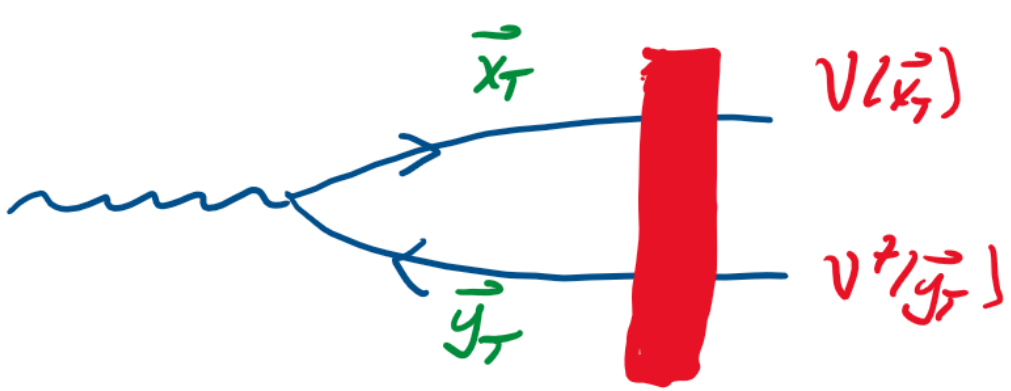
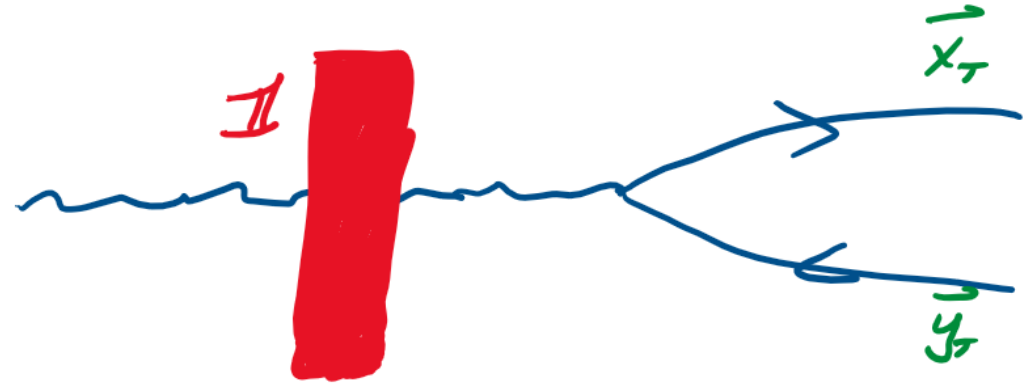
Splitting Before

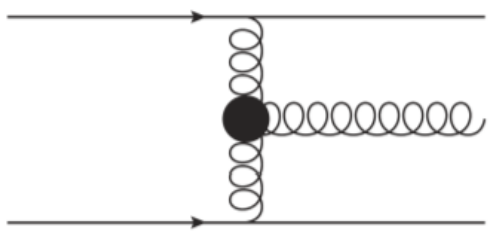


$\mathcal{H}^{g^+ \to g\bar{g}}(\vec{x}_T, \vec{y}_T)$ $\left[V(\vec{x}_T)\, V^\dagger(\vec{y}_T) \right]$

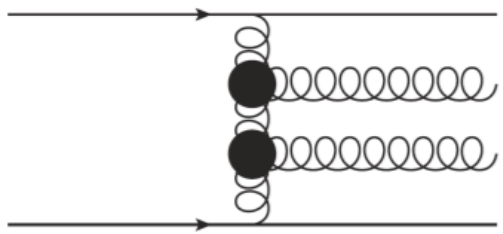
Splitting After



$-\mathcal{H}^{g^+ \to g\bar{g}}(\vec{x}_T, \vec{y}_T)$ $\left[\mathbb{I} \right]$



$\gg$

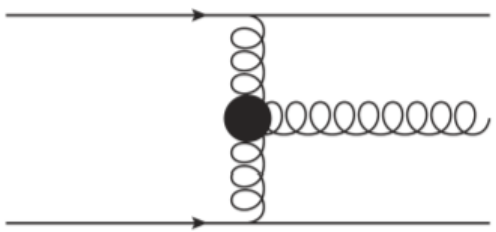


$\gg \cdots$

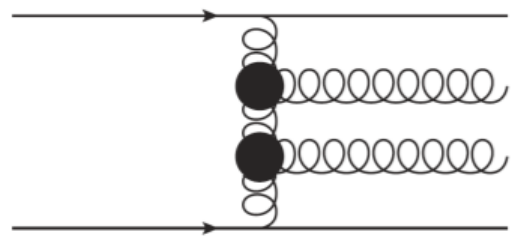
$\langle N_g \rangle \ll 1$

$\langle N_G \rangle_{LO} \sim \alpha_s \, \Delta Y$

$\langle N_G \rangle_{NLO} \sim (\alpha_s \, \Delta Y)^2$



$\sim$



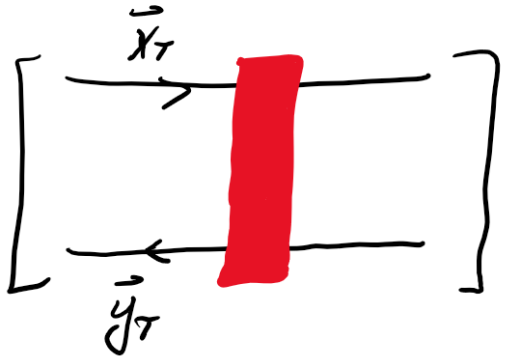
$\sim \cdots$

$\langle N_g \rangle \gtrsim 1$

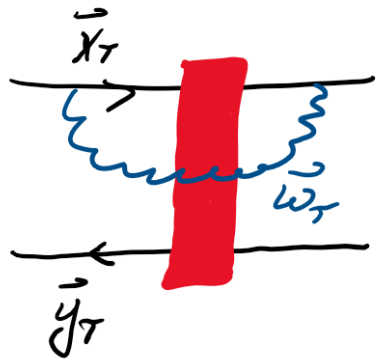
$\langle N_G \rangle_{LO} \sim \alpha_s \, \Delta Y$

