SIDIS process: $e(l) + P(p) \to e'(l') + h(P_h) + X$

- Since Sivers and Collins function are defined in the CM frame of p and p' where

$$p^\mu = \frac{\sqrt{S_h}}{2}(1,0,0,1) = \sqrt{\frac{S_h}{2}}\,\bar{n}^\mu_{CM} \qquad \bar{n}^\mu_{CM} = \frac{1}{\sqrt{2}}(1,0,0,1) = [1^+, 0^-, 0_\perp]$$

$$P_h^\mu = \frac{\sqrt{S_h}}{2}(1,0,0,-1) = \sqrt{\frac{S_h}{2}}\,n^\mu_{CM} \qquad n^\mu_{CM} = \frac{1}{\sqrt{2}}(1,0,0,-1) = [0^+, 1^-, 0_\perp]$$

$$S_h = (p+P_h)^2 = 2p\cdot P_h$$

In this frame, q^μ has transverse momentum, without loss of generality, we could choose the transverse momentum along x-direction, i.e.,

$$q^\mu = (q_0, q_T, q_L) \qquad q^2 = -Q^2 = q_0^2 - q_T^2 - q_L^2$$

This frame will be called CM frame

we'll assume $q_0 < 0$, $q_T < 0$, $q_L < 0$

- Hadron frame is the frame where experiments (close to) try to present the results

In hadron frame, we should have

$$p^\mu \propto (1,0,0,1)$$

$$P_h^\mu \propto (a,b,0,c)$$

$$q^\mu = (0,0,0,-Q)$$

How could one transform from CM frame to hadron frame

① Boost along x-direction, such that $q^\mu_{CM} \Rightarrow$ no transverse component

② Boost along z-direction, such that q^μ has no "zero"-component ($q^0 \to 0$)

after above two boosts, both p^μ and P_h^μ will have transverse component

③ Boost along x-direction again such that p^μ has no transverse component

The result can be found in mathematica file, we only write the final result here!

$$P^\mu = \frac{\sqrt{S_h}}{2Q}\,(q_{0,CM} - q_{L,CM})\,(1,0,0,1)$$

$$P_h^\mu = \frac{\sqrt{S_h}}{2Q}\,\frac{1}{q_{0,CM} - q_{L,CM}}\,(Q^2+q_T^2,\ -2q_T Q,\ 0,\ -Q^2+q_T^2)$$

$$q^\mu = (0,0,0,-Q)$$

NOTE:

$$x_B = \frac{Q^2}{2P\cdot q} \Rightarrow \frac{Q^2}{x_B} = \underset{\text{CM frame}}{2P\cdot q} = 2\frac{\sqrt{S_h}}{2}(q_{0,CM} - q_{L,CM}) = \sqrt{S_h}\,(q_{0,CM} - q_{L,CM}) \Rightarrow$$

$$z_h = \frac{P\cdot P_h}{P\cdot q} \Rightarrow S_h = 2P\cdot P_h = z_h\, 2P\cdot q = \frac{z_h}{x_B} Q^2$$

$$q_{0,CM} - q_{L,CM} = \frac{Q^2}{x_B\sqrt{S_h}} = \frac{Q^2}{x_B\sqrt{\frac{z_h}{x_B}}\,Q} = \frac{Q}{\sqrt{z_h x_B}}$$

Thus

$$P^\mu = \frac{Q}{2x_B}(1,0,0,1) = \frac{Q}{\sqrt{2}\,x_B}\bar{n}^\mu \qquad \left(\text{here } \bar{n}^\mu \equiv \bar{n}^\mu_{CM} = \frac{1}{\sqrt{2}}(1,0,0,1)\right)$$

$$P_h^\mu = \frac{z_h}{2Q}(Q^2+q_T^2,\ -2q_T Q,\ 0,\ -Q^2+q_T^2)$$

denote $q_\perp = |q_{T,CM}| =: -q_T$ (note: from beginning we assume $q_{T,CM} < 0$)

