Lecture 3: Factorization & Resummation in SCET

Tues. Aug 6, 2019 (pm)

Have to compute matching condition:

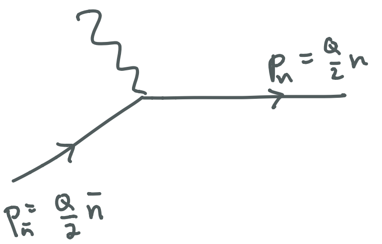
$$j^\mu_{QCD} = \bar q \gamma^\mu q \longrightarrow j^\mu_{SCET} \sum_{\substack{n_1 n_2 \\ \tilde p_1, \tilde p_2}} C_2(p_1,p_2,\mu)\, O^\mu_{n_1 n_2}(\tilde p_1,\tilde p_2)$$

$$O^\mu_{2\, n_1 n_2}(\tilde p_1,\tilde p_2) = \bar\chi_{n_1,p_1}\gamma_\perp^\mu \chi_{n_2,p_2} \xrightarrow{\text{redef}} \bar\chi_{n_1,p_1} Y_{n_1}^\dagger \gamma_\perp^\mu Y_{n_2}\chi_{n_2}$$

where $\chi_{n,p} = [W_n^\dagger \xi_n]_p$

Match matrix elements using any external states w/ nonzero overlap w/ operator.

e.g. DIS



Tree level:

$$\langle q(p_n)|\, j^\mu \,| q(p_{\bar n})\rangle = \bar u_n \gamma_\perp^\mu u_{\bar n}$$

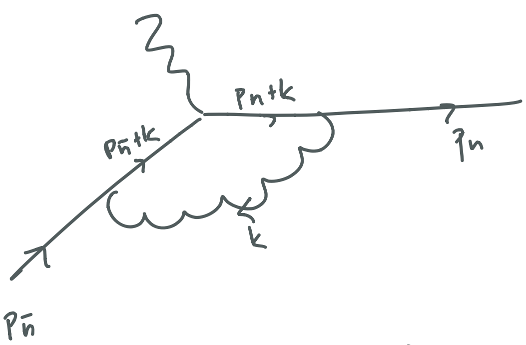
SCET:

$$\langle q_n |\, \bar\chi_n \gamma_\perp^\mu \chi_{\bar n} \,| q_{\bar n}\rangle = \text{same}$$

$$\Rightarrow C_2 = 1.$$

1-loop:

QCD:



$$= \bar u_n\, i g\mu^\epsilon \gamma^\alpha t^a \int \frac{d^Dk}{(2\pi)^D} \frac{i(\not p_n + \not k)}{(p_n+k)^2+i\epsilon}\gamma^\mu \frac{i(\not p_{\bar n}+\not k)}{(p_{\bar n}+k)^2+i\epsilon}\, i g\mu^\epsilon \gamma_\alpha t^a\, u_{\bar n} \frac{-i}{k^2+i\epsilon}$$

$$= -ig^2\mu^{2\epsilon} C_F \int \frac{d^Dk}{(2\pi)^D}\frac{\bar u_n\gamma^\alpha(\not p_n+\not k)\gamma^\mu(\not p_{\bar n}+\not k)\gamma_\alpha u_{\bar n}}{(k^2+i\epsilon)(k^2+2p_n\cdot k+i\epsilon)(k^2+2p_{\bar n}\cdot k+i\epsilon)}$$

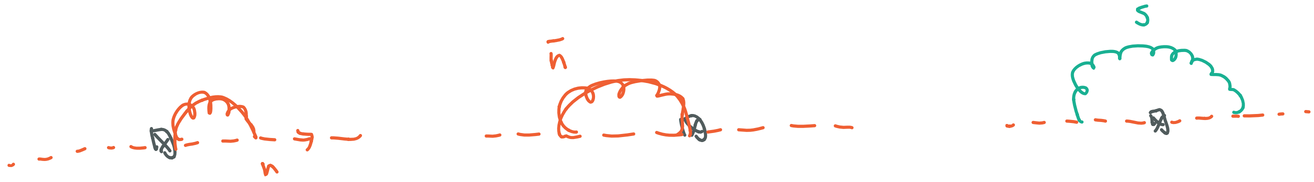
$\downarrow \ldots$ [exercise; remember $\left(\frac{1}{\sqrt{Z_2}}\right)^2$  $Z_2 = 1+\frac{\alpha_s C_F}{4\pi}\left(\frac{1}{\epsilon_{UV}}-\frac{1}{\epsilon_{IR}}\right)$]

$$\overset{\overline{MS}}{=} \frac{\alpha_s C_F}{4\pi}\left[-\frac{2}{\epsilon_{IR}^2} - \frac{2\ln\frac{\mu^2}{Q^2}+3}{\epsilon_{IR}} - \ln^2\frac{\mu^2}{Q^2} - 3\ln\frac{\mu^2}{Q^2} - 8 + \frac{\pi^2}{6}\right]$$

