Three-dimensional structure of the nucleon

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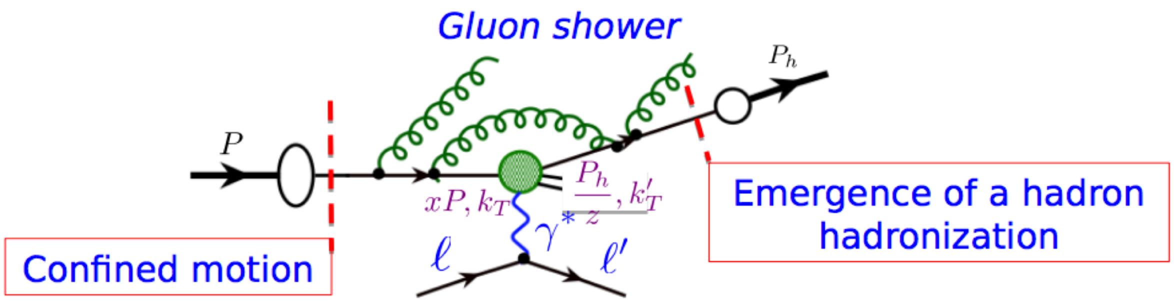
Why QCD evolution is interesting?

Study of evolution gives us insight on different aspects and origin of confined motion of partons, gluon radiation, parton fragmentation



Evolution allows to connect measurements at very different scales.

TMD evolution has also a universal non-perturbative part. The result of evolution cannot be uniquely predicted using evolution equations untill the non-perturbative part is reliably extracted from the data.

Courtesy of A. Prokudin

Parton distribution: energy dependence

- Experiments operate in very different kinematic ranges
  - Typical hard scale Q is different: Q ~ 1 – 3 GeV in SIDIS, Q ~ 4 – 90 GeV in pp
  - Also center-of-mass energy is different
- Such energy dependence (evolution) has to be taken into account for any reliable QCD description/prediction
- Both collinear PDFs and TMDs depend on the energy scale Q at which they are measured, such dependences are governed by QCD evolution equations

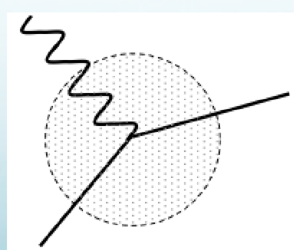
Collinear PDFs

$$F(x, Q)$$

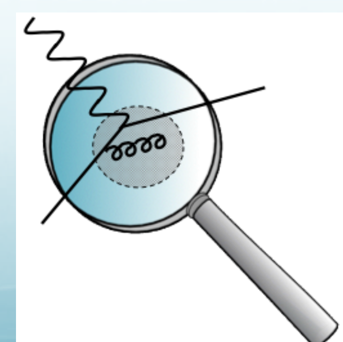
TMDs

$$F(x, k_\perp; Q)$$

Q_0^2



$Q^2 > Q_0^2$

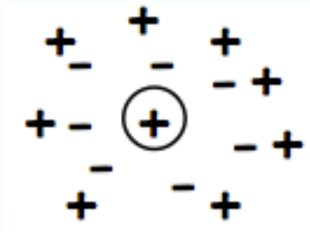


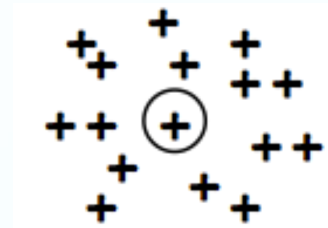
Divergence and evolution

- Divergence leads to evolution
  - Ultraviolet divergence: renormalization group equation, e.g. running of coupling constant
  - Collinear divergence: DGLAP evolution of collinear parton distribution function, fragmentation function, semi-inclusive jet function
  - Rapidity divergence (light-cone singularity): TMD evolution

Understanding QCD: running coupling (asymptotic freedom)

- Rough qualitative picture: due to gluon carrying color charges
  - Value of the strong coupling α_s depends on the distance (i.e., energy)





Screening: $\alpha_{em}(r) \uparrow \quad \text{as} \quad r \downarrow$

Anti-screening: $\alpha_s(r) \downarrow \quad \text{as} \quad r \downarrow$

Asymptotic Freedom ⇔ antiscreening

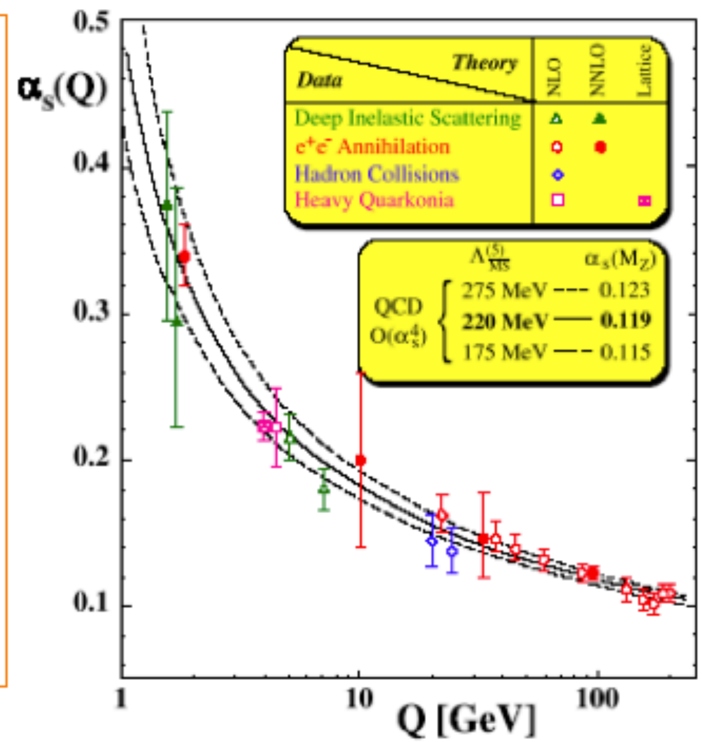
$$\text{QCD: } \frac{\partial \alpha_s(Q^2)}{\partial \ln Q^2} = \beta(\alpha_s) < 0$$

Compare

$$\text{QED: } \frac{\partial \alpha_{EM}(Q^2)}{\partial \ln Q^2} = \beta(\alpha_{EM}) > 0$$

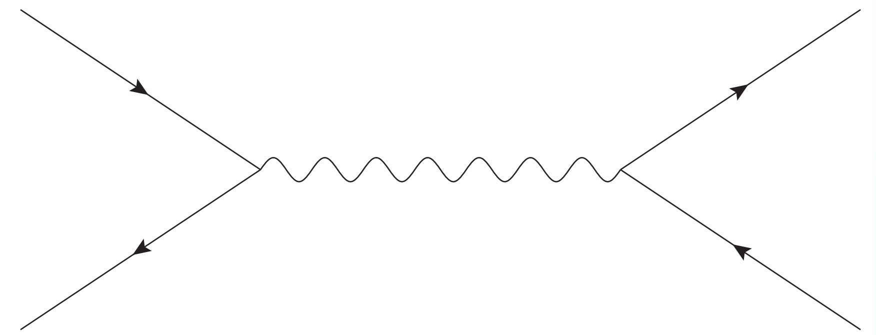
D.Gross, F.Willczek, Phys.Rev.Lett 30,(1973)
H.Politzer, Phys.Rev.Lett 30, (1973)

2004 Nobel Prize in Physics

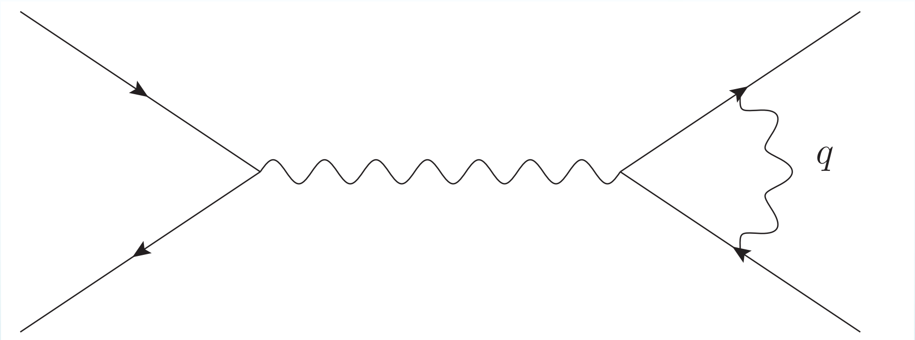


Why does the coupling constant run?

- Leading order calculation is simple: tree diagrams – always finite



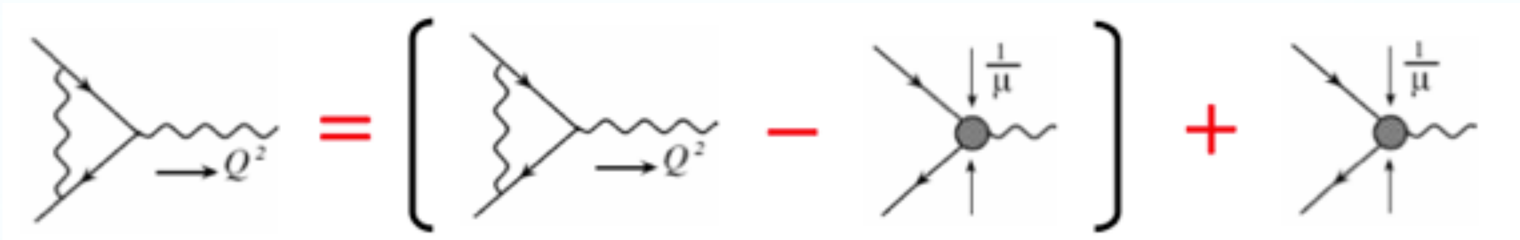
- Study a higher order Feynman diagram: one-loop, the diagram is divergent as $q \to \infty$



- Make sense of the result: redefine the coupling constant to be physical

Renormalization (Redefine the coupling constant)

- Renormalization
  - UV divergence due to "high momentum" states
  - Experiments cannot resolve the details of these states

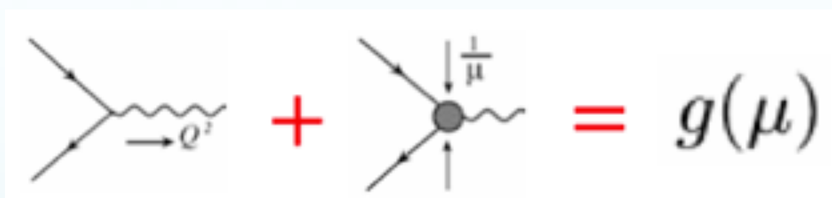


Low momentum state

High momentum state

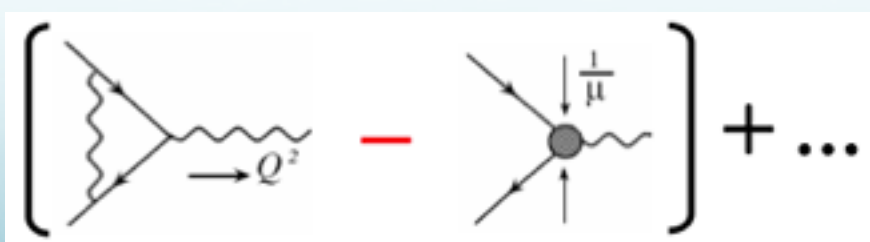
- Combine the "high momentum" states with leading order