thus

$$P_h^\mu = \frac{z_h}{2Q}(Q^2+q_\perp^2,\ 2q_\perp Q,\ 0,\ -Q^2+q_\perp^2)$$

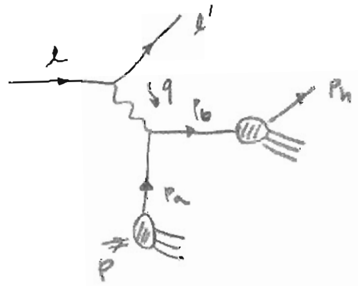
$\Rightarrow$ in hadron frame $\quad P_{h\perp} = \frac{z_h}{2Q}2q_\perp Q = z_h\, q_\perp = z_h\,\underbrace{|q_{T,CM}|}_{\text{transverse momentum of } q^\mu \text{ in CM-frame!!}}$

$$P_h^\mu = \frac{z_h}{2Q}\left(Q^2+q_\perp^2\ ,\ 2q_\perp Q,\ 0,\ -Q^2+q_\perp^2 \right)$$

$$= \frac{z_h}{2Q}\left\{ (Q^2, 0, 0, -Q^2) + (q_\perp^2, 0, 0, q_\perp^2) + (0, 2q_\perp Q, 0, 0) \right\}$$

$$= \frac{z_h}{2Q}\left\{ \sqrt{2}\, Q^2 n^\mu + \sqrt{2}\, q_\perp^2\, \bar{n}^\mu + 2Q\, q_\perp^\mu \right\}$$

$$P_h^\mu = \frac{z_h q_\perp^2}{\sqrt{2}\, Q}\bar{n}^\mu + \frac{z_h Q}{\sqrt{2}} n^\mu + z_h q_\perp^\mu$$



$$P_h = z\, p_b + p_\perp \Rightarrow p_b = \frac{P_h - p_\perp}{z}$$

$$(2\pi)^4 \delta^4(p_a + q - p_b)$$

$$p_a + q = p_b$$

$$p_a^\mu = xP^\mu + k_\perp^\mu = \frac{x}{x_B}\frac{Q}{\sqrt{2}}\bar{n}^\mu + k_\perp^\mu$$

$$p_b^\mu = \frac{P_h^\mu - p_\perp^\mu}{z} = \frac{z_h}{z}\frac{q_\perp^2}{\sqrt{2}Q}\bar{n}^\mu + \frac{z_h}{z}\frac{Q}{\sqrt{2}}n^\mu + \left(\frac{z_h}{z}q_\perp^\mu - \frac{p_\perp^\mu}{z}\right)$$

$$q^\mu = (0, 0, 0, -Q) = \frac{1}{\sqrt{2}}(-Q)\bar{n}^\mu + \frac{Q}{\sqrt{2}}n^\mu$$

$$\delta^4(p_a+q-p_b) = \delta(p_a^+ + q^+ - p_b^+)\,\delta(p_a^- + q^- - p_b^-)\,\delta^2(\perp\text{-part})$$

$$= \delta\left(\frac{x}{x_B}\frac{Q}{\sqrt{2}} - \frac{Q}{\sqrt{2}} - \frac{z_h}{z}\frac{q_\perp^2}{\sqrt{2}Q}\right)\delta\left(0 + \frac{Q}{\sqrt{2}} - \frac{z_h}{z}\frac{Q}{\sqrt{2}}\right)$$

$$\times\, \delta^2\left(\vec{k}_\perp - \left(\frac{z_h}{z}q_\perp - \frac{p_\perp}{z}\right)\right)$$

★ only neglect $\frac{q_\perp^2}{Q^2}$, recover usual $x \to x_B$

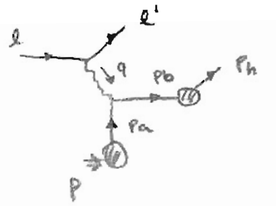
$$\approx \delta\left(\frac{x}{x_B}\frac{Q}{\sqrt{2}} - \frac{Q}{\sqrt{2}}\right)\delta\left(\frac{Q}{\sqrt{2}} - \frac{z_h}{z}\frac{Q}{\sqrt{2}}\right)\delta^2\left(k_\perp - \frac{P_{h\perp} - p_\perp}{z}\right)$$

$$= \frac{x_B}{Q/\sqrt{2}}\,\delta(x - x_B) \times \frac{z}{Q/\sqrt{2}}\,\delta(z - z_h) \times z^2\,\delta^2(z k_\perp + p_\perp - P_{h\perp})$$

$$= \frac{2 x_B z^3}{Q^2}\,\delta(x - x_B)\,\delta(z - z_h)\,\delta^2(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp})$$

$$= \frac{2 x_B z_h^3}{Q^2}\,\delta(x - x_B)\,\delta(z - z_h)\,\delta^2(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp})$$