$\langle N_G \rangle_{NLO} \sim (\alpha_s \, \Delta Y)^2$

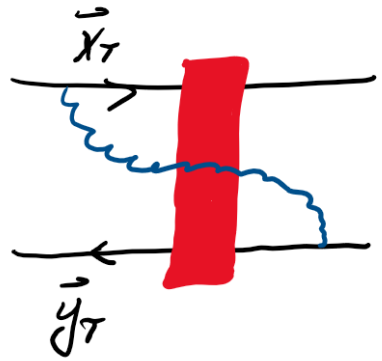
$$\frac{d}{dy}$$



$=$

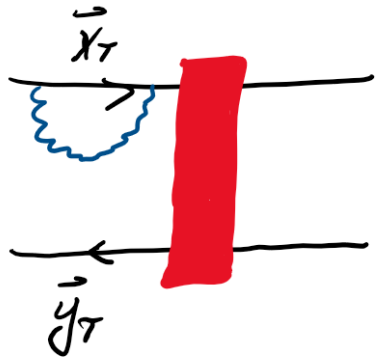


$+$

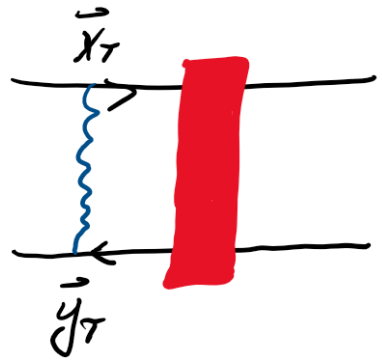


$+$ mirror images

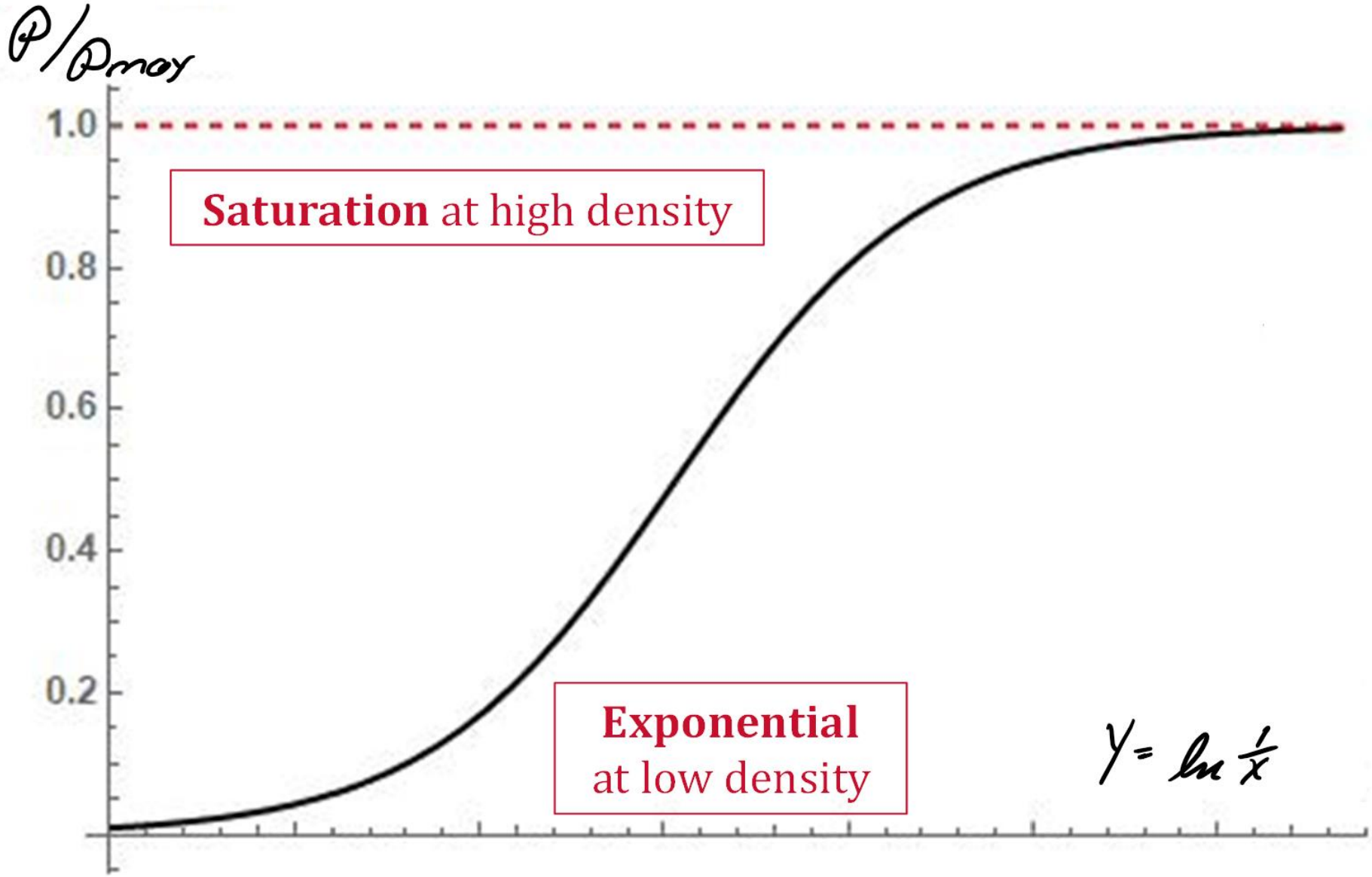
$-$

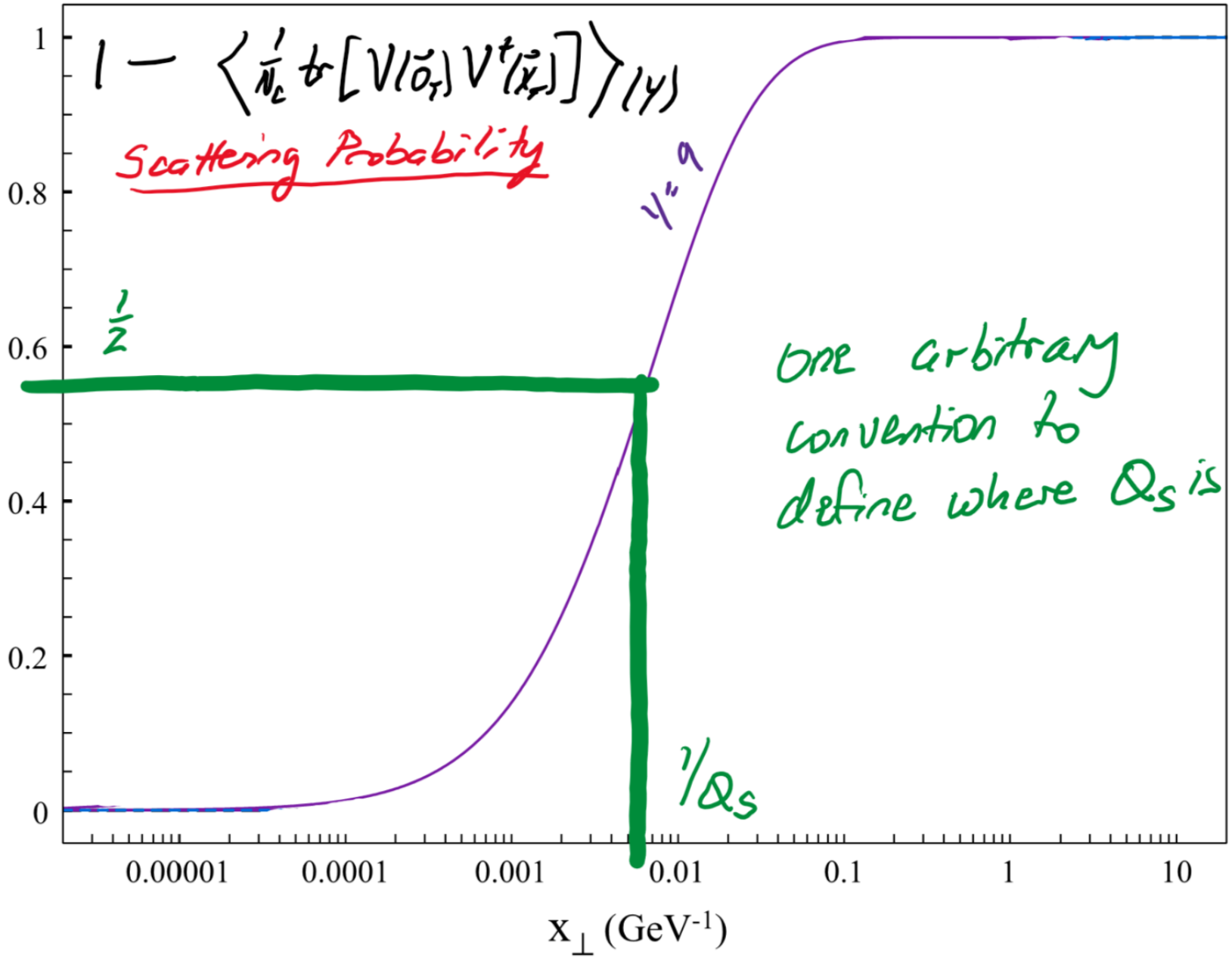


