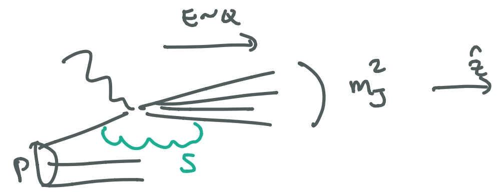
Lecture 2: SCET Lagrangian

Tues. Aug 6, 2019 (am)

What dofs do we need to describe jets? (or collimated beam radiation or energetic boosted hadrons...)



light-cone directions $n=(1,\hat{z})$ $\bar{n}=(1,-\hat{z})$ $n^2=\bar{n}^2=0$ $n\cdot\bar{n}=2$

light-cone coords: $p^\mu = \frac{\bar{n}\cdot p}{2} n^\mu + \frac{n\cdot p}{2}\bar{n}^\mu + p_\perp^\mu$

so $p^2 = \bar{n}\cdot p\, n\cdot p + p_\perp^2$

write $p^\mu = (p^-, p^+, p_\perp)$ where $p^- = \bar{n}\cdot p = E+p_z$
$p^+ = n\cdot p = E-p_z$

if m_J^2 measured to be small $\ll Q^2$ it forces kinematics:

$$p_J^2 = p^+ p^- + p_\perp^2 \sim m^2$$

$p^- \sim Q$

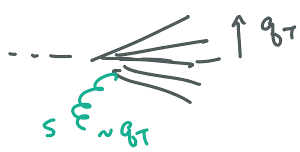
$\Rightarrow p^+ \sim \frac{m^2}{Q}$ $p_\perp \sim m$ $\rightarrow$ collinear $p_c \sim (Q, \frac{m^2}{Q}, m)$
$\sim Q(1,\lambda^2,\lambda)$ $\lambda\sim\frac{m}{Q}$

But can radiate wide-angle soft gluons w/o changing jet mass:

soft $p_s \sim Q(\lambda^2,\lambda^2,\lambda^2)$
$\sim (\frac{m^2}{Q},\frac{m^2}{Q},\frac{m^2}{Q})$ $\Rightarrow$ "$\mathrm{SCET_I}$"

In a jet mass factorization theorem, c & s only "talk" through p^+ momentum.