LO:



Renormalized coupling

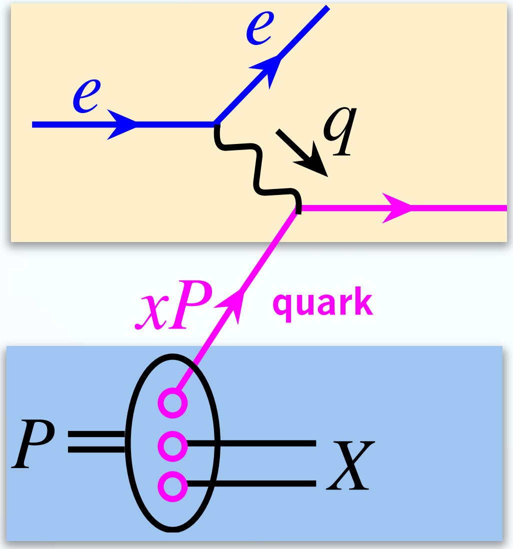
NLO:



No UV divergence!

QCD factorization

- Take deep inelastic scattering as an example



$\hat{\sigma}_{q,g}(Q)$

$Q = \sqrt{|q^2|}$

$f_{\mathrm{q,g/proton}}(x)$

Parton Distribution Functions (PDFs):
Probability for finding a parton in a proton with momentum fraction x

$$\sigma_{\mathrm{proton}}(Q) = f_{\mathrm{q,g/proton}}(x) \otimes \hat{\sigma}_{q,g}(Q)$$

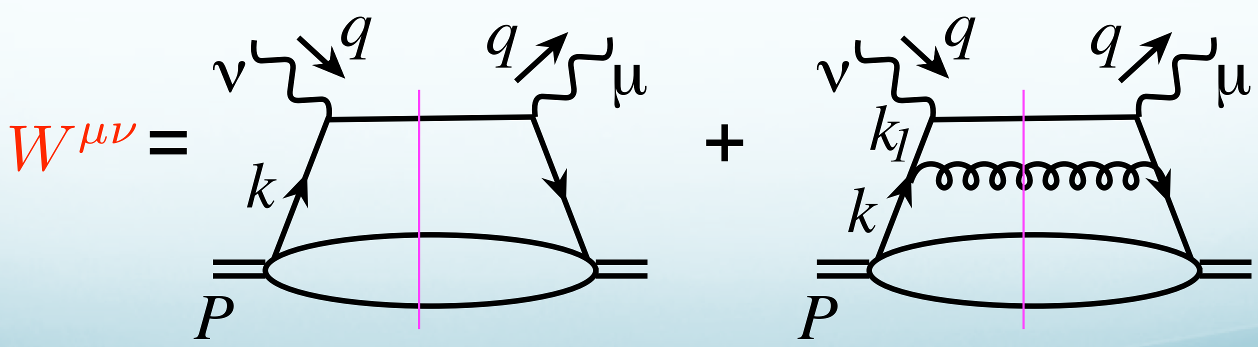
measured **extracted** **calculable**

- Proton structure: encoded in PDFs
- QCD dynamics at high-energy scale *Q*

What about higher order?

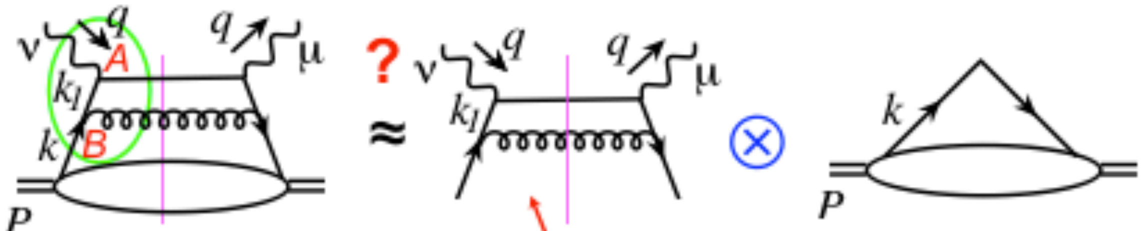
- pQCD calculations: understand and make sense of all kinds of divergences
  - Ultraviolet (UV) divergence $k \to \infty$: renormalization (redefine coupling constant)
  - Collinear divergence $k // P$: redefine the PDFs and FFs
  - Soft divergence $k \to 0$: usually cancel between real and virtual diagrams for collinear PDFs/FFs; do not cancel for TMDs, leads to new evolution equations

- Going beyond the leading order of the DIS, we face another divergence



QCD dynamics beyond tree level

- Going beyond leading order calculation



Collinear divergence!!! (from $k_1^2 \sim 0$)

$$\Rightarrow \int d^4k_1 \frac{i}{k_1^2+i\epsilon}\frac{-i}{k_1^2-i\epsilon} \Rightarrow \infty$$

$$k_1^2 = (k+k_g)^2 = 2EE_g(1-\cos\theta)$$

- $k_1^2 \sim 0$ intermediate quark is on-shell

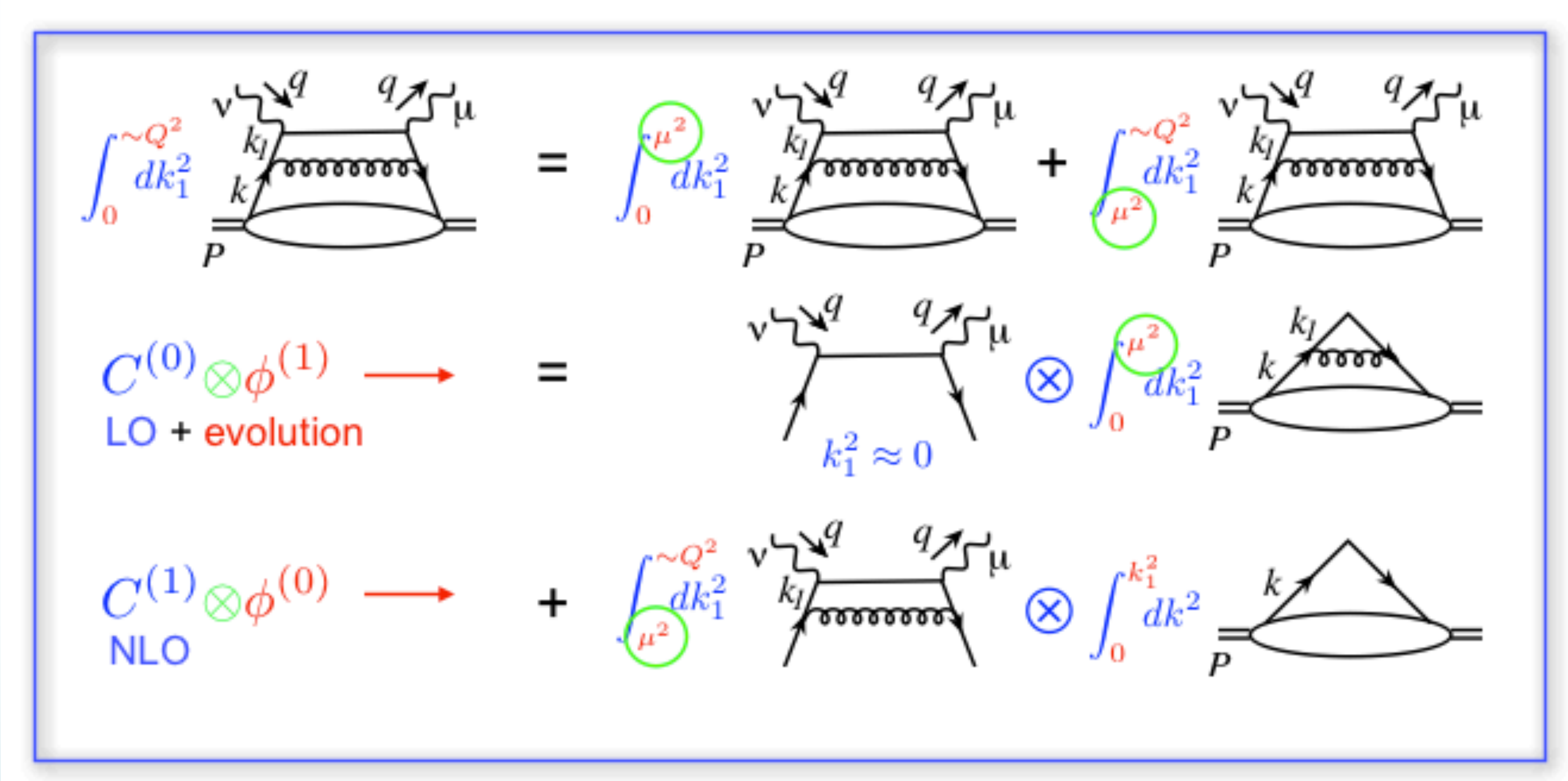
  $t_{AB} \rightarrow \infty$

- gluon radiation takes place long before the photon-quark interaction
  $\Longrightarrow$ a part of PDF

