I. EXERCISE #1: EIKONAL SOFT GLUON RADIATION

Consider the amount of radiation produced in the high-energy scattering of two quarks. One quark moves with large momentum p^+ along the $+z$ light cone; the other moves with large momentum q^- along the $-z$ light cone. We want to study the cross section to produce a gluon with momentum k at mid-rapidity. That is, we are interested in the kinematic limit $k^+ \ll p^+$ and $k^- \ll q^-$.

There are 5 diagrams contributing to this process, labeled as diagrams $A - E$ below.

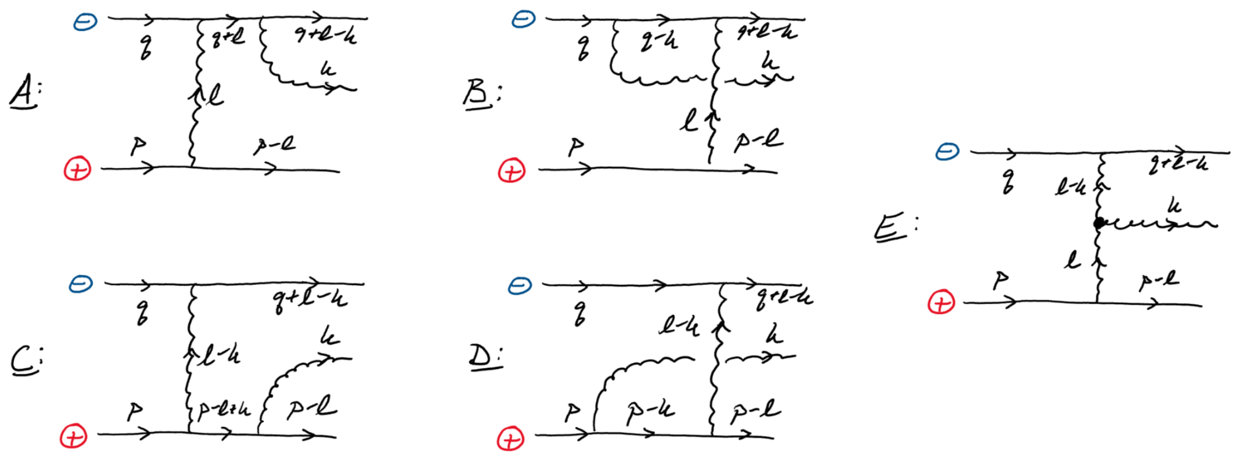

FIG. 1: Feynman diagrams contributing to the process $qq \to qqG$ at lowest order.

A. Calculation

First consider QED rather than QCD, where we have photons instead of gluons in diagrams $A - E$.

1. Compute diagrams $A + B$ for QED using the approximation of eikonal kinematics.

2. Obtain diagrams $C + D$ for QED by analogy with diagrams $A + B$.

3. What is the total cross section to radiate soft photons in QED in the eikonal approximation?

Then return to the real case of QCD at hand.

1. Recompute diagrams $A + B$ for QCD

2. The sum of diagrams $A - E$ can be shown to give

$$\mathcal{M} = 4ig^3 f^{abc}[t^b] \otimes [t^c] \frac{s}{\ell_T^2} \left[\frac{k_T^\mu}{k_T^2} - \frac{(k-\ell)_T^\mu}{(k-\ell)_T^2} \right] \epsilon_\mu^*(k). \tag{1}$$

   What is the cross section to radiate the soft gluon k in QCD? How does it depend on the rapidity $y = \frac{1}{2} \ln \frac{k^+}{k^-}$?

B. Discussion

1. What does it take to make a classical charge radiate electromagnetic power? How does this explain what you saw in the case of QED?

2. What makes the answer different in the case of QCD? Is this difference unique to QCD?

3. How important is the total radiative correction when the collision energy is large?

4. What do you expect would happen if you computed *another* eikonal gluon emission: $qq \to qqGG$?

II. EXERCISE #2: EVOLUTION OF THE GLUON DENSITY

One approximate description of how the gluon density radiated as in Exercies #1 depends on energy is given by the Gribov-Levin-Ryskin / Mueller-Qiu equation. Under a separability ansatz, the GLR-MQ equation can be written as an ordinary differential equation:

$$\frac{d\rho}{dY} = \alpha_s a \rho - \frac{\alpha_s^2 b}{Q^2} \rho^2 \tag{2}$$

with a, b some positive dimensionless constants and $Y \equiv \ln \frac{s}{Q^2} = \ln \frac{1}{x}$ the total rapidity interval of the collision.

A. Calculation

- What is the solution of the GLR-MQ equation in the weak coupling limit $\alpha_s \ll 1$?
- What is the full solution, including the nonlinear term?
- What is the asymptotic behavior of the gluon density ρ at large x and at small x?

B. Discussion

- What is the physical difference between the two regimes seen above?
- How large does the gluon density ρ have to be for the nonlinear term to become important, even at weak coupling?
- What does this imply for pQCD in the nonlinear regime?
- What does this imply for the small-x "radiation regime" of proton structure?