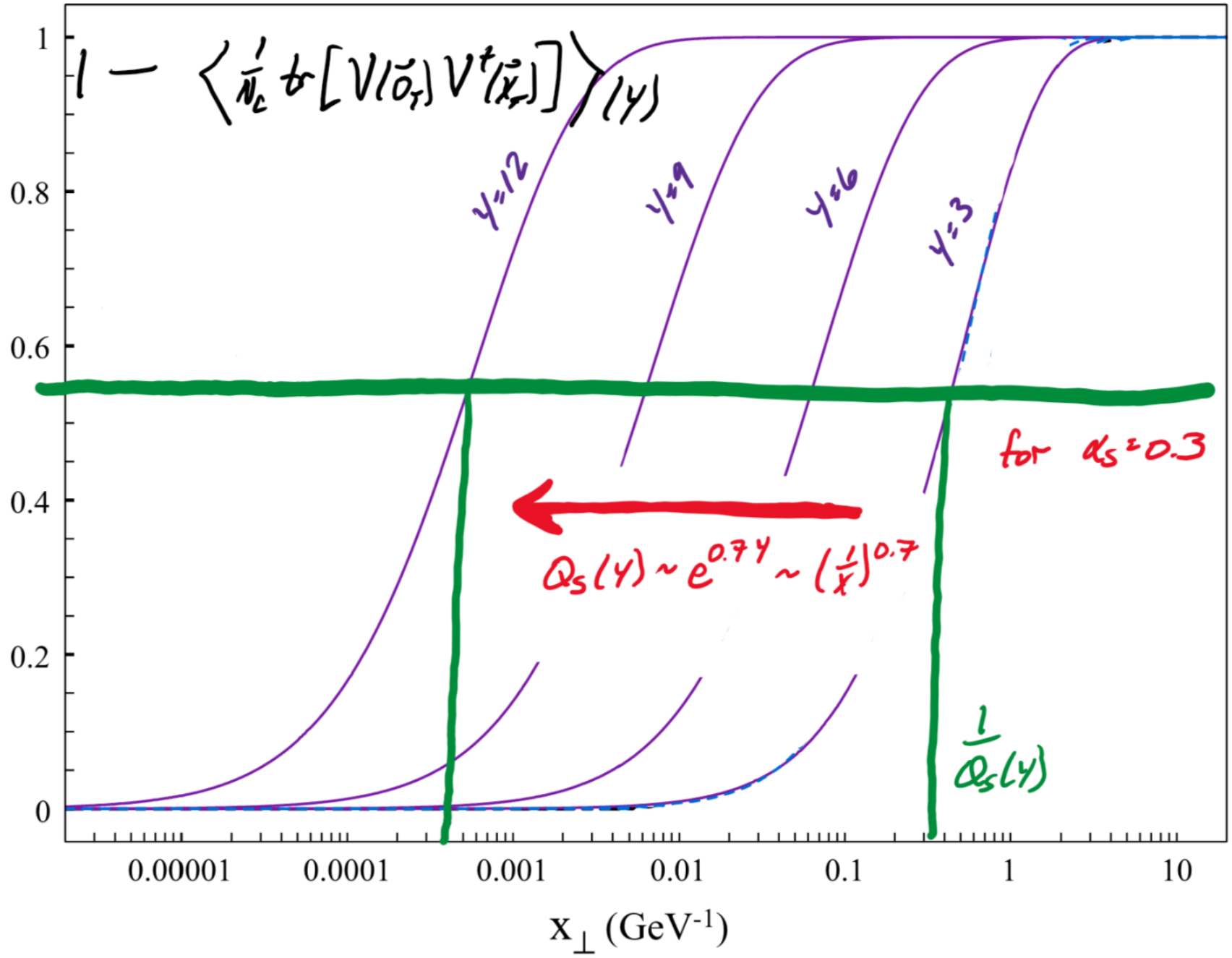
$+$

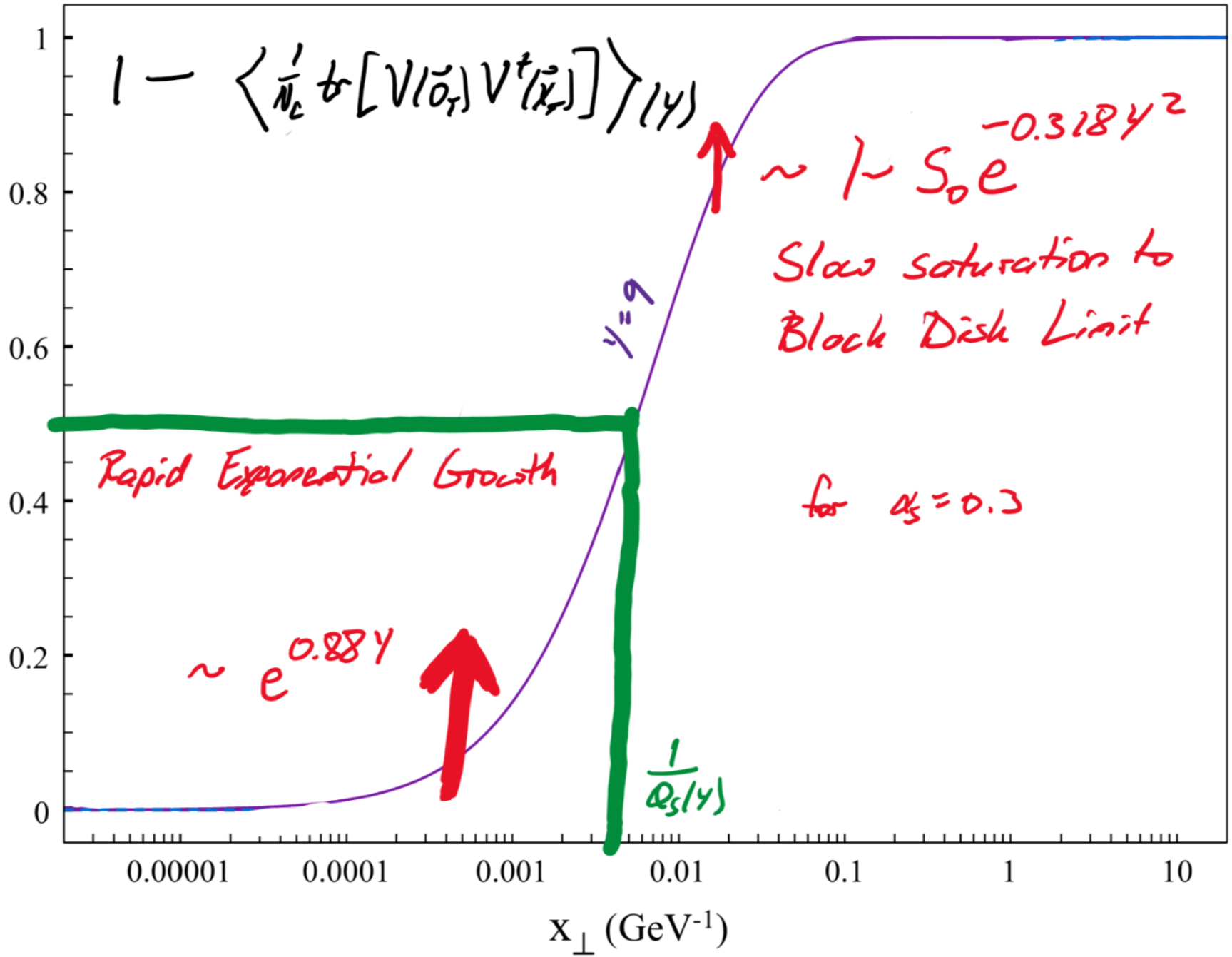


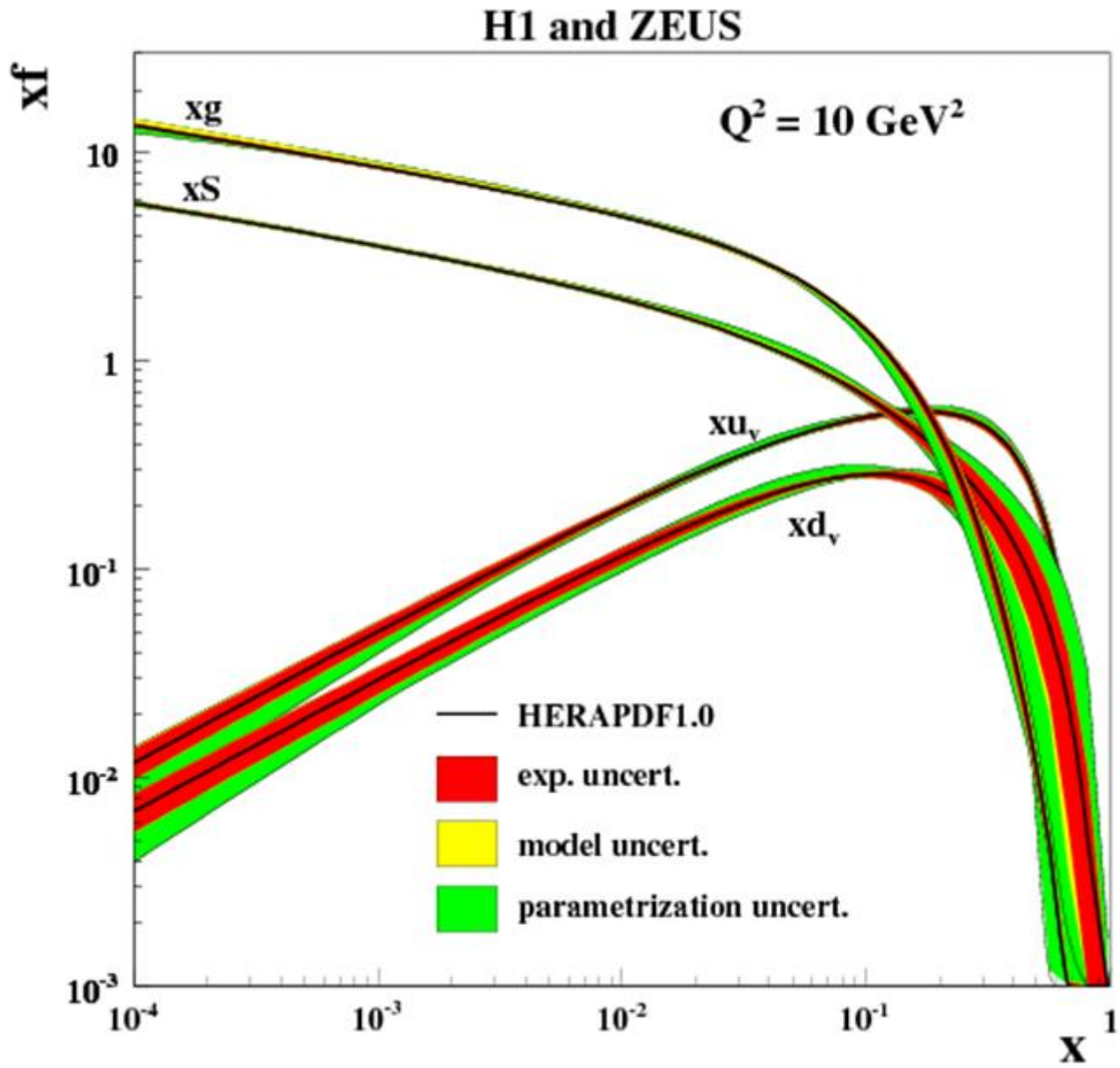
