Vacuum Stability & Symmetry Breaking in Left-Right Symmetric Model

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- It also features heavy right-handed Majorana neutrinos, and thus explains small masses of neutrinos via see-saw mechanism.

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- λ_h becomes negative at a scale of around 10^{10} GeV, making the SM vacuum unstable. Isidori, Ridolfi, Strumia (Nucl.Phys '01)
- This motivates us to to ensure the stability of the scalar Higgs potential in LRSM as a candidate for beyond SM.

Particle Content of LRSM

	$SU(2)_L$	$SU(2)_R$	$U(1)_{B-L}$
$Q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	2	1	$\frac{1}{3}$
$Q_R \equiv \begin{pmatrix} u_R \\ d_R \end{pmatrix}$	1	2	$\frac{1}{3}$
$\psi_L \equiv \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	2	1	-1
$\psi_R \equiv \begin{pmatrix} N \\ e_R \end{pmatrix}$	1	2	-1
$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}$	2	2	0
$\Delta_L = \begin{pmatrix} \frac{1}{\sqrt{2}}\Delta_L^+ & \Delta_L^{++} \\ \Delta_L^0 & -\frac{1}{\sqrt{2}}\Delta_L^+ \end{pmatrix}$	3	1	2
$\Delta_R = \begin{pmatrix} \frac{1}{\sqrt{2}}\Delta_R^+ & \Delta_R^{++} \\ \Delta_R^0 & -\frac{1}{\sqrt{2}}\Delta_R^+ \end{pmatrix}$	1	3	2

Scalar Potential

$$
\begin{aligned}
V \;=\; & -\mu_1^2 \mathrm{Tr}[\phi^\dagger\phi] - \mu_2^2\left(\mathrm{Tr}[\tilde{\phi}\phi^\dagger] + \mathrm{Tr}[\tilde{\phi}^\dagger\phi]\right) - \mu_3^2\left(\mathrm{Tr}[\Delta_L\Delta_L^\dagger] + \mathrm{Tr}[\Delta_R\Delta_R^\dagger]\right) + \lambda_1 \mathrm{Tr}[\phi^\dagger\phi]^2 \\
& +\lambda_2\left(\mathrm{Tr}[\tilde{\phi}\phi^\dagger]^2 + \mathrm{Tr}[\tilde{\phi}^\dagger\phi]^2\right) + \lambda_3 \mathrm{Tr}[\tilde{\phi}\phi^\dagger]\mathrm{Tr}[\tilde{\phi}^\dagger\phi] + \lambda_4 \mathrm{Tr}[\phi^\dagger\phi]\left(\mathrm{Tr}[\tilde{\phi}\phi^\dagger] + \mathrm{Tr}[\tilde{\phi}^\dagger\phi]\right) \\
& +\rho_1\left(\mathrm{Tr}[\Delta_L\Delta_L^\dagger]^2 + \mathrm{Tr}[\Delta_R\Delta_R^\dagger]^2\right) + \rho_2\left(\mathrm{Tr}[\Delta_L\Delta_L]\mathrm{Tr}[\Delta_L^\dagger\Delta_L^\dagger] + \mathrm{Tr}[\Delta_R\Delta_R]\mathrm{Tr}[\Delta_R^\dagger\Delta_R^\dagger]\right) \\
& +\rho_3 \mathrm{Tr}[\Delta_L\Delta_L^\dagger]\mathrm{Tr}[\Delta_R\Delta_R^\dagger] + \rho_4\left(\mathrm{Tr}[\Delta_L\Delta_L]\mathrm{Tr}[\Delta_R^\dagger\Delta_R^\dagger] + \mathrm{Tr}[\Delta_L^\dagger\Delta_L^\dagger]\mathrm{Tr}[\Delta_R\Delta_R]\right) \\
& +\alpha_1 \mathrm{Tr}[\phi^\dagger\phi]\left(\mathrm{Tr}[\Delta_L\Delta_L^\dagger] + \mathrm{Tr}[\Delta_R\Delta_R^\dagger]\right) + \alpha_3\left(\mathrm{Tr}[\phi\phi^\dagger\Delta_L\Delta_L^\dagger] + \mathrm{Tr}[\phi^\dagger\phi\Delta_R\Delta_R^\dagger]\right) \\
& +\alpha_2\left(\mathrm{Tr}[\Delta_L\Delta_L^\dagger]\mathrm{Tr}[\tilde{\phi}\phi^\dagger] + \mathrm{Tr}[\Delta_R\Delta_R^\dagger]\mathrm{Tr}[\tilde{\phi}^\dagger\phi] + \mathrm{H.c.}\right) \\
& +\beta_1\left(\mathrm{Tr}[\phi\Delta_R\phi^\dagger\Delta_L^\dagger] + \mathrm{Tr}[\phi^\dagger\Delta_L\phi\Delta_R^\dagger]\right) + \beta_2\left(\mathrm{Tr}[\tilde{\phi}\Delta_R\phi^\dagger\Delta_L^\dagger] + \mathrm{Tr}[\tilde{\phi}^\dagger\Delta_L\phi\Delta_R^\dagger]\right) \\
& +\beta_3\left(\mathrm{Tr}[\phi\Delta_R\tilde{\phi}^\dagger\Delta_L^\dagger] + \mathrm{Tr}[\phi^\dagger\Delta_L\tilde{\phi}\Delta_R^\dagger]\right) ,
\end{aligned}
$$

