,g}(\Delta^2) \, \gamma^\mu p'^\nu + B_{q,g}(\Delta^2) \, p'^\mu \frac{i\sigma_{\mu\alpha}}{2M} \Delta^\alpha - (\mu \leftrightarrow \nu) \,] u(p)$$

- **Dependence on Δ measures non-local correlations associated with angular momentum.**
- **To be extracted from DVCS (for example) and vector boson production.**
- **More generally, the full set of TMDs and GPDs offer the promise of a three dimensional picture of the nucleon.**

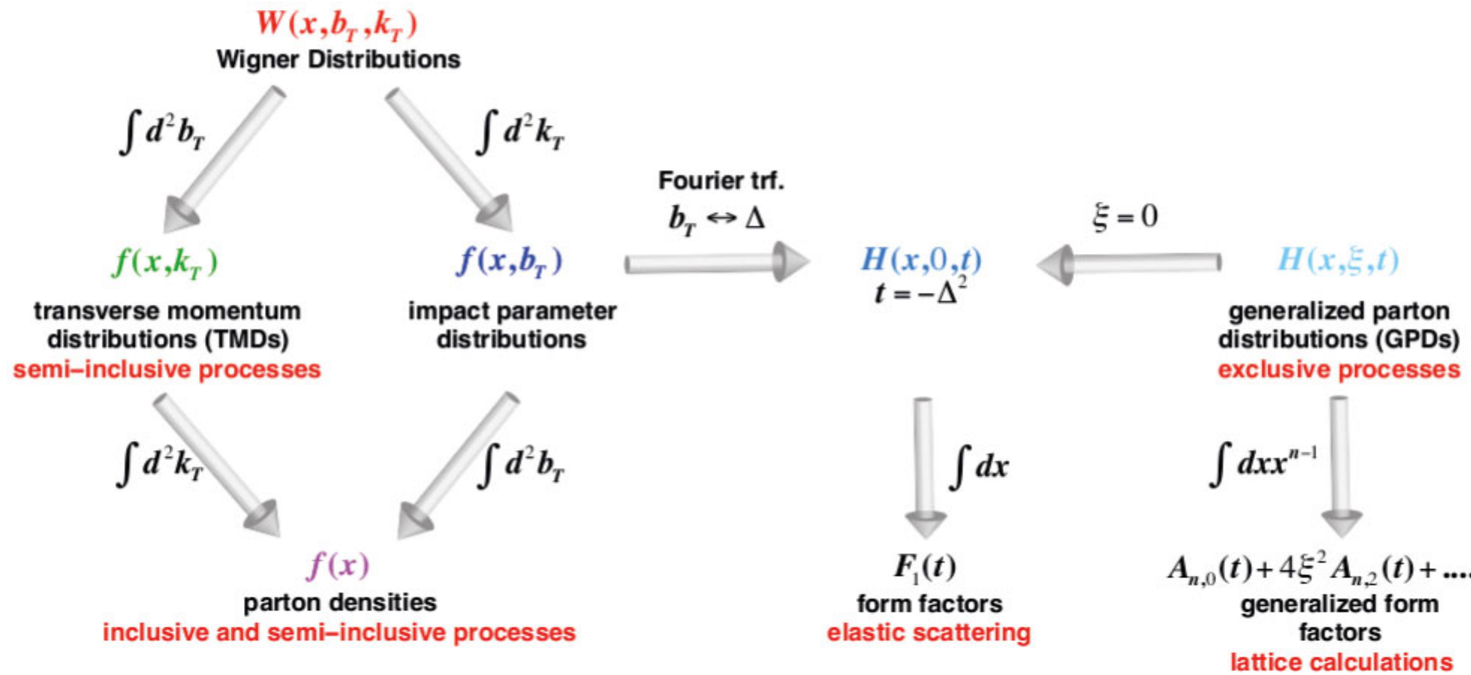
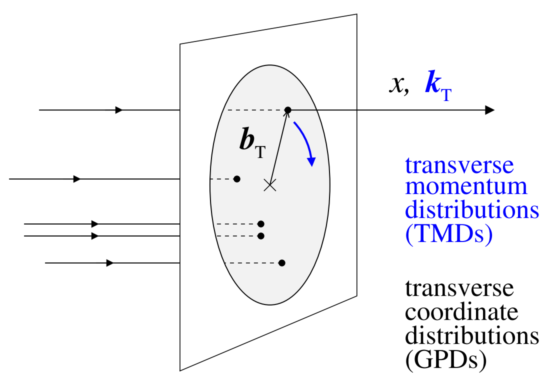
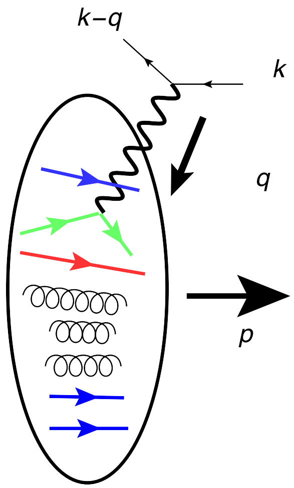


Fig. 6. Connections between different quantities describing the distribution of partons inside the proton. The functions given here are for unpolarized partons in an unpolarized proton; analogous relations hold for polarized quantities.