Partonic diagram has both long- and short-distance physics

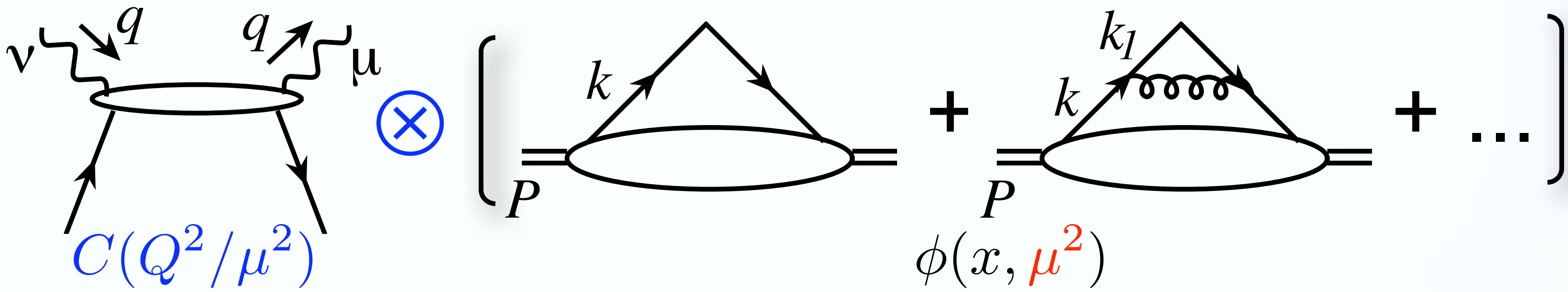
QCD factorization: beyond parton model

- Systematic remove all the long-distance physics into PDFs

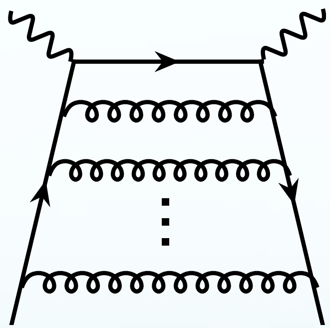


Scale-dependence of PDFs

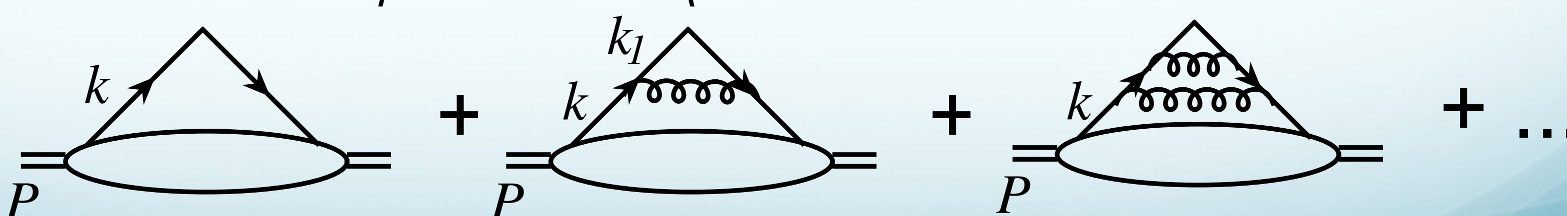
- Logarithmic contributions into parton distributions



- Going to even higher orders: QCD resummation of single logs



$\phi(x, \mu^2) =$

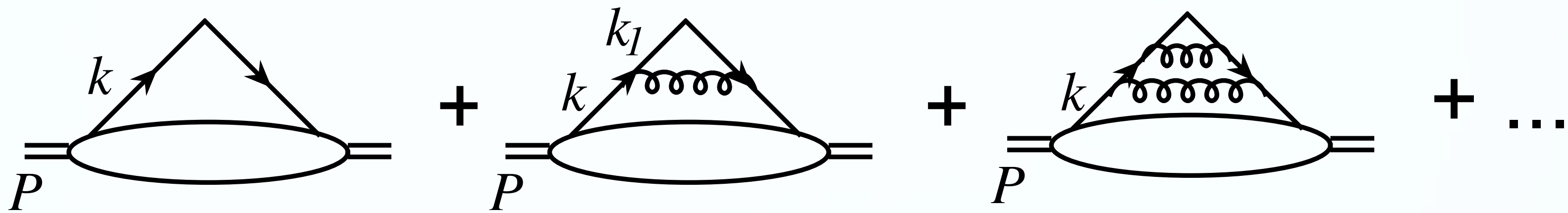


$$\alpha_s \ln \frac{\mu^2}{\Lambda^2} \qquad \left(\alpha_s \ln \frac{\mu^2}{\Lambda^2}\right)^2$$

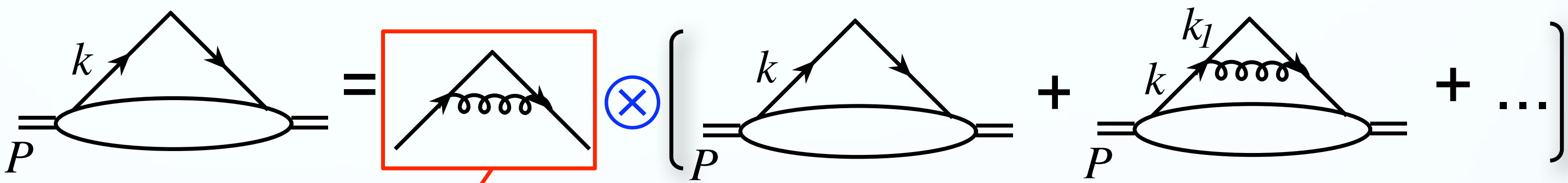
DGLAP evolution = resummation of single logs

- Evolution = Resum all the gluon radiation

$\phi(x, \mu^2)$ =



$\phi(x, \mu^2)$ -



→ **DGLAP Equation**

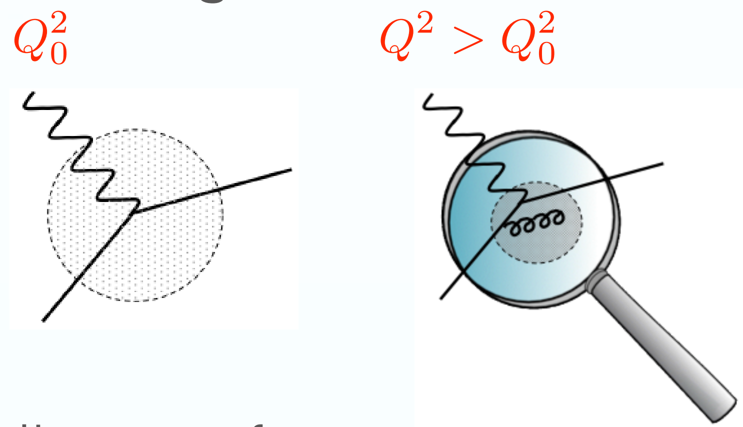
Evolution kernel
splitting function

$$\frac{\partial}{\partial \ln \mu^2} \phi_i(x, \mu^2) = \sum_j \boxed{P_{ij}(\frac{x}{x'})} \otimes \phi_j(x', \mu^2)$$

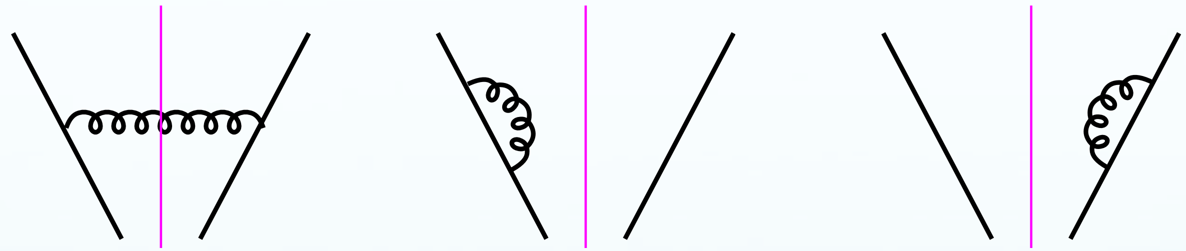
- By solving the evolution equation, one resums all the single logarithms of type $$\left(\alpha_s \ln \frac{\mu^2}{\Lambda^2}\right)^n$$

Evolutions of PDFs

- Perturbative change:



- Feynman diagrams for unpolarized PDFs



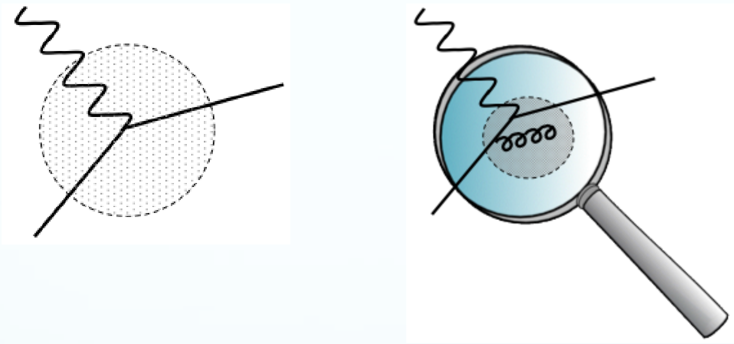
$$\frac{q(x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \left[P_{qq}(z)\, q(\xi,\mu_F) \right]$$

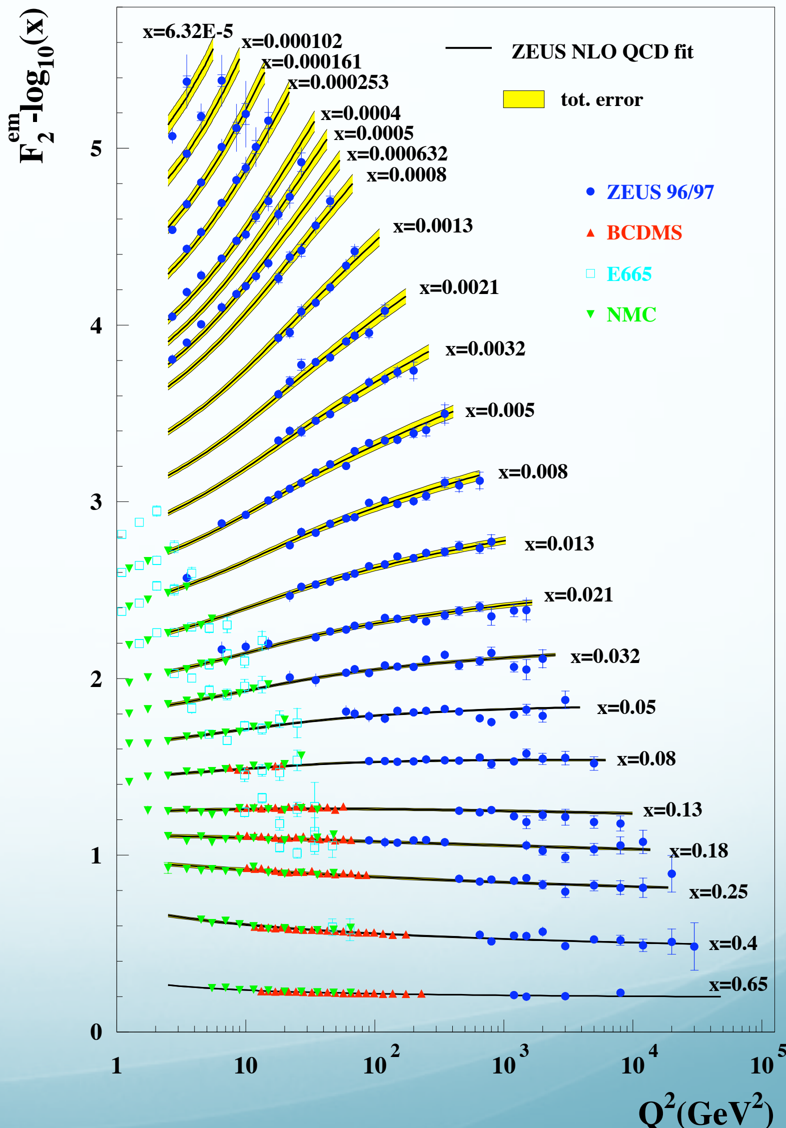
$$P_{qq}(z) = C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z) \right]$$

