

Status of Muon $g - 2$

Luchang Jin

UConn / RBRC

Sept 26, 2019

Brookhaven Forum 2019: Particle Physics and Cosmology in the 2020's

Brookhaven National Lab

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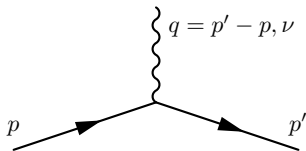
Christoph Lehner (BNL)

Antonin Portelli (Edinburgh)

Cheng Tu (UConn)

and **the RBC/UKQCD collaborations**

- **Introduction**
- Phenomenology determination
- Lattice calculation
- Perspect of near future lattice results

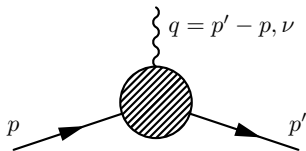


$$\vec{\mu} = g \frac{e}{2m} \vec{L}$$

Dirac equation implies:

$$\bar{u}(p') \gamma_\nu u(p)$$

$$g = 2$$



$$\bar{u}(p') \left(F_1(q^2) \gamma_\nu + i \frac{F_2(q^2) [\gamma_\nu, \gamma_\rho] q_\rho}{4m} \right) u(p)$$

(Euclidean space time)

$$a = F_2(q^2 = 0) = \frac{g - 2}{2}$$

- The quantity a is called the anomalous magnetic moments.
- Its value comes from quantum correction.

Muon $g - 2$: Fermilab E989, J-PARC E34 5 / 39

SM (Model HLbL) 11659181.3 ± 4.0

BNL E821 Exp 11659208.9 ± 6.3

Diff (Exp - SM) 27.6 ± 7.5

3.7σ deviations

New Physics?



INT Workshop INT-19-74W

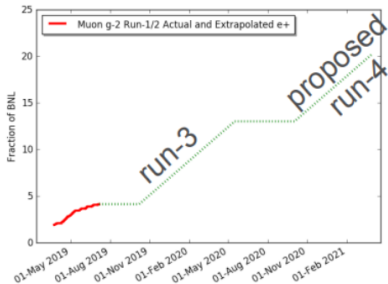
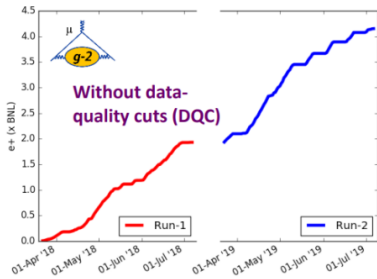
Hadronic contributions to $(g - 2)_\mu$

September 9 - 13, 2019



D. Hertzog's talk at Seattle INT workshop.

Compared to BNL totals



- Next Collaboration meeting is mid November

- MEANWHILE: RUN-3 Starts in October !!!



- J-PARC muon $g - 2$ experiments in preparation.

	$a_\mu \times 10^{10}$	
QED 5-loops	11658471.8853 ± 0.0036	Aoyama, et al, 2012
Weak 2-loops	15.36 ± 0.10	Gnendiger et al, 2013
HVP (LO)	692.5 ± 2.7	RBC-UKQCD and FJ17 combined
	693.26 ± 2.46	KNT18
	693.9 ± 4.0	DHMZ19
HVP (NLO)	-9.93 ± 0.07	Fred Jegerlehner, 2017
HVP (NNLO)	1.22 ± 0.01	Fred Jegerlehner, 2017
HLbL	10.3 ± 2.9	Fred Jegerlehner, 2017
	10.5 ± 2.6	Glasgow Consensus, 2007
SM Theory	11659181.3 ± 4.0	
BNL E821 Exp	11659208.9 ± 6.3	
Exp – SM	27.6 ± 7.5	

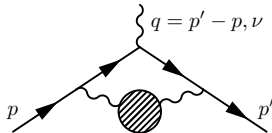
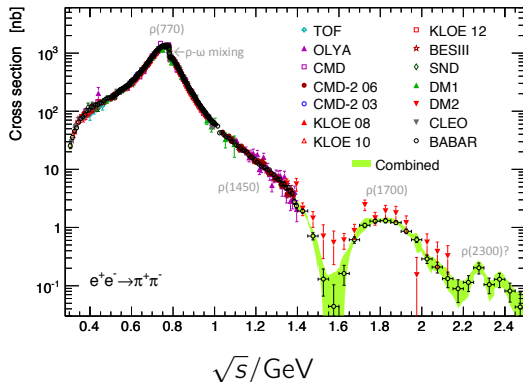
- Introduction
- **Phenomenology determination**
- Lattice calculation
- Perspect of near future lattice results

C. Bouchiat and L. Michel, J. Phys. Radium22(1961) 121

Z. Zhang's talk at Seattle INT workshop.

$$a_{\mu}^{\text{HVP LO}} = \frac{\alpha^2}{3\pi^2} \int_{4m_{\pi}^2}^{\infty} ds \frac{K(s)}{s} R(s), \quad R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

The Dominant $\pi^+\pi^-$ Channel (1)



KNT18; D. Nomura's talk at Seattle INT workshop.