Normalization of cross-section



$$S_{ep} = (P+l)^2 = 2P\cdot l$$

$$Q^2 = -q^2 = -(l-l')^2$$

$$x_B = \frac{Q^2}{2P\cdot q} \qquad z_h = \frac{P\cdot P_h}{P\cdot q} \qquad y = \frac{P\cdot q}{P\cdot l} = \frac{Q^2}{x_B S_{ep}}$$

$$d\sigma = \int dx\, d^2k_\perp\, f_1(x, k_\perp^2) \int dz\, d^2p_\perp\, D_1(z, p_\perp^2)\, d\hat{\sigma}$$

$$d\hat{\sigma} = \frac{1}{2\hat{s}} \overline{|M|^2}\, dPS \qquad \hat{s} = (xP+l)^2 \approx x S_{ep}$$

$$dPS = \frac{d^3l'}{(2\pi)^3 2E'} \frac{d^3P_b}{(2\pi)^3 2E_b} (2\pi)^4 \delta^4(p_a+q-p_b)$$

$$\Downarrow \quad P_b \approx \frac{P_h}{z}$$

$$= \frac{d^3l'}{(2\pi)^3 2E'} \frac{d^3P_h}{(2\pi)^3 2E_h} \frac{1}{z^2} (2\pi)^4 \delta^4(p_a+q-p_b)$$

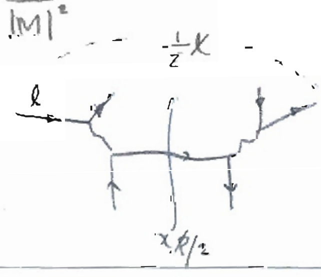
$$\frac{d^3l'}{2E'} = d^4l'\, \delta(l'^2) = \frac{\pi Q^2}{2x_B^2 S_{ep}} dx_B\, dQ^2 = \frac{\pi Q^2}{2x_B} dx_B\, dy$$

$$\frac{d^3P_h}{2E_h} = d^4P_h\, \delta(P_h^2) = dP_h^+ dP_h^-\, d^2P_{h\perp}\, \underbrace{\delta(2P_h^+P_h^- - P_{h\perp}^2)}_{\frac{1}{2P_h^-}\delta\left(P_h^+ - \frac{P_{h\perp}^2}{2P_h^-}\right)} = \frac{dP_h^-}{2P_h^-} d^2P_{h\perp} = \frac{dz_h}{2z_h} d^2P_{h\perp}$$

$$\frac{d\sigma}{dx_B\, dy\, dz_h\, d^2P_{h\perp}} = \int dx\, d^2k_\perp\, f_1(x,k_\perp^2) \int dz\, d^2p_\perp\, D_1(z,p_\perp^2)$$

$$* \left(\frac{\pi Q^2}{2x_B} \frac{1}{2z_h}\right) \frac{1}{(2\pi)^6} \frac{1}{z^2} (2\pi)^4 \delta^4(p_a+q-p_b)$$

$$* \frac{1}{2\hat{s}} \overline{|M|^2}$$

$\overline{|M|^2} \sim$  $\left(\frac{1}{2}\right)$ from spin-average

In the frame where $q^\mu = (0, 0, 0, -Q)$, one could always express

$$l^\mu = \frac{Q}{2}(\cosh\psi,\ \sinh\psi\cos\phi,\ \sinh\psi\sin\phi,\ -1)$$

$$l'^\mu = \frac{Q}{2}(\cosh\psi,\ \sinh\psi\cos\phi,\ \sinh\psi\sin\phi,\ +1) \qquad q^\mu = l^\mu - l'^\mu$$