PDFs also depends on the scale of the probe

- Increase the energy scale, one sees parton picture differently

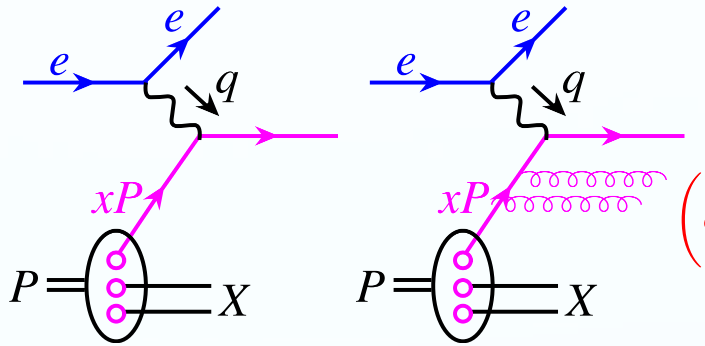
Q_0^2 $\qquad$ $Q^2 > Q_0^2$





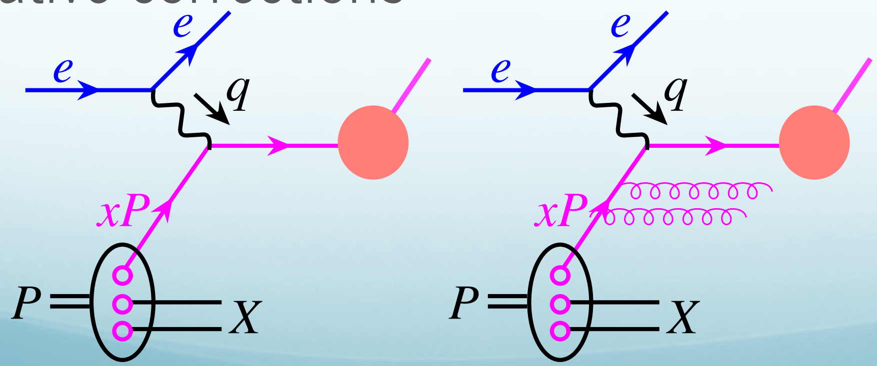
QCD evolution: meaning

- Evolution = include important perturbative corrections
  - DGLAP evolution of collinear PDFs: what it does is to resum the so-called single logarithms in the higher order perturbative calculations



$$\left(\alpha_s \ln \frac{Q^2}{\mu^2}\right)^n$$

- TMD factorization works in the situation where there are two observed momenta in the process, Q>>qt: what it does is to resum the so-called double logarithms in the higher order perturbative corrections



$$\left(\alpha_s \ln^2 \frac{Q^2}{q_T^2}\right)^n$$

Energy dependence of TMDs

- Experiments operate in very different kinematic ranges
  - Typical hard scale Q is different: Q ~ 1 – 3 GeV in SIDIS, Q ~ 4 – 90 GeV in pp
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Collinear PDFs

$$F(x, Q)$$

TMDs

$$F(x, k_\perp; Q)$$

Another important concept: gauge link

- The correlator is not gauge invariant (without gauge link)

$$\Phi_{ij}(p,P,S) = \frac{1}{(2\pi)^4}\int d^4\xi\, e^{ip\cdot\xi}\langle P,S|\,\bar{\psi}_j(0)\,\psi_i(\xi)\,|P,S\rangle$$

not invariant under $\psi(\xi) \to e^{i\alpha(\xi)}\,\psi(\xi)$

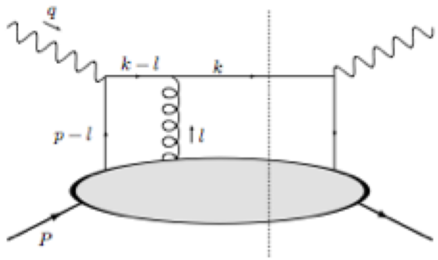
$$\Phi_{ij}(p,P,S) = \frac{1}{(2\pi)^4}\int d^4\xi\, e^{ip\cdot\xi}\langle P,S|\,\bar{\psi}_j(0)\,U_{[0,\xi]}\,\psi_i(\xi)\,|P,S\rangle$$

$$U(\xi_1,\xi_2) \to e^{i\alpha(\xi_1)}\,U(\xi_1,\xi_2)\,e^{-i\alpha(\xi_2)}.$$

$$U_{[a,b]} = \mathcal{P}\exp\left[-ig\int_a^b d\eta^\mu A_\mu(\eta)\right]$$

How gauge link appears

- Take DIS as an example



(a)

$$2MW_{\mu\nu}^{(a)} \sim \int d^4 l \int \frac{d^4\eta}{(2\pi)^4}\, e^{il\cdot(\eta-\xi)} \langle P,S|\overline{\psi}(0)\gamma_\mu\gamma^+\gamma_\alpha \frac{\not{k}-\not{l}}{(k-l)^2+i\epsilon}\gamma_\nu g A^\alpha(\eta)\psi(\xi)|P,S\rangle$$

$$i\,\frac{\not{k}-\not{l}}{(k-l)^2+i\epsilon} \approx i\,\frac{k^-\gamma^+}{-2l^+k^-+i\epsilon} \approx \frac{i}{2}\,\frac{\gamma^+}{-l^+ + i\epsilon}$$

eikonal approximation

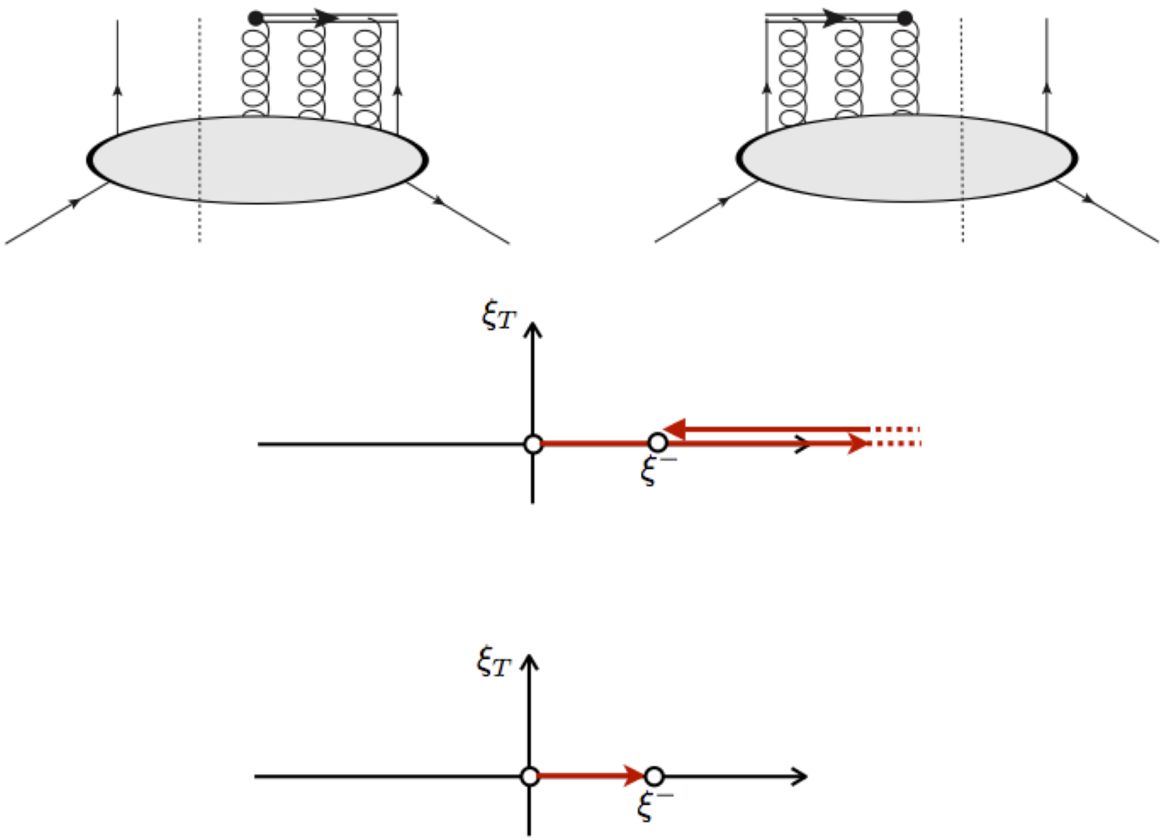
$$2MW_{\mu\nu}^{(a)} \sim \int \frac{d\eta^-}{2\pi} \int dl^+ e^{il^+(\eta^- - \xi^-)} \langle P,S|\overline{\psi}(0)\gamma_\mu\gamma^+ \frac{\gamma^-\gamma^+}{2}\gamma_\nu (ig)\frac{A^+(\eta)}{-l^+ + i\epsilon}\psi(\xi)|P,S\rangle\Big|_{\substack{\eta^+=\xi^+,\\ \boldsymbol{\eta}_T=\boldsymbol{\xi}_T}}$$

$$2MW_{\mu\nu}^{(a)} \sim \langle P,S|\overline{\psi}(0)\,\gamma_\mu\gamma^+\,\gamma_\nu\,(-ig)\int_{\infty^-}^{\xi^-} d\eta^-\; A^+(\eta)\,\psi(\xi)|P,S\rangle\Big|_{\substack{\eta^+=\xi^+=0\\ \boldsymbol{\eta}_T=\boldsymbol{\xi}_T=0}}$$

