QCD carpentry: 1D structure of the nucleon

Nobuo Sato
ODU/JLab

CFNS summer school
Stony Brook, 2019

Part IV: Phenomenology

- The Bayesian framework for QCD global analysis

The parent distribution

"If we could make an infinite number of measurements, then we could describe exactly the distribution of the data points. **This is not possible in practice, but we can hypothesize the existence of such a distribution that determines the probability of getting any particular observation in a single measurement. This distribution is called parent distribution.** Similarly we can hypothesize that the measurements we have make are samples from the parent distribution and they form the sample distribution. In the limit of an infinite number of measurements, the sample distribution becomes the parent distribution"

Data reduction and error analysis for the physical sciences
Bevington and Robison

The Bayes theorem

- Consider a quantity f to be inferred from data

The Bayes theorem

- Consider a quantity f to be inferred from data

- The goal is to estimate $\mathcal{P}(f|data)$

The Bayes theorem

- Consider a quantity f to be inferred from data

- The goal is to estimate $\mathcal{P}(f|data)$

- This is achieved by the **Bayes theorem**

$$\underbrace{\mathcal{P}(f|data)}_{\text{posterior}} = \underbrace{\frac{1}{Z}}_{\text{evidence}} \underbrace{\mathcal{L}(data|f)}_{\text{likelihood}} \underbrace{\pi(f)}_{\text{prior}}$$

Likelihoods and priors

- The **likelihood** function is typically chosen to be Gaussian

$$\mathcal{L}(data|f) = \exp\left[-\frac{1}{2}\sum_i \left(\frac{d_i - \mathrm{model}_i(f)}{\delta d_i}\right)^2\right]$$

Likelihoods and priors

- The **likelihood** function is typically chosen to be Gaussian

$$\mathcal{L}(data|f) = \exp\left[-\frac{1}{2}\sum_i \left(\frac{d_i - \mathrm{model}_i(f)}{\delta d_i}\right)^2\right]$$

- The **prior** function allows to restrict forbidden values for f i.e.

$$\pi(f) = \begin{cases} 1 & \mathrm{condition}(f) == \mathrm{True} \\ 0 & \mathrm{condition}(f) == \mathrm{False} \end{cases}$$

Likelihoods and priors

- The **likelihood** function is typically chosen to be Gaussian

$$\mathcal{L}(data|f) = \exp\left[-\frac{1}{2}\sum_i \left(\frac{d_i - \mathrm{model}_i(f)}{\delta d_i}\right)^2\right]$$

- The **prior** function allows to restrict forbidden values for f i.e.

$$\pi(f) = \begin{cases} 1 & \mathrm{condition}(f) == \mathrm{True} \\ 0 & \mathrm{condition}(f) == \mathrm{False} \end{cases}$$

- $\mathcal{P}(f|data)$ depends on what is chosen for $\mathcal{L}$ and π

Parametrization

- In practice f needs to be represented parametrized e.g

$$f(x) = Nx^a(1-x)^b(1+c\sqrt{x}+dx+...)$$
$$f(x) = Nx^a(1-x)^b\mathrm{NN}(x;\{w_i\})$$
$$f(x) = \mathrm{NN}(x;\{w_i\}) - \mathrm{NN}(1;\{w_i\})$$

Parametrization

- In practice f needs to be represented parametrized e.g

$$f(x) = Nx^a(1-x)^b(1 + c\sqrt{x} + dx + \ldots)$$
$$f(x) = Nx^a(1-x)^b\mathrm{NN}(x;\{w_i\})$$
$$f(x) = \mathrm{NN}(x;\{w_i\}) - \mathrm{NN}(1;\{w_i\})$$

- The Bayes theorem is implemented as

$$\boldsymbol{a} = (N, a, b, c, d, \ldots)$$
$$\mathcal{P}(\boldsymbol{a}|d) = \frac{1}{Z}\mathcal{L}(d|\boldsymbol{a})\pi(\boldsymbol{a})$$
$$\mathcal{L}(d|\boldsymbol{a}) = \exp\left[-\frac{1}{2}\sum_i\left(\frac{d_i - \mathrm{model}_i(f(\boldsymbol{a}))}{\delta d_i}\right)^2\right]$$

Expectation values and variances

- Having the parent distribution we can compute

$$\mathrm{E}[\mathcal{O}] = \int d^n a \;\; \mathcal{P}(\boldsymbol{a}|data) \;\; \mathcal{O}(\boldsymbol{a})$$
$$\mathrm{V}[\mathcal{O}] = \int d^n a \;\; \mathcal{P}(\boldsymbol{a}|data) \;\; (\mathcal{O}(\boldsymbol{a}) - \mathrm{E}[\mathcal{O}])^2$$

- $\mathcal{O}$ is any function of $\boldsymbol{a}$. e.g

$$\mathcal{O}(\boldsymbol{a}) = f(x;\boldsymbol{a})$$
$$\mathcal{O}(\boldsymbol{a}) = \int_x^1 \frac{d\xi}{\xi} C(\xi) f\left(\frac{x}{\xi};\boldsymbol{a}\right)$$

Expectation values and variances

- Typically $n \gg 1$

Expectation values and variances

- Typically $n \gg 1$
- $\mathcal{P}(\boldsymbol{a}|data)$ is computationally expensive

Expectation values and variances

- Typically $n \gg 1$
- $\mathcal{P}(\boldsymbol{a}|data)$ is computationally expensive
- For $\mathcal{O} = f(x)$, an n–dim integration is needed for each $x \rightarrow$ Not practical!

Expectation values and variances

- Typically $n \gg 1$
- $\mathcal{P}(\boldsymbol{a}|data)$ is computationally expensive
- For $\mathcal{O} = f(x)$, an n–dim integration is needed for each $x \rightarrow$ Not practical!
- The challenge: how to compute $\mathrm{E}[\mathcal{O}], \mathrm{V}[\mathcal{O}]$?
  - **Maximum likelihood**
  - **Monte Carlo approach**

Maximum Likelihood

- Estimation of expectation value

$$\mathrm{E}[\mathcal{O}] = \int d^n a \;\; \mathcal{P}(\boldsymbol{a}|data) \;\; \mathcal{O}(\boldsymbol{a}) \simeq \mathcal{O}(\boldsymbol{a}_0)$$

Maximum Likelihood

- Estimation of expectation value

$$\mathrm{E}[\mathcal{O}] = \int d^n a \;\; \mathcal{P}(\boldsymbol{a}|data) \;\; \mathcal{O}(\boldsymbol{a}) \simeq \mathcal{O}(\boldsymbol{a}_0)$$

- $\boldsymbol{a}_0$ is estimated from optimization algorithm

$$\max\left[\mathcal{P}(\boldsymbol{a}|data)\right] = \mathcal{P}(\boldsymbol{a}_0|data)$$
$$\max\left[\mathcal{L}(data|\boldsymbol{a})\pi(\boldsymbol{a})\right] = \mathcal{L}(data|\boldsymbol{a}_0)\pi(\boldsymbol{a}_0)$$

Maximum Likelihood

- Estimation of expectation value

$$\mathrm{E}[\mathcal{O}] = \int d^n a \;\; \mathcal{P}(\boldsymbol{a}|data) \;\; \mathcal{O}(\boldsymbol{a}) \simeq \mathcal{O}(\boldsymbol{a}_0)$$

- $\boldsymbol{a}_0$ is estimated from optimization algorithm

$$\max\left[\mathcal{P}(\boldsymbol{a}|data)\right] = \mathcal{P}(\boldsymbol{a}_0|data)$$
$$\max\left[\mathcal{L}(data|\boldsymbol{a})\pi(\boldsymbol{a})\right] = \mathcal{L}(data|\boldsymbol{a}_0)\pi(\boldsymbol{a}_0)$$

- For Gaussian likelihood it is χ^2 minimization

$$\min\left[-2\log\left(\mathcal{L}(data|\boldsymbol{a})\pi(\boldsymbol{a})\right)\right] = -2\log\left(\mathcal{L}(data|\boldsymbol{a}_0)\pi(\boldsymbol{a}_0)\right)$$
$$= \chi^2(\boldsymbol{a}_0) - 2\log\left(\pi(\boldsymbol{a}_0)\right)$$

Hessian method : eigen direction decomposition

$$
\begin{aligned}
\mathcal{P}(\boldsymbol{a}|data) &\propto \exp\left(-\frac{1}{2}\chi^2(\boldsymbol{a})\right) \propto \exp\left(-\frac{1}{2}\chi^2(\boldsymbol{a}_0) - \frac{1}{2}\Delta\chi^2(\boldsymbol{a}))\right) \\
&\propto \exp\left(-\frac{1}{2}\Delta\chi^2(\boldsymbol{a}))\right) \\
&\propto \exp\left(-\frac{1}{2}\Delta\boldsymbol{a}^T\, H\, \Delta\boldsymbol{a}\right) + O(\Delta a^3) \\
&\propto \exp\left(-\frac{1}{2}\sum_k\left(t_k\frac{\hat{\boldsymbol{e}}_k^T}{\sqrt{w_k}}\right) H \sum_l\left(t_l\frac{\hat{\boldsymbol{e}}_l}{\sqrt{w_l}}\right)\right) + O(\Delta a^3) \\
&\propto \exp\left(-\frac{1}{2}\sum_k t_k^2\right) + O(\Delta a^3) \\
&\propto \prod_k \exp\left(-\frac{1}{2}t_k^2\right) + O(\Delta a^3)
\end{aligned}
$$

The posterior distribution "factorizes" along each eigen direction

Maximum Likelihood + Hessian method

- Estimation of variance

$$\mathrm{V}[\mathcal{O}] = \int d^n a \;\; \mathcal{P}(\boldsymbol{a}|data) \;\; \left(\mathcal{O}(\boldsymbol{a}) - \mathrm{E}[\mathcal{O}]\right)^2$$

$$\simeq \prod_k \int dt_k \frac{e^{-\frac{1}{2}t_k^2}}{\sqrt{2\pi}} \sum_{lm} \frac{\partial \mathcal{O}}{\partial t_l} \frac{\partial \mathcal{O}}{\partial t_m} t_l t_m$$

$$= \sum_k \left(\frac{\partial \mathcal{O}}{\partial t_k}\right)^2 \simeq \sum_k \left[\frac{\mathcal{O}(t_k = 1) - \mathcal{O}(t_k = -1)}{2}\right]^2$$

Maximum Likelihood + Hessian method

- Estimation of variance

$$\mathrm{V}[\mathcal{O}] = \int d^n a \;\; \mathcal{P}(\boldsymbol{a}|data) \;\; (\mathcal{O}(\boldsymbol{a}) - \mathrm{E}[\mathcal{O}])^2$$

$$\simeq \prod_k \int dt_k \frac{e^{-\frac{1}{2}t_k^2}}{\sqrt{2\pi}} \sum_{lm} \frac{\partial \mathcal{O}}{\partial t_l} \frac{\partial \mathcal{O}}{\partial t_m} t_l t_m$$

$$= \sum_k \left(\frac{\partial \mathcal{O}}{\partial t_k}\right)^2 \simeq \sum_k \left[\frac{\mathcal{O}(t_k = 1) - \mathcal{O}(t_k = -1)}{2}\right]^2$$

- It relies on
  - linear approximation for $\mathcal{O}(\boldsymbol{a})$
  - Gaussian factorization of the posterior

The tolerance criterion

- In QCD global analysis it is common to find discrepancies among datasets

The tolerance criterion

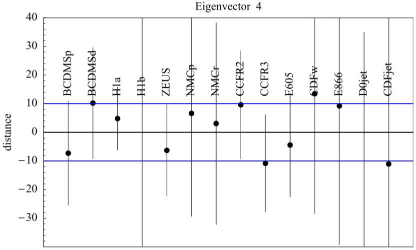
- In QCD global analysis it is common to find discrepancies among datasets
- The variance is then scaled by a tolerance factor T

$$\mathrm{V}[\mathcal{O}] \simeq T^2 \sum_k \left[\frac{\mathcal{O}(t_k = 1) - \mathcal{O}(t_k = -1)}{2} \right]^2$$

The tolerance criterion

- In QCD global analysis it is common to find discrepancies among datasets
- The variance is then scaled by a tolerance factor T

$$\mathrm{V}[\mathcal{O}] \simeq T^2 \sum_k \left[\frac{\mathcal{O}(t_k = 1) - \mathcal{O}(t_k = -1)}{2} \right]^2$$



$T \simeq 10$

Why do we need T?

Why do we need T?

- Consider observable m and measurements $(m_1, \delta m_1), (m_2, \delta m_2)$

Why do we need T?

- Consider observable m and measurements $(m_1, \delta m_1), (m_2, \delta m_2)$
- The χ^2 function is given by

$$\chi^2(m) = \left(\frac{m - m_1}{\delta m_1}\right)^2 + \left(\frac{m - m_2}{\delta m_2}\right)^2$$

Why do we need T?

- Consider observable m and measurements $(m_1, \delta m_1), (m_2, \delta m_2)$
- The χ^2 function is given by

$$\chi^2(m) = \left(\frac{m - m_1}{\delta m_1}\right)^2 + \left(\frac{m - m_2}{\delta m_2}\right)^2$$

- The maximum likelihood and Hessian gives

$$\mathrm{E}[m] = \frac{m_1 \delta m_2^2 + m_2 \delta m_1^2}{\delta m_1^2 + \delta m_2^2} \qquad \mathrm{V}[m] = \frac{\delta m_2^2 \delta m_1^2}{\delta m_2^2 + \delta m_1^2}$$

Why do we need T?

- Consider observable m and measurements $(m_1, \delta m_1), (m_2, \delta m_2)$
- The χ^2 function is given by

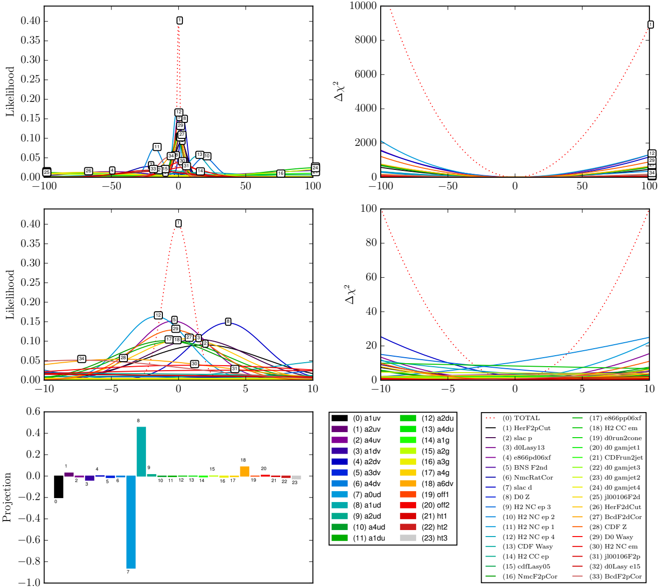
$$\chi^2(m) = \left(\frac{m - m_1}{\delta m_1}\right)^2 + \left(\frac{m - m_2}{\delta m_2}\right)^2$$

- The maximum likelihood and Hessian gives

$$\mathrm{E}[m] = \frac{m_1 \delta m_2^2 + m_2 \delta m_1^2}{\delta m_1^2 + \delta m_2^2} \qquad \mathrm{V}[m] = \frac{\delta m_2^2 \delta m_1^2}{\delta m_2^2 + \delta m_1^2}$$

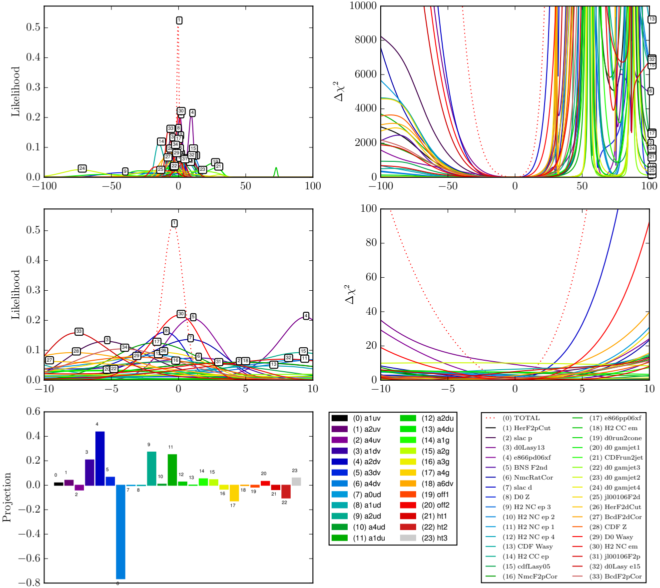
- $\mathrm{V}[m]$ is independent of $|m_1 - m_2|$