$+$ mirror images

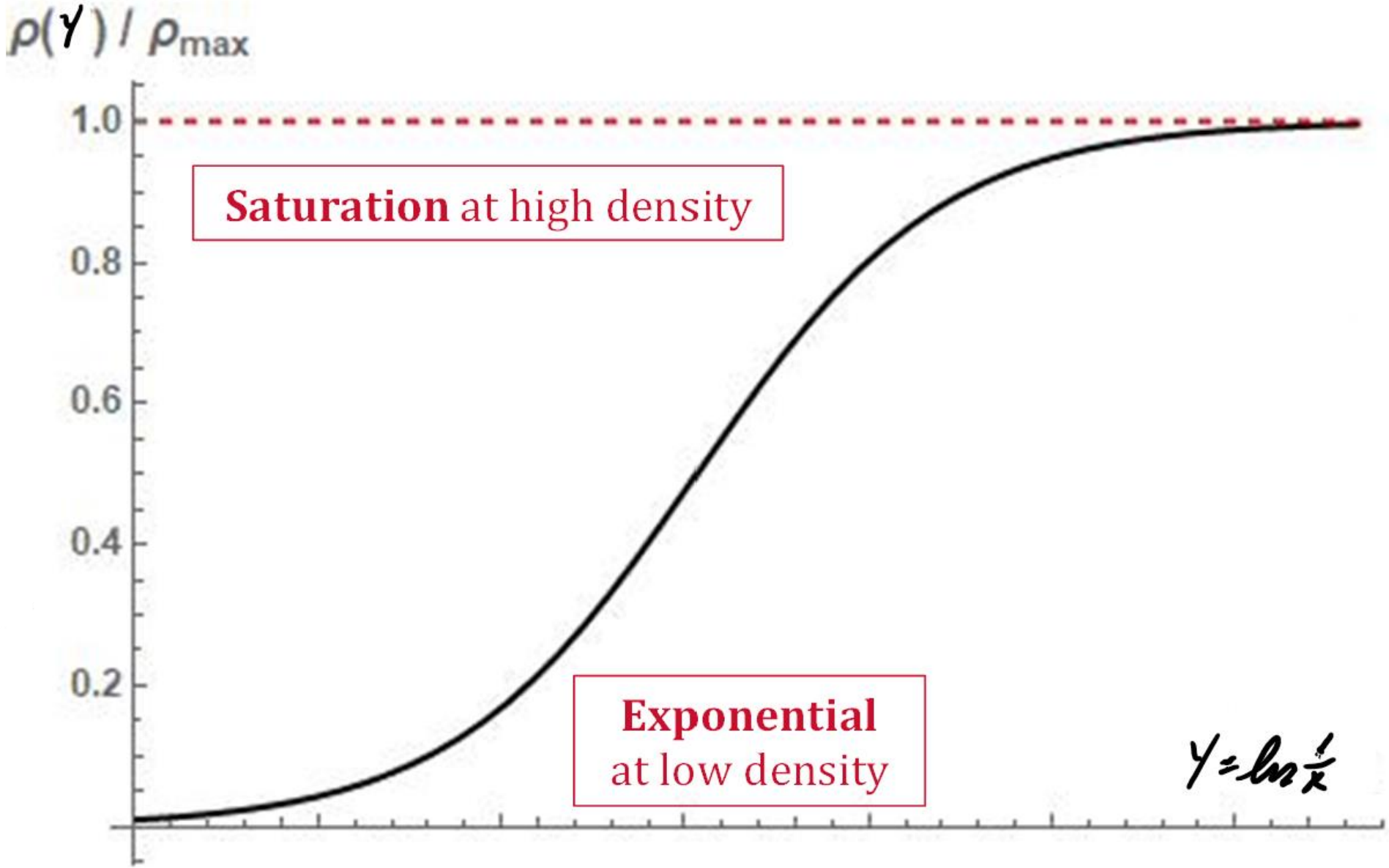

P/Pmax
1.0
0.8
0.6
0.4
0.2
Saturation at high density
Exponential
at low density
y = ln 1/x