They can be resumed to all orders

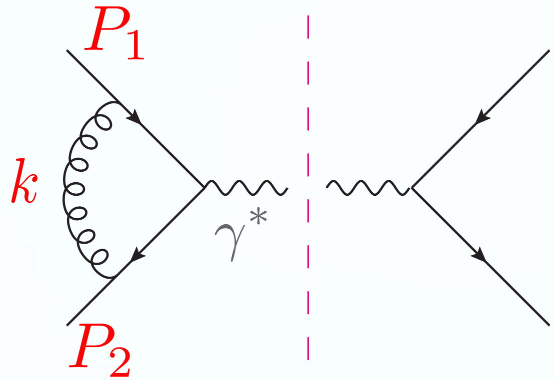
- Gauge link: eikonal line

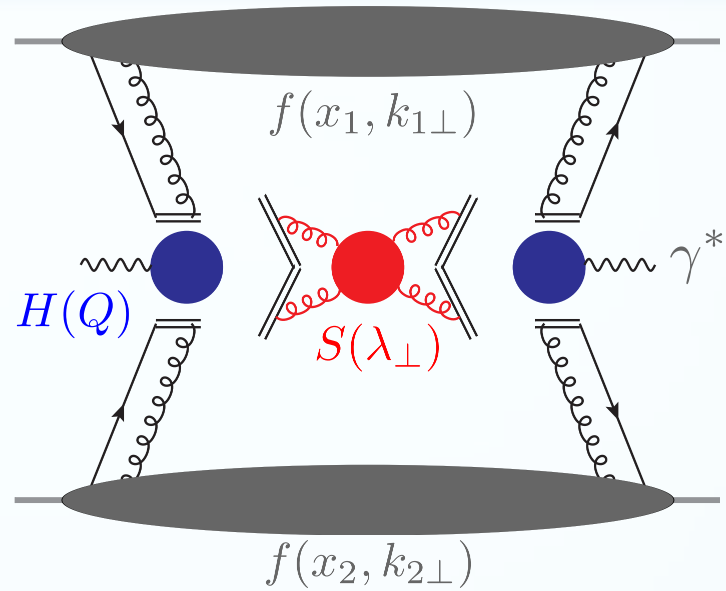
$$\Phi(x,S) \sim \langle P,S|\,\bar{\psi}(0)\,U_{[0,\infty^-]}\,U_{[\infty^-,\xi^-]}\psi(\xi)\,|P,S\rangle$$



TMD factorization in a nut-shell

- Drell-Yan: $p + p \to [\gamma^* \to \ell^+\ell^-] + X$





Factorization of regions:
(1) k//P_1, (2) k//P_2, (3) k soft, (4) k hard

- Factorized form and mimic "parton model"

$$\frac{d\sigma}{dQ^2 dy d^2 q_\perp} \propto \int d^2k_{1\perp}\, d^2k_{2\perp}\, d^2\lambda_\perp H(Q) f(x_1, k_{1\perp}) f(x_2, k_{2\perp}) S(\lambda_\perp) \delta^2(k_{1\perp} + k_{2\perp} + \lambda_\perp - q_\perp)$$

$$= \int \frac{d^2b}{(2\pi)^2} e^{iq_\perp \cdot b} H(Q) f(x_1, b) f(x_2, b) S(b)$$

$$F(x, b) = f(x, b)\sqrt{S(b)}$$

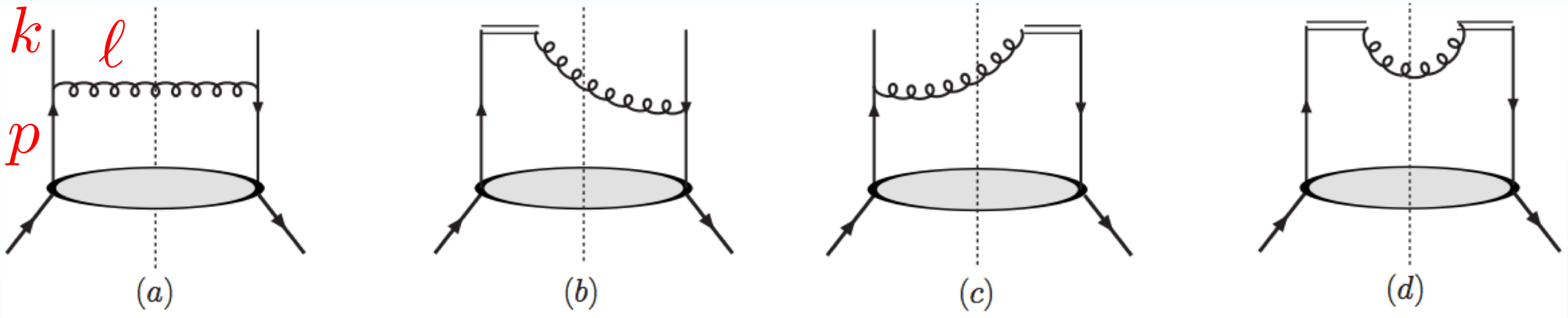
$$= \int \frac{d^2b}{(2\pi)^2} e^{iq_\perp \cdot b} H(Q) F(x_1, b) F(x_2, b)$$

mimic "parton model"

Divergence and evolution

- Divergence leads to evolution
  - Ultraviolet divergence: renormalization group equation, e.g. running of coupling constant
  - Collinear divergence: DGLAP evolution of collinear parton distribution function, fragmentation function, semi-inclusive jet function
  - Rapidity divergence (light-cone singularity): TMD evolution

- What is rapidity divergence?



$$f_{q/q}(z, k_\perp^2) = \frac{\alpha_s}{2\pi^2}\frac{1}{k_\perp^2}\left[\frac{2z}{1-z} + (1-z)\right]$$

$$z = \frac{k^+}{p^+} \qquad z \to 1 \Leftrightarrow \ell^+ \to 0 \qquad y = \frac{1}{2}\ln\frac{\ell^+}{\ell^-} \to -\infty$$

Different ways to regularize rapidity divergences

- There are different ways to regularize rapidity divergences
  - Off-light-cone — Collins, Soper 79, ...
  - δ-regulator — Chiu, Fuhrer, Hoang, Kelley, Manohar, 09, Echevarria, Idilbi, Scimemi, 11, ...
  - Analytic regulator — Becher, Bell, 11, ...
  - Rapidity regulator — Chiu, Jain, Neill, Rothstein, 11, 12, ...
  - Exponential regulator — Li, Neill, Zhu, 16, ...

- Rapidity regulator

$$W_n = \sum_{\text{perms}} \exp\left[-\frac{gw^2}{\bar{n}\cdot\mathcal{P}} \frac{|\,\bar{n}\cdot\mathcal{P}_g\,|^{-\eta}}{\nu^{-\eta}} \bar{n}\cdot A_n\right]$$

$$S_n = \sum_{\text{perms}} \exp\left[-\frac{gw}{n\cdot\mathcal{P}} \frac{|\,2\mathcal{P}_{g3}\,|^{-\eta/2}}{\nu^{-\eta/2}} n\cdot A_s\right]$$

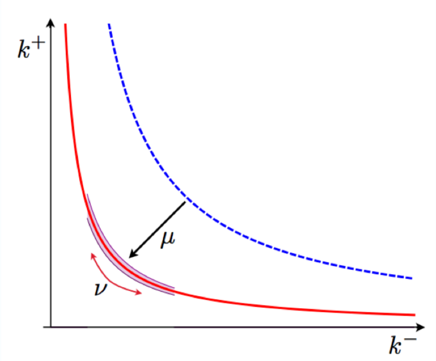
$$\int \frac{dk^+}{k^+} \to \int \frac{dk^+}{k^+} \left|\frac{\nu}{p^+}\right|^{\eta}$$

$$f_{q/q}(z,k_\perp^2) = \frac{\alpha_s}{2\pi^2}\Gamma(1+\epsilon)e^{\gamma_E\epsilon}\frac{1}{\mu^2}\left(\frac{\mu^2}{k_\perp^2}\right)^{1+\epsilon}\left[\frac{2z}{(1-z)^{1+\eta}}\left(\frac{\nu}{p^+}\right)^{\eta} + (1-\epsilon)(1-z)\right]$$

TMD evolution in b-space

- Quark TMD at one loop

$$f_{q/q}(x,b) = \frac{\alpha_s}{2\pi} C_F \left\{ \left[\frac{2}{\eta} \left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{\mu_b^2} \right) + \frac{2}{\epsilon} \ln\frac{\nu}{p^+} + \frac{3}{2}\frac{1}{\epsilon} \right] \delta(1-x) + \left(-\frac{1}{\epsilon} - \ln\frac{\mu^2}{\mu_b^2} \right) P_{qq}(x) + \left[2\ln\frac{\mu^2}{\mu_b^2} \ln\frac{\nu}{p^+} + \frac{3}{2}\ln\frac{\mu^2}{\mu_b^2} \right] \delta(1-x) + (1-x) \right\}$$



- Soft factor

$$S(b) = \frac{\alpha_s}{2\pi} C_F \left\{ \frac{4}{\eta} \left(-\frac{1}{\epsilon} - \ln\frac{\mu^2}{\mu_b^2} \right) + \frac{2}{\epsilon^2} + \frac{2}{\epsilon} \left(\ln\frac{\mu^2}{\mu_b^2} - \ln\frac{\nu^2}{\mu_b^2} \right) + \left[-2\ln\frac{\mu^2}{\mu_b^2} \ln\frac{\nu^2}{\mu_b^2} + \ln^2\frac{\mu^2}{\mu_b^2} - \frac{\pi^2}{6} \right] \right\}$$

$$\mu_b = 2e^{-\gamma_E}/b$$

- Interesting features
  - Rapidity divergence cancels in $F_{q/q}^{\rm sub}(x,b) = f_{q/q}(x,b)\sqrt{S(b)}$
  - $f_{q/q}(x, b)$ and $S(b)$ lives in the same $\mu \sim \mu_b$, but different rapidity scale $\nu \sim p^+$, μ_b

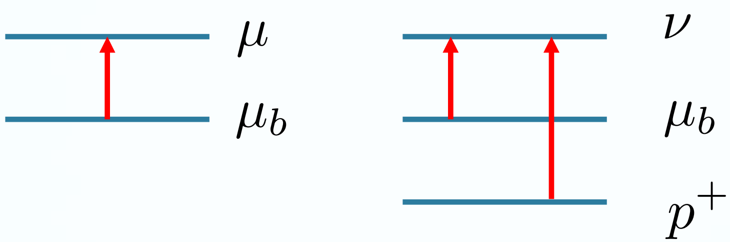
- Two evolution equations: μ-RG and ν-RG

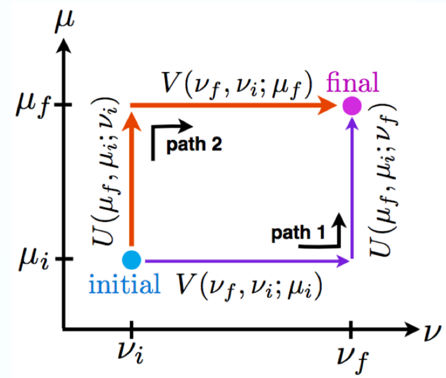
$$\mu\frac{d}{d\mu}\ln f_{q/q}(x,b) = \gamma_\mu^f \qquad \mu\frac{d}{d\mu}\ln S(b) = \gamma_\mu^S$$

$$\nu\frac{d}{d\nu}\ln f_{q/q}(x,b) = \gamma_\nu^f \qquad \nu\frac{d}{d\nu}\ln S(b) = \gamma_\nu^S$$

TMD evolution in b-space

- Solution of TMD evolution equations





- The well-known CSS solution

$$F(x, b; Q_f) = F(x, b; Q_i) \exp\left\{-\int_{Q_i}^{Q_f} \frac{d\mu}{\mu}\left(A \ln\frac{Q_f^2}{\mu^2} + B\right)\right\}\left(\frac{Q_f^2}{Q_i^2}\right)^{-\int_{c/b}^{Q_i} \frac{d\mu}{\mu} A}$$

$$A = \sum_{n=1} A^{(n)} \left(\frac{\alpha_s}{\pi}\right)^n, \qquad B = \sum_{n=1} B^{(n)} \left(\frac{\alpha_s}{\pi}\right)^n$$

Only valid for small b

Collins-Sopoer-Sterman papers
Kang, Xiao, Yuan, PRL 11,
Aybat, Rogers, Collins, Qiu, 12,
Aybat, Prokudin, Rogers, 12,
Sun, Yuan, 13,
Echevarria, Idilbi, Schafer, Scimemi, 13,
Echevarria, Idilbi, Kang, Vitev, 14,
Kang, Prokudin, Sun, Yuan, 15, 16, ...