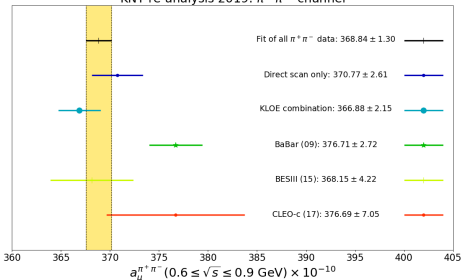
DHMZ19; Z. Zhang's talk at Seattle INT workshop.

- KNT18: $a_\mu^{\text{HVP LO}} = 693.26(2.46) \times 10^{-10}$.
- DHMZ19: $a_\mu^{\text{HVP LO}} = 693.9(4.0) \times 10^{-10}$.

BaBar & KLOE tension

Contribution to $(g-2)_\mu$ from $\pi^+\pi^-$ channel

KNT re-analysis 2019: $\pi^+\pi^-$ channel



Combined Results Fit [<0.6 GeV] + Data [$0.6-1.8$ GeV]

Take into account the correlation of 62% (based on pseudo-data samples) of the two regions

\sqrt{s} range [GeV]	$a_\mu^{\text{had}} [10^{-10}]$ All data	$a_\mu^{\text{had}} [10^{-10}]$ All but BABAR	$a_\mu^{\text{had}} [10^{-10}]$ All but KLOE
threshold - 1.8	$506.9 \pm 1.9_{\text{total}}$	$505.0 \pm 2.1_{\text{total}}$	$510.6 \pm 2.2_{\text{total}}$

2017: $507.1 \pm 2.6_{\text{total}}$

⇒ The difference “All but BABAR” and “All but KLOE” = 5.6 to be compared with 1.9 uncertainty with “All data”

- The local error inflation is not sufficient to amplify the uncertainty
- Global tension (normalisation/shape) not previously accounted for
- Potential underestimated uncertainty in at least one of the measurements?
- Other measurements not precise enough and are in agreement with BABAR or KLOE

⇒ Given the fact we do not know which dataset is problematic, we decide to

- Add half of the discrepancy (2.8) as an additional uncertainty (correcting the local PDG inflation to avoid double counting)
- Take the mean value “All but BABAR” and “All but KLOE” as our central value

Future experiments at Belle II, CMD-3, SND can provide more insight.

Bogdan Malaescu's talk at Seattle INT workshop.

Proposal: *conservative merging* of model-independent HVP combination results

- Basic requirements for the *merging* procedure:
 - *Conservative* (see tensions between experimental data and differences between combinations based on same datasets)
 - *Accounting for correlations between different channels* (understood meaning of systematic uncertainties and identified 15 common ones, DHMZ since arXiv:1010.4180)
 Yields *unavoidable increase of total uncertainty* $693.9 \pm 1.0 \pm 3.4 \pm \boxed{1.6} \pm 0.1_{\psi} \pm 0.7_{\text{QCD}}$
- Proposed *merging* procedure:
 - *Central value*: simple average of the DHMZ and KNT sums of channels
 (the DHMZ and KNT central values are, *by chance*, very similar)
 - *Experimental uncertainties*: in each channel/mass range use max(DHMZ, KNT) and see by how much to increase the corresponding DHMZ uncertainty (sq. difference); enhance the DHMZ *sum of channels (with correlations)* by these amounts (sq. sum)
 - Use $|\text{DHMZ}(\text{ch.}) - \text{KNT}(\text{ch.})| / 2$ as extra systematic in each channel; independent between channels (sign of algebraic difference fluctuates for various channels)
 - o) $\pi\pi$ BABAR/KLOE systematic: $\max(\text{DHMZ B./K. syst.}, |\text{DHMZ}(\pi\pi) - \text{KNT}(\pi\pi)| / 2)$
 (stay conservative, but avoid double-counting the effect of this B./K. tension)
 - o) $\pi^+\pi^-\pi^0$: do not include this systematic (difference understood: 1st/2nd order interp.)

G. Colangelo et al., Phys.Rev.Lett. 118 (2017) no.23, 232001

M. Hoferichter et al., JHEP 1810 (2018) 141; Mainz PRD 100 (2019) no.3, 034520

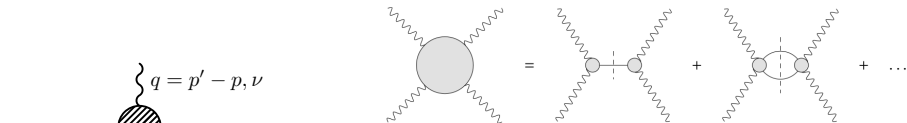


Figure 2. Intermediate states in the direct channel: pion pole and two-pion cut.