Real life global analysis of PDFs CJ15



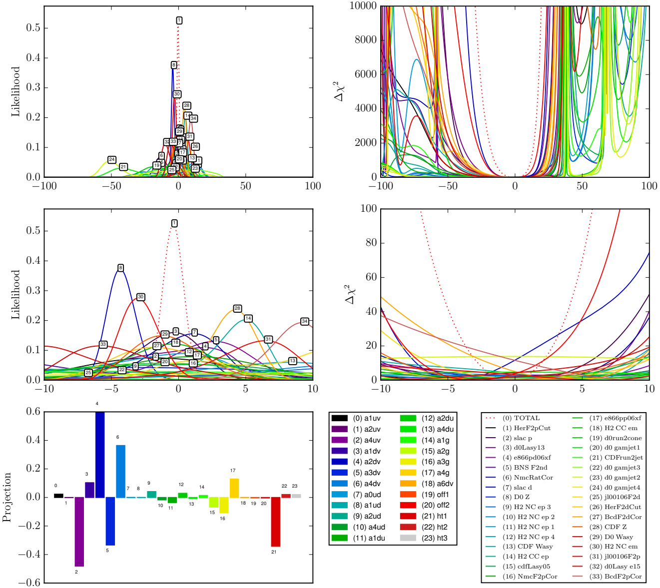
- eigen direction 1
- likelihood behaves as gaussian
- one can see which parameters and datasets are relevant

Real life global analysis of PDFs CJ15



- eigen direction 13
- likelihood is less gaussian
- some datasets are in tension

Real life global analysis of PDFs CJ15



- eigen direction 16
- likelihood is less gaussian
- some datasets are in tension

Maximum Likelihood + Hessian method

- **pros**
  - Very practical. Most the PDF groups use this method
  - It is computationally inexpensive
  - f and its eigen directions can be precalculated/tabulated

Maximum Likelihood + Hessian method

- **pros**
  - Very practical. Most the PDF groups use this method
  - It is computationally inexpensive
  - f and its eigen directions can be precalculated/tabulated
- **cons**
  - Assumes local gaussian approximation of the likelihood
  - Assumes linear approximation of the observables $\mathcal{O}$ around $\boldsymbol{a}_0$
  - These assumptions are strictly valid for linear models.
  - Hessian matrix is numerically unstable if flat directions are present
  - To deal with incompatible data one needs to apply the tolerance

Monte Carlo Methods

- Recall that we are interested in computing

$$\mathrm{E}[\mathcal{O}] = \int d^n a \;\; \mathcal{P}(\boldsymbol{a}|data) \;\; \mathcal{O}(\boldsymbol{a})$$

$$\mathrm{V}[\mathcal{O}] = \int d^n a \;\; \mathcal{P}(\boldsymbol{a}|data) \;\; (\mathcal{O}(\boldsymbol{a}) - \mathrm{E}[\mathcal{O}])^2$$

Monte Carlo Methods

- Recall that we are interested in computing

$$\mathrm{E}[\mathcal{O}] = \int d^n a \;\; \mathcal{P}(\boldsymbol{a}|data) \;\; \mathcal{O}(\boldsymbol{a})$$
$$\mathrm{V}[\mathcal{O}] = \int d^n a \;\; \mathcal{P}(\boldsymbol{a}|data) \;\; (\mathcal{O}(\boldsymbol{a}) - \mathrm{E}[\mathcal{O}])^2$$

- MC methods attempts to do this using MC sampling

$$\mathrm{E}[\mathcal{O}] \simeq \sum_k w_k \mathcal{O}(\boldsymbol{a}_k)$$
$$\mathrm{V}[\mathcal{O}] \simeq \sum_k w_k (\mathcal{O}(\boldsymbol{a}_k) - \mathrm{E}[\mathcal{O}])^2$$

Monte Carlo Methods

- Recall that we are interested in computing

$$\mathrm{E}[\mathcal{O}] = \int d^n a \;\; \mathcal{P}(\boldsymbol{a}|data) \;\; \mathcal{O}(\boldsymbol{a})$$
$$\mathrm{V}[\mathcal{O}] = \int d^n a \;\; \mathcal{P}(\boldsymbol{a}|data) \;\; (\mathcal{O}(\boldsymbol{a}) - \mathrm{E}[\mathcal{O}])^2$$

- MC methods attempts to do this using MC sampling

$$\mathrm{E}[\mathcal{O}] \simeq \sum_k w_k \mathcal{O}(\boldsymbol{a}_k)$$
$$\mathrm{V}[\mathcal{O}] \simeq \sum_k w_k (\mathcal{O}(\boldsymbol{a}_k) - \mathrm{E}[\mathcal{O}])^2$$

- $\{w_k, \boldsymbol{a}_k\}$ is the **sample distribution** of the **posterior distribution** $\mathcal{P}(\boldsymbol{a}|data)$

MC Method 1: **data resampling**

- Distorted data sets with gaussian noise

$$d_{k,i}^{(\mathrm{pseudo})} = d_i^{(\mathrm{exp})} + \sigma_i^{(\mathrm{exp})} R_{k,i}$$

i : i–th data point

k : k–th pseudo data set index

$R_{k,i}$: random number from normal distribution

MC Method 1: **data resampling**

- Distorted data sets with gaussian noise

$$d_{k,i}^{(\mathrm{pseudo})} = d_i^{(\mathrm{exp})} + \sigma_i^{(\mathrm{exp})} R_{k,i}$$

i : i–th data point

k : k–th pseudo data set index

$R_{k,i}$: random number from normal distribution

- Fit each pseudo data $k = 1, .., N$ to obtain parameter vectors $\boldsymbol{a}_k$
The **sample distribution** of $\mathcal{P}(\boldsymbol{a}|data)$ is approximately

$$\{w_k = 1/N, \boldsymbol{a}_k\}$$

MC Method 2: Hybrid Markov Chain Monte Carlo

- **The basic idea**

→ This is an MCMC based algorithm (random walks + rejection sampling)

→ The random walks are optimized by solving Hamilton's equations.

→ The parameters $\boldsymbol{a}$ are the "coordinates" and a conjugate vector $\boldsymbol{p}$ e.g. "momentum" is defined

→ An initial "state" is defined by a random coordinate vector $\boldsymbol{a}_0$ and a random momentum vector $\boldsymbol{p}_0$.

→ A new state is proposed by solving a Hamiltonian using the leap frog method

$$H(\boldsymbol{p}, \boldsymbol{a}) = \frac{\boldsymbol{p}^2}{2m} - \log(\mathcal{L}(\boldsymbol{a}))$$

- **pros**

→ It provides a faithful **sampling distribution**

- **cons**

→ the number of steps and step size of the leap frog must be tuned.

→ Cannot be parallelized

MC Method 3: nested resampling

- **The basic idea**: compute

$$Z = \int \mathcal{L}(\mathrm{data}|\boldsymbol{a})\pi(\boldsymbol{a})d^n a = \int_0^1 \mathcal{L}(X)dX$$

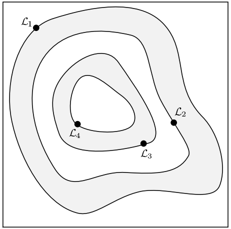
→ The algorithm traverses ordered isolikelihood contours in the variable X such that X follows the progression $X_i = t_i X_{i-1}$

→ The variable t_i is estimated statistically

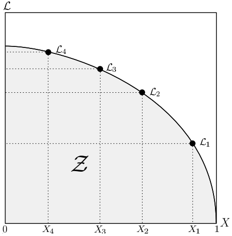
→ The algorithm can be optimized iteration to iteration. One can sample only in the regions where the likelihood is larger → "importance sampling"

→ The nested sampling is parallelizable

$\mathcal{L}(\mathrm{data}|\boldsymbol{a})$ in $\boldsymbol{a}$ space



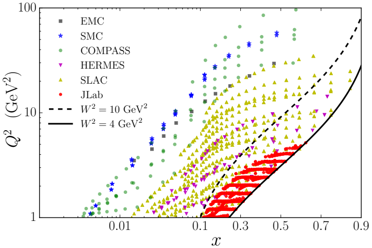
$\mathcal{L}(X)$ in X space

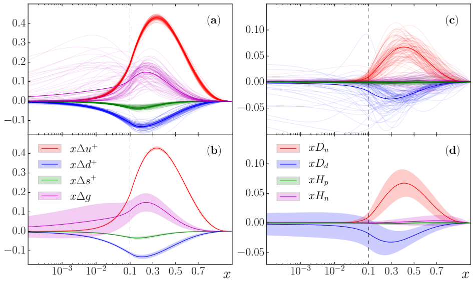


JAM Jefferson Lab Angular Momentum Collaboration

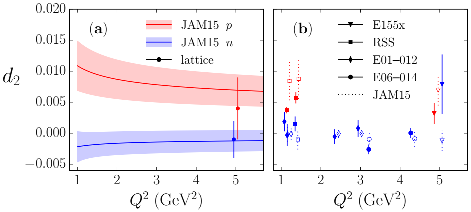
Polarized PDFs: inclusive polarized DIS

NS, Melnitchouk, Kuhn, Ethier, Accardi (PRD 93,074005)



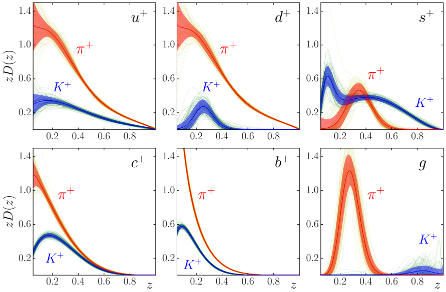


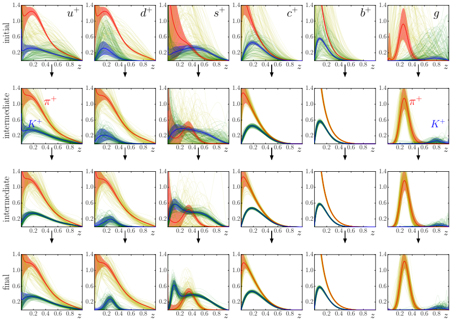
- → Inclusion of all the JLab 6GeV data
- → Determination of twist 3 g_2 (not power suppresed)
- → Extraction of d_2 matrix element



Fragmentation Functions: SIA

NS, Ethier, Melnitchouk, Hirai, Kumano, Accardi (PRD 94, 114004)





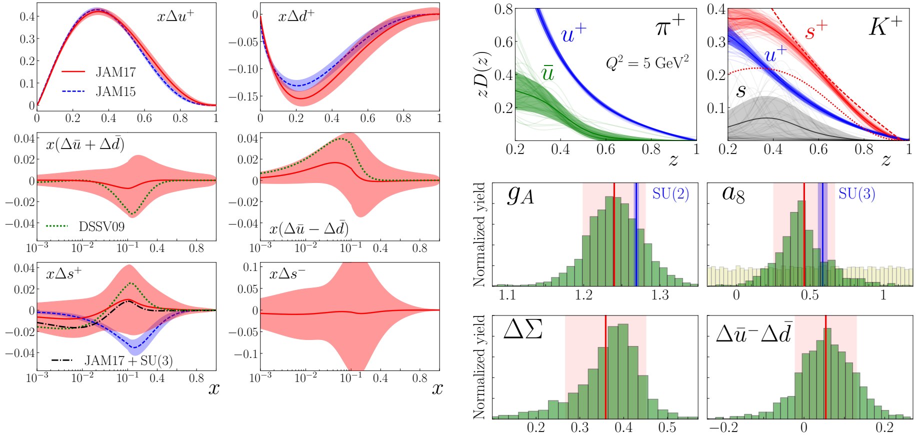
→ Inclusion of all the global data from Belle and Babar up to LEP data at $Q = M_z$

→ Fits were done for pion and kaon samples

→ We only extracted $D_q^+ = D_q + D_{\bar{q}}$

Combined ΔPDF and FF: pDIS+pSIDIS+SIA

Ethier, NS, Melnitchouk (PRL 119, 132001)

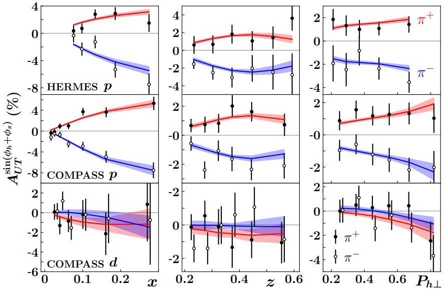


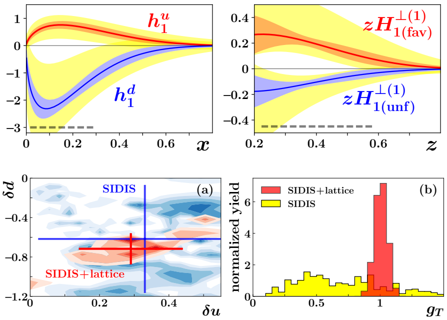
→ First simultaneous extraction of polarized PDFs and FFs

→ Extraction of the polarized strange distribution without SU(3) constraints

SIDIS+Lattice analysis of nucleon tensor charge

Lin, Melnitchouk, Prokudin, NS, Shows (PRLett 120, 152502)

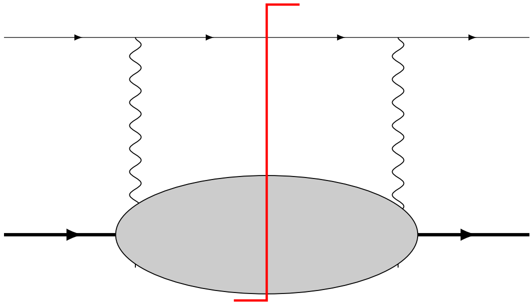




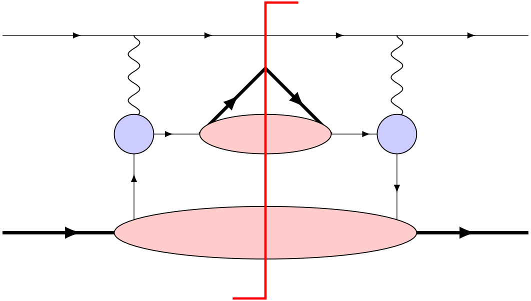
→ Extraction of transversity and Collins FFs from SIDIS A_{UT}+Lattice g_T

→ In the absence of Lattice, SIDIS at present has no significant constraints on g_T → this will change with the upcoming JLab12 measurements

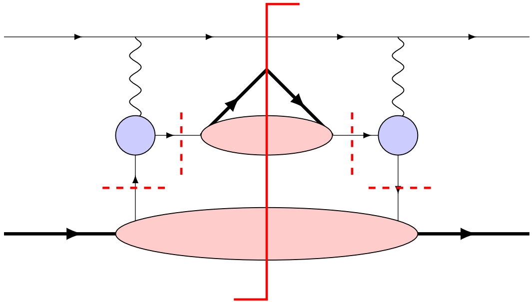
Factorization in SIDIS $e + p \to e' + h + X$



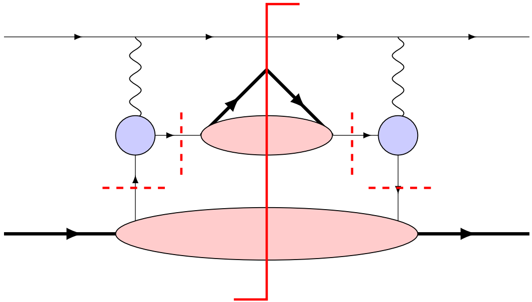
Factorization in SIDIS $e + p \to e' + h + X$



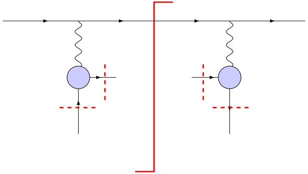
Factorization in SIDIS $e + p \to e' + h + X$



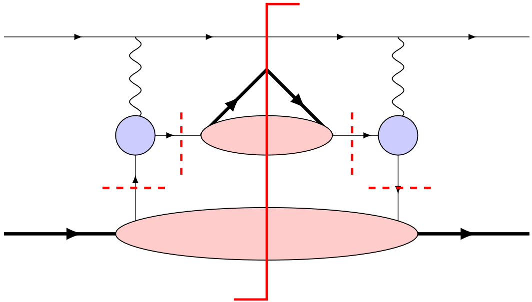
Factorization in SIDIS $e + p \to e' + h + X$



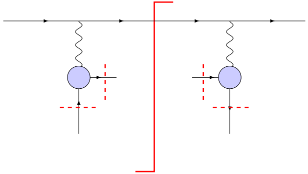
partonic cross section

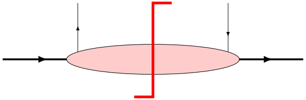


Factorization in SIDIS $e + p \to e' + h + X$

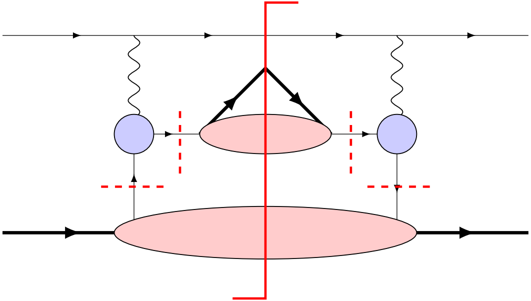


partonic cross section **parton distribution functions**

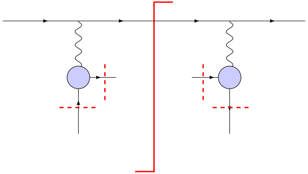




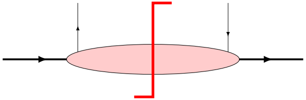
Factorization in SIDIS $e + p \to e' + h + X$