where $\cosh\psi = \frac{2x_B s_{ep}}{Q^2} - 1 = \frac{2}{y} - 1$

$$l^\mu = \frac{Q}{2}\left[\cosh\psi\, T^\mu + \sinh\psi\cos\phi\, X^\mu + \sinh\psi\sin\phi\, Y^\mu - Z^\mu\right]$$

$$l'^\mu = \frac{Q}{2}\left[\cosh\psi\, T^\mu + \sinh\psi\cos\phi\, X^\mu + \sinh\psi\sin\phi\, Y^\mu + Z^\mu\right]$$

$$L^{\mu\nu} = \frac{1}{2}\mathrm{Tr}[\not{l}\gamma^\mu \not{l}'\gamma^\nu]$$

$$= \frac{1}{2}\times 4(l^\mu l'^\nu + l^\nu l'^\mu - g^{\mu\nu} l\cdot l')$$

$$= 2(l^\mu l'^\nu + l^\nu l'^\mu - g^{\mu\nu} l\cdot l')$$

$$\frac{2L^{\mu\nu}}{Q^2} = (1+\cosh^2\psi)\underbrace{(X^\mu X^\nu + Y^\mu Y^\nu)}_{-g_\perp^{\mu\nu}} + 2\sinh^2\psi\, T^\mu T^\nu + \sinh 2\psi\cos\phi\,(T^\mu X^\nu + T^\nu X^\mu)$$

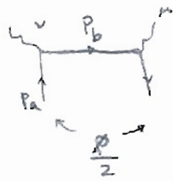
$$+ \sinh 2\psi\sin\phi\,(T^\mu Y^\nu + T^\nu Y^\mu) + \sinh^2\psi\sin 2\phi\,(X^\mu Y^\nu + X^\nu Y^\mu) + \sinh^2\psi\cos 2\phi\,(X^\mu X^\nu - Y^\mu Y^\nu)$$

$$\cosh\psi = \frac{2}{y} - 1$$

$$1+\cosh^2\psi = 1 + \left(\frac{2}{y}-1\right)^2 = \frac{4}{y^2}(1 - y + y^2/2)$$

$$\sinh^2\psi = \cosh^2\psi - 1 = \left(\frac{2}{y}-1\right)^2 - 1 = \frac{4}{y^2}(1-y)$$

hadronic tensor



should be $\mathrm{Tr}[\frac{\not{p}_a}{2}\gamma^\mu \not{p}_b \gamma^\nu] \sim \frac{x}{z}\,\mathrm{Tr}[\frac{\not{P}}{2}\gamma^\mu \not{P}_h \gamma^\nu]$

$$W_{UU}^{\mu\nu} = \mathrm{Tr}[\frac{\not{P}}{2}\gamma^\mu \not{P}_h \gamma^\nu]\; f_1 * D_1$$

$$W_{UT}^{\mu\nu} = \mathrm{Tr}[\gamma_\alpha \gamma^\mu \not{P}_h \gamma^\nu]\, \epsilon^{\alpha\beta\rho\sigma} P_\beta k_{\perp\rho} S_{\perp\sigma}\; \frac{1}{M} f_{1T}^{\perp} * D_1$$

NOTE: to project D_1, we need $\not{P}_h$, NOT "$\not{n}$" $\hookrightarrow (1,0,0,-1)$

However, $$P_h^\mu = \frac{z_h Q}{\sqrt{2}} n^\mu + \frac{z_h q_\perp^2}{\sqrt{2}Q}\bar{n}^\mu + z_h q_\perp^\mu$$
$$= \frac{z_h Q}{\sqrt{2}} n^\mu + O(q_\perp)$$

again, keep the leading contribution (consistent with TMD)

$$W_{UU}^{\mu\nu} = \mathrm{Tr}[\frac{\not{P}}{2}\gamma^\mu \not{n} \gamma^\nu] * \frac{z_h Q}{\sqrt{2}}\; f_1 * D_1$$

$$W_{UT}^{\mu\nu} = \mathrm{Tr}[\gamma_\alpha \gamma^\mu \not{n} \gamma^\nu] * \frac{z_h Q}{\sqrt{2}}\, \epsilon^{\alpha\beta\rho\sigma} P_\beta k_{\perp\rho} S_{\perp\sigma}\; \frac{1}{M} f_{1T}^{\perp} * D_1$$