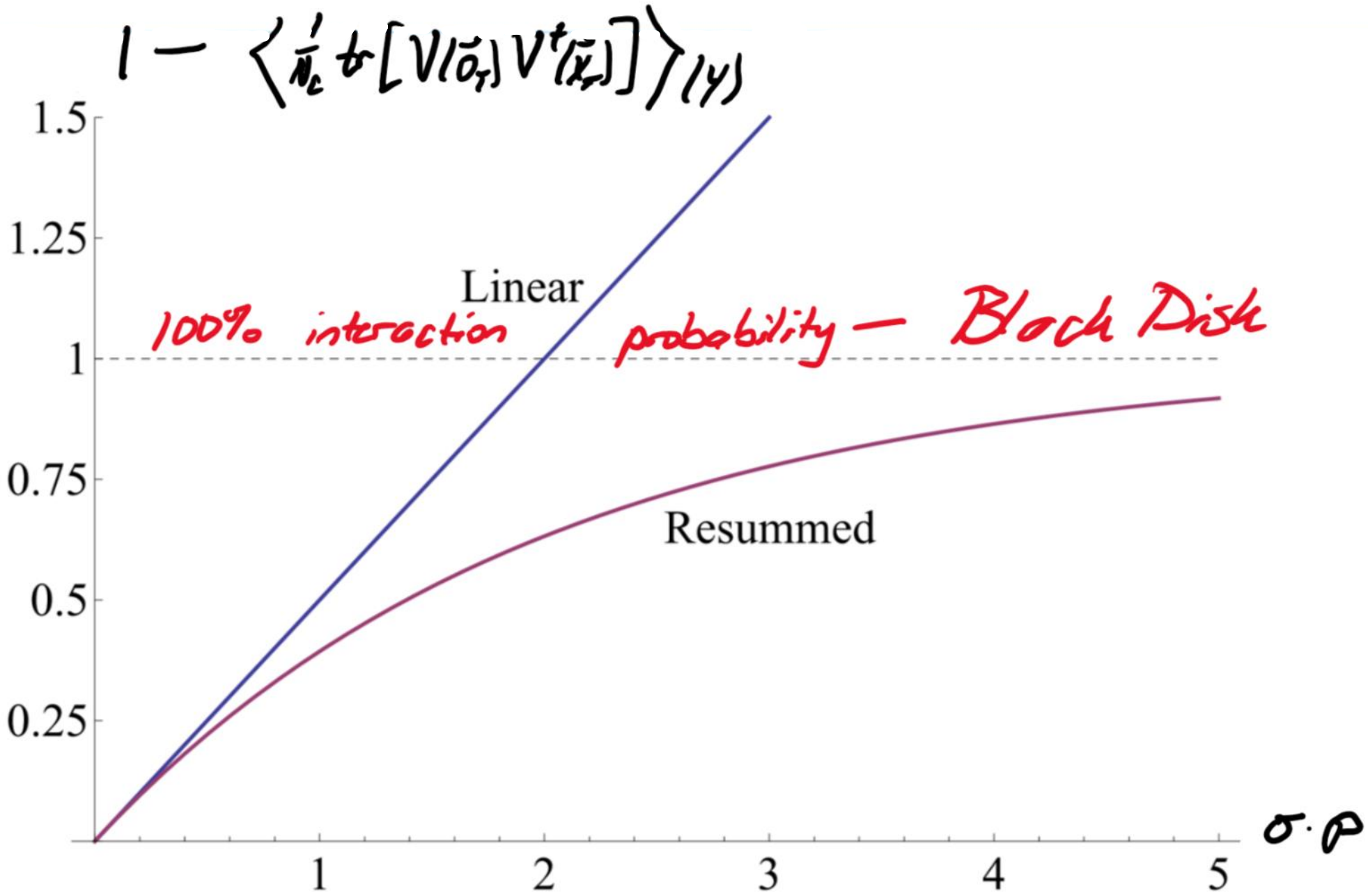

$1 - \left\langle \frac{1}{N_c} \mathrm{tr}[V(\vec{x}_\perp) V^\dagger(\vec{y}_\perp)] \right\rangle_{(Y)}$
Scattering Probability
$Y = 9$
$\frac{1}{2}$
$1/Q_s$
one arbitrary convention to define where Q_s is
1
0.8
0.6
0.4
0.2
0
0.00001
0.0001
0.001
0.01
0.1
1
10
$x_\perp$ (GeV^{-1})