$D = 4-2\epsilon$

no $\frac{1}{\epsilon_{UV}}$: j^μ not renormalized due to current conservation

SCET: 3 diagrams to compute:



(wf renorm (Z_2 same as in QCD)

In pure DR all diags scaleless $\Rightarrow 0$

Technically

$$A_n + A_{\bar{n}} + A_s \propto \left(\frac{2}{\epsilon_{UV}^2} - \frac{2}{\epsilon_{IR}^2} + \left(3 - \ln\frac{\mu^2}{Q^2}\right)\left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}}\right)\right)$$

$\Rightarrow$ QCD - SCET $\Rightarrow$

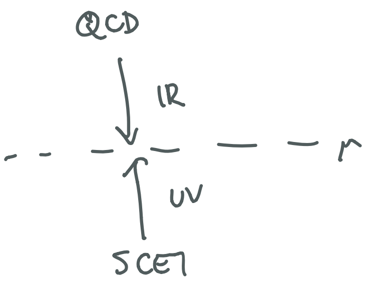
$$C_2 = 1 + \frac{\alpha_s C_F}{4\pi}\left(-\ln^2\frac{\mu^2}{Q^2} - 3\ln\frac{\mu^2}{Q^2} - 8 + \frac{\pi^2}{6}\right)$$

($Q^2 \to -Q^2$ e^+e^- or pp)

$$Z_2 = 1 + \frac{\alpha_s C_F}{4\pi}\left(-\frac{2}{\epsilon_{UV}^2} - \frac{3}{\epsilon_{UV}} - \frac{2}{\epsilon_{UV}}\ln\frac{\mu^2}{Q^2}\right)$$

$\hookrightarrow$ IR div in QCD turns into UV div in SCET

$\Rightarrow$ RGE to resum IR logs

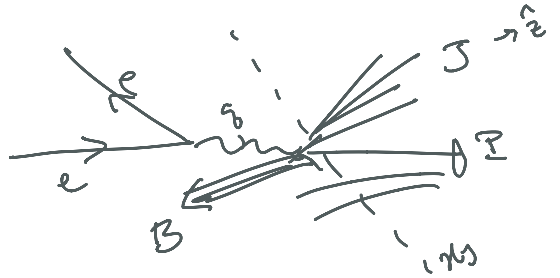


Exercise: put in explicit IR regulator and repeat, e.g. off shellness $p_n^2, p_{\bar{n}}^2 \neq 0$.

Consider measurement of DIS "thrust":

or "1-jettiness"

$$\tau_1 = \frac{2}{Q^2} \sum_{i \in X} \min\{q_B \cdot p_i,\, q_J \cdot p_i\}$$



here $q_B = xP$

$q_J = q + xP$

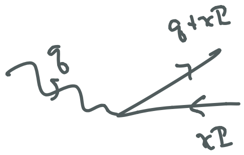
(other choices possible) [1303.6952]

(Breit frame)

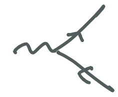
$$\tau_1 = \frac{1}{Q} \sum_{i \in X} \min\{\bar{n}_z \cdot p_i,\, n_z \cdot p_i\} = \text{DIS thrust}$$

$$= 1 - \frac{2}{Q} \sum_{i \in X_J} p_z^i$$

[exercise]



exact partonic limit $\tau_1 \to 0$.