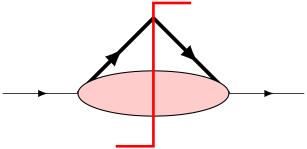
partonic cross section



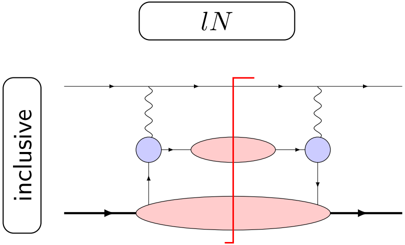
parton distribution functions



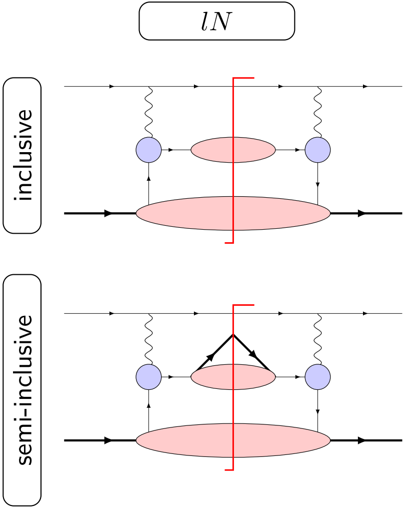
parton to hadron fragmentation function



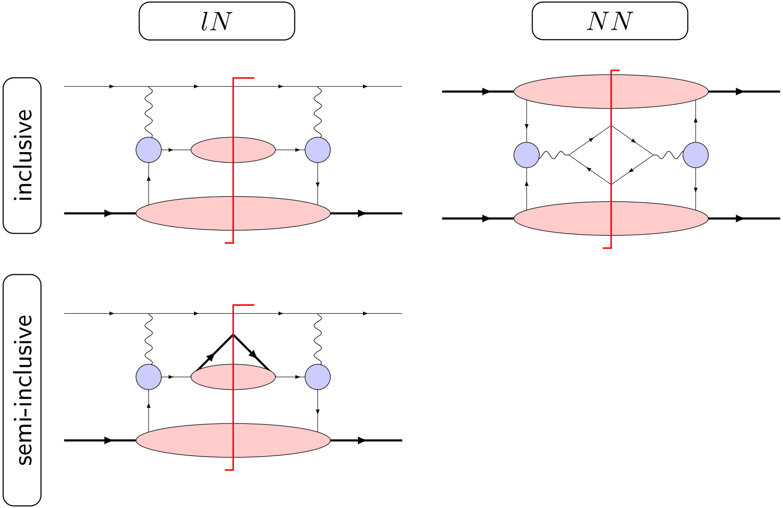
Universality of factorization



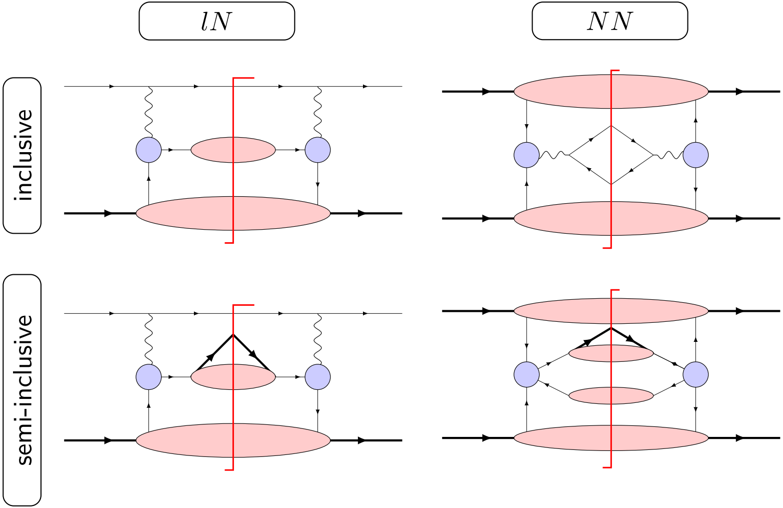
Universality of factorization



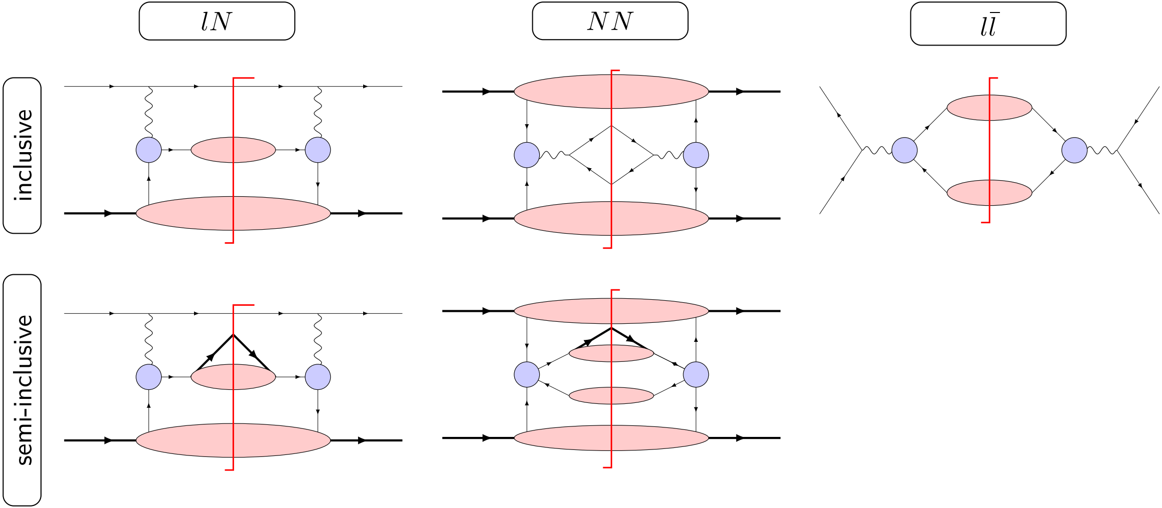
Universality of factorization



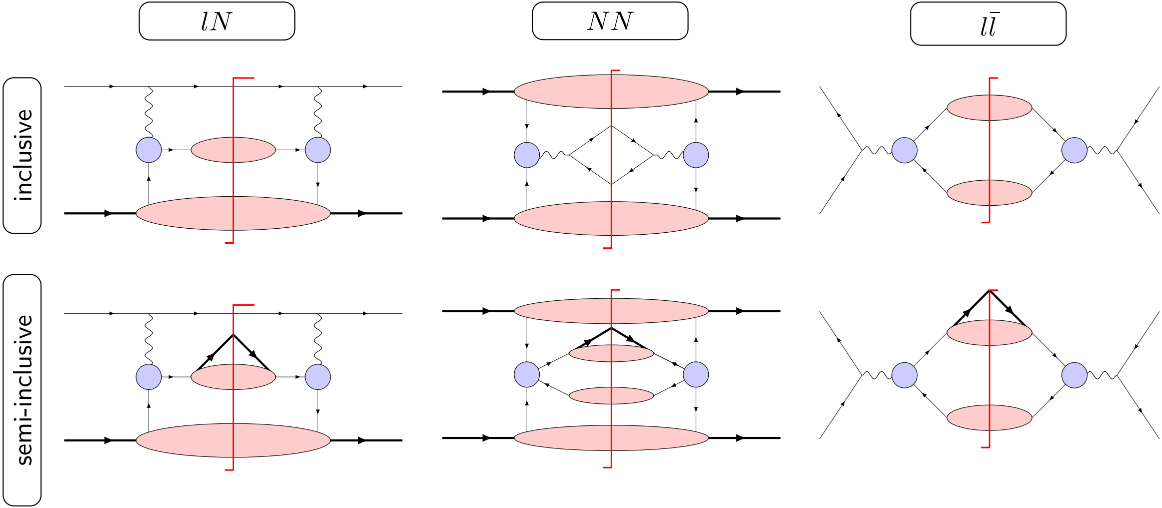
Universality of factorization



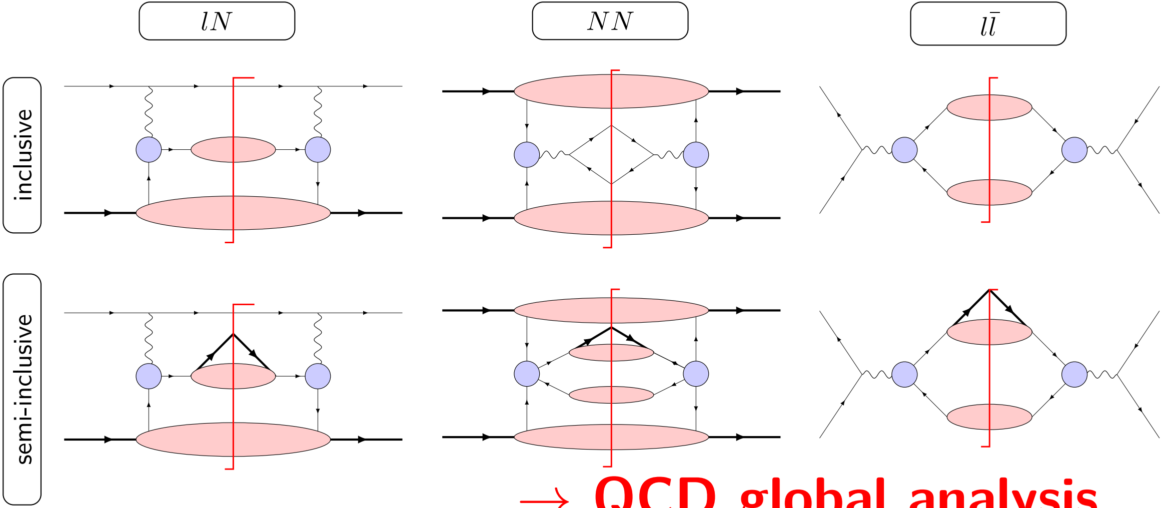
Universality of factorization



Universality of factorization

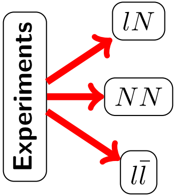


Universality of factorization

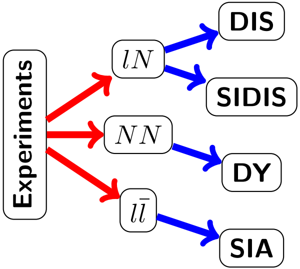


→ QCD global analysis

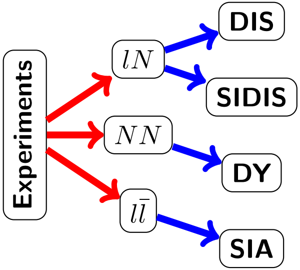
Global analysis in a nutshell

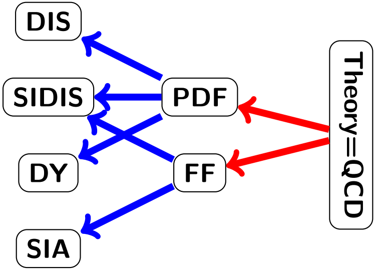


Global analysis in a nutshell

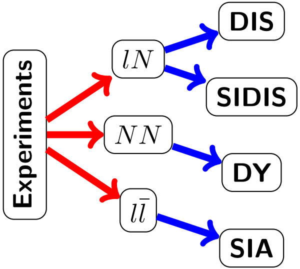


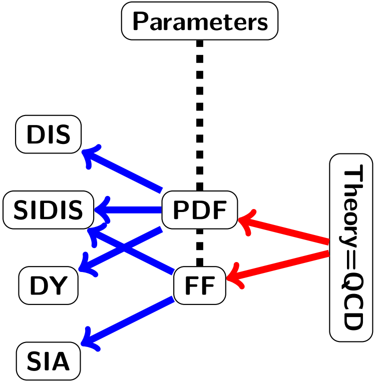
Global analysis in a nutshell



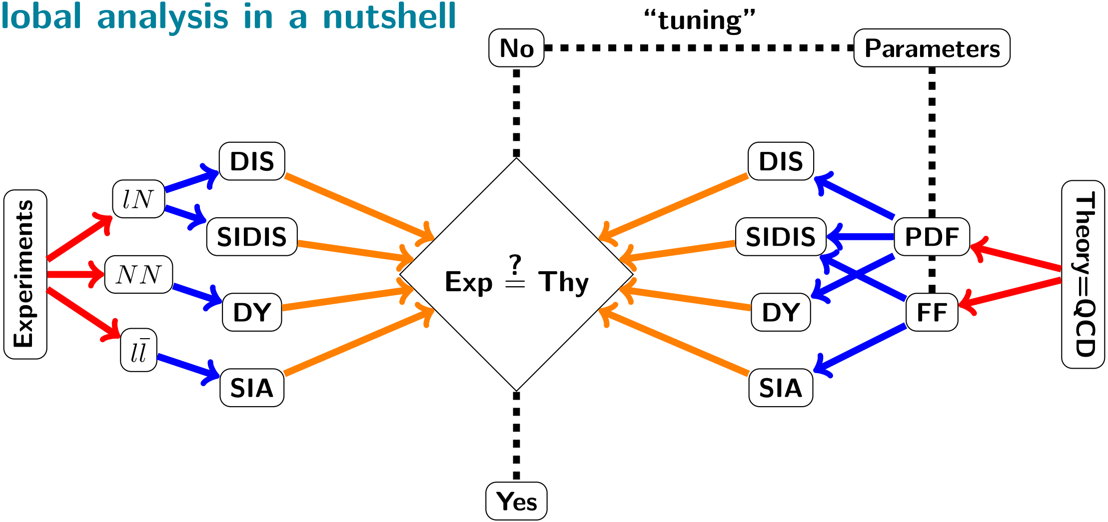


Global analysis in a nutshell

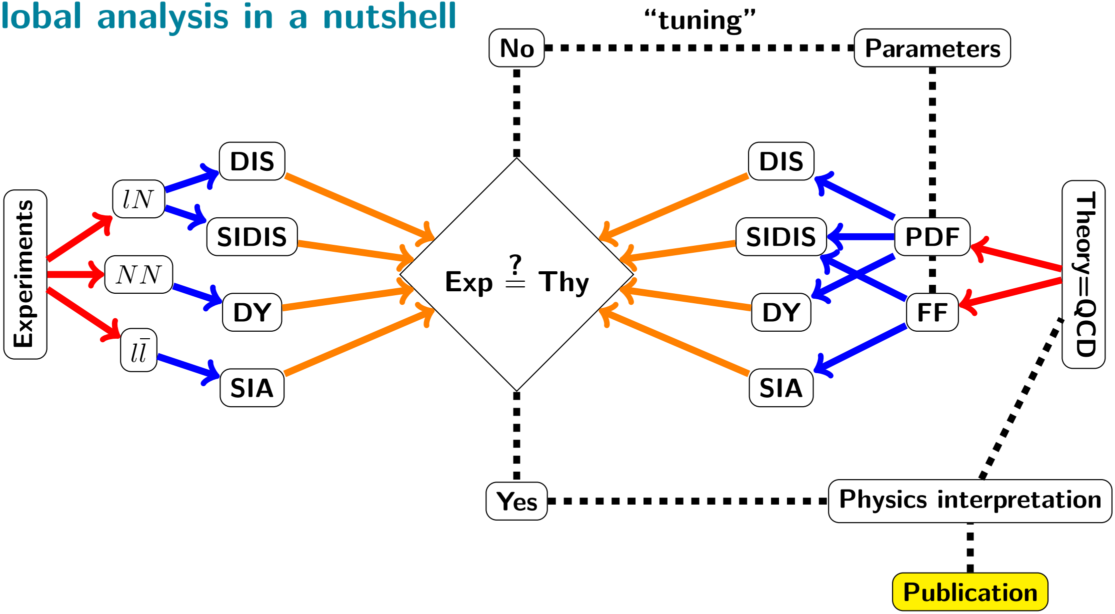




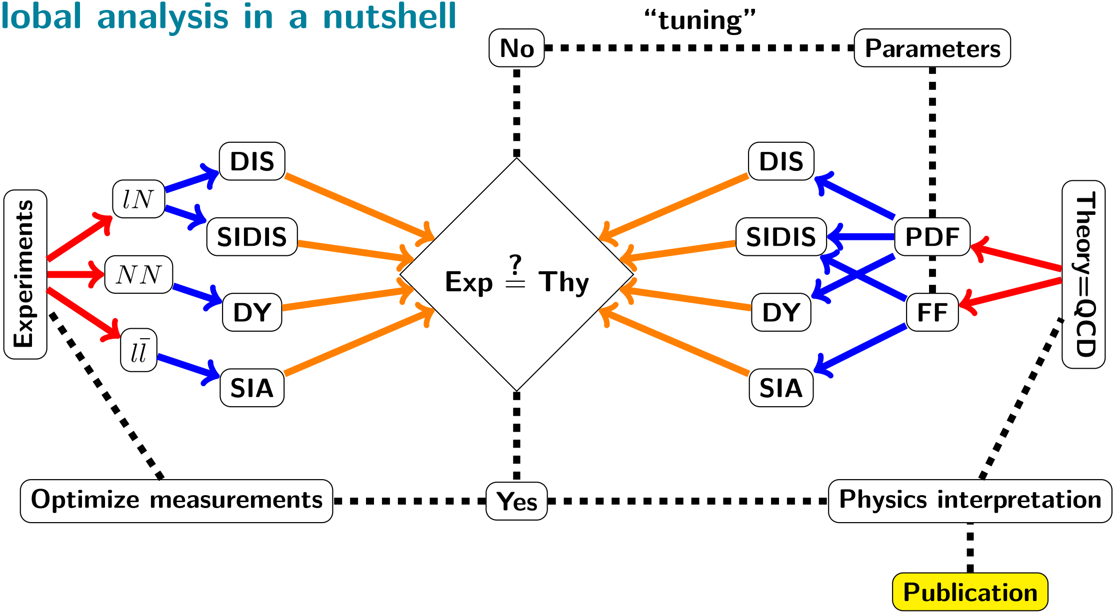
Global analysis in a nutshell

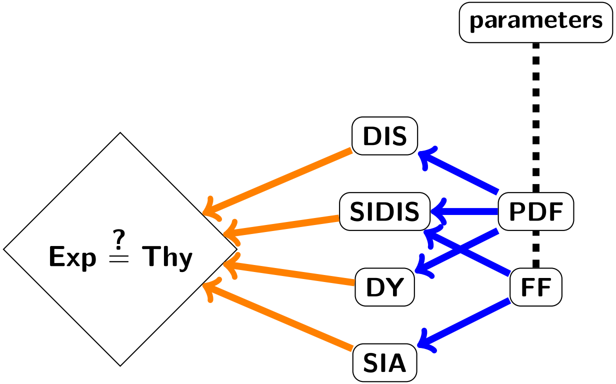


Global analysis in a nutshell

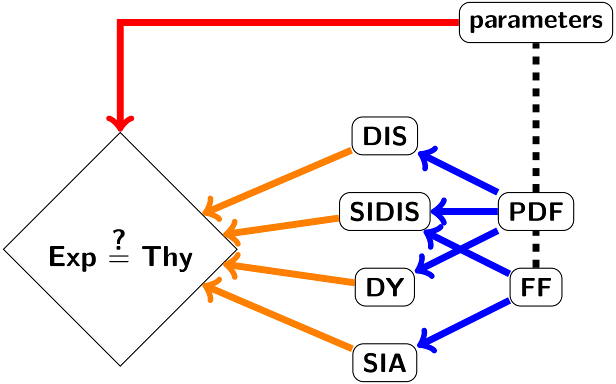


Global analysis in a nutshell