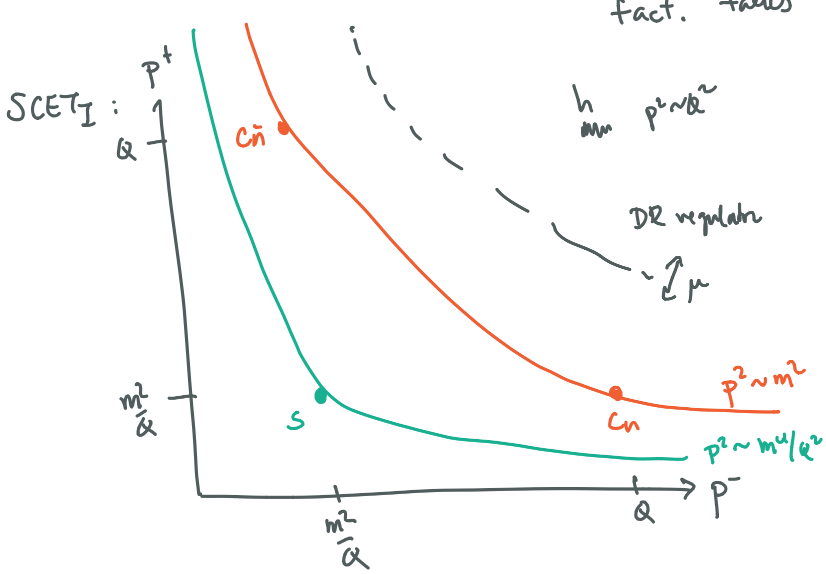
If we measure $p_\perp \sim q_T$

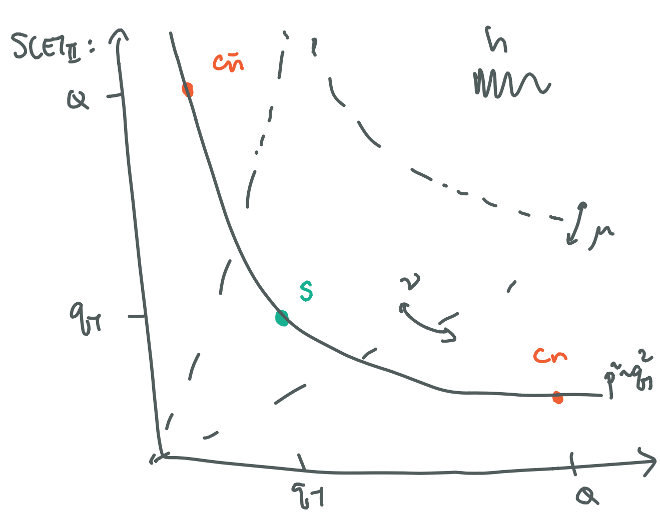


$p_c \sim (Q, \frac{q_T^2}{Q}, q_T) \sim Q(1,\lambda^2,\lambda)$ $\lambda\sim\frac{q_T}{Q}$

$p_s \sim (q_T, q_T, q_T) \sim Q(\lambda,\lambda,\lambda)$ $\Rightarrow$ "$\mathrm{SCET_{II}}$"

fact. "talks" through $\perp$ component



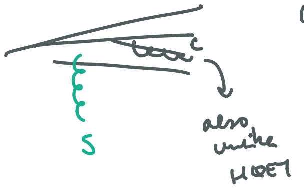


need 2 regulators DR & rapidity reg.

Momentum labels:

(SCET$_{\mathrm{I}}$ for now)

Mimic HQET:



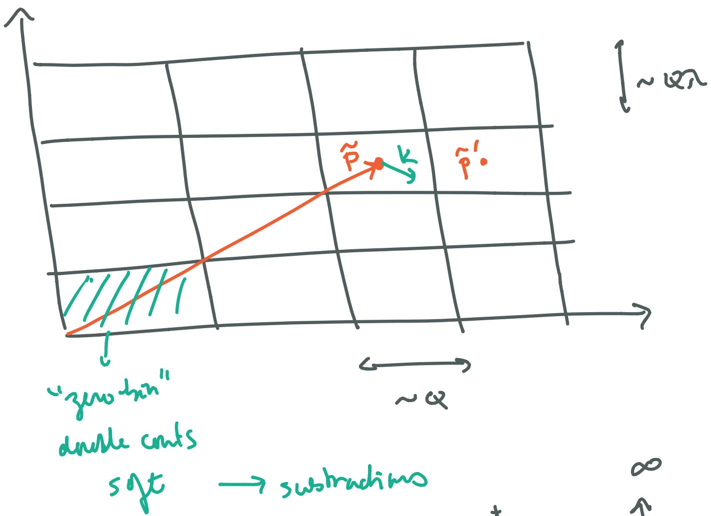
collinear momenta

$p_c = \tilde{p}_c + k$ ↳ residual $\sim Q\lambda^2$ — like HQET

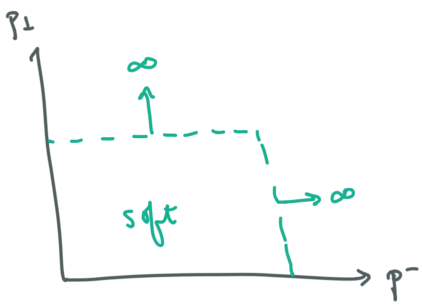
↓ "label"