(analogous to back-to-back jets in e^+e^- )

full QCD:

$$\frac{d\sigma}{dx\, dQ^2\, d\tau} = L_{\mu\nu}(x, Q^2)\, W^{\mu\nu}(x, Q^2, \tau)$$

$$W^{\mu\nu} = \sum_X \langle P | J^{\mu\dagger} | X \rangle \langle X | J^\nu | P \rangle\, (2\pi)^4 \delta(P + q - p_X)\, \delta(\tau - \tau(X))$$

$$J^\mu(x) = \bar{q} \gamma^\mu q$$

$$\downarrow$$

$$\sum_{n_1 n_2} \int d^3\tilde{p}_1\, d^3\tilde{p}_2\, e^{i(\tilde{p}_1 - \tilde{p}_2)\cdot x}\, C^\mu_{\alpha\beta}(\tilde{p}_1, \tilde{p}_2)\, O^{\alpha\beta}_{q\bar{q}}(\tilde{p}_1, \tilde{p}_2; x)$$

where $O^{\alpha\beta}_{q\bar{q}} = \bar{\chi}^\alpha_{n_1,p_1}(x)\, \chi^\beta_{n_2,p_2}(x)$

$$\downarrow \text{ redef}$$

$$\bar{\chi}^{(0)\alpha}_{n_1,p_1}\, Y^\dagger_{n_1} Y_{n_2}\, \chi^\beta_{n_2,p_2}$$

$$W^{\mu\nu} = \int d^4x\, e^{iq\cdot x} \sum_{\substack{n_1 n_2 \\ n_1' n_2'}} \int d^3\tilde{p}_1\, d^3\tilde{p}_2\, d^3\tilde{p}_1'\, d^3\tilde{p}_2'\, e^{i(\tilde{p}_2' - \tilde{p}_1')\cdot x} \int d\tau_J\, d\tau_B\, d\tau_S\, \delta(\tau - \tau_J - \tau_B - \tau_S)$$

$$\times \langle P_{n_B} | \bar{C}^{\beta\alpha}_\mu(\tilde{p}_1, \tilde{p}_2)\, \bar{\chi}^\beta_{n_2,\tilde{p}_2}\, Y^\dagger_{n_2} Y_{n_1}\, \chi^\alpha_{n_1,\tilde{p}_1}(x)\, \delta\!\left(\tau_J - \frac{n_J \cdot \hat{p}^{n_J}}{Q}\right) \delta\!\left(\tau_B - \frac{n_B \cdot \hat{p}^{n_B}}{Q}\right)$$

$(\bar{C} = \gamma^0 C^\dagger \gamma^0)$

$$\times\, \delta\!\left(\tau_S - \frac{n_J \cdot \hat{p}^s_J}{Q} - \frac{n_B \cdot \hat{p}^s_B}{Q}\right) C^{\alpha'\beta'}_\nu(\tilde{p}_1', \tilde{p}_2')\, \bar{\chi}^{\alpha'}_{n_1',\tilde{p}_1'}\, Y^\dagger_{n_1'} Y_{n_2'}\, \chi^{\beta'}_{n_2',\tilde{p}_2'}(0) | P_{n_B} \rangle$$

$\vdots$ (momentum conservation, $n_{J,B}$ coll & soft decoupling)

$$= Q^2 \int d\tau_J\, d\tau_B\, d\tau_S\; \delta(\tau - \tau_J - \tau_B - \tau_S)\; \bar{C}^{\beta\alpha}_{\mu}\, C^{\alpha'\beta'}_{\nu} \int d^2\tilde{p}_b$$

$$\times \langle 0 | [Y^\dagger_{n_B} Y_{n_J}](0)\; \delta(Q\tau_S - n_J\cdot \hat{p}^S_J - n_B\cdot \hat{p}^S_B)\, [Y^\dagger_{n_J} Y_{n_B}](0) | 0\rangle$$

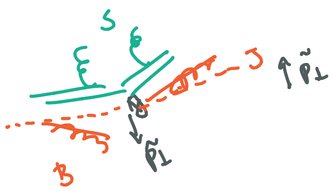
$$\times \Big\{ \langle P_{n_B} | \bar{\chi}^{\beta}_{n_B}(0)\; \delta(Q\tau_B - n_B\cdot \hat{p}^{n_B})\; \delta(\bar{n}_B\cdot q + \bar{n}_B\cdot \mathcal{P})\; \delta^2(\tilde{p}_\perp - \mathcal{P}_\perp)\, \chi^{\beta'}_{n_B}(0) | P_{n_B}\rangle$$

$$\times \langle 0 | \chi^{\alpha}_{n_J}(0)\; \delta(Q\tau_J - n_J\cdot \hat{p}^{n_J})\; \delta(\bar{n}_J\cdot q + \bar{n}_J\cdot \mathcal{P})\; \delta^2(\tilde{p}_\perp + \mathcal{P}_\perp)\, \bar{\chi}^{\alpha'}_{n_J}(0) | 0\rangle$$