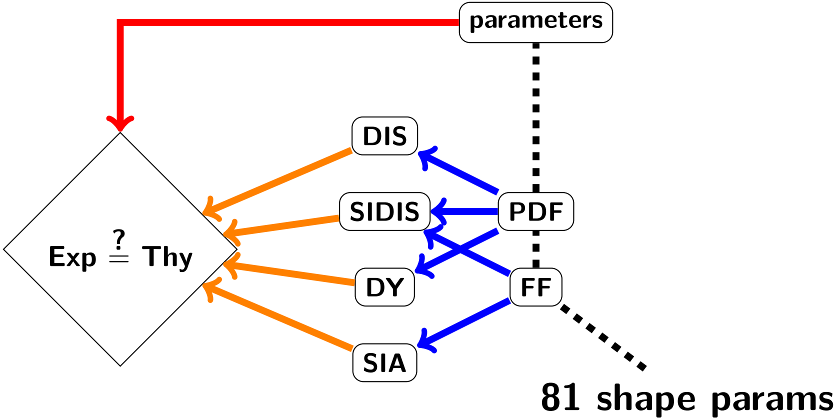


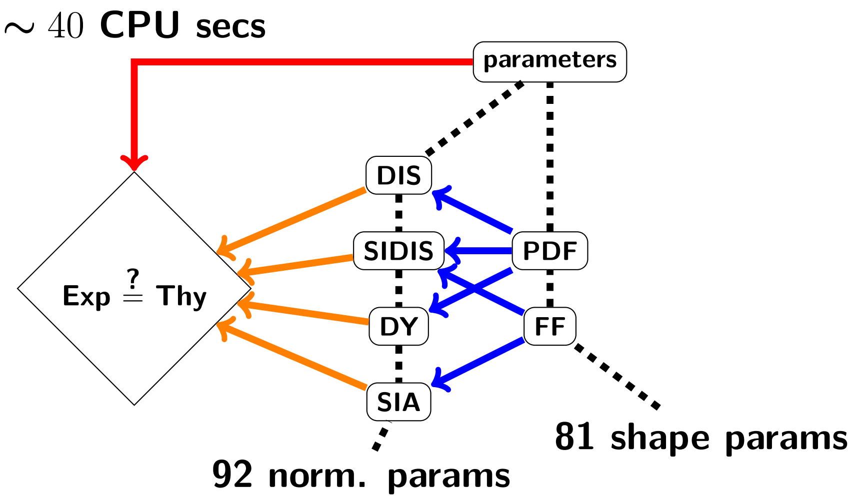

parameters
DIS
SIDIS
PDF
DY
FF
SIA
Exp $\stackrel{?}{=}$ Thy

~ 40 **CPU secs**

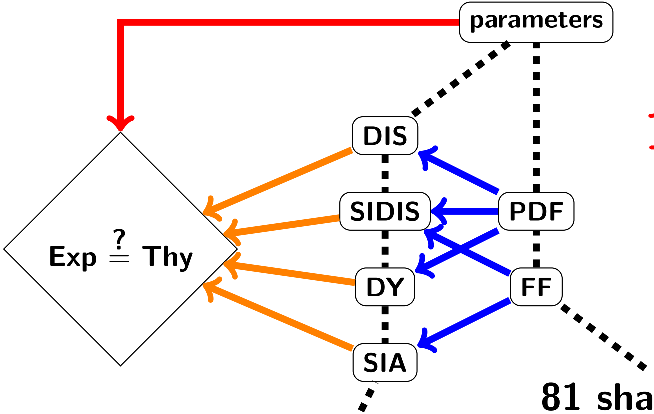


~ 40 **CPU secs**




~ 40 CPU secs
parameters
DIS
SIDIS
PDF
DY
FF
SIA
Exp $\stackrel{?}{=}$ Thy
81 shape params
92 norm. params

~ 40 **CPU secs**



173 parameters

81 shape params

92 norm. params

JAM19: *Strange quark suppression from a simultaneous Monte Carlo analysis of parton distributions and fragmentation functions*

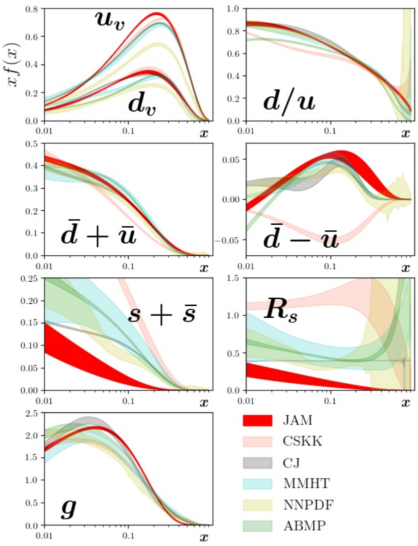
NS, Carlota Andres, Jake Ethier,
Wally Melnitchouk

arXiv:1905.03788

JAM Jefferson Lab Angular Momentum Collaboration

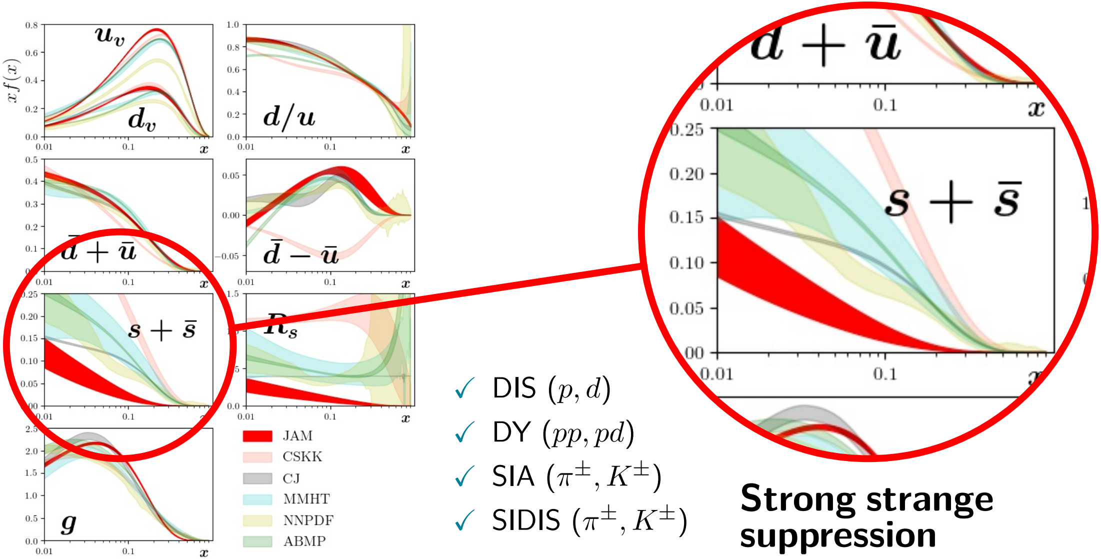
The results...

New PDFs



- ✓ DIS (p, d)
- ✓ DY (pp, pd)
- ✓ SIA $(\pi^{\pm}, K^{\pm})$
- ✓ SIDIS $(\pi^{\pm}, K^{\pm})$

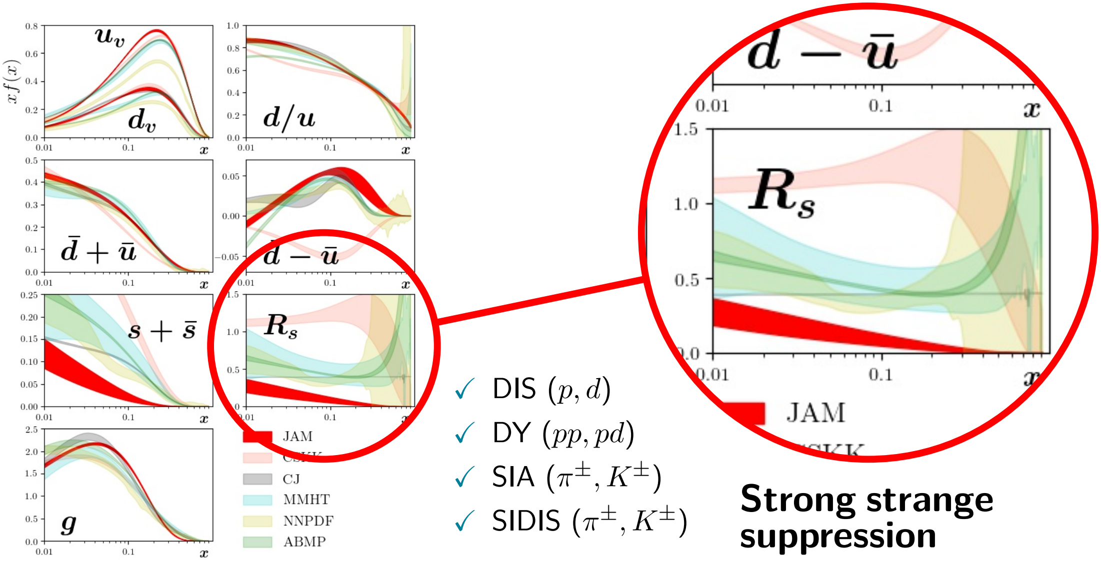
New PDFs



- ✓ DIS (p, d)
- ✓ DY (pp, pd)
- ✓ SIA $(\pi^{\pm}, K^{\pm})$
- ✓ SIDIS $(\pi^{\pm}, K^{\pm})$

Strong strange suppression

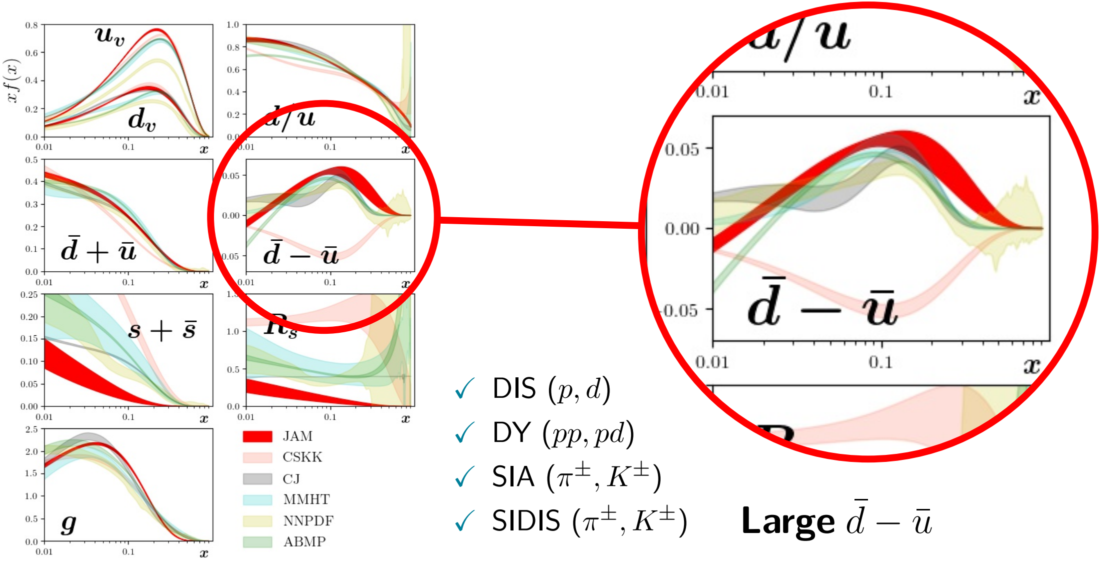
New PDFs



- ✓ DIS (p, d)
- ✓ DY (pp, pd)
- ✓ SIA $(\pi^{\pm}, K^{\pm})$
- ✓ SIDIS $(\pi^{\pm}, K^{\pm})$

Strong strange suppression

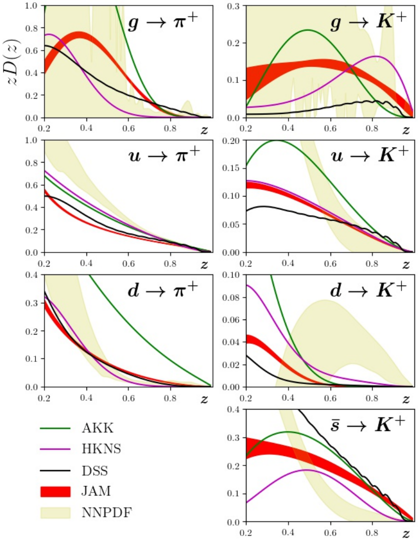
New PDFs



- ✓ DIS (p, d)
- ✓ DY (pp, pd)
- ✓ SIA $(\pi^{\pm}, K^{\pm})$
- ✓ SIDIS $(\pi^{\pm}, K^{\pm})$