TMD evolution contains non-perturbative component

- Fourier transform back to the momentum space, one needs the whole b region (large b): need some non-perturbative extrapolation
  - Many different methods/proposals to model this non-perturbative part

$$F(x,k_\perp;Q) = \frac{1}{(2\pi)^2}\int d^2b e^{ik_\perp\cdot b}F(x,b;Q) = \frac{1}{2\pi}\int_0^\infty db\, bJ_0(k_\perp b)F(x,b;Q)$$

Collins, Soper, Sterman 85, ResBos, Qiu, Zhang 99, Echevarria, Idilbi, Kang, Vitev, 14, Aidala, Field, Gamberg, Rogers, 14, Sun, Yuan 14, D'Alesio, Echevarria, Melis, Scimemi, 14, Rogers, Collins, 15, …

- Eventually evolved TMDs in b-space

$$F(x,b;Q) \approx C\otimes F(x,c/b^*) \times \exp\left\{-\int_{c/b^*}^{Q}\frac{d\mu}{\mu}\left(A\ln\frac{Q^2}{\mu^2}+B\right)\right\} \times \exp\left(-S_{\text{non-pert}}(b,Q)\right)$$

longitudinal/collinear part

transverse part

- ✓ Non-perturbative: fitted from data
- ✓ The key ingredient – ln(Q) piece is spin-independent

Since the polarized scattering data is still limited kinematics, we can use unpolarized data to constrain/extract the key ingredient for the non-perturbative part

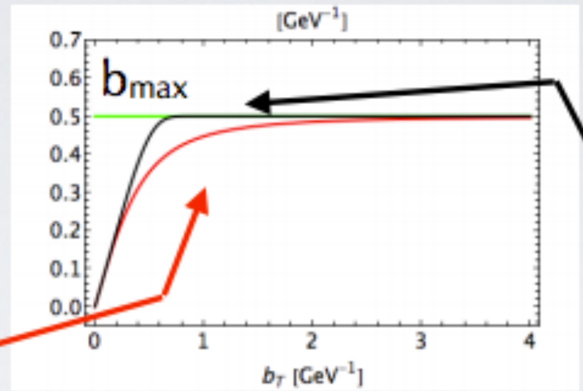
Different treatments at large b

- In terms of b* prescription (see also other proposals Qiu, Vogelsang)

b* prescription

$$\mu_b = \frac{2e^{-\gamma_E}}{b^*}$$

$$b^* = \frac{b_T}{\sqrt{1+\frac{b_T^2}{b_{max}^2}}}$$



exp prescription

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T^\dagger}$$

$$b_T^\dagger = b_{max}\left\{1-\exp\left[-\frac{b_T^4}{b_{max}^4}\right]\right\}^{\frac{1}{4}}$$

- Non-perturbative Sudakov factor

$$\exp\left[-g_2 b^2 \ln(Q/Q_0) + \cdots\right]$$ CSS, Echevarria, Idlibi, Kang, Vitev, 14, ...

$$\exp\left[-g_2 \ln(b/b^*) \ln(Q/Q_0) + \cdots\right]$$

$$\frac{1}{2}\ln\left(1+\frac{b^2}{b_{max}^2}\right)$$

Aidala, Field, Gamberg, Rogers, 1401.2654, Sun, Isaacson, Yuan, Yuan, 1406.3073

$$\exp\left\{-g_0(b_{max})\left[1-\exp\left(-\frac{C_F\alpha_s(\mu_{b_*})b^2}{\pi g_0(b_{max})b_{max}}\right)\right]\right\}$$ Collins, Rogers, 1412.3820

TMD evolves

- Just like collinear PDFs, TMDs also depend on the scale of the probe = evolution

Collinear PDFs

$F(x, Q)$

- ✓ DGLAP evolution
- ✓ Resum $\left[\alpha_s \ln(Q^2/\mu^2)\right]^n$
- ✓ Kernel: purely perturbative

TMDs

$F(x, k_\perp; Q)$

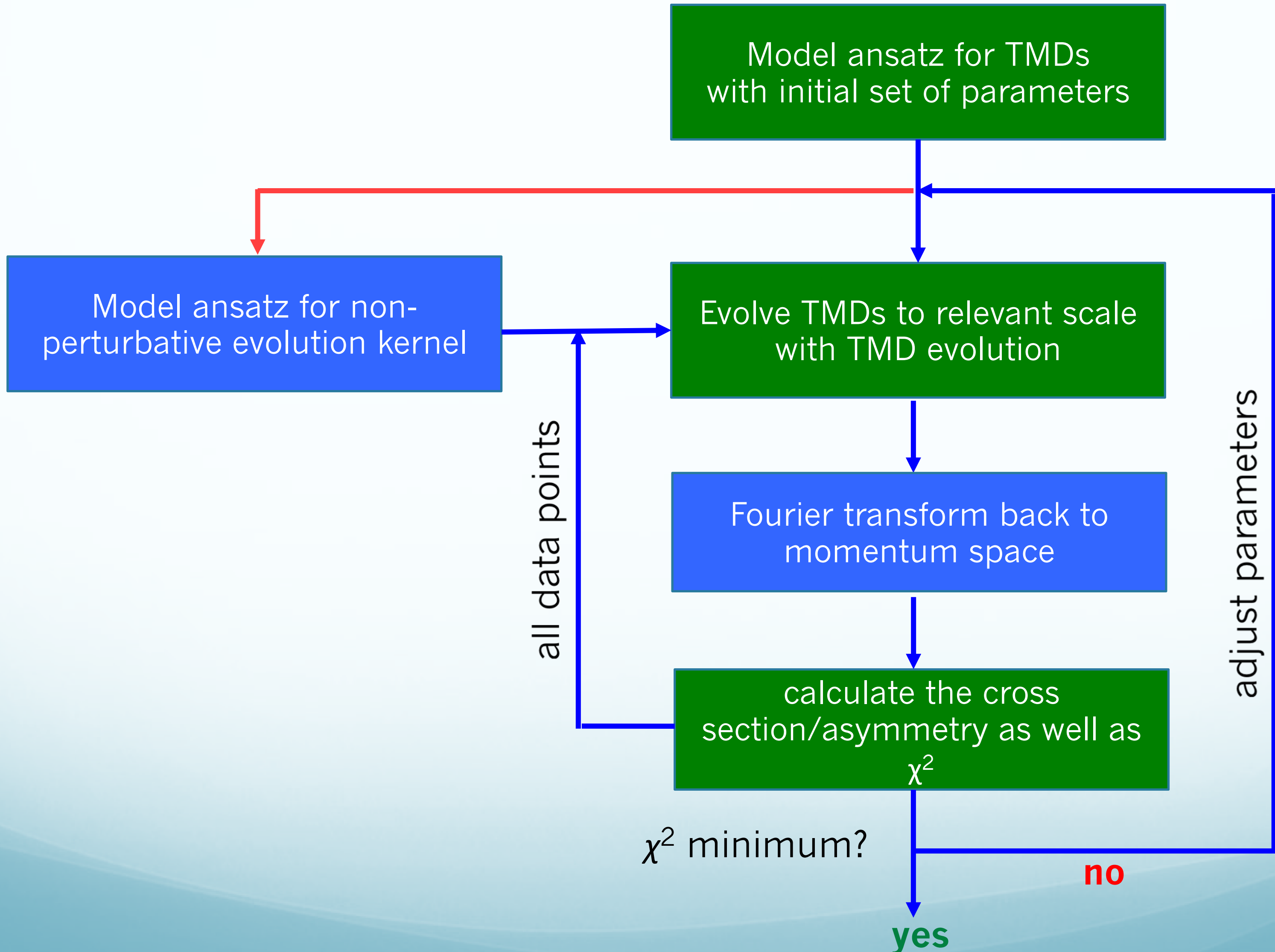
- ✓ Collins-Soper/rapidity evolution equation
- ✓ Resum $\left[\alpha_s \ln^2(Q^2/k_\perp^2)\right]^n$
- ✓ Kernel: can be non-perturbative when $k_\perp \sim \Lambda_{\mathrm{QCD}}$

$$F(x, Q_i) \downarrow R^{\mathrm{coll}}(x, Q_i, Q_f) \downarrow F(x, Q_f)$$

$$F(x, k_\perp, Q_i) \downarrow R^{\mathrm{TMD}}(x, k_\perp, Q_i, Q_f) \downarrow F(x, k_\perp, Q_f)$$

TMD global analysis

- Outline of a TMD global analysis: numerically more heavy



SIDIS structure functions

hep-ph/0611265

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \\
&\quad \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
&\qquad +\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
&\qquad +S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&+S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&+|\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\quad +\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\quad +\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
&+|\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&\quad +\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}, \qquad (2.7)
\end{aligned}
$$

Structure functions related to TMDs

- Sivers term

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right],$$

- Collins term

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right]$$

- etc: total 18 terms

$$\mathcal{C}\big[w\, f\, D\big] = x \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big)\, w(\boldsymbol{p}_T,\boldsymbol{k}_T)\, f^a(x,p_T^2)\, D^a(z,k_T^2),$$