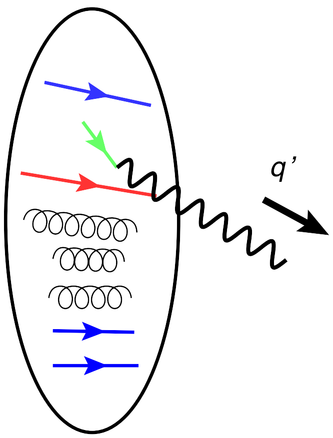
Eur. Phys. J. A (2017) **53**: 71



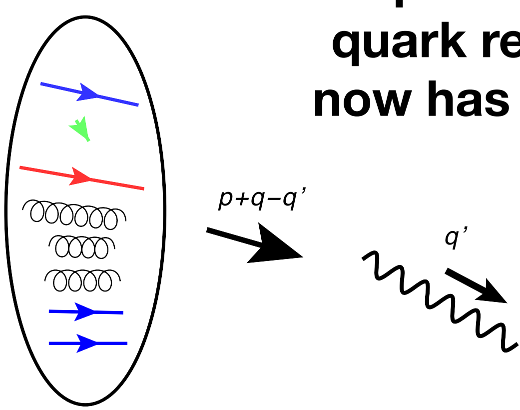
Here's one way of thinking about how it works . . .



Step 1: Electron exchanges virtual photon with a quark



Step 2: Quark travels some distance, then re-emits another photon



Step 3: The photon escapes, but the quark remains in the proton, which now has a different total momentum

In all this, varying the spin of the election and/or the proton gives information on the spin of the quark. These give a family of generalized parton distributions.

An unprecedented example: the pressure distribution (a "gravitational form factor"). At 6 GeV from CLAS, with 12 pending (in red):

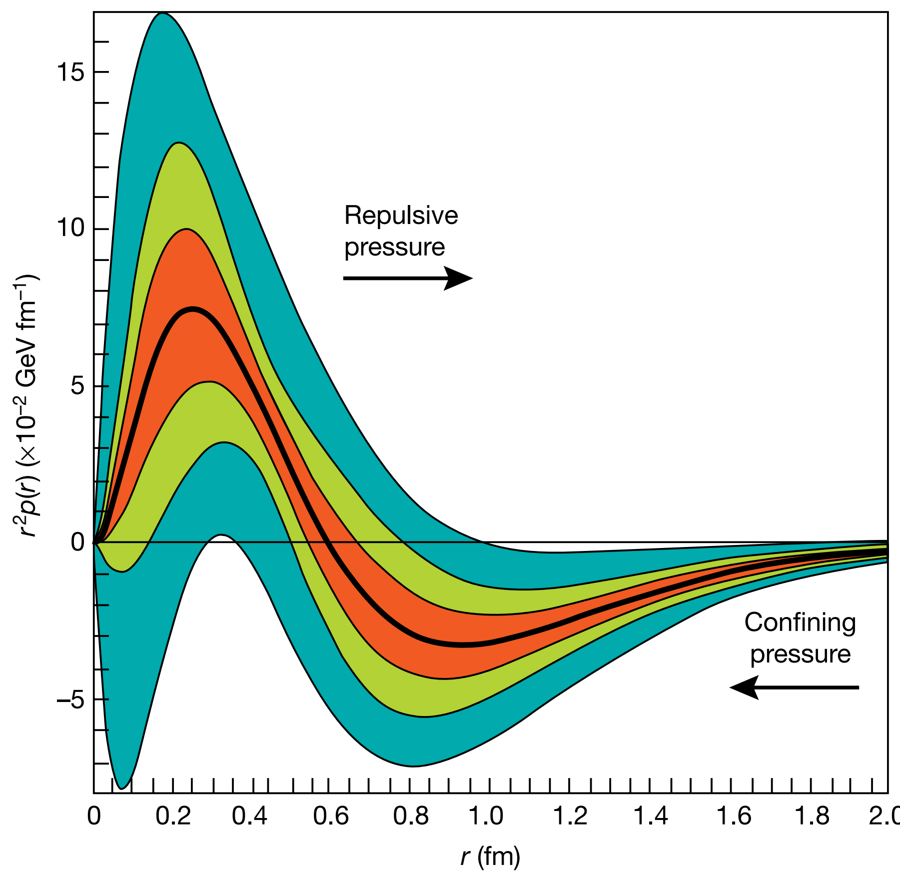


Fig. 1 | Radial pressure distribution in the proton.