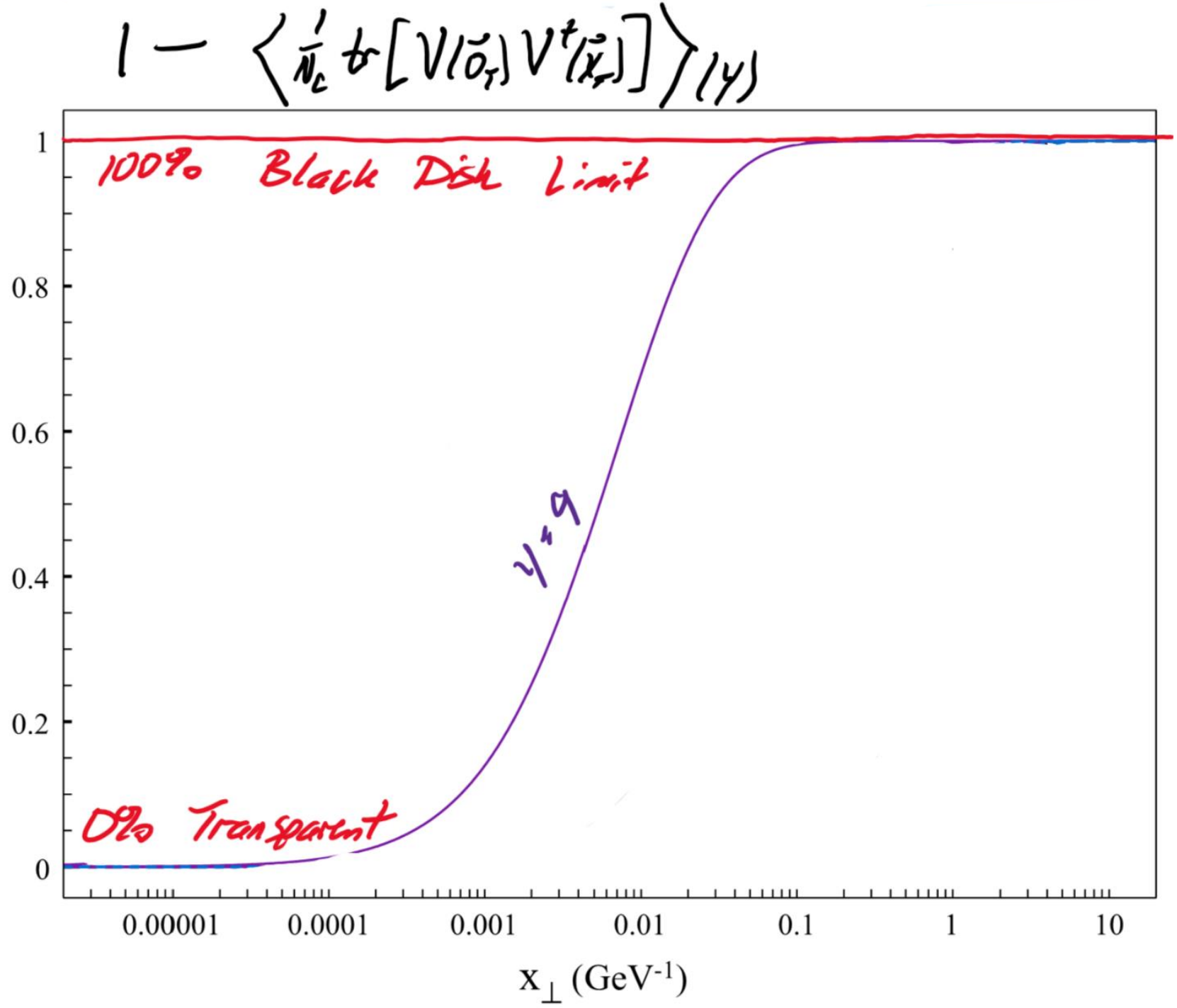

$1 - \left\langle \frac{1}{N_c} \mathrm{tr}[V(\vec{x}_\perp) V^\dagger(\vec{y}_\perp)] \right\rangle_{(Y)}$
Y=12
Y=9
Y=6
Y=3
for $\alpha_s = 0.3$
$Q_s(Y) \sim e^{0.7Y} \sim \left(\frac{1}{x}\right)^{0.7}$
$\frac{1}{Q_s(Y)}$
1
0.8
0.6
0.4
0.2
0
0.00001
0.0001
0.001
0.01
0.1
1
10
$x_\perp$ (GeV^{-1})

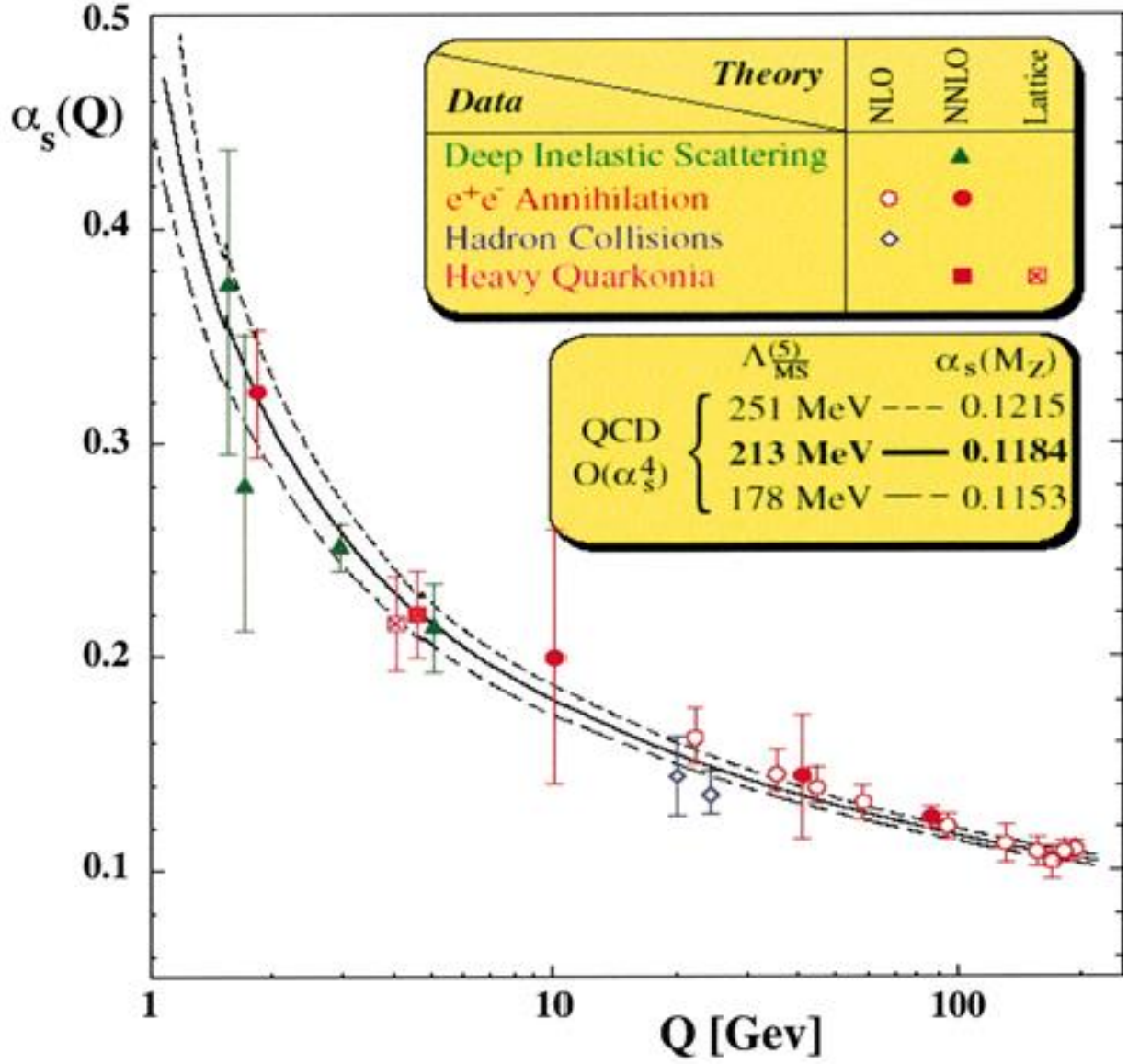

$1 - \left\langle \frac{1}{N_c} \mathrm{tr}[V(\vec{x}_\perp) V^\dagger(\vec{y}_\perp)] \right\rangle(Y)$
$\sim 1 - S_0 e^{-0.318 Y^2}$
Slow saturation to Black Disk Limit
for $\alpha_s = 0.3$
$Y = 9$
Rapid Exponential Growth
$\sim e^{0.88 Y}$
$\frac{1}{Q_s(Y)}$
1
0.8
0.6
0.4
0.2
0
0.00001
0.0001
0.001
0.01
0.1
1
10
$x_\perp$ (GeV^{-1})