Calculate in detail

$$W_{UU}^{\mu\nu} = \frac{Q}{\sqrt{2}x_B}\frac{z_h Q}{\sqrt{2}}\,\mathrm{Tr}[\not{\bar{n}}\gamma^\mu \not{n}\gamma^\nu]\; f_1 * D_1$$
$$= \frac{z_h}{x_B}\, 2Q^2 \underbrace{(-g^{\mu\nu} + \bar{n}^\mu n^\nu + \bar{n}^\nu n^\mu)}_{-g_\perp^{\mu\nu}}\; f_1 * D_1$$
$$= \frac{z_h}{x_B}\, 2Q^2\, (-g_\perp^{\mu\nu})\; f_1 * D_1$$

$$W^{\mu\nu}_{UT} = 4(g_\alpha{}^\mu n^\nu - n_\alpha g^{\mu\nu} + g_\alpha{}^\nu n^\mu) * \frac{z_h Q}{\sqrt{2}} \frac{Q}{\sqrt{2} x_B} \epsilon^{\alpha \bar{n} k_\perp s_\perp} \frac{1}{M} f_{1T}^\perp * D_1$$

$$= \frac{z_h}{x_B} 2Q^2 \Big(\epsilon^{\bar{n} n s_\perp k_\perp} (-g^{\mu\nu}) + \underbrace{\epsilon^{\mu \bar{n} k_\perp s_\perp} n^\nu + \epsilon^{\nu \bar{n} k_\perp s_\perp} n^\mu}_{I^{\mu\nu}} \Big) \frac{1}{M} f_{1T}^\perp D_1$$

NOTE: $I^{\mu\nu} * \begin{pmatrix} X^\mu X^\nu + Y^\mu Y^\nu \\ T^\mu X^\nu + T^\nu X^\mu \\ T^\mu Y^\nu + T^\nu Y^\mu \\ X^\mu Y^\nu + X^\nu Y^\mu \\ X^\mu X^\nu - Y^\mu Y^\nu \end{pmatrix} \Rightarrow$ "zero"

only worry about $T^\mu T^\nu = \frac{1}{\sqrt{2}}(\bar{n}^\mu + n^\mu)(\bar{n}^\nu + n^\nu)\frac{1}{\sqrt{2}}$

$$= \frac{1}{2}(\bar{n}^\mu \bar{n}^\nu + n^\mu n^\nu + \bar{n}^\mu n^\nu + \bar{n}^\nu n^\mu)$$

$$I^{\mu\nu} * T^\mu T^\nu = \frac{1}{2} * [2\epsilon^{n \bar{n} k_\perp s_\perp}] = \epsilon^{\bar{n} n s_\perp k_\perp}$$

$$\epsilon^{\bar{n} n s_\perp k_\perp} (-g^{\mu\nu}) T^\mu T^\nu = -\epsilon^{\bar{n} n s_\perp k_\perp}$$

Thus $(-g^{\mu\nu} \epsilon^{\bar{n} n s_\perp k_\perp} + I^{\mu\nu}) T^\mu T^\nu = 0 \Rightarrow W^{\mu\nu}_{UT} T_\mu T_\nu = 0$

only need to care about $(-g_{\mu\nu})$ - term, in other words

$$W^{\mu\nu}_{UT} \to \frac{z_h}{x_B} 2Q^2 \epsilon^{\bar{n} n s_\perp k_\perp} (-g^{\mu\nu}) * \frac{1}{M} f_{1T}^\perp D_1$$