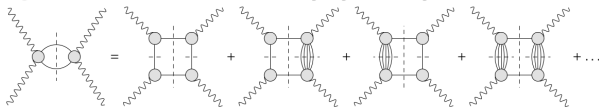


Figure 3. Two-pion contributions to HLbL. Further crossed diagrams are not shown explicitly.

- Pion pole + pion box + rescattering + ...
Estimate/bound short distance contribution.
- “Not in a form as compact as for HVP”.
Need many inputs from experiments for lattice.

G. Colangelo's talk at Seattle INT workshop.

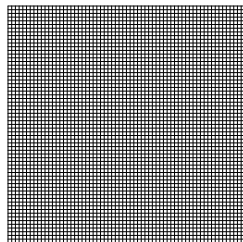
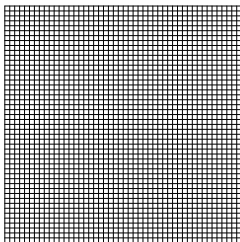
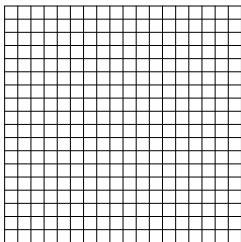
Contribution	PdRV(09)	N/JN(09)	J(17)	White Paper
π^0, η, η' -poles	114 ± 13	99 ± 16	95.45 ± 12.40	93.8 ± 4.0
π, K -loop/box	-19 ± 19	-19 ± 13	-20 ± 5	-16.4 ± 0.2
S -wave $\pi\pi$	—	—	—	-8 ± 1
scalars	-7 ± 7	-7 ± 2	-5.98 ± 1.20	} -2 ± 3
tensors	—	—	1.1 ± 0.1	
axials	15 ± 10	22 ± 5	7.55 ± 2.71	
q -loops / SD	2.3	21 ± 3	22.3 ± 5.0	10 ± 10
total	105 ± 26	116 ± 39	100.4 ± 28.2	$85 \pm XX$

- In unit of 10^{-11} for this slides.
- PdRV = Prades, de Rafael, Vainshtein (“Glasgow consensus”).
- N = Nyffeler, J = Jegerlehner.
- Uncertainties added in quadrature: $XX=12$.
- Uncertainties added linearly: $XX=21$.

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The QCD partition function in Euclidean space time:

$$Z = \int [\mathcal{D}U_\mu] e^{-S_G[U]} \det(D[m_l, U])^2 \det(D[m_s, U]) \quad (1)$$



(Left) 19×19 Go board (Middle) 48×48 (Right) 64×64

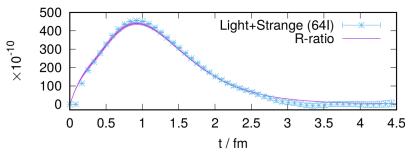
The configuration is stored in position space. The reason is that the action is local in position space. Working in position makes the calculation simpler.

This is in contrast to analytical perturbative calculation, where interaction only happens occasionally. So it is advantageous to work in momentum space, where the propagator can be diagonalized.

T. Blum 2003; D. Bernecker, H. Meyer 2011.

$$C(t) = \frac{1}{3} \sum_{\vec{x}} \sum_{j=0,1,2} \langle J_j(\vec{x}, t) J_j(0) \rangle$$

$$a_\mu^{\text{HVP LO}} = \sum_{t=0}^{+\infty} w(t) C(t)$$



- In Euclidean space-time, $C(t)$ decreases exponentially as t increases.

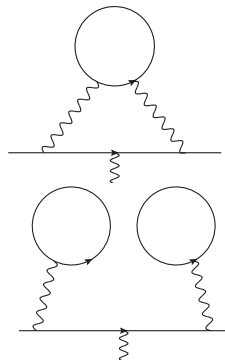
For $t \sim 1$ fm, $C(t) \sim e^{-m_\rho t}$.

For $t \rightarrow \infty$, $C(t) \sim e^{-2m_\pi t}$.

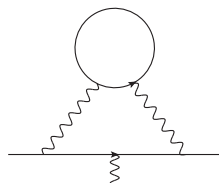
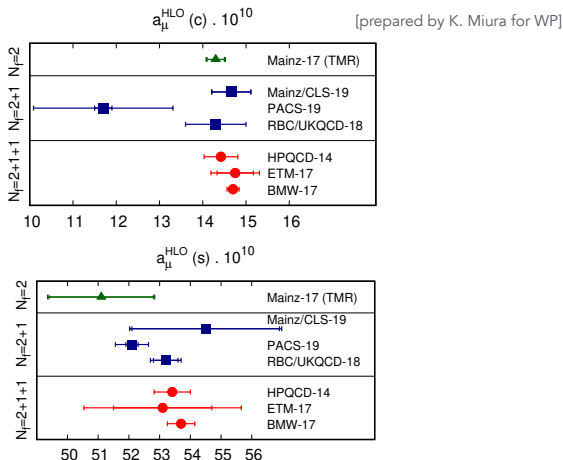
Lattice statistical error: $\delta C(t) \sim e^{-m_\pi t}$.