Nature, 2019

Two More Factorizations: First,

Transverse momentum factorization

- The classic extension of collinear factorization.
- For Drell-Yan and DIS, (see Collins (2011))

$$\frac{d\sigma_{AB\to V}}{dQ^2 d^2\mathrm{Q}_T} = \frac{1}{S}\,\sigma^{(0)}_{ab\to V}(Q^2)\ \int dx_a\, dx_b\ h^{(\mathrm{j})}_{ab}\left(\frac{Q^2}{x_a x_b S}, \alpha_s(\mu)\right)\ \sum_{a,b}$$

$$\times \int dx_a d^2\mathrm{k}_{t,a}\, f_{a/A}(x_a, \mathrm{k}_{t,a}, \mu)\ \int dx_b d^2\mathrm{k}_{t,b}\, f_{b/B}(x_b, \mathrm{k}_{t,b}, \mu)$$

$$\times\ \delta^2\left(\mathrm{Q}_T + \mathrm{k}_{t,a} + \mathrm{k}_{t,b} + \mathrm{k}_{t,s}\right) + Y_{\mathrm{j}}\,.$$

- Interpetations and limitations
  - f's: are now TMDs
  - h: short distance (off shell by order Q
  - Corrections nonsingular for $Q_T \to 0$
  - Q_T is fixed already at the hard scattering.
  - In general does not extend to pairs of hadrons in pp – final state interactions don't decouple.
  - This failure of "universality" is an opening to new physical phenomena, not a limitation. (Collins, Qiu, Rogers, Muelders . . .)

- The double sfactorization leads to evolution of double logs. For example (thanks to Ted Rogers) . . . the cross section as an inverse transform:

W term

$$\frac{d\sigma}{dq_t^2} \sim \int d^2\mathbf{b}_T \, e^{-i\mathbf{b}_T\cdot\mathbf{q}_T} \times$$

Collinear OPE

$\alpha_s(\sim 1/b_T)$

$$\times \int_{x_1}^{1} \frac{d\hat{x}_1}{\hat{x}_1} \tilde{C}_{f/j}(x_1/\hat{x}_1, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}_1, \mu_b) \times$$

$$\times \int_{x_2}^{1} \frac{d\hat{x}_2}{\hat{x}_2} \tilde{C}_{\bar{f}/j'}(x_2/\hat{x}_2, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j'/\bar{P}}(\hat{x}_2, \mu_b) \times$$

Perturbative Logs

$$\times \exp\left\{ \int_{\mu_b}^{Q} \frac{d\mu'^2}{\mu'^2} \left[\mathcal{B}(g(\mu')) + \ln\frac{Q^2}{\mu'^2} \mathcal{A}(g(\mu')) \right] \right\} \times$$

Non-perturbative Large b_T

$$\times \exp\left\{ -g_1(x_1, b_T) - g_2(x_2, b_T) - 2g_K(b_T) \ln\frac{Q}{Q_0} \right\}$$

Hadron 1 Intrinsic — *Hadron 2 Intrinsic* — *NP soft Evolution*

+ Y term

Ex: Matching Prescription:

$$\mathbf{b}_*(\mathbf{b}_T) \equiv \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}$$

$$\mu_b \equiv C_1/|\mathbf{b}_*(b_T)|$$

(Collins, Soper, Sterman (CSS) formalism (1981-1985)… (many similar formalisms))

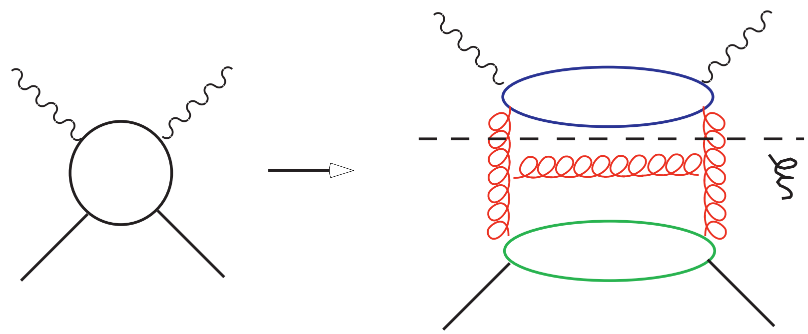
- The "many similar formalisms" include those based on soft-collinear effective theory, and with different large-distance regularizations. The similarities don't rule out lively discussion. Much of this has to do with the important nonperturbative factors and their evolution. (In 2014: Aidala, Field, Gamberg, Rogers and Sun, Yuan, Yuan)

And second . . .