We thus have

$$L_{\mu\nu} W^{\mu\nu}_{UU} = \frac{Q^2}{2} * \frac{z_h}{x_B} 2Q^2 * 2 \quad f_1 * D_1 \quad [1+\cosh^2\psi]$$

$$L_{\mu\nu} W^{\mu\nu}_{UT} = \frac{Q^2}{2} * \frac{z_h}{x_B} 2Q^2 * \epsilon^{\bar{n} n S_\perp k_\perp} * 2 * \frac{1}{M} f_{1T}^{\perp} D_1 \quad [1+\cosh^2\psi]$$

also photon propagator $\left(\frac{1}{Q^2}\right)^2$

$$\frac{d\sigma}{dx_B\, dy\, dz_h\, d^2P_{h\perp}} = \int dx\, d^2k_\perp \int dz\, d^2p_\perp \begin{pmatrix} f_1 \, D_1 \\ \frac{1}{M} f_{1T}^{\perp} D_1 \end{pmatrix} * e^4 e_q^2$$

$$* \left(\frac{\pi Q^2}{2x_B} \frac{1}{2z_h}\right) \frac{1}{(2\pi)^6} \frac{1}{z^2} (2\pi)^4 \delta^4(p_a+q-p_b)$$

$$* \frac{1}{2x S_{ep}} * \left(\frac{1}{Q^2}\right)^2 * \begin{pmatrix} \frac{Q^2}{2} * \frac{z_h}{x_B} 2Q^2 * 2 \\ \frac{Q^2}{2} * \frac{z_h}{x_B} 2Q^2 * 2\, \epsilon^{\bar{n} n S_\perp k_\perp} \end{pmatrix} * \frac{x}{z} * \underbrace{[1+\cosh^2\psi]}_{\frac{4}{y^2}(1-y+y^2/2)}$$

$$\Downarrow$$

$$\delta^4(p_a+q-p_b) = \frac{2x_B z_h^3}{Q^2} \delta(x-x_B)\, \delta(z-z_h)\, \delta^2(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp})$$

$$= e_q^2 \frac{4\pi \alpha_{em}^2}{Q^2 y} \left[1-y+y^2/2\right] * \left[f_1 * D_1 + \epsilon^{\bar{n} n S_\perp k_\perp} \frac{f_{1T}^{\perp}}{M} D_1 \right]$$

Summary:

$$\frac{d\sigma}{dx_B\, dy\, dz_h\, d^2P_{h\perp}} = \sum_q e_q^2 \frac{4\pi\alpha_{em}^2}{Q^2 y}\left[1-y+y^2/2\right]\left[f_1 \otimes D_1 + \epsilon^{\bar{n}n S_\perp k_\perp} \frac{1}{M} f_{1T}^\perp \otimes D_1 \right]$$

where $f_1 \otimes D_1 = \int d^2k_\perp\, d^2p_\perp\, \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp})\, f_1(x_B, k_\perp^2)\, D_1(z_h, p_\perp^2)$

$$\epsilon^{\bar{n}nS_\perp k_\perp}\frac{1}{M} f_{1T}^\perp \otimes D_1 = \int d^2k_\perp\, d^2p_\perp\, \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp})\, \frac{1}{M} f_{1T}^\perp(x_B, k_\perp^2)\, D_1(z_h, p_\perp^2)\, \epsilon^{\bar{n}nS_\perp k_\perp}$$

$$\epsilon^{\bar{n}nS_\perp k_\perp} = \epsilon^{\bar{n}n\alpha\beta} S_{\perp\alpha} k_{\perp\beta}$$

$$= \epsilon_\perp^{\alpha\beta} S_{\perp\alpha} k_{\perp\beta}$$

where $\epsilon^{\bar{n}n\alpha\beta} \equiv \epsilon_\perp^{\alpha\beta} = \begin{cases} +1 & \alpha=1,\ \beta=2 \\ -1 & \alpha=2,\ \beta=1 \end{cases}$

define

$$I^\beta = \int d^2k_\perp\, d^2p_\perp\, \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp})\, \frac{1}{M} f_{1T}^\perp(x_B, k_\perp^2)\, D_1(z_h, p_\perp^2)\, k_\perp^\beta$$

after integration over $d^2k_\perp\, d^2p_\perp$, only left vector $P_{h\perp}$

$$= I * P_{h\perp}^\beta$$

$$I = \frac{I^\beta P_{h\perp\beta}}{-\vec{P}_{h\perp}^2} = \frac{\overbrace{k_\perp \cdot P_{h\perp}}^{-\vec{k}_\perp\cdot\vec{P}_{h\perp}}}{-\vec{P}_{h\perp}^2}(\cdots) = \frac{\vec{k}_\perp\cdot\vec{P}_{h\perp}}{\vec{P}_{h\perp}^2}$$

