

What can I cover in 3 1-hr lectures + discussion?

I Intro to EFTs

- overview of SCET
- Fermi theory
- HQET
- SCET

General References:

Google "Iain Stewart SCET lectures"

QCD ↔ SCET comparisons / dictionaries
 CL & G. Stewart hep-ph/0611041
 + Almeida, Ellis,
 Sung & Watson 1401.4460

A. Manohar Intro to EFTs 1804.05863

II SCET Lagrangian

- label & residual momenta
- momentum bins & dim reg.

- SCET_I Lagrangian
- collinear Wilson lines
- BPS field redefinition
- soft Wilson lines

→ invention of SCET
 hep-ph/0005275

0011336
 0107001
 0109045

Deer Florig like
 B, F, Proj, Stewart
 BS
 BPS

III Factorization & Resummation using SCET

- 2-jet matching
- DIS 1-jettiness / thrust
- jet, beam & soft functions
- large logs
- RG Evolution

Discussion ?

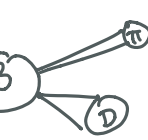
What is SCET?

↓
Soft Collinear Effective Theory

• an EFT of QCD applicable to processes with energetic, light-like degrees of freedom

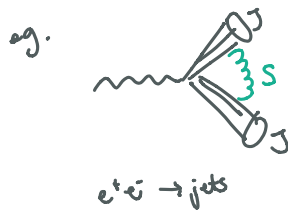
$E \gg m$
(collinear)

e.g. jets

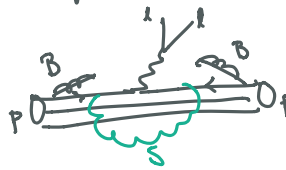


light hadrons

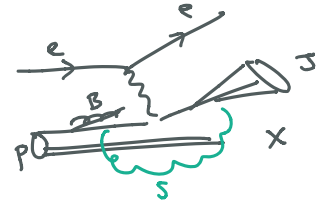
that exchange/radiate soft wide-angle radiation



$e^+e^- \rightarrow \text{jets}$



$pp \rightarrow \text{jets} X$



$ep \rightarrow e' X$

• invented 2000-2002...

mainly B physics

2000's

→ jet physics

2010's

→ heavy-ion physics
TMD
small- x
EW / dark matter

2020's

• EFT organization of many concepts (tools in QCD factorization)

- power expansion in $\lambda \sim \frac{m}{E}, \frac{q_T}{Q}$

- factorized modes for soft and collinear dots separated in virtuality or rapidity scales

- systematic organization of power corrections

- RG evolution to sum large logs

- identify universal nonperturbative effects in terms of field theory matrix elements

• can do all these in "direct" QCD but often easier in EFT

e.g. SCET \Rightarrow N³LL resummations

jet & TMD physics at subleading power

rigorous proof of universal NP corrections

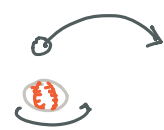
new approaches to jet substructure, NLLs, TMDs, small x , heavy ions...

So ... what is an EFT?

A1: Every theory is an EFT.

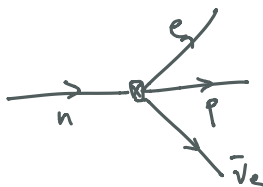
A2: Field theory containing

- dots accessible kinematically at relevant energy/distance scales
 - operators in \mathcal{L} up to power/dimension needed for desired accuracy
e.g. 1 particle Newtonian mechanics for baseball but need seams to describe curveball
 - symmetries known to be obeyed experimentally or from "full theory"
 - regularization/renormalization scheme (e.g. DR)
- EFT does not have to be renormalizable! can have UV cutoff Λ_{UV}



Example 1: Fermi theory

β decay:



$$\mathcal{L}_{EFT} = \sum_{ij} \frac{C_{ij}}{\Lambda^2} \bar{p} \Gamma_i n \bar{e} \Gamma_j \nu_e + h.c.$$

can be $\in \{1, \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5, \sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]\}$