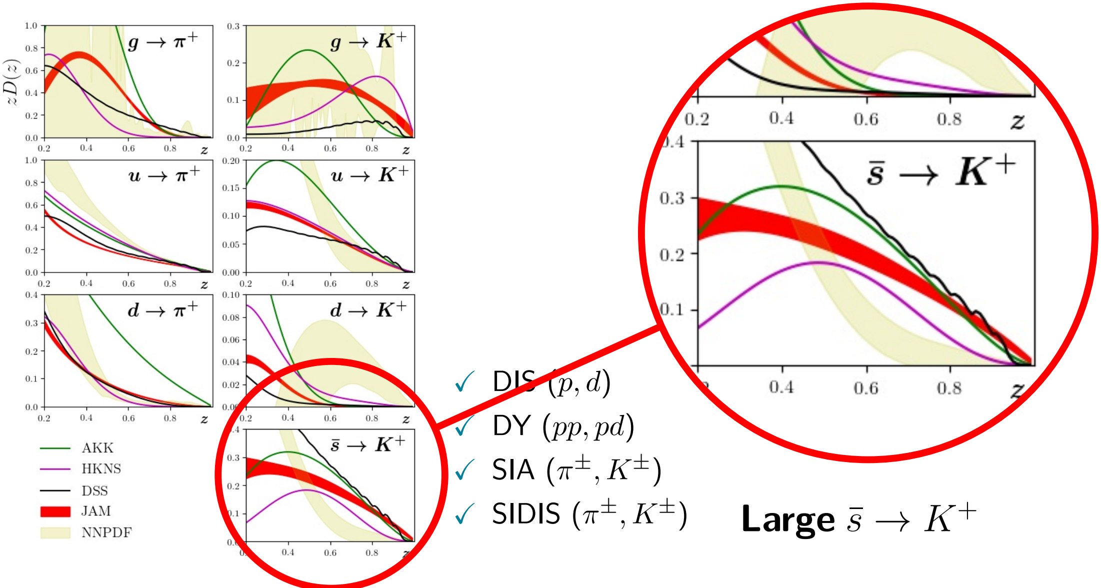
Large $\bar{d} - \bar{u}$

New $\pi\&K$ FFs



- ✓ DIS (p, d)
- ✓ DY (pp, pd)
- ✓ SIA $(\pi^{\pm}, K^{\pm})$
- ✓ SIDIS $(\pi^{\pm}, K^{\pm})$

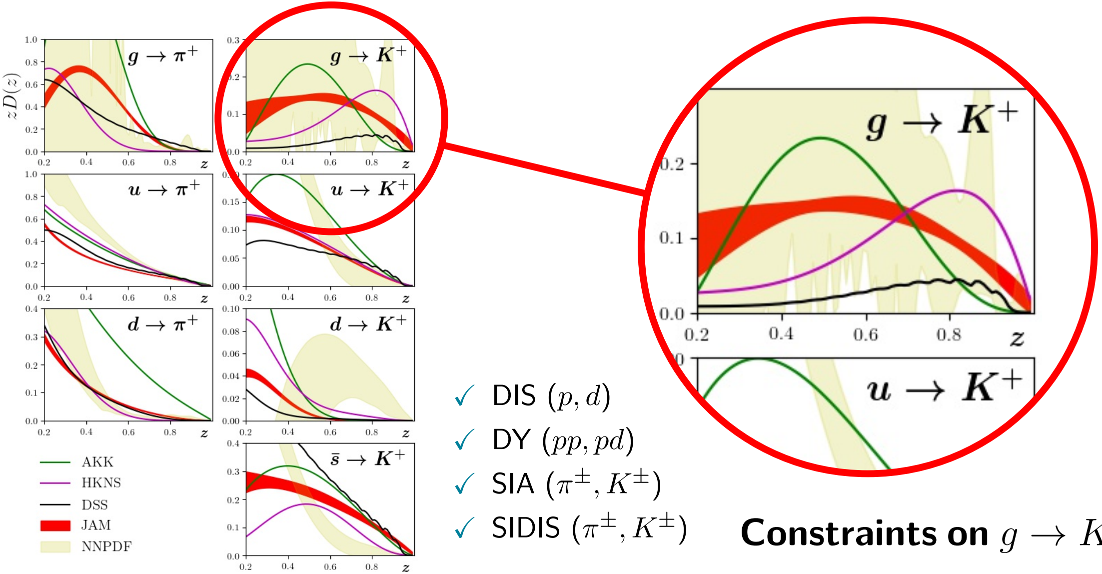
New $\pi\&K$ FFs



- ✓ DIS (p, d)
- ✓ DY (pp, pd)
- ✓ SIA $(\pi^\pm, K^\pm)$
- ✓ SIDIS $(\pi^\pm, K^\pm)$

Large $\bar{s} \to K^+$

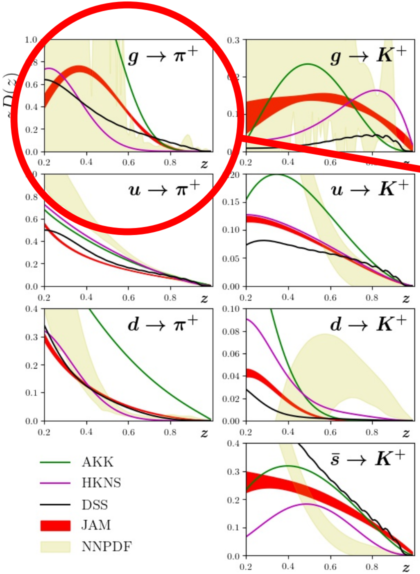
New $\pi\&K$ FFs

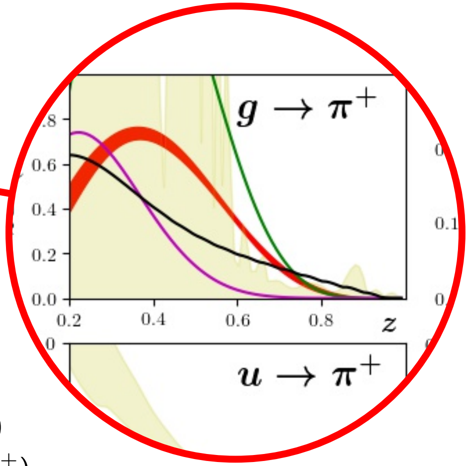


- ✓ DIS (p, d)
- ✓ DY (pp, pd)
- ✓ SIA $(\pi^\pm, K^\pm)$
- ✓ SIDIS $(\pi^\pm, K^\pm)$

Constraints on $g \to K^+$

New $\pi\&K$ FFs

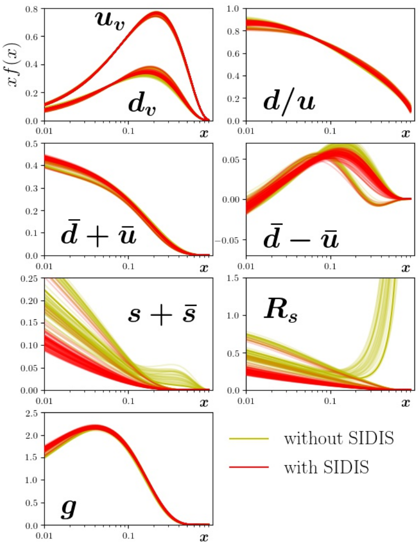




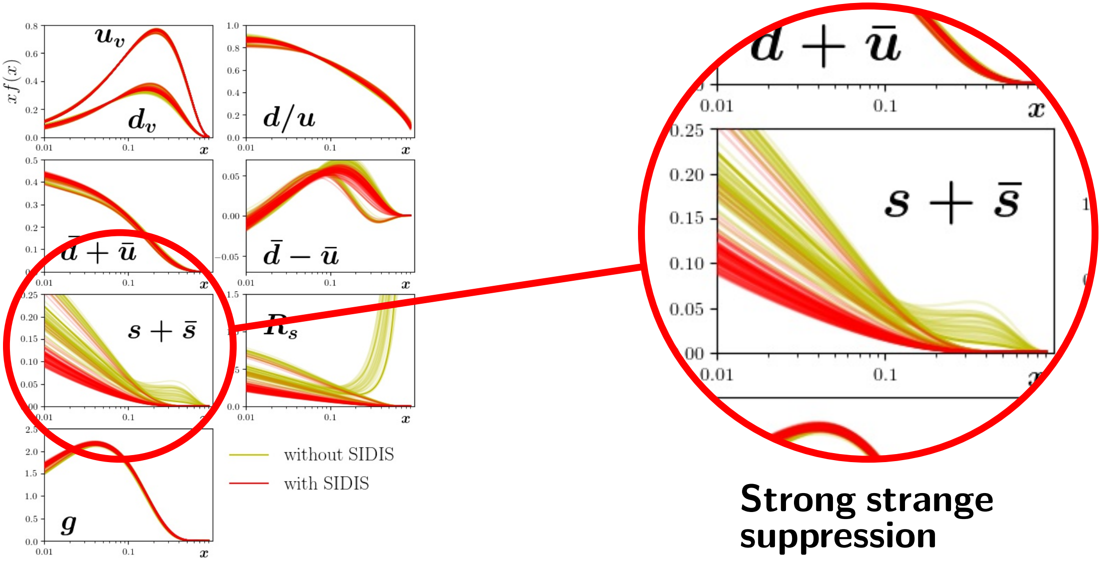
- ✓ DIS (p, d)
- ✓ DY (pp, pd)
- ✓ SIA $(\pi^{\pm}, K^{\pm})$
- ✓ SIDIS $(\pi^{\pm}, K^{\pm})$