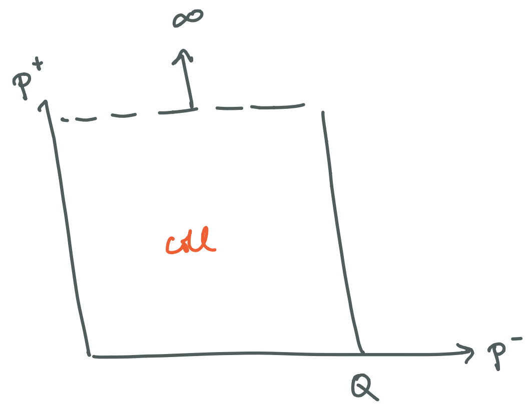
$\tilde{p}_c = \underbrace{\bar{n}\cdot\tilde{p}_c}_{\sim Q} \frac{n^\mu}{2} + \underbrace{\tilde{p}_\perp}_{\sim Q\lambda}$ ↳ unlike HQET

momentum space:



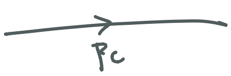
DR regulator:



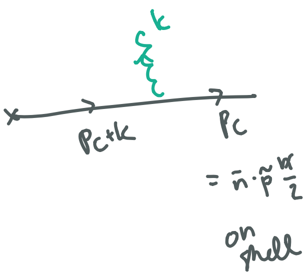


again, matching & renormalization take care of differences QCD − SCET.

Feynman rules?

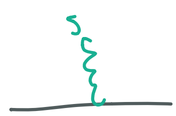


$$= \frac{i(\tilde{\not{p}}+\not{k})}{(p+k)^2+i\varepsilon} = \frac{i\,\bar{n}\cdot\tilde{p}}{\bar{n}\cdot\tilde{p}\;n\cdot k + \tilde{p}_\perp^2 + i\varepsilon}\frac{\not{n}}{2}$$



$$\sim \frac{i\,\bar{n}\cdot\tilde{p}}{\bar{n}\cdot\tilde{p}\;n\cdot k + i\varepsilon}\frac{\not{n}}{2}\gamma_\mu T^A u_n \to \frac{i}{n\cdot k + i\varepsilon}\Big(\underbrace{-\gamma_\mu\frac{\not{n}}{2}}_{\not{n}u_n=0} + n_\mu\Big)T^A u_n$$

$\Rightarrow$



$\sim \underbrace{i g n^\mu T^A}_{\text{eikonal vertex}}$

Construct SCET Lagrangian

1) $\mathcal{L}_{QCD} = \bar{\psi}\, i \not{D} \psi$

coll quarks $\quad \psi(x) = \sum_{\tilde{p}} e^{-i\tilde{p}\cdot x}\, \psi_{n,\tilde{p}}(x)$

coll + soft gluons $\quad A^\mu(x) \longrightarrow A_c^\mu(x) + A_s^\mu(x)$

$$A_c^\mu(x) = \sum_{\tilde{q}} e^{-i\tilde{q}\cdot x} A^\mu_{n,q}(x)$$

$\downarrow$

label sums required due to

unlike HQET

$$\Rightarrow \mathcal{L} = \sum_{\tilde{p},\tilde{p}'} e^{i(\tilde{p}'-\tilde{p})\cdot x}\, \bar{\psi}_{n,p'} \left(i\not{D} + \tilde{\not{p}}\right) \psi_{n,p}$$

2) project out "large" & "small" spinor components

$$P_n = \frac{\not{n}\bar{\not{n}}}{4} \qquad P_{\bar{n}} = \frac{\bar{\not{n}}\not{n}}{4}$$

$$\xi_{n,p} \equiv P_n \psi_{n,p} \qquad \Xi_{n,p} \equiv P_{\bar{n}} \psi_{n,p} \qquad \psi_{n,p} = \xi_{n,p} + \Xi_{n,p}$$

$$\Rightarrow \mathcal{L} = \sum_{\tilde{p},\tilde{p}'} e^{i(\tilde{p}'-\tilde{p})\cdot x} \left(\bar{\xi}_{n,p'} + \bar{\Xi}_{n,p'}\right)\left(i\not{D} + \tilde{\not{p}}\right)\left(\xi_{n,p} + \Xi_{n,p}\right) \qquad \text{[exercise]}$$