Drell-Yan: 48 structure functions

arXiv: 0809.2262

$$
\begin{aligned}
\frac{d\sigma}{d^4q\,d\Omega} = \frac{\alpha_{em}^2}{F\,q^2}\times \\
&\Big\{\Big((1+\cos^2\theta)\,F_{UU}^1 + (1-\cos^2\theta)\,F_{UU}^2 + \sin 2\theta\cos\phi\,F_{UU}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{UU}^{\cos 2\phi}\Big) \\
&+ S_{aL}\Big(\sin 2\theta\sin\phi\,F_{LU}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{LU}^{\sin 2\phi}\Big) \\
&+ S_{bL}\Big(\sin 2\theta\sin\phi\,F_{UL}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{UL}^{\sin 2\phi}\Big) \\
&+ |\vec{S}_{aT}|\Big[\sin\phi_a\Big((1+\cos^2\theta)\,F_{TU}^1 + (1-\cos^2\theta)\,F_{TU}^2 + \sin 2\theta\cos\phi\,F_{TU}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TU}^{\cos 2\phi}\Big) \\
&\qquad + \cos\phi_a\Big(\sin 2\theta\sin\phi\,F_{TU}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TU}^{\sin 2\phi}\Big)\Big] \\
&+ |\vec{S}_{bT}|\Big[\sin\phi_b\Big((1+\cos^2\theta)\,F_{UT}^1 + (1-\cos^2\theta)\,F_{UT}^2 + \sin 2\theta\cos\phi\,F_{UT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{UT}^{\cos 2\phi}\Big) \\
&\qquad + \cos\phi_b\Big(\sin 2\theta\sin\phi\,F_{UT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{UT}^{\sin 2\phi}\Big)\Big] \\
&+ S_{aL}\,S_{bL}\Big((1+\cos^2\theta)\,F_{LL}^1 + (1-\cos^2\theta)\,F_{LL}^2 + \sin 2\theta\cos\phi\,F_{LL}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{LL}^{\cos 2\phi}\Big) \\
&+ S_{aL}\,|\vec{S}_{bT}|\Big[\cos\phi_b\Big((1+\cos^2\theta)\,F_{LT}^1 + (1-\cos^2\theta)\,F_{LT}^2 + \sin 2\theta\cos\phi\,F_{LT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{LT}^{\cos 2\phi}\Big) \\
&\qquad + \sin\phi_b\Big(\sin 2\theta\sin\phi\,F_{LT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{LT}^{\sin 2\phi}\Big)\Big] \\
&+ |\vec{S}_{aT}|\,S_{bL}\Big[\cos\phi_a\Big((1+\cos^2\theta)\,F_{TL}^1 + (1-\cos^2\theta)\,F_{TL}^2 + \sin 2\theta\cos\phi\,F_{TL}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TL}^{\cos 2\phi}\Big) \\
&\qquad + \sin\phi_a\Big(\sin 2\theta\sin\phi\,F_{TL}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TL}^{\sin 2\phi}\Big)\Big] \\
&+ |\vec{S}_{aT}|\,|\vec{S}_{bT}|\Big[\cos(\phi_a+\phi_b)\Big((1+\cos^2\theta)\,F_{TT}^1 + (1-\cos^2\theta)\,F_{TT}^2 + \sin 2\theta\cos\phi\,F_{TT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TT}^{\cos 2\phi}\Big) \\
&\qquad + \cos(\phi_a-\phi_b)\Big((1+\cos^2\theta)\,\bar{F}_{TT}^1 + (1-\cos^2\theta)\,\bar{F}_{TT}^2 + \sin 2\theta\cos\phi\,\bar{F}_{TT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,\bar{F}_{TT}^{\cos 2\phi}\Big) \\
&\qquad + \sin(\phi_a+\phi_b)\Big(\sin 2\theta\sin\phi\,F_{TT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TT}^{\sin 2\phi}\Big) \\
&\qquad + \sin(\phi_a-\phi_b)\Big(\sin 2\theta\sin\phi\,\bar{F}_{TT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,\bar{F}_{TT}^{\sin 2\phi}\Big)\Big]\Big\}. \qquad (57)
\end{aligned}
$$

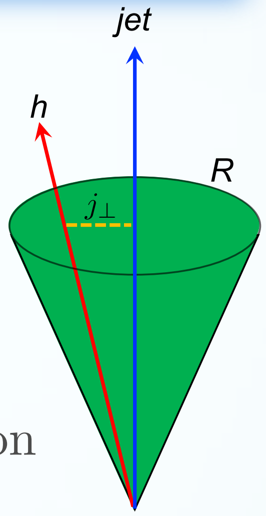
TMD hadron distribution inside the jet?

- Definition

$$F(z_h, j_\perp; p_T) = \frac{d\sigma^h}{dp_T d\eta dz_h d^2 j_\perp} \Big/ \frac{d\sigma}{dp_T d\eta}$$

$$z_h = p_T^{\rm h}/p_T^{\rm jet}$$

$j_\perp$: hadron transverse momentum with respect to the jet direction



- Factorization formalism

Kang, Liu, Ringer, Xing, 1705.08443

$$\frac{d\sigma}{dp_T d\eta dz_h d^2 j_\perp} \propto \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab\to c} \otimes \mathcal{G}_c^h(z, z_h, \omega_J R, j_\perp, \mu)$$

- Re-factorization of semi-inclusive fragmenting jet function

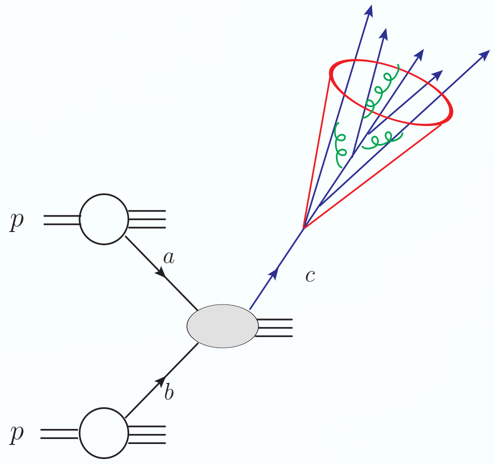
$$\mathcal{G}_c^h(z, z_h, \omega_J R, \boldsymbol{j}_\perp, \mu) = \mathcal{H}_{c\to i}(z, \omega_J R, \mu) \int d^2\boldsymbol{k}_\perp d^2\boldsymbol{\lambda}_\perp \delta^2\left(z_h \boldsymbol{\lambda}_\perp + \boldsymbol{k}_\perp - \boldsymbol{j}_\perp\right)$$
$$\times D_{h/i}(z_h, \boldsymbol{k}_\perp, \mu, \nu) S_i(\boldsymbol{\lambda}_\perp, \mu, \nu R)$$

What's different for hadron in the jet?

- Soft radiation has to happen inside the jet
  - For single inclusive jet production, first we produce a high-pt jet
  - This process only involves hard-collinear factorization, and such a process is not sensitive to any soft radiation
  - This is the usual standard "collinear factorization"

$$\int_0^\infty \frac{dy}{y} \Rightarrow \int_0^{\tan^2 \frac{R}{2}} \frac{dy}{y}$$

$$y \sim \frac{\ell^+}{\ell^-}$$



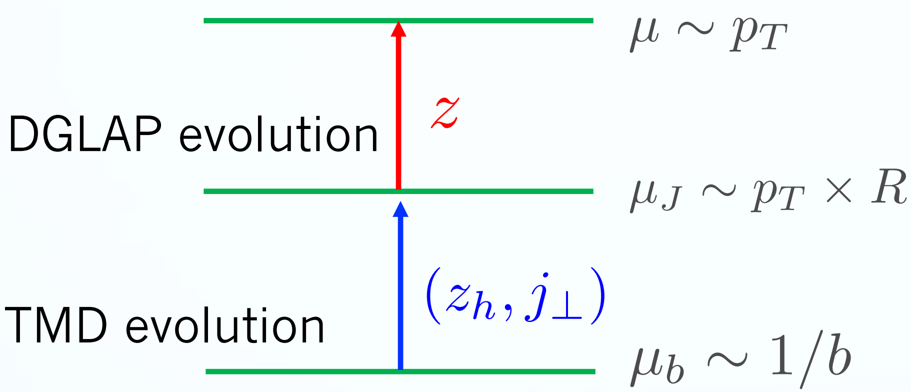
  - Once such a high-pt jet is produced, we further observe a hadron inside the jet
  - At this step, we measure the relative transverse momentum of hadron w.r.t the jet. For such a step, soft radiation matters
  - However, only those soft radiation that happens inside the jet matters
  - Restricts soft radiation to be within the jet: cuts half of the rapidity divergence

- Rapidity divergence cancel between restricted "soft factor" and TMD FFs
  - At least up to this order, the combined evolution is the same as the usual TMD evolution in SIDIS, DY, e+e-; justify the use of same TMD evolution here

$$\sqrt{S(b)}\,D_c^h(z_h, b)_{\mathrm{e^+e^-}} \Rightarrow S(b, R)\,D_c^h(z_h, b)_{\mathrm{pp}}$$

TMD + DGLAP evolution

- Evolution structure



Resum ln(R)

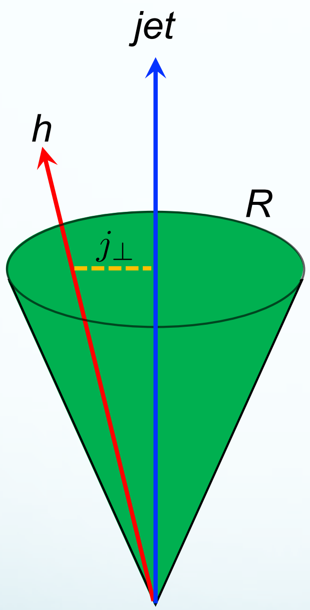
Evolve TMD FFs from $\boldsymbol{\mu}_b$ to pT*R

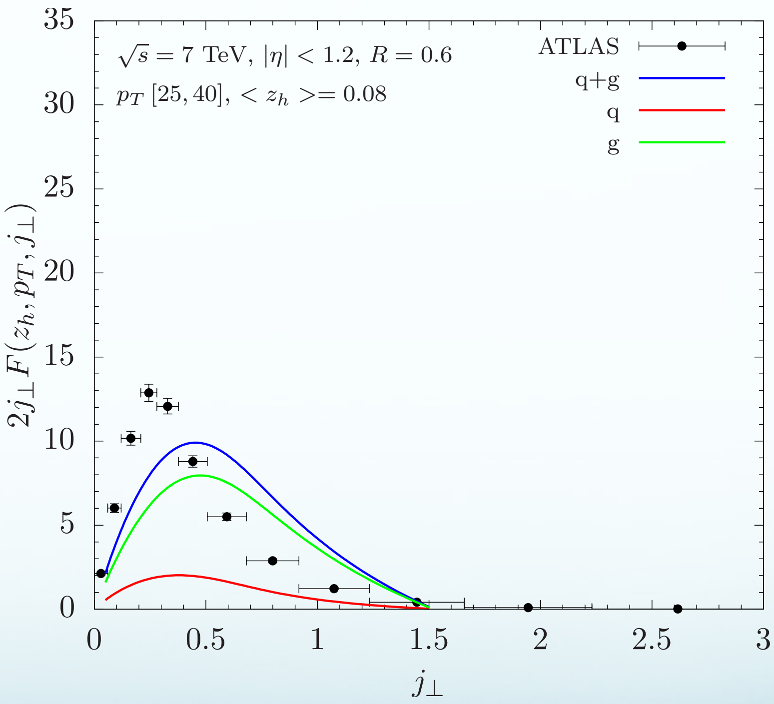
- TMD FFs thus are related to the usual TMD FFs in SIDIS at scale pT*R

- Thus hadron TMD distribution inside the jet could be used to test the universality of TMD FFs from SIDIS, e+e- processes

Hadron TMD distribution inside jets

- Unpolarized p+p collisions: very sensitive to gluon TMDs
- If we want to be able to compare gluon TMDs in p+p and e+p, then p+p measurements are essentially necessary



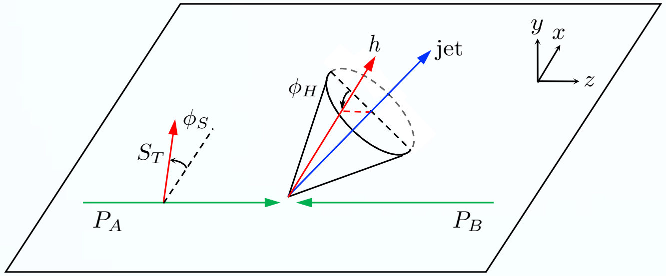


- Issue of non-global logarithms (NGLs)?