Constraints on $g \to \pi^+$

Impact of SIDIS data on PDFs

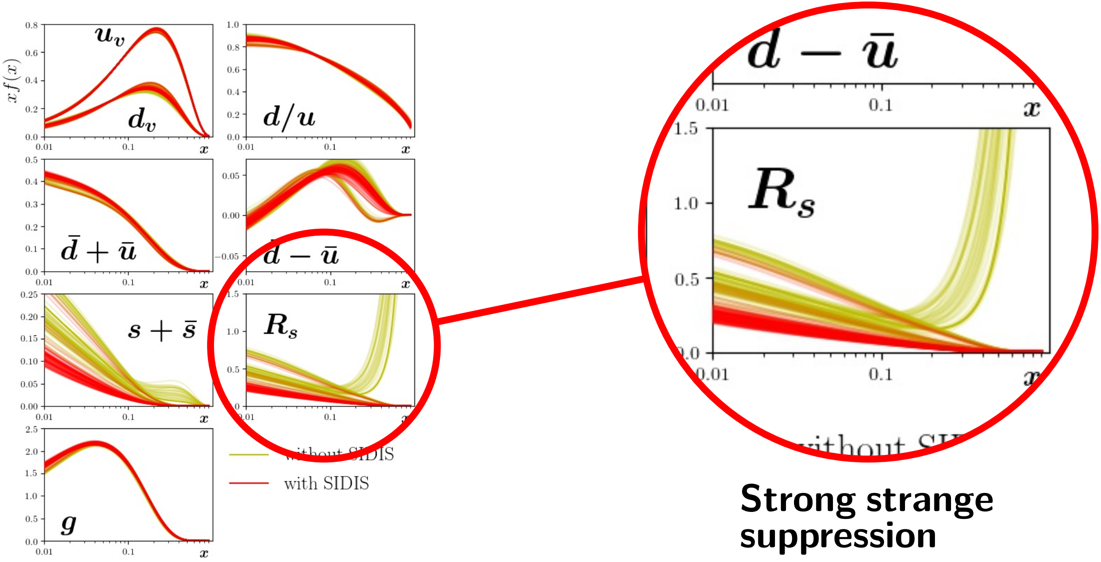


Impact of SIDIS data on PDFs



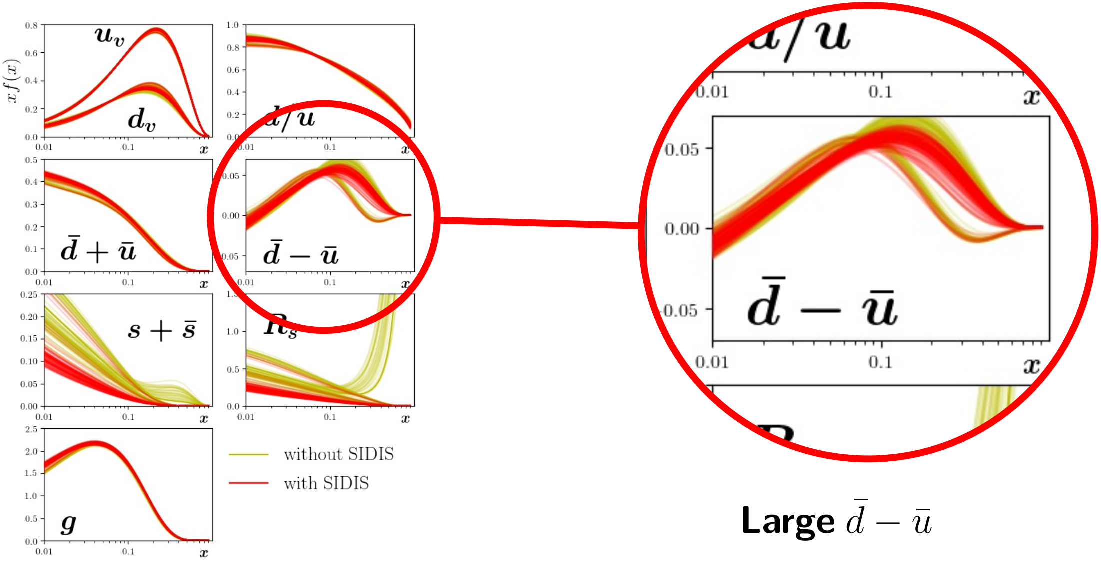
Strong strange suppression

Impact of SIDIS data on PDFs



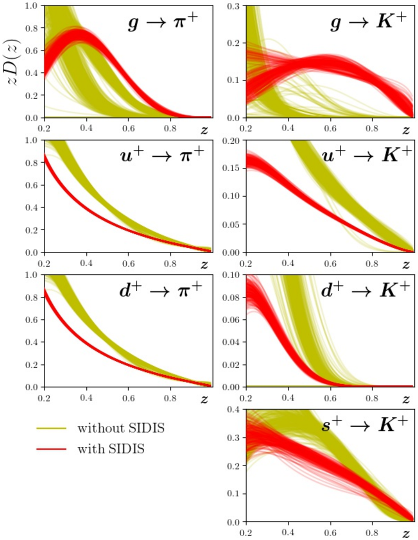
Strong strange suppression

Impact of SIDIS data on PDFs

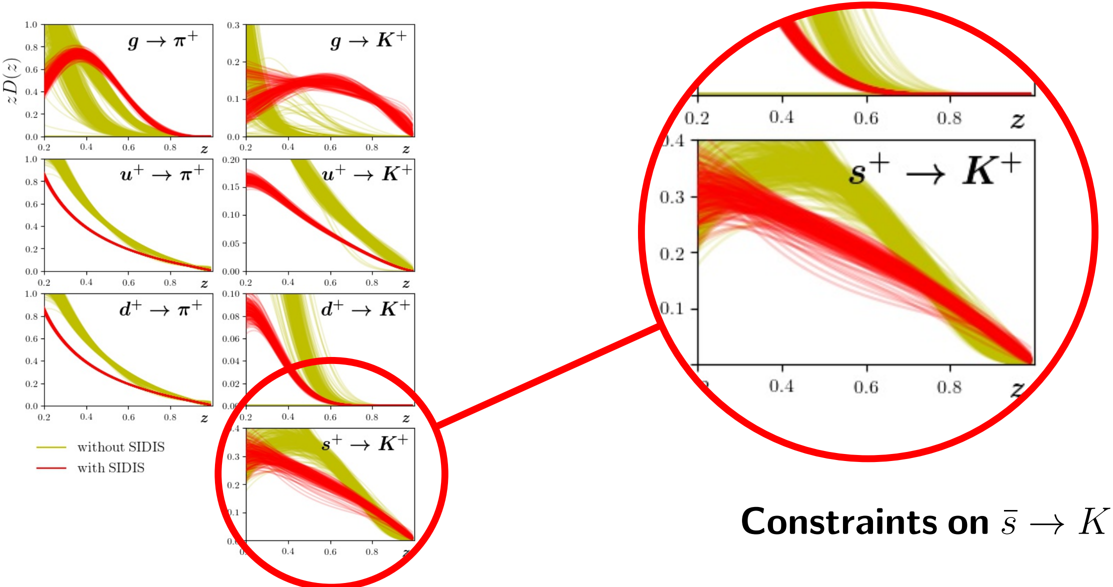


Large $\bar{d} - \bar{u}$

Impact of SIDIS data on FFs

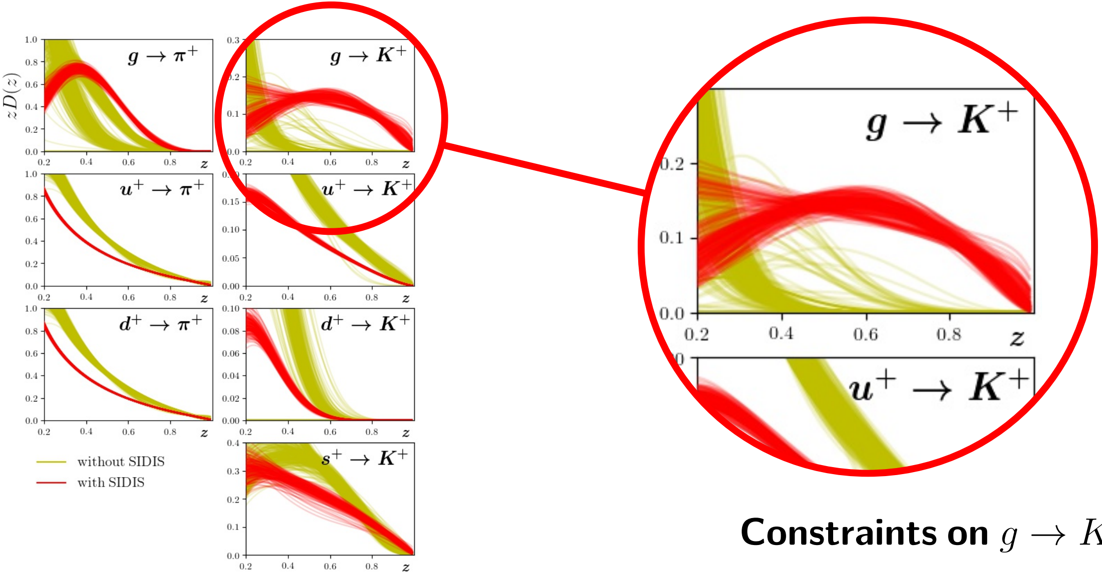


Impact of SIDIS data on FFs



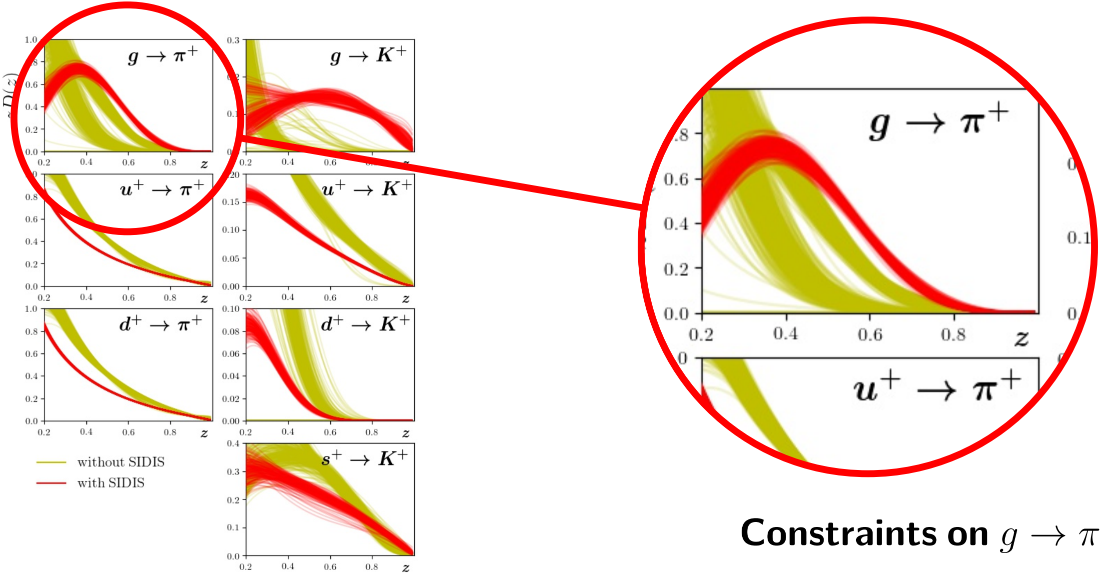
Constraints on $\bar{s} \to K^+$

Impact of SIDIS data on FFs



Constraints on $g \to K^+$

Impact of SIDIS data on FFs

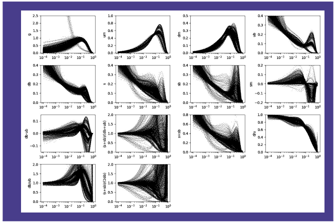


Constraints on $g \to \pi^+$

... ok, so how we get this?

Multi-step strategy

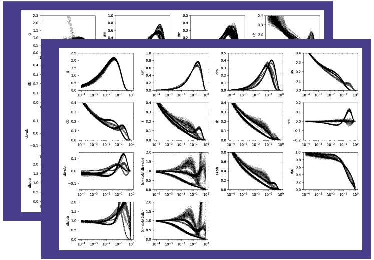
PDFs



+DIS (No HERA)

Multi-step strategy

PDFs

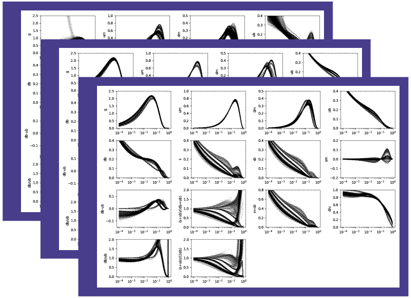


+DIS (No HERA)

+DIS HERA

Multi-step strategy

PDFs



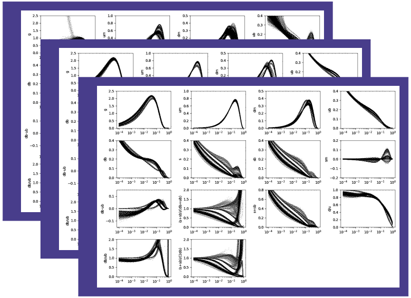
+DIS (No HERA)

+DIS HERA

+DY

Multi-step strategy

PDFs

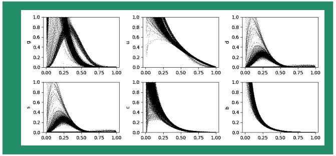


+DIS (No HERA)

+DIS HERA

+DY

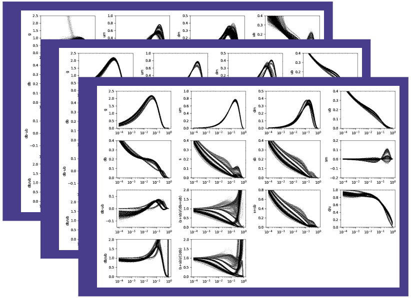
pion FFs



+SIA pions

Multi-step strategy

PDFs

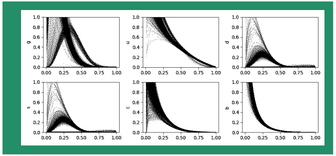


+DIS (No HERA)

+DIS HERA

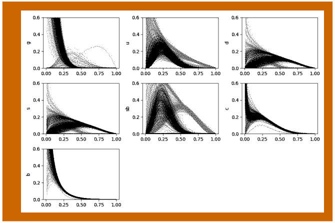
+DY

pion FFs



+SIA pions

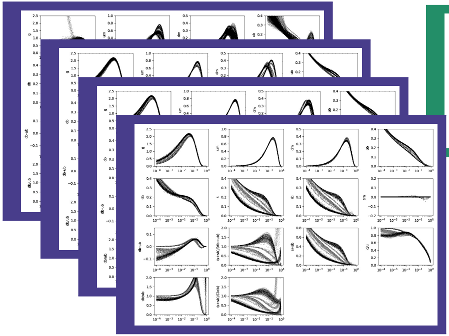
kaon FFs



+SIA kaons

Multi-step strategy

PDFs

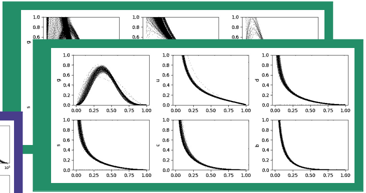


+DIS (No HERA)

+DIS HERA

+DY

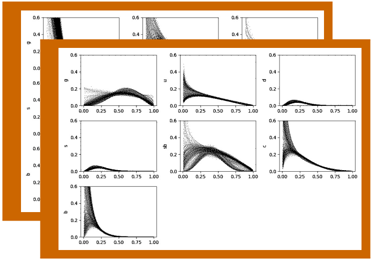
pion FFs



+SIA pions

+SIDIS pions

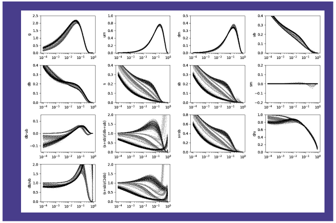
kaon FFs



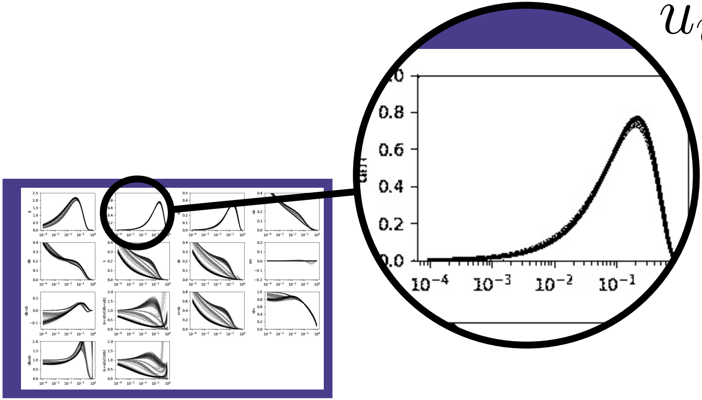
+SIA kaons

+SIDIS kaons

Discriminating multiple solutions

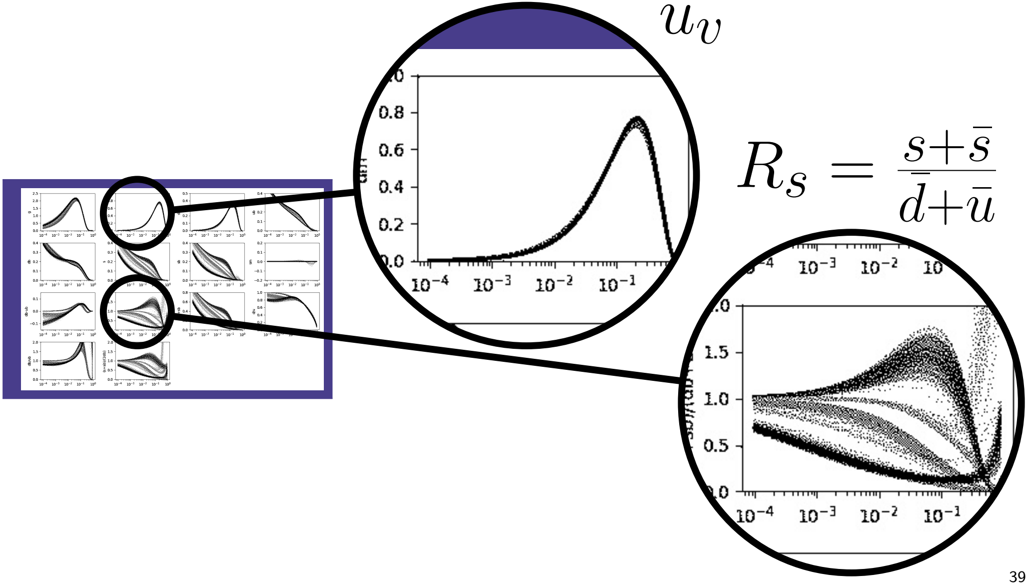


Discriminating multiple solutions



u_V

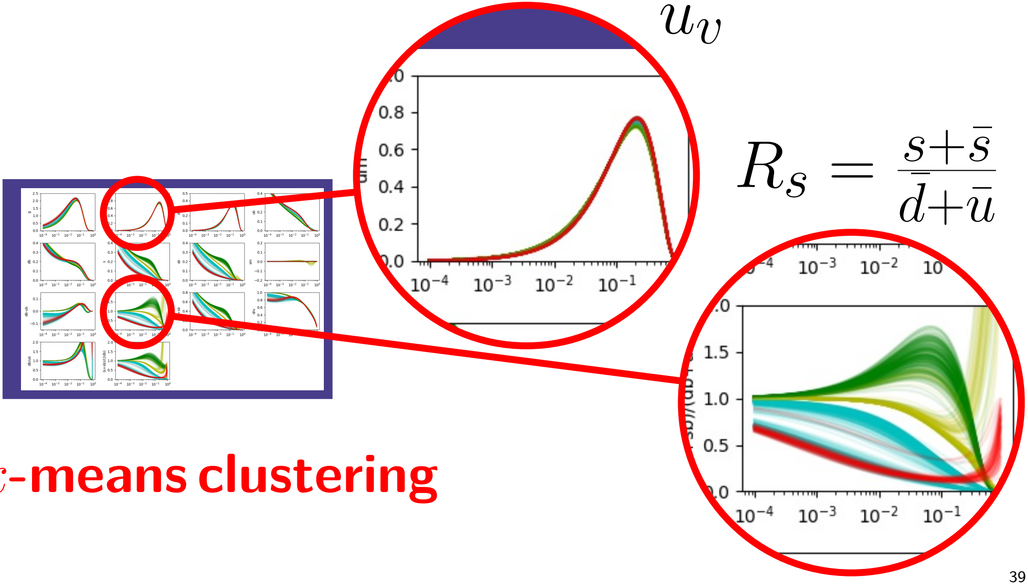
Discriminating multiple solutions



u_v

$$R_s = \frac{s+\bar{s}}{\bar{d}+\bar{u}}$$

Discriminating multiple solutions

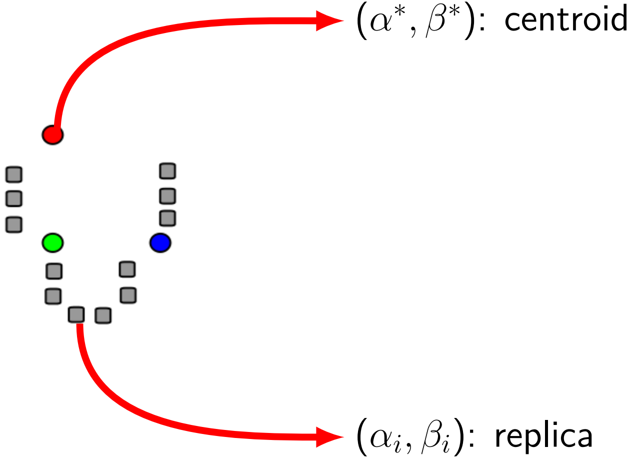


$$R_s = \frac{s+\bar{s}}{\bar{d}+\bar{u}}$$

k-means clustering

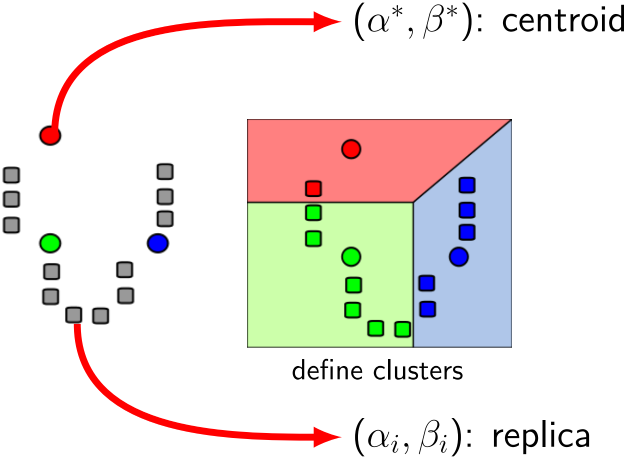
k-means clustering: 2D example (α, β)

e.g $f(x) = x^{\alpha}(1-x)^{\beta}$



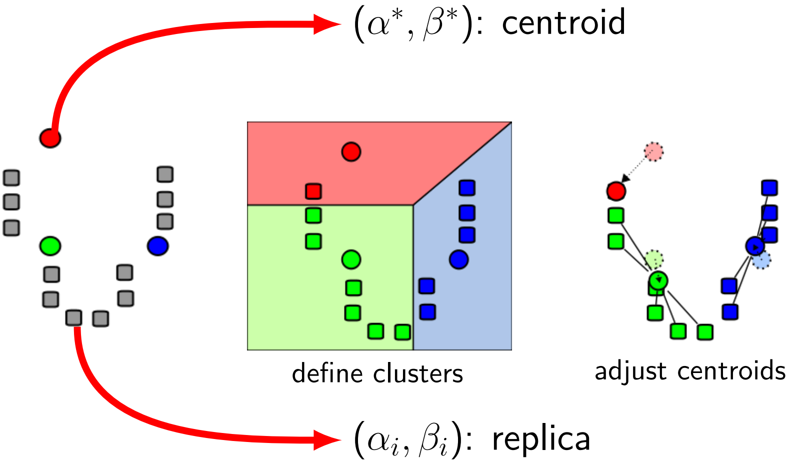
k-means clustering: 2D example (α, β)

e.g $f(x) = x^{\alpha}(1-x)^{\beta}$



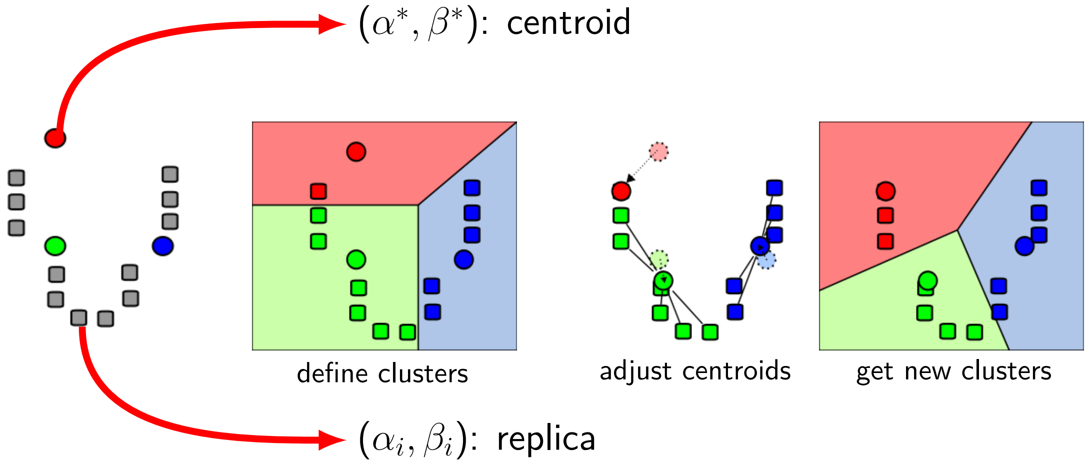
k-means clustering: 2D example (α, β)

e.g $f(x) = x^\alpha(1-x)^\beta$



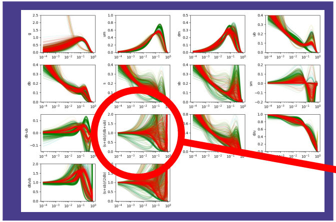
k-means clustering: 2D example (α, β)