16 x 16

Now know full theory: SM

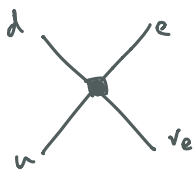


$$\mathcal{A} = \frac{g^2}{8} V_{ud} \frac{i}{k^2 - M_W^2} \bar{u} \gamma_\mu (1 - \gamma_5) d \bar{e} \gamma^\mu (1 - \gamma_5) \nu_e$$

↓ for $k \ll M_W$

$$-\frac{i}{M_W^2} \left(1 + \frac{k^2}{M_W^2} - \frac{1}{2} \left(\frac{k^2}{M_W^2} \right)^2 + \dots \right)$$

at leading order generates "effective operator"



(still need NP matrix elems. in \mathcal{P}, n)

$$\hat{\mathcal{O}} = \bar{u} \gamma_\mu (1 - \gamma_5) d \bar{e} \gamma^\mu (1 - \gamma_5) \nu_e$$

↓
"V-A"

also known experimentally in Fermi Th.

$$\mathcal{L}_{eff} = -\frac{G_F}{\sqrt{2}} V_{ud} \hat{\mathcal{O}}$$

with the SM prediction

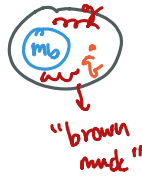
$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g^2}{2M_W^2}}$$

On this type of EFT, we "integrate out" a heavy particle(s) ⇒ also SMEFT.

On the type we shall consider, we integrate out only large momentum modes of some given particle (e.g. HQET, NRQCD, SCET)

Example 2: HQET (Heavy Quark Effective Theory)

consider B meson



$$P_Q = m_Q \gamma + k$$

↓ $k \ll m_Q$

$v = (1, \vec{0})$ in B rest frame

Goal: "integrate out" m_Q fluctuations while leaving $k \sim \Lambda_{QCD}$ fluctuations in EFT
 what do we expect in HQET?

heavy quark propagator

$$\frac{i(P_Q + m_Q)}{P_Q^2 - m_Q^2 + i\epsilon} = i \frac{m_Q \cancel{v} + k + m_Q}{m_Q^2 + 2m_Q v \cdot k + k^2 - m_Q^2 + i\epsilon}$$

small (pointing to k)

$$= i \frac{1 + \cancel{\gamma} v}{2v \cdot k + i\epsilon} \rightarrow \text{indep. of } m_Q!$$

Flavor Symmetry
 $c \leftrightarrow b$

gluon vertex

$$\not{\alpha} \not{k} \not{\alpha} \sim \frac{1 + \cancel{\gamma} v}{2} \cancel{\gamma}_\mu \frac{1 + \cancel{\gamma} v}{2} = \frac{1 + \cancel{\gamma} v}{2} \left(\frac{1 - \cancel{\gamma} v}{2} \gamma_\mu + \cancel{\gamma}_\mu \right) = \frac{1 + \cancel{\gamma} v}{2} \cancel{\gamma}_\mu \frac{1 + \cancel{\gamma} v}{2}$$

$\Rightarrow \cancel{\gamma}_\mu \rightarrow \cancel{\gamma}_\mu$ in HQET \Rightarrow Spin Symmetry
e.g. $B \leftrightarrow B^*$

heavy quark symmetry: relates e.g. $B \rightarrow D^* e \nu$ decay rates to $D^* e \nu$
 corrections suppressed by $\frac{1}{m_Q}$.

\Rightarrow even though we know the full theory (QCD), HQET makes approx. symmetries manifest and simplifies many calculations.