$$+ \chi \leftrightarrow \bar{\chi} \Big\}$$

$$= \frac{d\sigma_0}{dx\, dQ^2} \int dt_J\, dt_B\, dk_S\; \delta\Big(\tau - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\Big)\, S(k_S,\mu)$$

$$\times \int d^2p_\perp\; J_q(t_J - \vec{p}_\perp^{\,2}, \mu)\, \big[H_q(Q^2,\mu)\, B_q(t_B, x, \vec{p}_\perp^{\,2}, \mu) + H_{\bar{q}}(Q^2,\mu)\, B_{\bar{q}}(t_B, x, \vec{p}_\perp^{\,2}, \mu) \big]$$



where $\dfrac{d\sigma_0}{dx\, dQ^2} = \dfrac{2\pi \alpha_{em}^2}{Q^4}\, [(1-y)^2 + 1]$ where $y = \dfrac{Q^2}{xs}$

Hard function:

$$H_{q,\bar{q}}(Q^2,\mu) = |C(Q^2,\mu)|^2 \cdot Q_f^2 \qquad (\gamma^* \text{ channel only})$$

Soft function:

$$S(k_S,\mu) = \frac{1}{N_C} \mathrm{Tr} \sum_{X_S} |\langle X_S | Y^\dagger_{n_J} Y_{n_B} | 0\rangle|^2\; \delta\Big(k_S - \sum_{i\in X_S} [\theta(n_B\cdot k - n_J\cdot k)\, n_J\cdot k + \theta(n_J\cdot k - n_B\cdot k)\, n_B\cdot k]\Big)$$

Jet function:

$$J_q(\tilde{p}^- k^+ + \tilde{p}_\perp^2, \mu) = \frac{(2\pi)^2}{N_C} \int \frac{dy^-}{2\tilde{p}^-}\, e^{ik^+ y^-/2}\, \mathrm{Tr} \langle 0 | \frac{\bar{n}\!\!\!/}{2} \chi_n\Big(y^- \frac{n}{2}\Big)\, \delta(\tilde{p}^- + \bar{n}\cdot \mathcal{P})\, \delta^2(\tilde{p}_\perp + \mathcal{P}_\perp)\, \bar{\chi}_n(0) | 0\rangle$$

$$(J_{\bar{q}} = J_q \quad \chi \leftrightarrow \bar{\chi})$$

Beam function:

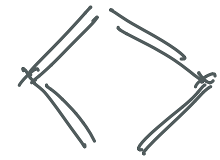
$$B_q\Big(\tilde{p}^- k^+, \frac{\tilde{p}^-}{P^-}, \tilde{p}_\perp^2, \mu\Big) = \frac{\theta(\tilde{p}^-)}{\tilde{p}^-} \int \frac{dy^-}{4\pi}\, e^{ik^+ y^-/2}$$

$$\times \langle P_n(P^-) | \bar{\chi}_n\Big(y^- \frac{n}{2}\Big) \frac{\bar{n}\!\!\!/}{2} \Big[\delta(\tilde{p}^- - \bar{n}\cdot \mathcal{P})\, \frac{1}{\pi}\, \delta(\tilde{p}_\perp^2 - \mathcal{P}_\perp^2)\, \chi_n(0) \Big] | P_n(P^-)\rangle$$

$$(B_{\bar{q}} \quad \chi \leftrightarrow \bar{\chi})$$

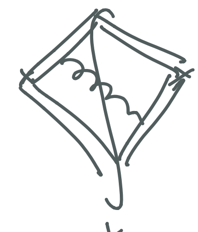
1-loop results:

$$H(Q^2,\mu) = 1 + \frac{\alpha_s(\mu)C_F}{2\pi}\left(-\ln^2\frac{\mu^2}{Q^2} - 3\ln\frac{\mu^2}{Q^2} - 8 + \frac{\pi^2}{6}\right)$$

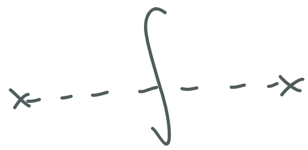


$$S(k_s,\mu) = \delta(k_s)$$

$$+ \frac{\alpha_s(\mu)C_F}{4\pi}\left\{\frac{\pi^2}{3}\delta(k) - \frac{16}{\mu}\left[\frac{\theta(k)\ln(k/\mu)}{k/\mu}\right]_+\right\}$$



$$J(t,\mu) = \delta(t)$$



$$+ \frac{\alpha_s(\mu)C_F}{4\pi}\left\{(7-\pi^2)\delta(t) - \frac{3}{\mu^2}\left[\frac{\theta(t)}{t/\mu^2}\right]_+ + \frac{4}{\mu^2}\left[\frac{\theta(t)\ln(t/\mu^2)}{t/\mu^2}\right]_+\right\}$$

etc.