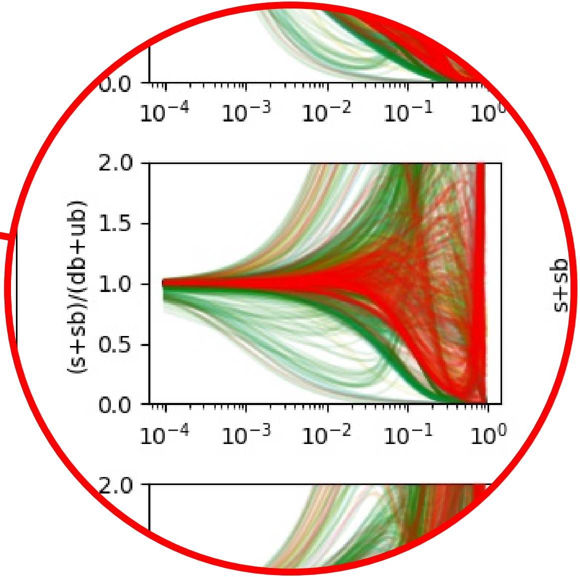
e.g $f(x) = x^{\alpha}(1-x)^{\beta}$



Constraints on R_s

PDFs

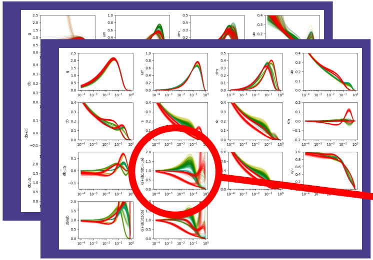


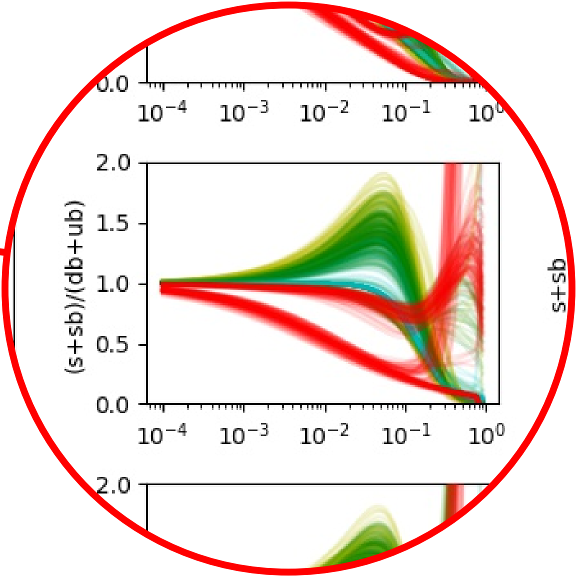


+DIS (No HERA)

Constraints on R_s

PDFs



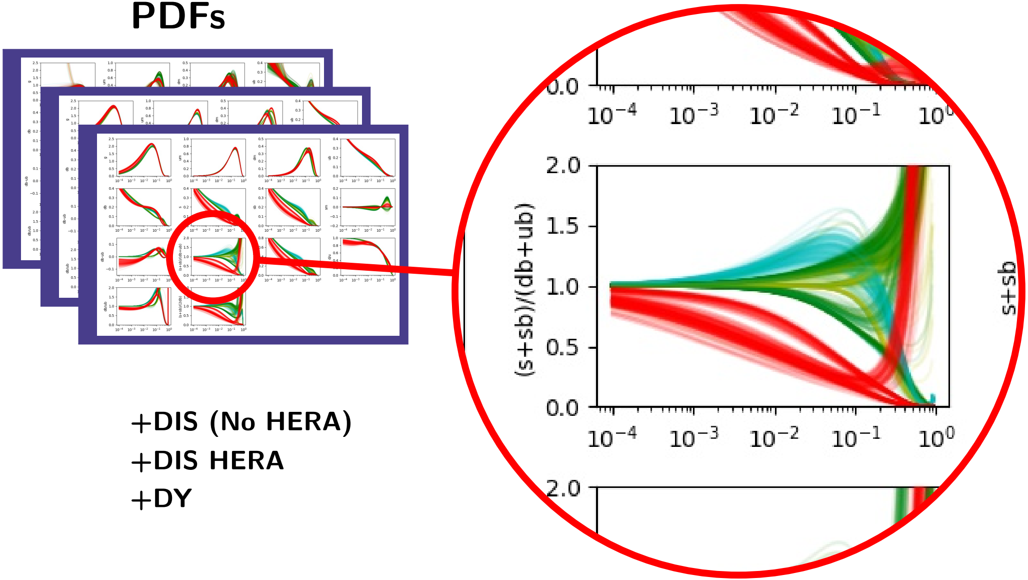


+DIS (No HERA)

+DIS HERA

Constraints on R_s

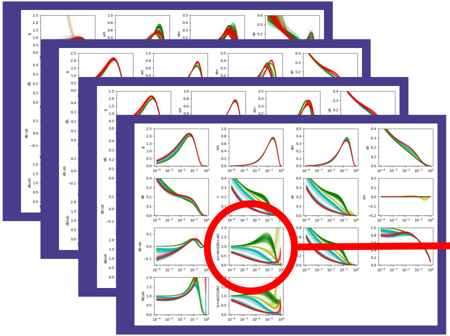
PDFs

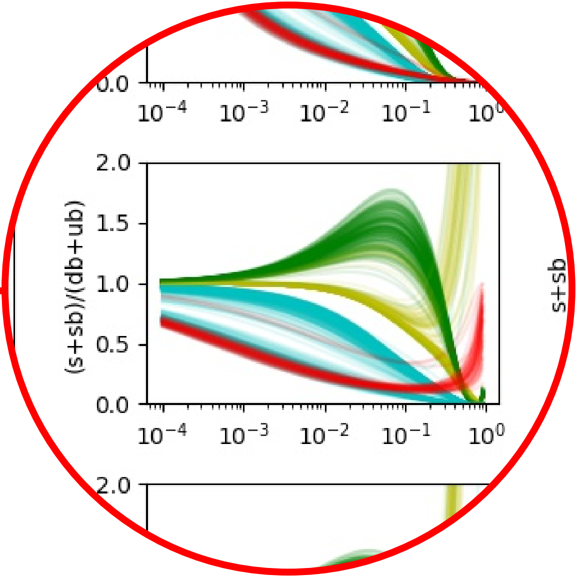


+DIS (No HERA)
+DIS HERA
+DY

Constraints on R_s

PDFs

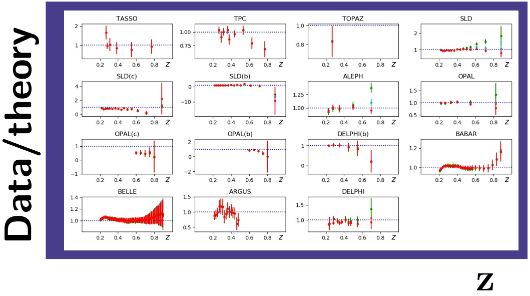




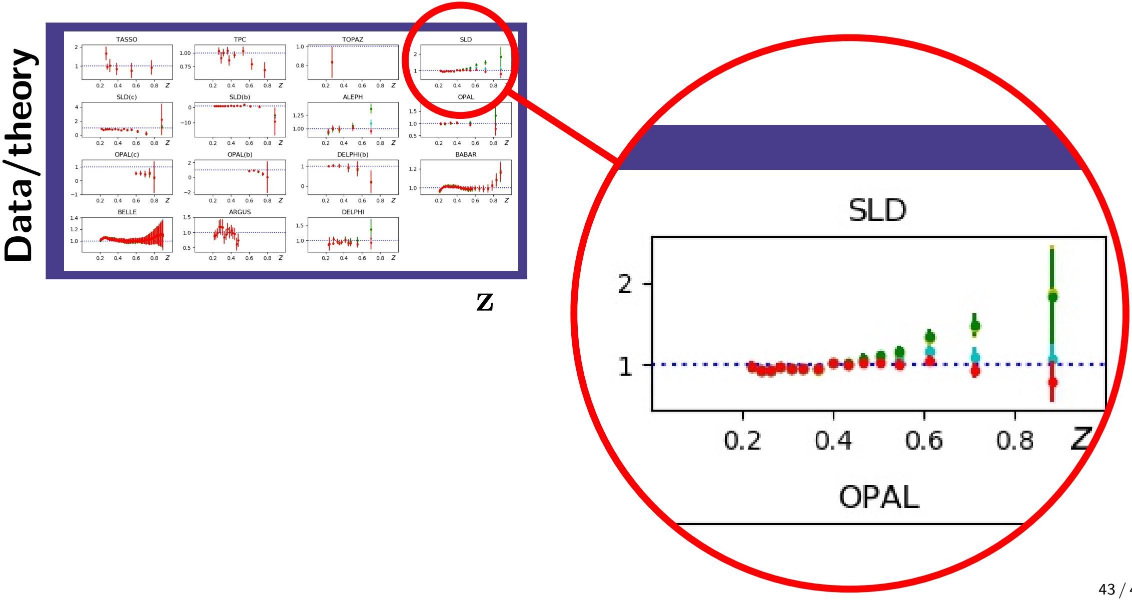
+DIS (No HERA)
+DIS HERA
+DY
+SIA + SIDIS

... So what makes R_s small?