$$= \sum_{\tilde{p}\tilde{p}'} e^{i(\tilde{p}'-\tilde{p})\cdot x} \Big[\bar{\xi}_{n,p'} \frac{\bar{\not{n}}}{2}\, in\cdot D\, \xi_{n,p} + \bar{\Xi}_{n,p'} \frac{\not{n}}{2} \left(i\bar{n}\cdot D + \bar{n}\cdot\tilde{p}\right) \Xi_{n,p}$$
$$+ \bar{\xi}_{n,p'} \left(i\not{D}_\perp + \not{p}_\perp\right) \Xi_{n,p} + \bar{\Xi}_{n,p'} \left(i\not{D}_\perp + \tilde{\not{p}}_\perp\right) \xi_{n,p} \Big]$$

3) integrate out $\Xi_{n,p}$: like heavy field w/ "mass" $\bar{n}\cdot\tilde{p} \sim Q$

EOM $\qquad \left(i\bar{n}\cdot D + \bar{n}\cdot\tilde{p}\right) \frac{\not{n}}{2}\, \Xi_{n,p} = -\left(i\not{D}_\perp + \tilde{\not{p}}_\perp\right) \xi_{n,p}$

$\frac{\bar{\not{n}}}{2} \times \quad \ldots$

$$\Rightarrow \boxed{\Xi_{n,p} = \frac{i\not{D}_\perp + \tilde{\not{p}}_\perp}{i\bar{n}\cdot D + \bar{n}\cdot\tilde{p}} \frac{\bar{\not{n}}}{2}\, \xi_{n,p}}$$

$$\Rightarrow \mathcal{L} = \sum_{\tilde{p}\cdot\tilde{p}'} e^{i(\tilde{p}-\tilde{p}')\cdot x}\, \bar{\xi}_{n,p'} \left[in\cdot D + \left(i\not{D}_\perp + \not{p}_\perp\right) \frac{1}{i\bar{n}\cdot D + \bar{n}\cdot\tilde{p}} \left(i\not{D}_\perp + \tilde{\not{p}}_\perp\right) \right] \frac{\bar{\not{n}}}{2}\, \xi_{n,p}$$

4) expand in λ, keep leading order

note $\quad A_c^\mu \sim Q(\overset{-}{1}, \overset{+}{\lambda^2}, \overset{\perp}{\lambda}) \quad$ by gauge inv.

$\quad\quad A_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$

define a "label operator" $\quad \mathcal{P}^\mu \phi_{n,p} = \tilde{p}^\mu \phi_{n,p}$

$$\mathcal{P}^\mu \phi^\dagger_{n,p} = -\tilde{p}^\mu \phi^\dagger_{n,p}$$

and a coll. cov. der. $\quad iD_c^\mu = \mathcal{P}^\mu + gA_c^\mu$

exercise

$$\Rightarrow \quad \mathcal{L}^{(0)}_{SCET} = e^{i\mathcal{P}\cdot x}\, \bar{\xi}_{n,p'} \left[\underbrace{in\cdot D}_{\text{only place } A_s^\mu \text{ appears}} + i\not{D}_\perp^c \frac{1}{i\bar{n}\cdot D_c} i\not{D}_\perp^c \right] \frac{\not{\bar{n}}}{2} \xi_{n,p}$$

n^μ

5) further simplify using

Wilson line $\quad W_n(x) = P \exp\left(ig \int_{-\infty}^{x} ds\ \bar{n}\cdot A_n(\bar{n}s) \right)$

obeys $\quad i\bar{n}\cdot D_c W_n = 0$

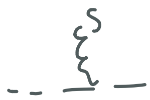
$$\Rightarrow \quad W_n^\dagger\, i\bar{n}\cdot D_c (W_n \cdot \hat{O}) = \bar{n}\cdot\mathcal{P}\, \hat{O}$$

$$\Rightarrow \quad i\bar{n}\cdot D_c = W \bar{\mathcal{P}} W^\dagger \qquad \text{operator eq.}$$

$$\Rightarrow \quad \mathcal{L} = e^{i\mathcal{P}\cdot x} \sum_{\tilde{p},\tilde{p}'} \bar{\xi}_{n,p'} \left(in\cdot D + i\not{D}_\perp^c W_n \frac{1}{\bar{n}\cdot\mathcal{P}} W_n^\dagger\, i\not{D}_\perp^c \right) \frac{\not{\bar{n}}}{2} \xi_{n,p}$$

eikonal vertex

collinear Feynman rules



s

↓

can be decoupled!