BFKL & high parton density

- "Multiperipheral" re-factorization for a DIS structure function.

$$F(x,Q^2) \;=\; \int_x^1 \frac{d\xi}{\xi}\, C\big(\frac{x}{\xi}, \mu^2/Q^2\big) G(\xi,Q^2) + \mathcal{O}\left(\frac{1}{Q^2}\right)$$
$$G(\xi,Q^2) \;=\; \int^Q d^2k_T\; \psi(\xi,k_T)$$



$$F(x,Q^2) \;=\; \int d^2k_T\; c\left(\frac{\xi}{\xi'}, Q, k_T\right)\; \psi(\xi',k_T) + \mathcal{O}\left(\frac{1}{\ln^2 x}\right)$$
$$\Downarrow$$
$$\xi\, \frac{d\psi(\xi,k_T)}{d\xi} \;=\; \int d^2k'_T\; \mathcal{K}(k_T - k'_T)\; \psi(\xi,k'_T)$$

- Roles of k_T and ξ exhanged – ξ' as factorization scale

- **Equation, ansatz and solution:** $(\tilde{\psi} \equiv (1/k_T^2)\psi \quad \bar{\alpha}_s \equiv \alpha_s/\pi)$

$$\xi \frac{d\tilde{\psi}(\xi, k_T)}{d\xi} = -\frac{\alpha_s N}{\pi^2} \int \frac{d^2 k_T'}{(k_T - k_T')^2} \left[\tilde{\psi}(\xi, k_T) - \frac{{k_T'}^2}{2k_T^2} \tilde{\psi}(\xi, k_T') \right] + \mathrm{NLO}$$

$$\tilde{\psi} \sim \xi^{-\omega} \left(\frac{k_T^2}{\mu^2} \right)^{\gamma - 1}$$

$$\Downarrow$$

$$\omega(\gamma) = \bar{\alpha}_s \chi_0(\gamma) \left[1 - \beta_0 \bar{\alpha}_s \ln \frac{k^2}{\mu^2} \right] + {\bar{\alpha}_s}^2 \chi_1(\gamma)$$

- **a fast growth for small $\xi \leftrightarrow x$.**
- **Good to recall that beyond NLO must generalize the factorization & equation**
- **Nuclear targets enhance these effects through the build-up of low-x partons.**

This dynamics will be a goal at the “EIC for nuclear targets

Effective theories for high parton density

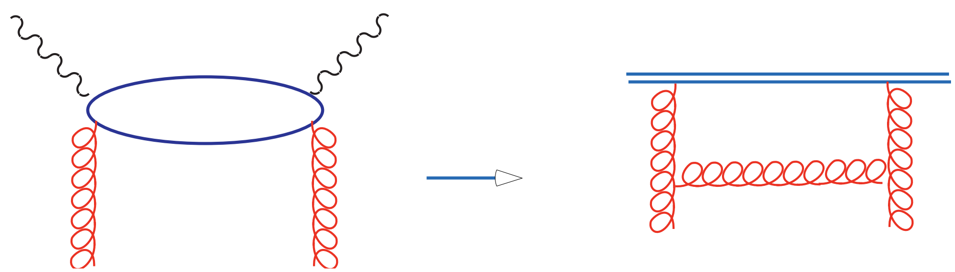
- One thing that’s special about BFKL: evolve to low $x \leftrightarrow$ high parton density at “fixed” (actually diffusing) virtuality $xG(x) \sim x^{-\omega}$

- Theory of dense, weakly-interacting partons ($\alpha_s \ll 1$)

- LO BFKL from scattering of recoilless sources (Wilson lines):

$W_+ - W_-$ scattering as an effective field theory (Balitsky 99)

$$W_\pm(x^\mp, x_t) = P \exp\left[\int_{-\infty}^{\infty} dx^\pm A^\mp(x)\right]$$



- Extensions to dipole splitting: BK and JIMWLK equations.

Each of these areas and much more will be explored, in ongoing programs and at the future Electron-Ion Collider.

It should be exciting

Enjoy the school and the adventures beyond!