- For $t \lesssim 1$ fm, $w(t) \sim t^4$.

For $t \rightarrow \infty$ ($m_\mu t \gg 1$), $w(t) \sim t^2$.



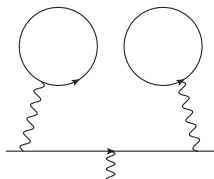
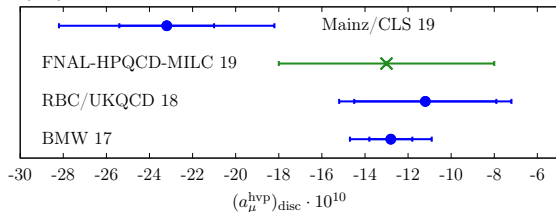
A. El-Khadra's talk at Seattle INT workshop.



- Charm and Strange quark connected is very accurate.

A. El-Khadra's talk at Seattle INT workshop.

[prepared by K. Miura for WP]

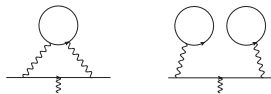
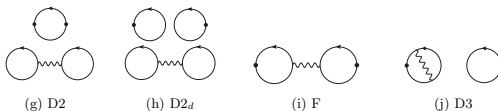
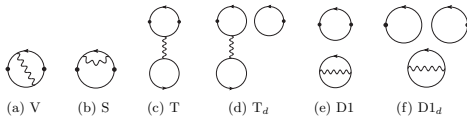
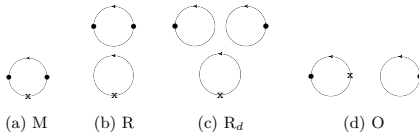


- Include u , d , s quark contribution.
- Strong suppression from charge factor.

$$\text{-- con: } e_u^2 + e_d^2 = 5/9$$

$$\text{-- discon: } (e_u + e_d)^2 = 1/9$$

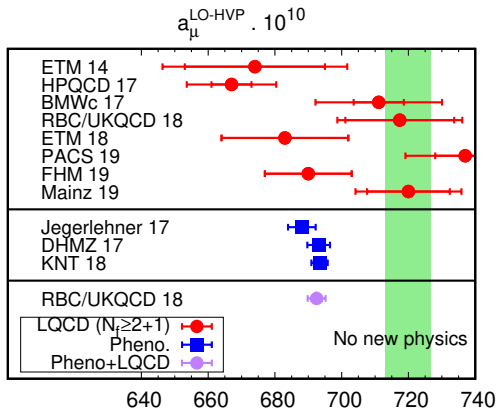
- Vanishes in the flavor $SU(3)$ limit $\Rightarrow \sim (m_{u,d} - m_s)^2$.

Isospin
limitQED
correctionsStrong
isospin
breaking

A. El-Khadra's talk at Seattle INT workshop.

[prepared by K. Miura for WP]

[V.]



Calculation of the Hadronic Vacuum Polarization Contribution to the Muon Anomalous Magnetic Moment

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(RBC and UKQCD Collaborations)

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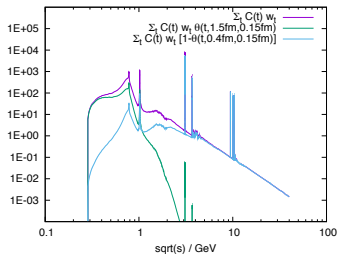
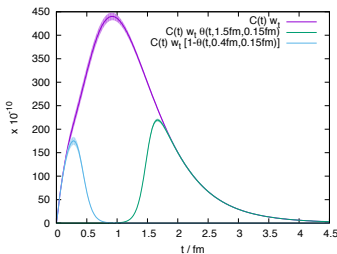
We present a first-principles lattice QCD + QED calculation at physical pion mass of the leading-order hadronic vacuum polarization contribution to the muon anomalous magnetic moment. The total contribution of up, down, strange, and charm quarks including QED and strong isospin breaking effects is $a_{\mu}^{\text{HVP LO}} = 715.4(18.7) \times 10^{-10}$. By supplementing lattice data for very short and long distances with R -ratio data, we significantly improve the precision to $a_{\mu}^{\text{HVP LO}} = 692.5(2.7) \times 10^{-10}$. This is the currently most precise determination of $a_{\mu}^{\text{HVP LO}}$.