Beam functions $\mathcal{B}_q(t,x,p_\perp^2,\mu)$ match onto ordinary PDFs:

$$f_q\Big(\underbrace{\frac{\tilde{p}^-}{P^-}}_{=x},\mu\Big) = \theta(\tilde{p}^-)\left\langle P_n(P^-)\right|\bar{\chi}_n(0)\frac{\bar{n}\!\!\!/}{2}\left[\delta(\tilde{p}^- - \bar{n}\cdot\mathcal{P})\chi_n(0)\right]\left|P_n(P^-)\right\rangle$$

matching: $$\mathcal{B}_q(t,x,\vec{p}_\perp^{\,2},\mu) = \sum_{j=q,g}\int_x^1 \frac{dz}{z}\,\mathcal{I}_{qj}(t,z,\vec{p}_\perp^{\,2},\mu)\,f_j\left(\frac{x}{z},\mu\right)$$

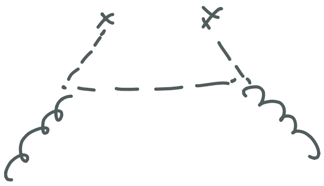
$$\mathcal{I}_{qq}(t,z,\vec{p}_\perp^{\,2},\mu) = \frac{1}{\pi}\delta(t)\delta(1-z)\delta(\vec{p}_\perp^{\,2})$$

$$+ \frac{\alpha_s C_F}{2\pi^2}\theta(z)\left\{\frac{2}{\mu^2}\left[\frac{\theta(t)\ln(t/\mu^2)}{t/\mu^2}\right]_+\delta(1-z)\delta(\vec{p}_\perp^{\,2})\right.$$

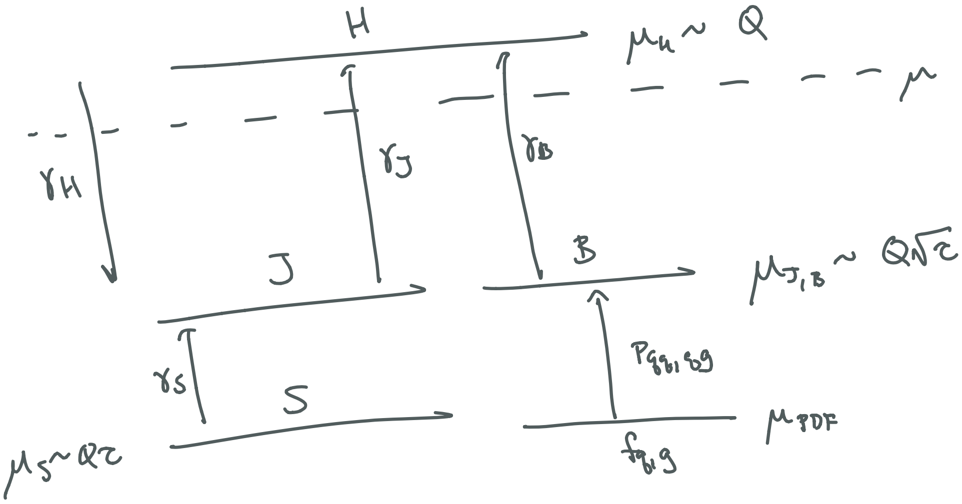
$$\left.+ \frac{1}{\mu^2}\left[\frac{\theta(t)}{t/\mu^2}\right]_+\left[P_{qq}(z) - \frac{3}{2}\delta(1-z)\right]\delta\left(\vec{p}_\perp^{\,2} - \frac{(1-z)t}{z}\right)\right.$$

$$+\,\delta(t)\,\delta(\vec p_\perp^{\,2})\left(\left[\frac{\theta(1-z)\ln(1-z)}{1-z}\right]_+(1+z^2)-\frac{\pi^2}{6}\delta(1-z)\right.$$
$$\left.+\,\theta(1-z)\left(1-z-\frac{1+z^2}{1-z}\ln z\right)\right)\Bigg\}$$

$$\mathcal{I}_{qg}(t,z,\vec p_\perp^{\,2},\mu)=\frac{\alpha_s(\mu)T_F}{2\pi^2}\,\theta(z)\left\{\frac{1}{\mu^2}\left[\frac{\theta(t)}{t/\mu^2}\right]_+P_{qg}(z)\,\delta\!\left(\vec p_\perp^{\,2}-\frac{(1-z)}{z}t\right)\right.$$
$$\left.+\,\delta(t)\,\delta(\vec p_\perp^{\,2})\left[P_{qg}(z)\ln\frac{1-z}{z}+2\,\theta(1-z)\,z(1-z)\right]\right\}$$