only coll-soft interaction is in $\quad \bar{\xi}_n (in\cdot D_s) \frac{\not{\bar{n}}}{2} \xi_n \qquad iD_s = i\partial^\mu + gA_s^\mu$

define soft wilson line

$$Y_n(x) = P\exp\left[ig \int_{-\infty}^{x} ds\ n\cdot A_s(ns) \right]$$

$$in\cdot D_s Y_n = 0$$

perform field redefinition $\quad \xi_n(x) = Y_n(x)\, \xi_n^{(0)}(x) \qquad A_n(x) = Y_n(x) A_n^{(0)}(x) Y_n^\dagger(x)$

$(\Rightarrow W_n = Y_n W_n^{(0)} Y_n^\dagger)$

$$\Rightarrow \quad \bar{\xi}_n \, in\cdot D_s \, \xi_n \longrightarrow \bar{\xi}_n^{(0)} Y_n^{\dagger} \, in\cdot D_s \, Y_n \frac{\bar{n}\!\!\!/}{2} \xi_n^{(0)}$$

$$= \bar{\xi}_n^{(0)} Y_n^{\dagger} Y_n \, in\cdot\partial \frac{\bar{n}\!\!\!/}{2} \xi_n^{(0)}$$

$$= \bar{\xi}_n^{(0)} \, in\cdot\partial \, \frac{\bar{n}\!\!\!/}{2} \xi_n^{(0)}$$

$\downarrow$

no soft gluons!

same in all-gluon $\mathcal{L}$

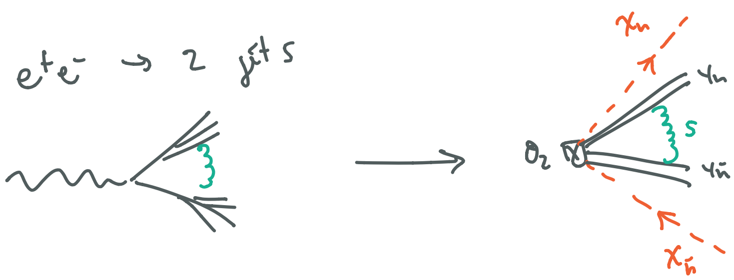
$\Rightarrow$ This means $\mathcal{L}_{SCET}$ is a bunch of decoupled copies of QCD

$$\mathcal{L}_{SCET} = \underbrace{\sum_n \mathcal{L}_{c,n}^{(0)}}_{\text{boosted QCD}} + \underbrace{\mathcal{L}_s}_{\text{full QCD}}$$

So you can compute interactions in each sector w/ full QCD Feynman rules, w/ currents tying the sectors together w/ Wilson line sources.

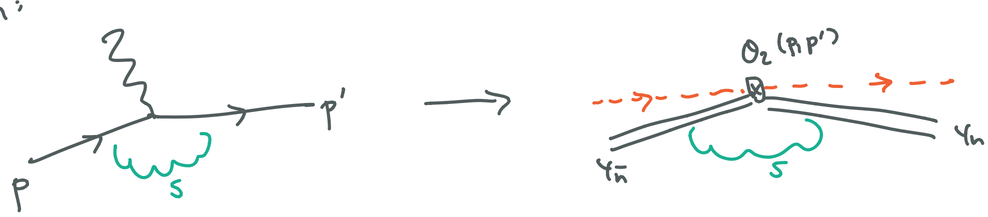
For hard scattering process usually need to perform current / operator matching also

e.g. $e^+e^- \to 2$ jets



Same operator:

DIS



also $pp \to 2$ jets