Thus $I^\beta = \frac{P_{h\perp}^\beta}{\vec{P}_{h\perp}^2}\int d^2\vec{k}_\perp\, d^2\vec{p}_\perp\, (\vec{k}_\perp\cdot\vec{P}_{h\perp})\, \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp})\, \frac{1}{M} f_{1T}^\perp D_1$

$\Downarrow$ define $\frac{\vec{P}_{h\perp}^\beta}{|\vec{P}_{h\perp}|} = \hat{P}_{h\perp}^\beta$

$$= \hat{P}_{h\perp}^\beta \int d^2k_\perp\, d^2p_\perp\, \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp})\, (\vec{k}_\perp\cdot\hat{P}_{h\perp})\, \frac{1}{M} f_{1T}^\perp D_1$$

$$\frac{d\sigma}{dx_B\, dy\, dz_h\, d^2P_{h\perp}} = \sum_q e_q^2 \frac{4\pi\alpha_{em}^2}{Q^2 y}\left(1-y+y^2/2\right)\left[f_1 \otimes D_1 + \epsilon^{\bar{n} n S_\perp \hat{P}_{h\perp}} \frac{1}{M} f_{1T}^\perp \otimes D_1\right]$$

$$\begin{aligned}
\epsilon^{\bar{n} n S_\perp \hat{P}_{h\perp}} &= \epsilon_\perp^{\alpha\beta} S_{\perp\alpha} \hat{P}_{h\perp\beta} \\
&= S_{\perp 1} \hat{P}_{\perp 2} - S_{\perp 2} \hat{P}_{\perp 1} \\
&= \cos\phi_s \sin\phi_h - \sin\phi_s \cos\phi_h \\
&= \sin(\phi_h - \phi_s)
\end{aligned}$$

$$\frac{d\sigma}{dx_B\, dy\, dz_h\, d^2P_{h\perp}} = \sum_q e_q^2 \frac{4\pi\alpha_{em}^2}{Q^2 y}\left(1-y+y^2/2\right)\left[f_1 \otimes D_1 + \sin(\phi_h-\phi_s)\frac{1}{M} f_{1T}^\perp \otimes D_1\right]$$

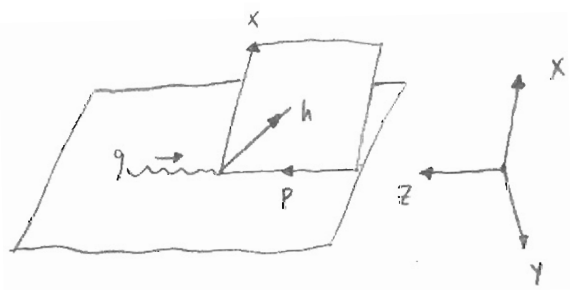
$$f_1 \otimes D_1 = \int f_1(x_B, k_\perp^2)\, D_1(z_h, p_\perp^2)$$

$$\frac{1}{M} f_{1T}^\perp \otimes D_1 = \int \frac{1}{M} f_{1T}^\perp(x_B, k_\perp^2)\, D_1(z_h, p_\perp^2)\, \vec{k}_\perp \cdot \hat{P}_{h\perp}$$

$$\int = \int d^2k_\perp\, d^2p_\perp\, \delta^2(z_h \vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp})$$

In fact, in this "hadron frame" : $\phi_h = 0$

our reference frame : hadron frame!



Trento convention: $\tilde{\phi}_h$, $\tilde{\phi}_s$ counted w.r.t leptonic plane!

$$\tilde{\phi}_h - \tilde{\phi}_s = -[\phi_h - \phi_s]$$

lepton plane   hadron plane

In Trento convention,

$$\frac{d\sigma}{dx_B\, dy\, dz_h\, d^2P_{h\perp}} = \sum_q e_q^2 \frac{4\pi \alpha_{em}^2}{Q^2 y} (1 - y + y^2/2) \left[f_1 \otimes D_1 + \sin(\tilde{\phi}_h - \tilde{\phi}_s) \underline{(-1)} \frac{1}{M} f_{1T}^{\perp} \otimes D_1 \right]$$