RG Evolution



Each function obeys an RGE eq:

$$\mu\frac{d}{d\mu}H(Q^2,\mu)=\gamma_H(\mu)\,H(Q^2,\mu)\qquad \gamma_H(\mu)=2\Gamma_{cusp}[\alpha_s(\mu)]\ln\frac{Q^2}{\mu^2}+\gamma_H[\alpha_s(\mu)]$$

[Laplace space $\quad\tilde f(\nu)=\int dt\,e^{-\nu t}f(t)$]:

$$\mu\frac{d}{d\mu}\tilde B(\nu,\mu)=\gamma_{\tilde B}(\nu,\mu)\,\tilde B(\nu,x,\mu)\qquad \gamma_{\tilde B}(\mu)=2\Gamma_{cusp}[\alpha_s(\mu)]\ln(\mu^2\nu e^{\gamma_E})+\gamma_B[\alpha_s(\mu)]$$

$$\mu\frac{d}{d\mu}\tilde J(\nu,\mu)=\gamma_{\tilde J}(\nu,\mu)\,\tilde J(\nu,\mu)\qquad =\gamma_{\tilde J}(\mu)$$

$$\mu\frac{d}{d\mu}\tilde S(\nu,\mu)=\gamma_{\tilde S}(\nu,\mu)\,\tilde S(\nu,\mu)\qquad \gamma_{\tilde S}(\mu)=-4\Gamma_{cusp}[\alpha_s(\mu)]\ln(\mu\nu e^{\gamma_E})+\gamma_S[\alpha_s(\mu)]$$

RG inv: $\quad\gamma_H+\gamma_B+\gamma_J+\gamma_S=0.$

Solutions:

$$H(Q^2,\mu) = H(Q^2,\mu_H)\,\exp\left(\int_{\mu_H}^{\mu} \frac{d\mu'}{\mu'}\,\gamma_H(\mu')\right)$$

$$\tilde{F}(\nu,\mu) = \tilde{F}(\nu,\mu_F)\,\exp\left(\int_{\mu_F}^{\mu} \frac{d\mu'}{\mu'}\,\gamma_{\tilde{F}}(\mu')\right) \qquad F = J, B, S$$

Achieves:

$$\tilde{\sigma}(\nu) \sim 1 + \frac{\alpha_s}{4\pi}\left(C_{12}\ln^2\nu + C_{11}\ln\nu + C_{10}\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left(C_{24}\ln^4\nu + C_{23}\ln^3\nu + \dots + C_{20}\right)$$

$\Gamma \sim \alpha_s$, $\gamma_F \sim 1$ $\Gamma \sim \alpha_s^2$, $\gamma_F \sim \alpha_s$ $\Gamma \sim \alpha_s^3$, $\gamma_F \sim \alpha_s^2$ ...

$$\downarrow$$

$$\sim \exp\left\{\frac{\alpha_s}{4\pi}\left(C_{12}\ln^2\nu + C_{11}\ln\nu + C_{10}\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left(C_{23}\ln^3\nu + C_{22}\ln^2\nu + C_{21}\ln\nu + C_{20}\right) + \dots\right\}$$

LL   NLL N^2LL N^3LL --

$\ln\nu \sim \frac{1}{\alpha_s}$ $\sim \frac{1}{\alpha_s}$ 1 α_s α_s^2 ---

Comparing most "traditional" resummations vs. SCET:

$\downarrow$

equiv of $\mu_{H,B,J,S}$ usually pre-fixed $\longrightarrow$ remain variable

$\Rightarrow$ improved results in mom. space & uncertainty estimations

Q —— H —— $\mu_H \sim Q$

$\frac{Q}{\sqrt{\nu}}$ —— J —— $\mu_J \sim Q\sqrt{\tau}$

$\frac{Q}{\nu}$ —— S —— $\mu_S \sim Q\tau$

$\longrightarrow$ hard to transform $\alpha_s\left(\frac{Q}{\nu}\right)$

similar issue in $q_T \leftrightarrow b$ space transforms for TMD cross sections