Pure lattice result and dispersive result with reduced $\pi\pi$ dependence (window method)

RBC-UKQCD PRL 121, 022003 (2018)

$$a_{\mu}^{\text{HVP LO}} = \sum_{t=0}^{+\infty} w(t) C(t)$$

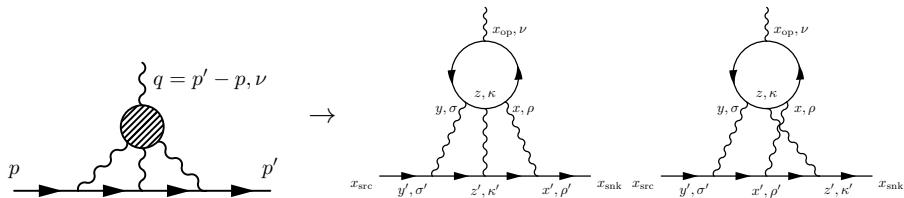
$$w(t) = w^{\text{SD}}(t) + w^{\text{W}}(t) + w^{\text{LD}}(t)$$



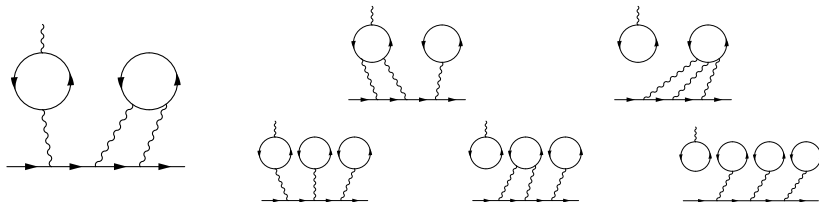
Most of $\pi\pi$ peak is captured by window from $t_0 = 0.4$ fm to $t_1 = 1.5$ fm, so replacing this region with lattice data reduces the dependence on BaBar versus KLOE data sets.

Babar & KLOE tension \Rightarrow syst error $5.6/2 = 2.8 \times 10^{-10}$.

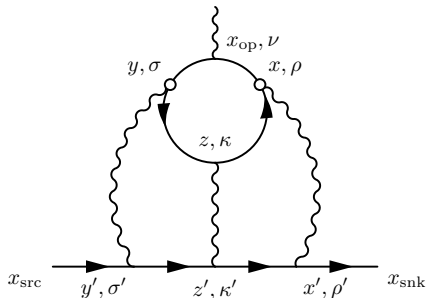
The window method is much less affected by this tension.



- Gluons and sea quark loops (not directly connected to photons) are included automatically to all orders!
- There are additional four different permutations of photons not shown.



RBC-UKQCD PRD 93, 014503 (2016)



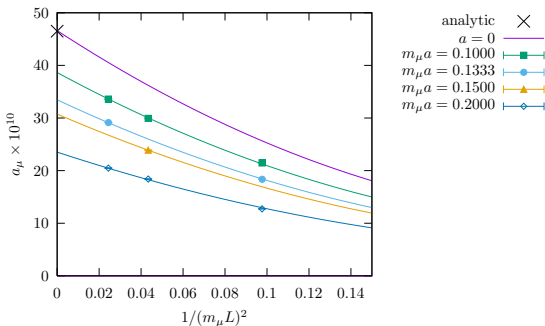
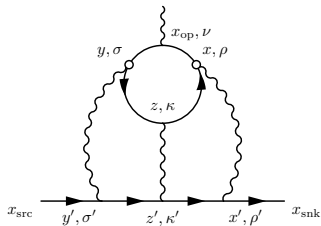
- Two point sources at x, y : randomly sample x and y .
- Importance sampling: focus on small $|x - y|$.
- Complete sampling for $|x - y| \leq 5a$ upto discrete symmetry.

$$\frac{a_\mu}{m_\mu} \bar{u}_{s'}(\vec{0}) \frac{\Sigma}{2} u_s(\vec{0}) = \sum_{r=x-y} \sum_z \sum_{x_{\text{op}}} \frac{1}{2} (\vec{x}_{\text{op}} - \vec{x}_{\text{ref}}) \times \bar{u}_{s'}(\vec{0}) i \vec{\mathcal{F}}^C(\vec{0}; x, y, z, x_{\text{op}}) u_s(\vec{0})$$

$$\vec{\mu} = \sum_{\vec{x}_{\text{op}}} \frac{1}{2} (\vec{x}_{\text{op}} - \vec{x}_{\text{ref}}) \times \vec{J}(\vec{x}_{\text{op}})$$

- Muon is plane wave, $x_{\text{ref}} = (x + y)/2$.
- Sum over time component for x_{op} .
- Only sum over $r = x - y$.