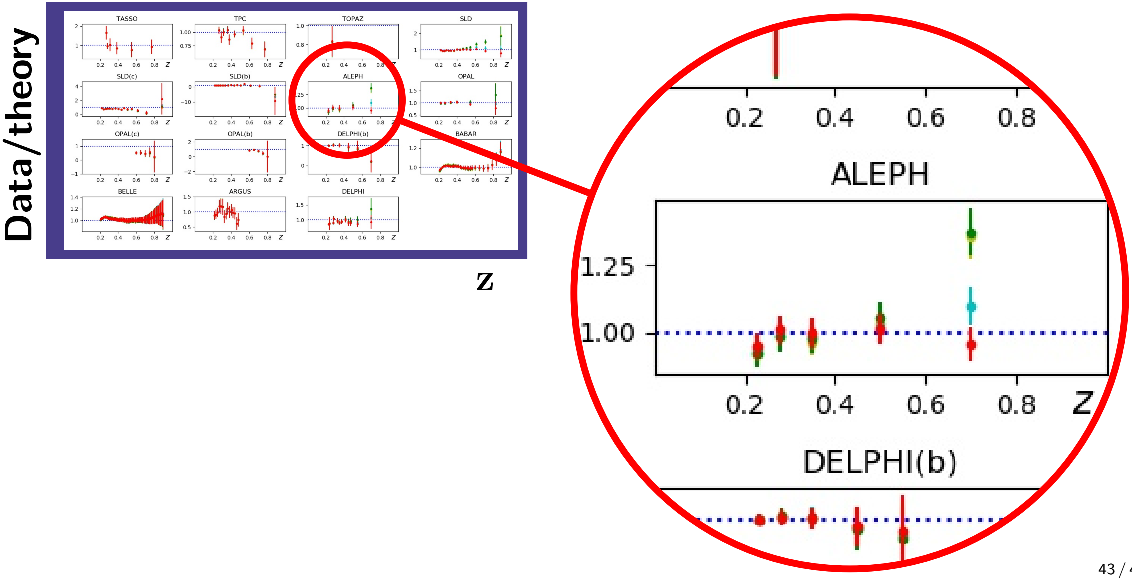
SIA Kaon (±) data



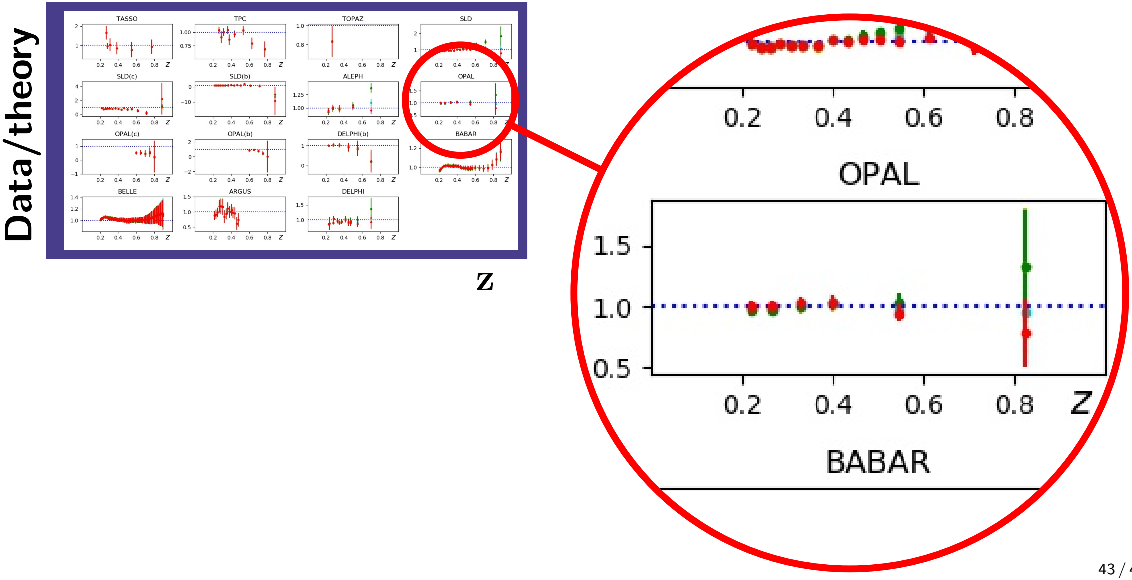
SIA Kaon (±) data



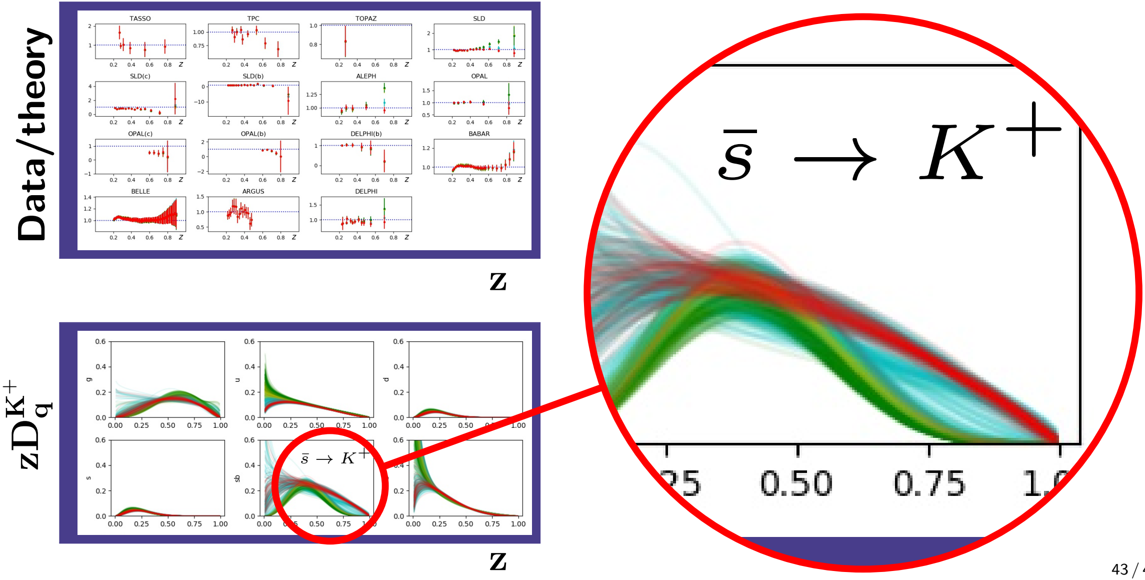
SIA Kaon (±) data



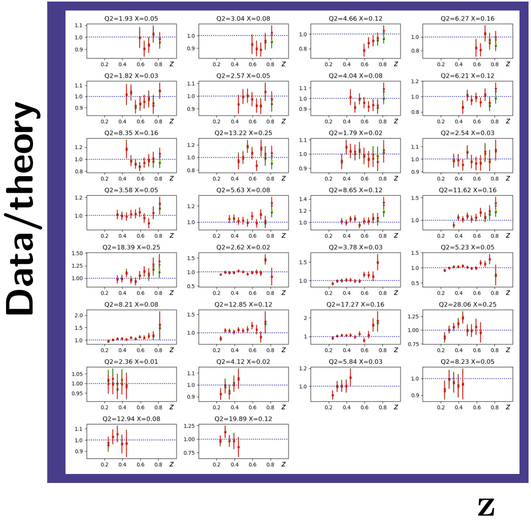
SIA Kaon (±) data



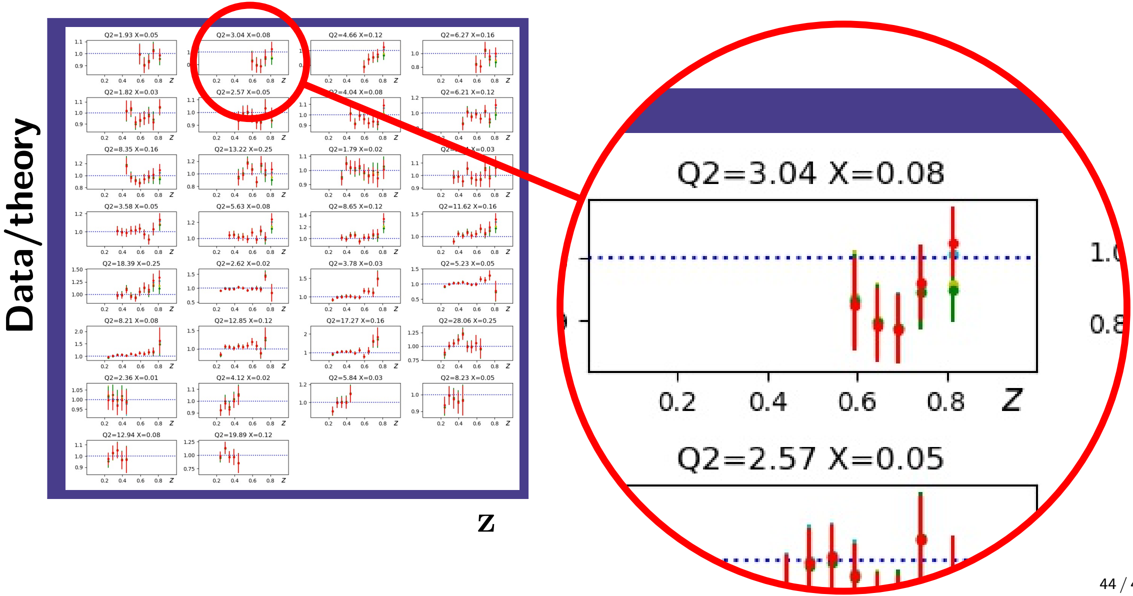
SIA Kaon (±) data



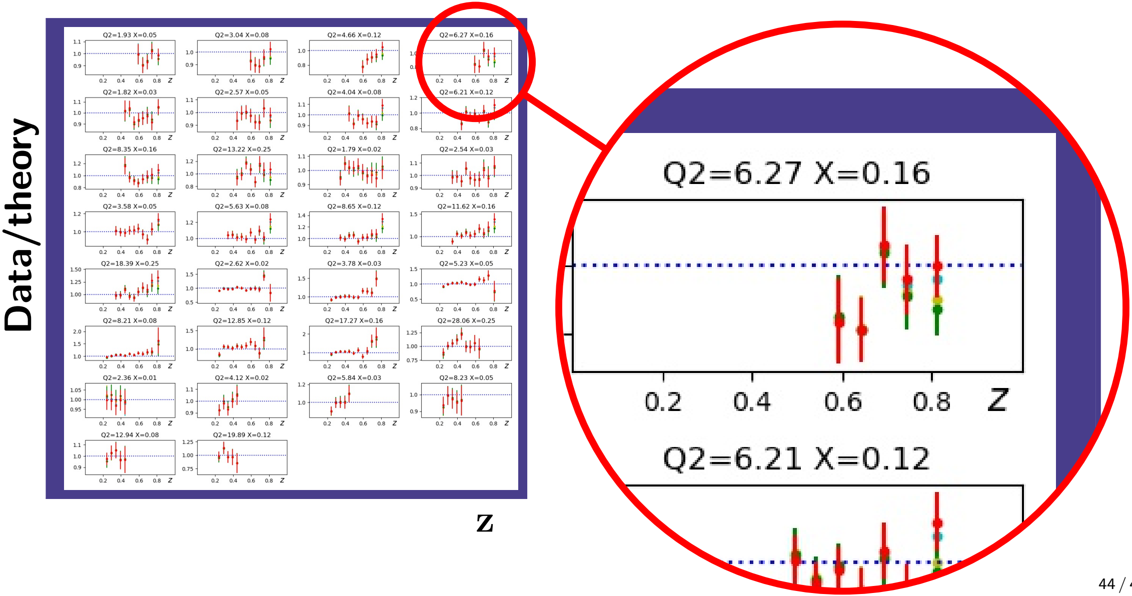
SIDIS Kaon (+) data



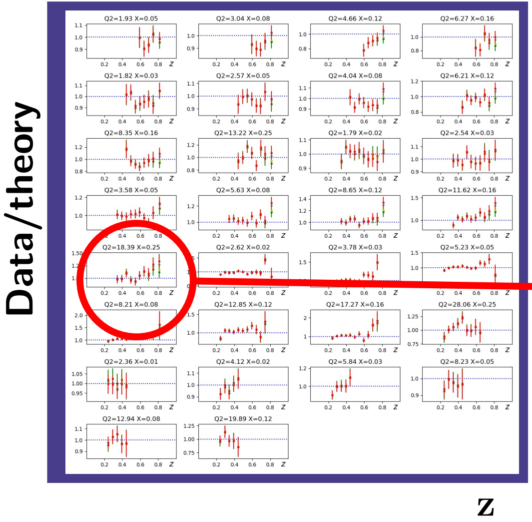
SIDIS Kaon (+) data

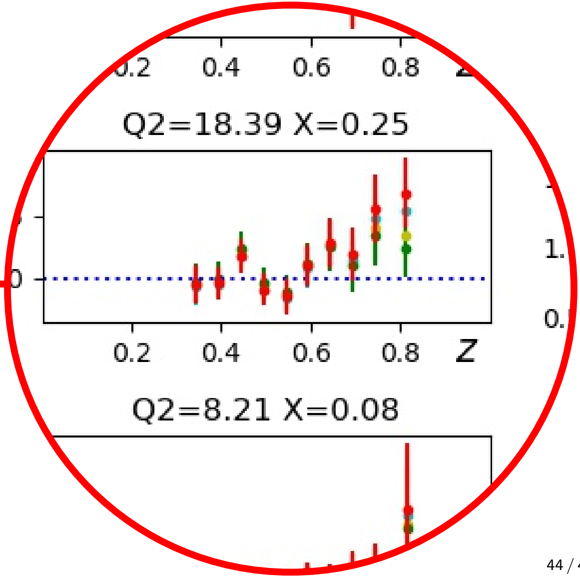


SIDIS Kaon (+) data

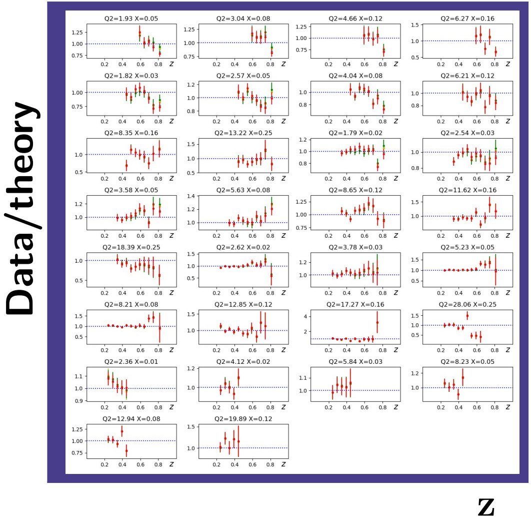


SIDIS Kaon (+) data

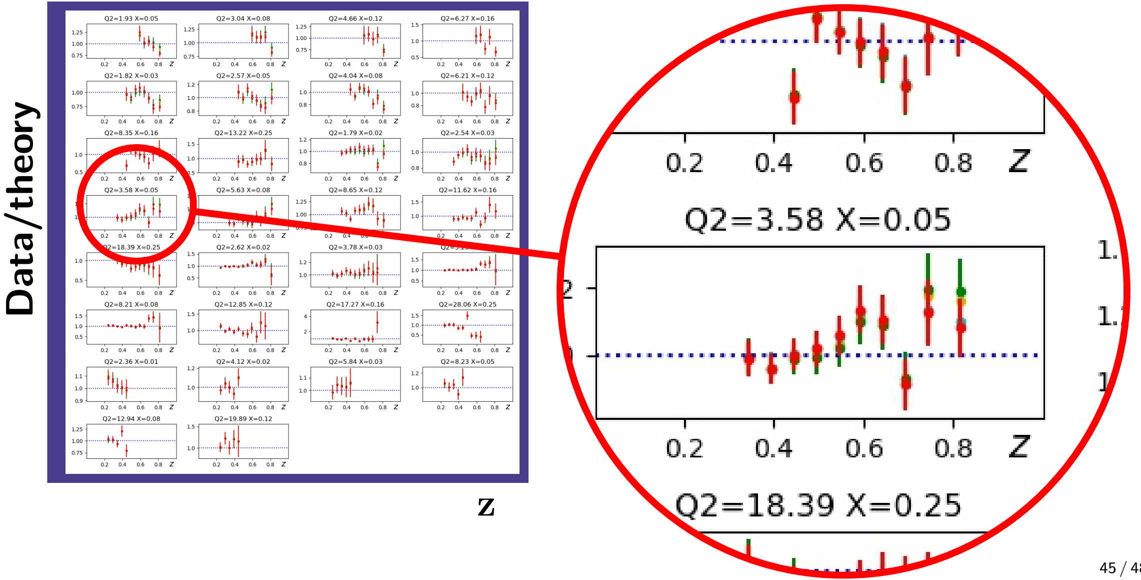




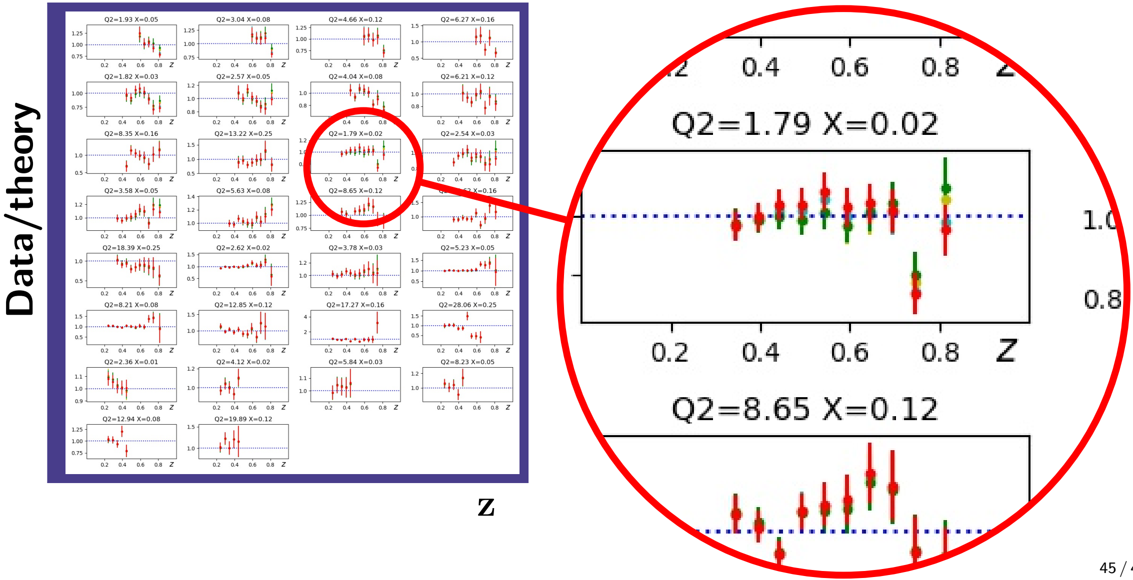
SIDIS Kaon (-) data



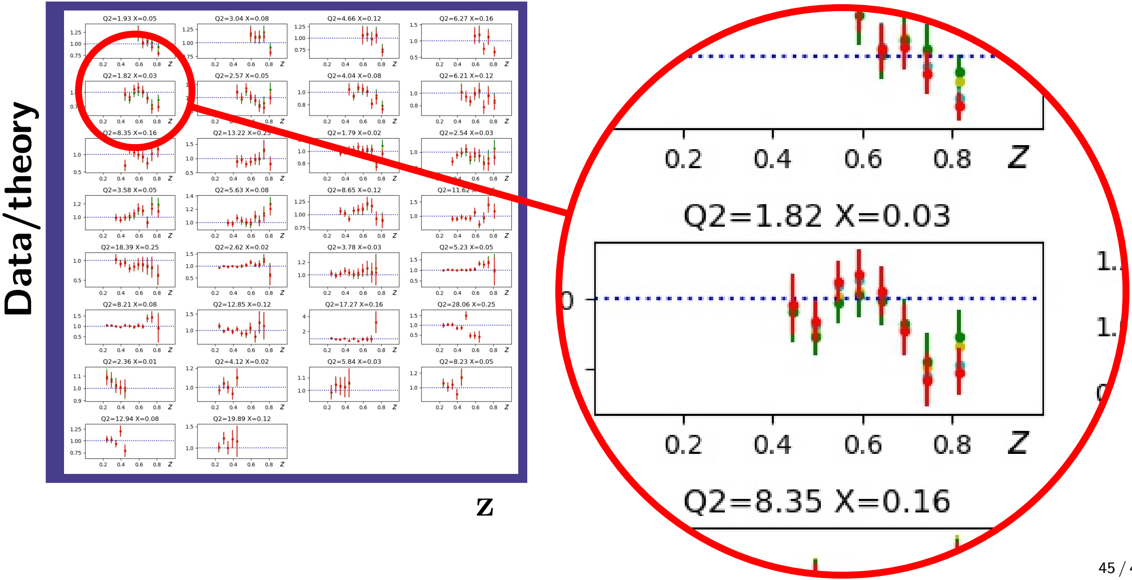
SIDIS Kaon (-) data



SIDIS Kaon (-) data



SIDIS Kaon (-) data

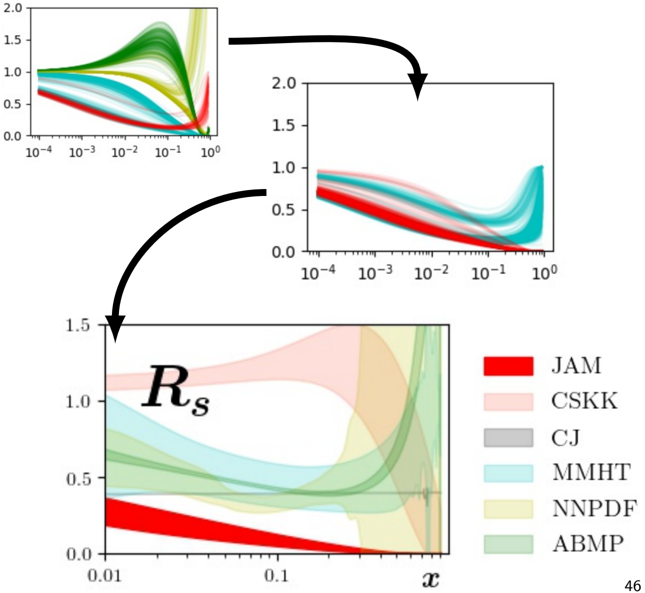


Selection criteria

- Apply k-means clustering
- Classify clusters by increasing order in "extended reduced Chi2"

$$\frac{\chi^2}{N_{\text{tot}}} + \sum_{\text{exp}} \frac{\chi^2_{\text{exp}}}{N_{\text{exp}}}$$

- Perform a new sampling with flat priors around best cluster



Summary and outlook