To construct HQET:

Step 1: heavy quark field $\psi(x) = e^{-im_Q v \cdot x} \psi_v(x)$ $\psi_v(x)$ velocity "label" : pulls out large momentum phase

Step 2: separate "large" & "small" spin components:

$$\psi_v = h_v + H_v$$

$$h_v = \frac{1 + \cancel{\gamma} v}{2} \psi_v \quad H_v = \frac{1 - \cancel{\gamma} v}{2} \psi_v$$

Step 3: plug in to QCD:

$$\mathcal{L} = \bar{\psi} (i \cancel{D} - m_Q) \psi$$

$$= \bar{\psi}_v e^{im_Q v \cdot x} (i \cancel{D} - m_Q) e^{-im_Q v \cdot x} \psi_v$$

$$= \bar{\psi}_v (i \cancel{D} + m_Q \cancel{v} - m_Q) \psi_v$$

use $(1 - \cancel{\gamma} v) h_v = 0$
 $(1 - \cancel{\gamma} v) H_v = 2H_v$

$$\begin{aligned}
 &= (\bar{h}_r + \bar{H}_r) [i \not{D} h_r + (i \not{D} - 2ma) H_r] \\
 &= \bar{h}_r i \not{D} h_r - \bar{H}_r (i \not{D} + 2ma) H_r \\
 &\quad + \bar{h}_r i \not{D}_1 H_r + \bar{H}_r i \not{D}_1 h_r
 \end{aligned}$$

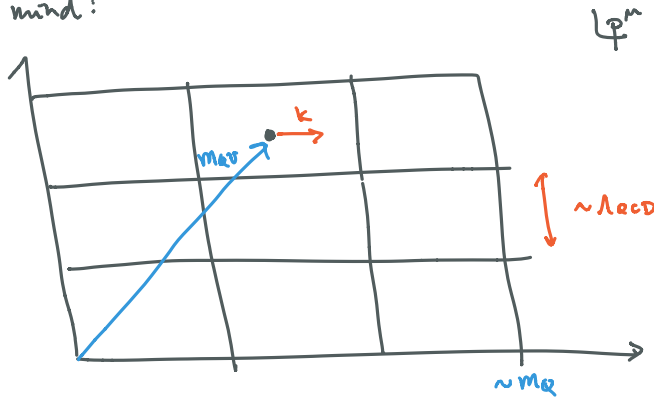
use $\frac{1+\not{v}}{2} i \not{D} \frac{1+\not{v}}{2} = \frac{1+\not{v}}{2} i \not{D}$
 where $X_1^\mu = X^\mu - X \cdot v v^\mu$

step 4: H_r is like a "heavy" particle w/ mass $2ma$
 \Rightarrow integrate out using EOM $(i \not{D} + 2ma) H_r = i \not{D}_1 h_r$
 $\Rightarrow \mathcal{L} = \bar{h}_r (i \not{D} + i \not{D}_1 \frac{1}{i \not{D} + 2ma} i \not{D}_1) h_r$
 $= \bar{h}_r (i \not{D} - \frac{1}{2ma} \not{D}_1 \not{D}_1 + \dots) h_r$
 $= \bar{h}_r (i \not{D} - \frac{\not{D}_1^2}{2ma} - g \frac{\sigma^{\mu\nu} G_{\mu\nu}}{2ma} + \dots) h_r$

\downarrow
 $\mathcal{O}(1)$
 reproduces leading order HQET Feynman rules

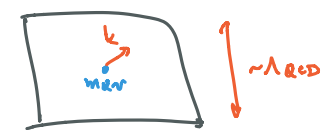
$\mathcal{O}(\frac{1}{ma})$
 breaks spin-flavor symmetry

Picture to have in mind:



Note: RPI (Reparameterization Invariance)
 HQET invariant under
 $v \rightarrow v + \frac{\epsilon}{mR}$
 $k \rightarrow k - \epsilon$

HQET lives in the box:



defined w/ a regulator:
 cutoff intuitive but bad for Lorentz/gauge invariance
 use dim reg & scale boundaries away:



HQET accurately reproduces logs in the box (IR)
 but not for large $k \gg \Lambda_{QCD}$
 and "wrong" UV structure as $k \rightarrow \infty$
 \Downarrow
 fixed by matching & renormalization
 \downarrow
 actually useful for RG to resum logs.