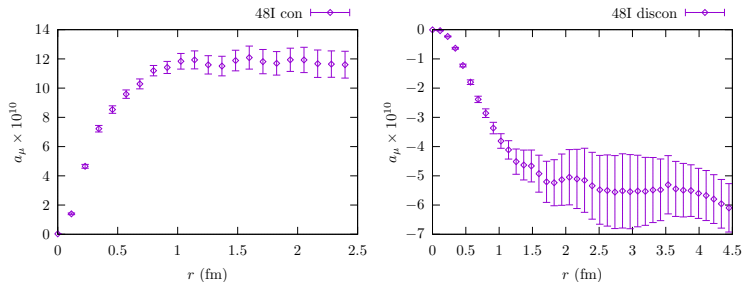
RBC-UKQCD PRD 93, 014503 (2016)



$$F_2(a, L) = F_2 \left(1 - \frac{c_1}{(m_\mu L)^2} + \frac{c'_1}{(m_\mu L)^4} \right) (1 - c_2 a^2 + c'_2 a^4) \rightarrow F_2 = 46.6(2) \times 10^{-10} \quad (19)$$

- **Pure QED computation.** Muon leptonic light by light contribution to muon $g - 2$. Phys.Rev. D93 (2016) 1, 014503. arXiv:1510.07100.
- Analytic results: $0.371 \times (\alpha/\pi)^3 = 46.5 \times 10^{-10}$.
- $\mathcal{O}(1/L^2)$ finite volume effect, because the photons are emitted from a conserved loop.

RBC-UKQCD PRL 118, 022005 (2017)



- Left: **connected diagrams**. Right: leading **disconnected diagrams**.
- $48^3 \times 96$ lattice, with $a^{-1} = 1.73$ GeV, $m_\pi = 139$ MeV, $m_\mu = 106$ MeV.
- We use Lanczos, AMA, and zMobius techniques to speed up the computations.
- 65 configurations are used. They each are separated by 20 MD time units.

$$a_\mu^{\text{cHLbL}} = (11.60 \pm 0.96) \times 10^{-10} \quad (30)$$

$$a_\mu^{\text{dHLbL}} = (-6.25 \pm 0.80) \times 10^{-10} \quad (31)$$

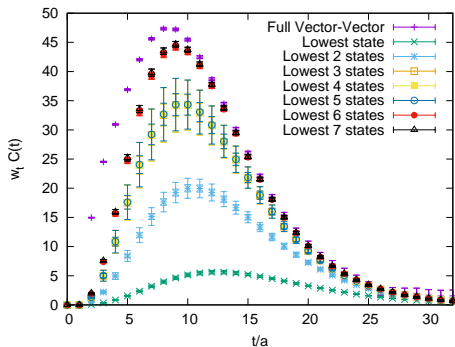
$$a_\mu^{\text{cHLbL}} = (5.35 \pm 1.35) \times 10^{-10} \quad (32)$$

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C. Lehner's talk at Seattle INT workshop. RBC-UKQCD prelim.

The Euclidean space-time correlator in finite volume

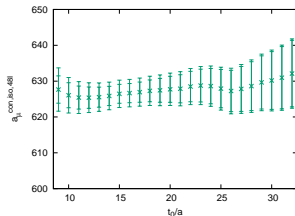
$$C(t) = \sum_n |\langle 0 | V | n \rangle|^2 e^{-E_n t}$$



48l, ud,conn,iso: $626.5(2.6)_{\text{stat}} \times 10^{-10}$

Previous stat error: 14.2×10^{-10}

- At large t
large statistical error
dominated by a few states.
- SD: Full Vector-Vector
LD: Sum of states
- Systematic can be strictly bounded.

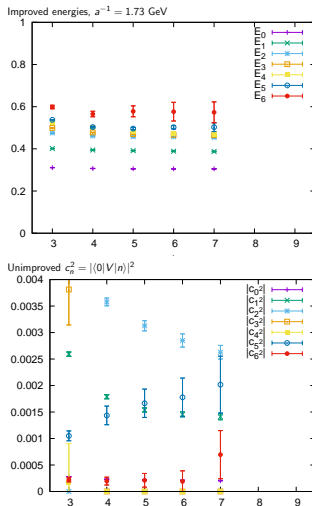


C. Lehner's talk at Seattle INT workshop. RBC-UKQCD prelim.

The Euclidean space-time correlator in finite volume

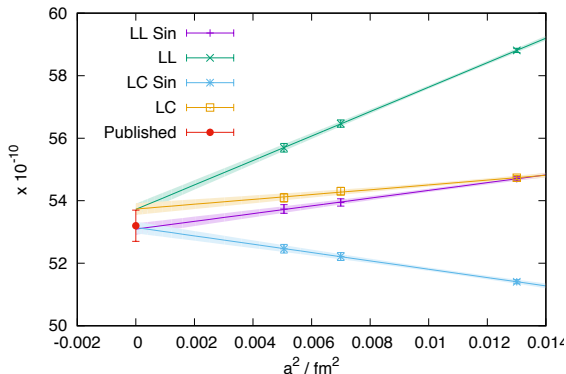
$$C(t) = \sum_n |\langle 0|V|n\rangle|^2 e^{-E_n t}$$

- Study the $N \times N$ correction matrix formed by N operators with the same quantum number:
 - local and smeared vector currents
 - 2 pion operator with 1,2,3,4 units of momentum
 - 4 pion operator
- Both E_n and $\langle 0|V|n\rangle$ can be extracted.