Dasgupta, Salam, 01, Banfi, Marchesini, Smye, 02, …

Collins asymmetry in p+p

- Collins asymmetry can also be studied through the azimuthal distribution of hadrons inside a jet in p+p collisions



$$p^{\uparrow}\left[\vec{S}_{\perp}(\phi_S)\right] + p \to [\text{jet}\, h(\phi_H)] + X$$

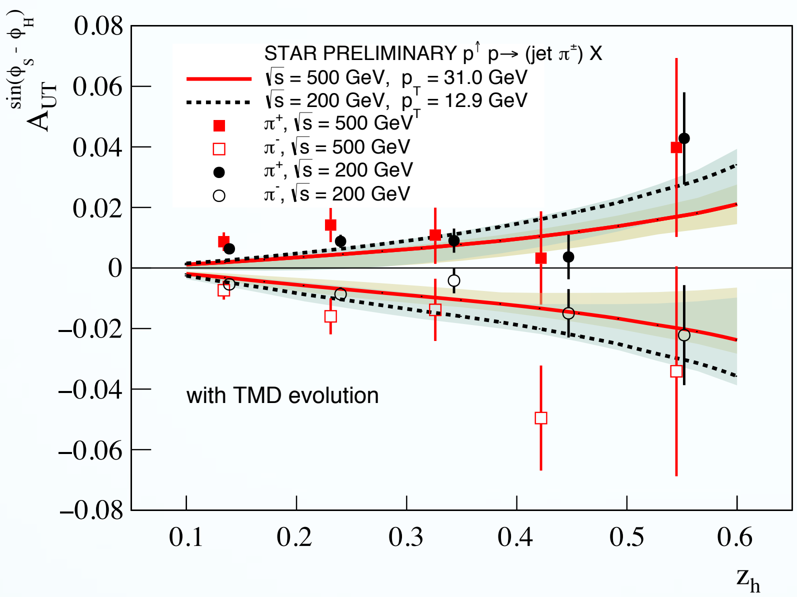
$$\frac{d\sigma}{dy d^2 p_{\perp}^{\text{jet}} dz d^2 j_T} = F_{UU} + \sin(\phi_S - \phi_H) F_{UT}^{\sin(\phi_S - \phi_H)}$$

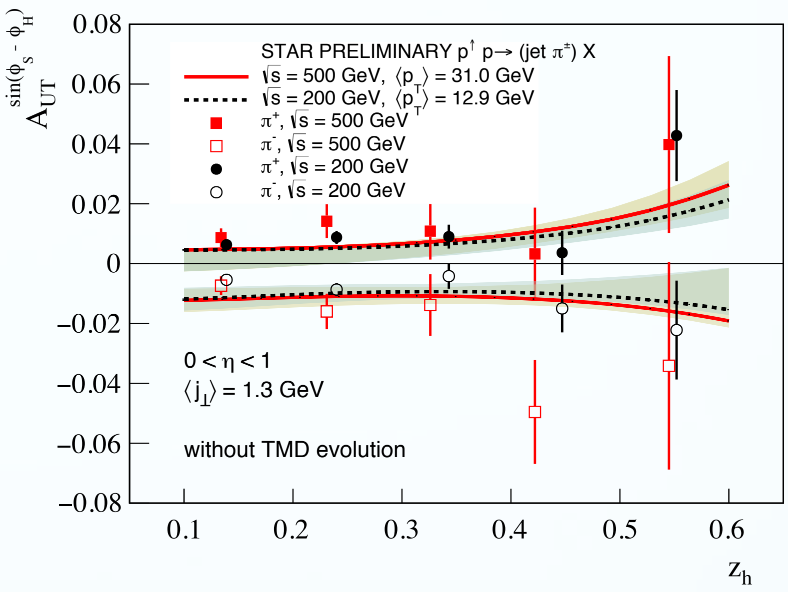
$$F_{UT}^{\sin(\phi_S - \phi_H)} \propto h_1^a(x_1) \otimes f_{b/B}(x_2) \otimes \frac{j_T}{z M_h} H_1^{\perp c}(z, j_T^2) \otimes H_{ab\to c}^{\text{Collins}}(\hat{s}, \hat{t}, \hat{u})$$

j_T : hadron transverse momentum with respect to the jet direction

- Such an asymmetry has been measured by STAR at RHIC
  - Could be used to test the universality of the Collins functions

Calculated Collins azimuthal asymmetry





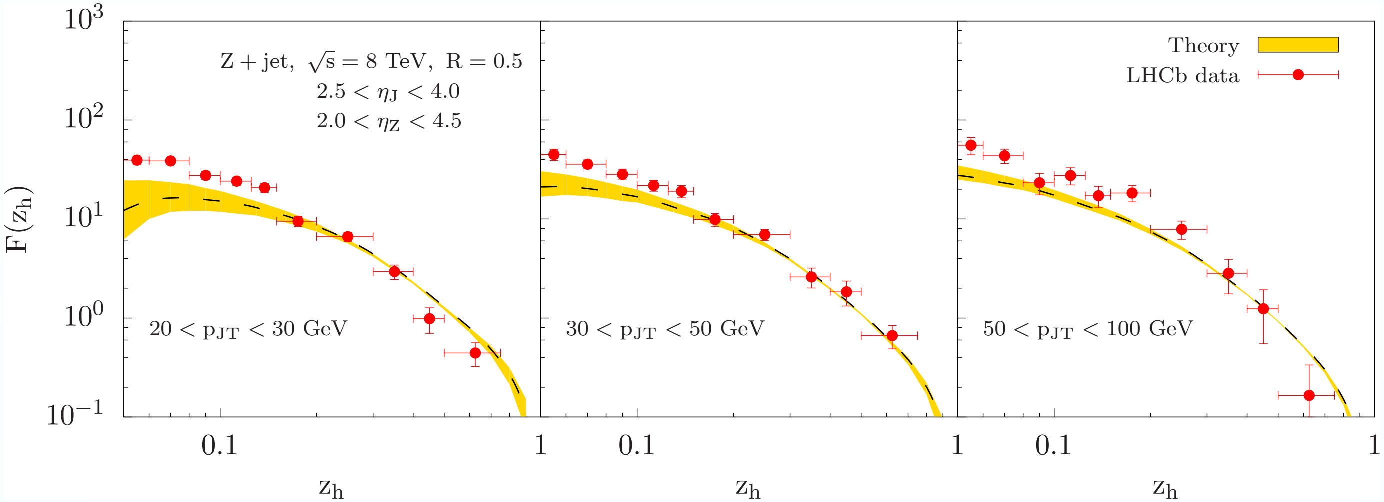
- *Test universality of Collins function between e+p, e+e, and p+p*
- *Test TMD evolution*

Kang, Prokudin, Ringer, Yuan, 1707.00913

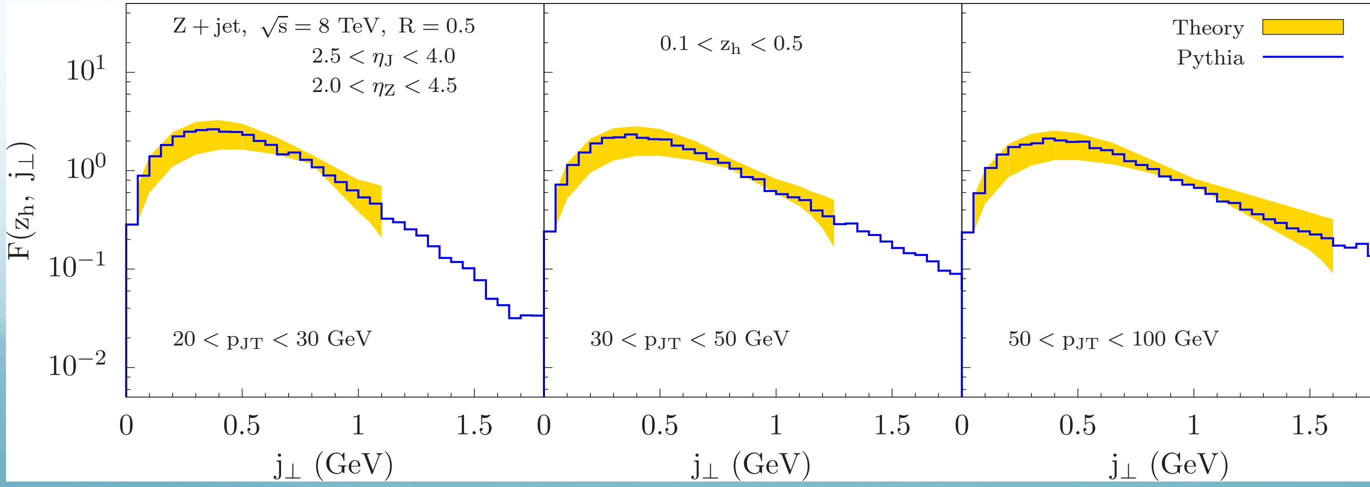
Jet fragmentation functions in Z+jet

- z_h distribution

Kang, Lee, Terry, Xing, arXiv:1906.07187



- For the reason mentioned, a direct comparison with LHCb data on j_T distribution does now work well



Summary

- TMDs open a new door for us to study structure of the nucleon and QCD dynamics, and much more

- Nucleon as a QCD “laboratory”: in particular topics/ideas that are similar to those in AMO/Condensed Matter Physics
  - Quantum correlation: spin-spin correlation, spin-orbit correlation, orbital motion, quantum phase interference effects …
  - 3D imaging of the nucleon at the most fundamental level
- Exciting opportunities: lots of experiments activities/measurements being/to be performed/planned in current and future experimental facilities