H1 and ZEUS
$Q^2 = 10\ GeV^2$
xf
xg
xS
xu_v
xd_v
HERAPDF1.0
exp. uncert.
model uncert.
parametrization uncert.
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x

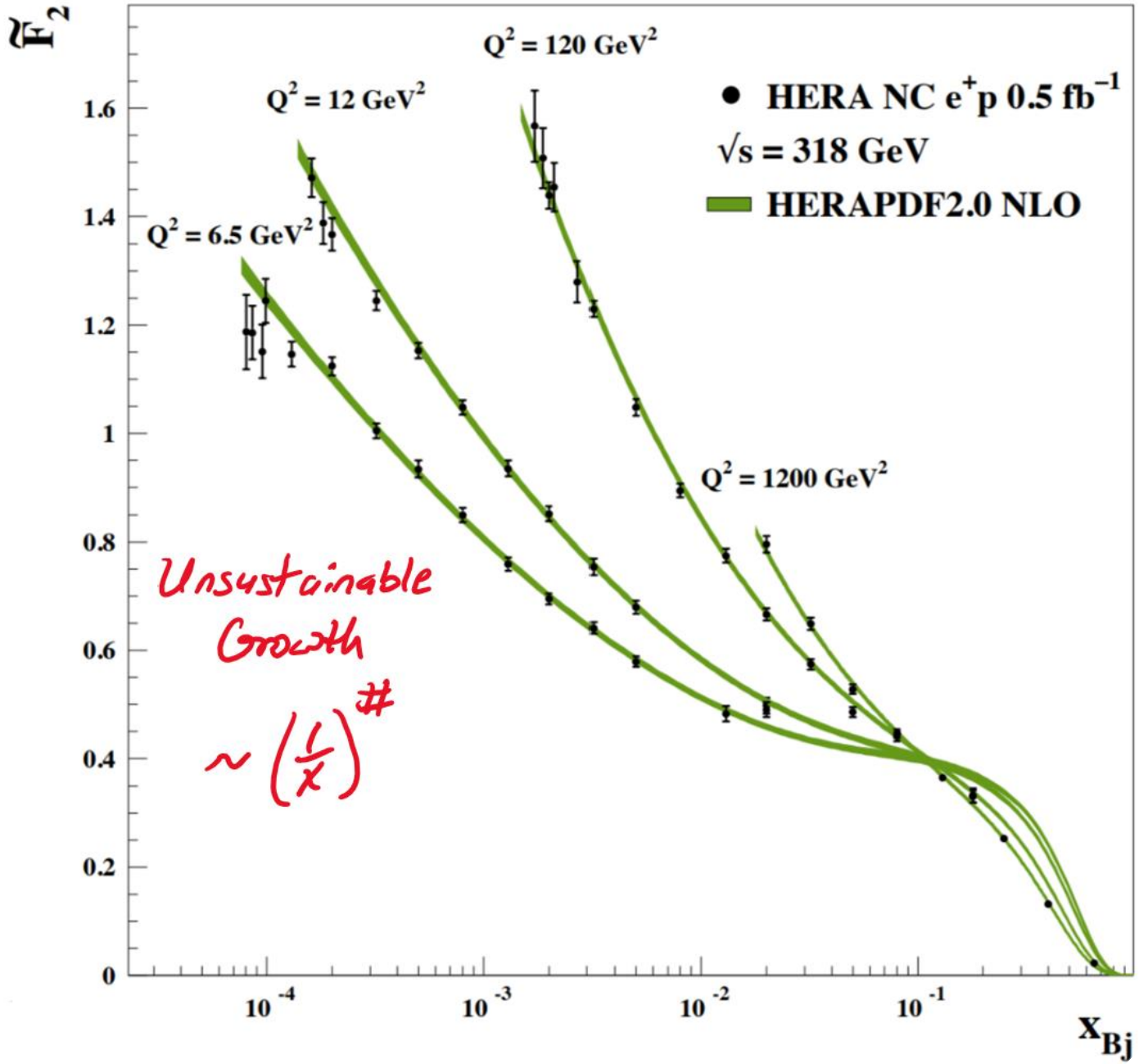

ρ(γ) / ρmax
1.0
0.8
0.6
0.4
0.2
Saturation at high density
Exponential
at low density
γ = ln 1/x


$1 - \left\langle \frac{1}{N_c} \mathrm{tr}[V(\vec{x}_1) V^\dagger(\vec{x}_2)] \right\rangle_{(Y)}$
1.5
1.25
1
0.75
0.5
0.25
Linear
100% interaction probability — Black Disk
Resummed
1
2
3
4
5
$\sigma \cdot \rho$


$1 - \left\langle \frac{1}{N_c} \mathrm{tr}[V(\vec{x}_1) V^\dagger(\vec{x}_2)] \right\rangle_{(y)}$
100% Black Disk Limit
0% Transparent
y=9
1
0.8
0.6
0.4
0.2
0
0.00001
0.0001
0.001
0.01
0.1
1
10
$x_\perp$ (GeV^{-1})


0.5
0.4
0.3
0.2
0.1
$\alpha_s(Q)$
1
10
100
Q [Gev]
Data
Theory
NLO
NNLO
Lattice
Deep Inelastic Scattering
e^+e^- Annihilation
Hadron Collisions
Heavy Quarkonia
$\Lambda^{(5)}_{\overline{MS}}$
$\alpha_s(M_Z)$
QCD
$O(\alpha_s^4)$
251 MeV --- 0.1215
213 MeV — 0.1184
178 MeV — - 0.1153

H1 and ZEUS


$\mathfrak{F}_2$
Q^2 = 120 GeV^2
Q^2 = 12 GeV^2
Q^2 = 6.5 GeV^2
Q^2 = 1200 GeV^2
HERA NC e^+p 0.5 fb^{-1}
$\sqrt{s}$ = 318 GeV
HERAPDF2.0 NLO
Unsustainable Growth
$\sim \left(\frac{1}{x}\right)^{\#}$
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
10^{-4}
10^{-3}
10^{-2}
10^{-1}
x_{Bj}