C. Lehner's talk at Seattle INT workshop. RBC-UKQCD prelim.

- Third lattice spacing for strange data ($a^{-1} = 2.77$ GeV with $m_\pi = 234$ MeV with sea light-quark mass corrected from global fit):

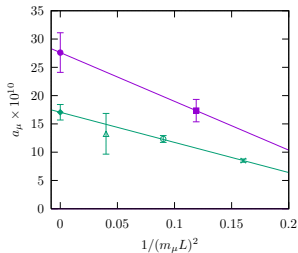


- For light quark need new ensemble at physical pion mass. Started run on Summit Machine at Oak Ridge this year ($a^{-1} = 2.77$ GeV with $m_\pi = 139$ MeV).

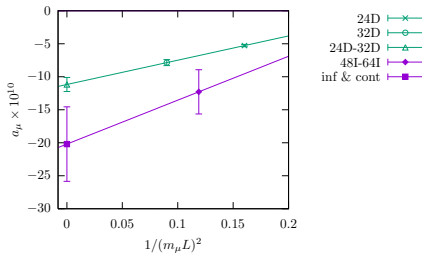
RBC-UKQCD prelim

- MDWF+Iwasaki: continuum limit (5.4fm)
- MDWF+DSDR: $a^{-1} = 1.015$ GeV: $24^3 \times 64$ (4.8fm), $32^3 \times 64$ (6.4fm), $48^3 \times 64$ (9.6fm).
- MDWF+DSDR: $a^{-1} = 1.371$ GeV: $32^3 \times 64$ (4.6fm).

$$F_2(a, L) = F_2 \left(1 - \frac{c_1}{(m_\mu L)^2} \right) (1 - c_2 a^2) \quad (43)$$



Connected diagrams



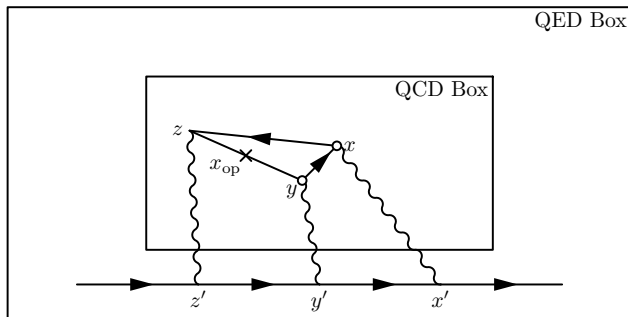
Disconnected diagrams

$$a_\mu^{\text{cHLbL}} = (27.61 \pm 3.51_{\text{stat}} \pm 0.32_{\text{sys}, a^2}) \times 10^{-10} \quad (44)$$

$$a_\mu^{\text{dHLbL}} = (-20.20 \pm 5.65_{\text{stat}}) \times 10^{-10} \quad (45)$$

$$a_\mu^{\text{HLbL}} = (7.41 \pm 6.32_{\text{stat}} \pm 0.32_{\text{sys}, a^2}) \times 10^{-10} \quad (46)$$

Mainz PoS LATTICE2016 164, RBC-UKQCD PRD 96, 034515 (2017)

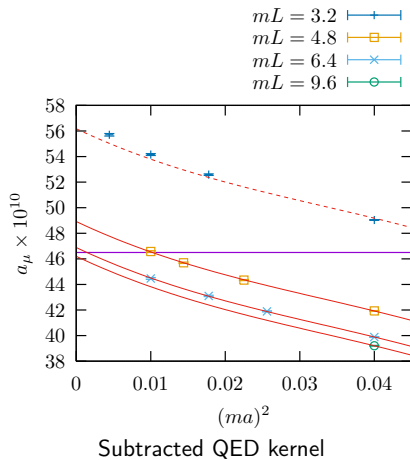
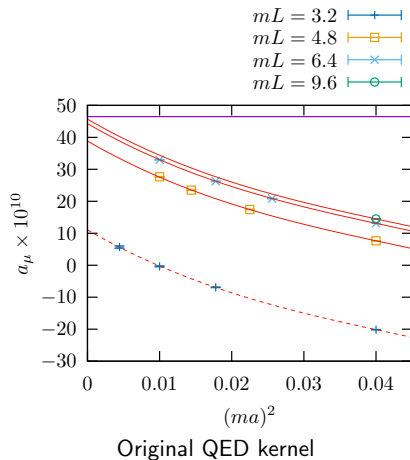


Subtracted QED kernel (use the current conservation condition)
 reduce discretization & finite volume & statistical error

$$\begin{aligned} \mathfrak{G}_{\rho,\sigma,\kappa}^{(2)}(x, y, z) \\ = \mathfrak{G}_{\rho,\sigma,\kappa}^{(1)}(x, y, z) - \mathfrak{G}_{\rho,\sigma,\kappa}^{(1)}(y, y, z) - \mathfrak{G}_{\rho,\sigma,\kappa}^{(1)}(x, y, y) + \mathfrak{G}_{\rho,\sigma,\kappa}^{(1)}(y, y, y) \end{aligned}$$

RBC-UKQCD PRD 96, 034515 (2017)

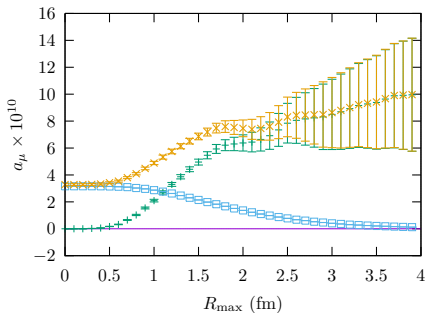
- Compare the two $\mathfrak{G}_{\rho,\sigma,\kappa}(x,y,z)$ in **pure QED computation**.



- Notice the vertical scales in the two plots are different.

RBC-UKQCD prelim

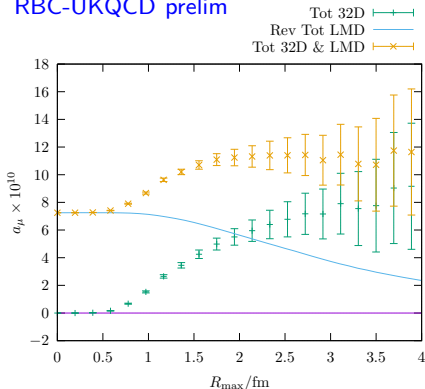
Tot 24DH —+—
 Rev Tot LMD —□—
 Tot 24DH & LMD —x—



- $R_{\max} = \max(|x-y|, |x-z|, |y-z|)$.
- $a^{-1} = 1.015$ GeV.
- 24DH: partial sum upto R_{\max} .
- Rev LMD:
reverse partial sum down to R_{\max} .
- 24DH & LMD:
the sum of the above two curves.

- Short distance part is given by lattice data.
- Long distance part is given by LMD model of π^0 pole.
- At 2.0 fm, the combination gives: $a_\mu^{\text{tot}} = 7.46(62)_{\text{stat}} \times 10^{-10}$.
 $a_\mu^{\text{con}} = 13.44(10)_{\text{stat}} \times 10^{-10}$ $a_\mu^{\text{discon}} = -5.70(58)_{\text{stat}} \times 10^{-10}$

RBC-UKQCD prelim



- $R_{\max} = \max(|x-y|, |x-z|, |y-z|)$.
- $a^{-1} = 1.015$ GeV.
- 32D: partial sum upto R_{\max} .
- Rev LMD:
reverse partial sum down to R_{\max} .
- 32D& LMD:
the sum of the above two curves.

- Short distance part is given by lattice data.

- Long distance part is given by LMD model of π^0 pole.

- At 2.5 fm, the combination gives: $a_\mu^{\text{tot}} = 11.40(1.27)_{\text{stat}} \times 10^{-10}$.

$$a_\mu^{\text{con}} = 29.19(0.73)_{\text{stat}} \times 10^{-10} \quad a_\mu^{\text{discon}} = -17.79(1.13)_{\text{stat}} \times 10^{-10}$$

	$a_\mu \times 10^{10}$	
HVP	692.5 ± 2.7	RBC-UKQCD and FJ17 combined
(LO)	693.26 ± 2.46	KNT18
	693.9 ± 4.0	DHMZ19
	$??? \pm 5.0$	Pure lattice results
	$??? \pm 2.0$	Hybrid approach (KNT & RBC-UKQCD)
		Weigh in on BaBar/KLOE
HLbL	10.3 ± 2.9	Fred Jegerlehner, 2017
	10.5 ± 2.6	Glasgow Consensus, 2007
	8.5 ± 2.1	Dispersive Seattle INT 2019
		± 1.2 if add error in quadrature.
	$7.41 \pm 6.32_{\text{stat}} \pm 0.32_{\text{sys}, a^2}$	RBC-UKQCD prelim (QED_L)
	$11.40 \pm 1.27_{\text{stat}} \pm ???_{\text{sys}}$	RBC-UKQCD prelim (QED_∞ & LMD)
		Accumulate stat & Estimate syst

Timeline for the White Paper

- 🕒 Earliest possible release date for Fermilab g-2 measurement:
15-20 December 2019
- 🕒 Post the WP on arXiv by:
1 Dec. 2019
- 🕒 Deadline for finalizing individual WP chapters:
1 Nov 2019
At this date the Overleaf chapters will be frozen.
- 🕒 Editorial board will release complete WP to authors for feedback on:
15 Nov. 2019
will need to receive feedback from authors within a week
- 🕒 Experimental and theoretical inputs used in WP must be published by:
15 Oct 2019
To make sure to be included in WP discussion, a paper to be posted in arXiv by same date.

Note: The WP will be posted on arXiv in December, even if the Fermilab experiment's release date is delayed.

Thank You!