Deshpande, Gunion, Kayser, Olness (PRD '91)

Maiezza, Senjanovic, Vasquez (PRD '17)

Vacuum Stability

- For the stability, the potential should be bounded in all field directions.

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- For the stability, the potential should be bounded in all field directions.
- In the large-field limit, terms with dimension d < 4 can be ignored in comparison to the quartic terms.
- Requiring $V_4(\phi_i) > 0$ as field values $\phi_i \to \infty$ is a strong condition for boundedness. (BFB)
- For applying BFB criterion, concepts of copositivity criteria and gauge orbit spaces can help simplify the analysis.

  Kim (JMP '84), Kannike (EPJC '12)

Gauge Orbit Spaces

- Consider the scalar potential of a theory with two Higgs fields ϕ and π charged under G and G' respectively:

$$
\begin{aligned}
V(\phi,\pi) \quad = \quad & -\mu_1^2(\phi_i^*\phi_i) - \mu_2^2(\pi_i^*\pi_i) + \lambda_1(\phi_i^*\phi_i)^2 + \lambda_2 f_{ijkl}\phi_i^*\phi_j\phi_k^*\phi_l \\
& +\rho_1(\pi_i^*\pi_i)^2 + \rho_2 g_{ijkl}\pi_i^*\pi_j\pi_k^*\pi_l + \cdots \\
& +\alpha_1(\phi_i^*\phi_i)(\pi_j^*\pi_j) + \cdots (\text{other terms coupling } (\phi,\pi))
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- The dimensionless ratios of invariants called orbit space parameters are defined:

$$A_n(\hat{\phi}) = \frac{f_{ijkl}\phi_i^*\phi_j\phi_k^*\phi_l}{(\phi_i^*\phi_i)^2} \quad B_n(\hat{\pi}) = \frac{g_{ijkl}\pi_i^*\pi_j\pi_k^*\pi_l}{(\pi_j^*\pi_j)^2}$$

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- Similarly, for coupled terms $C_n(\hat{\phi},\hat{\pi})$ can be defined but normalized by $\phi_i^*\phi_i\pi_j^*\pi_j$.

Gauge Orbit Spaces

- The potential can be written as:

$$
\begin{aligned}
V(\phi,\pi) \quad = \quad & -\mu_1^2|\phi|^2 - \mu_2^2|\pi|^2 + |\phi|^4 A(\lambda,\hat{\phi}) + |\pi|^4 B(\rho,\hat{\pi}) \\
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- where,

$$|\phi|^2 = \phi_i^*\phi_i, \quad |\pi|^2 = \pi_i^*\pi_i, \quad \hat{\phi} = \frac{\phi}{|\phi|}, \quad \hat{\pi} = \frac{\pi}{|\pi|}$$

$$A(\lambda,\hat{\phi}) = \lambda_1 + \lambda_2 A_1(\hat{\phi}) + \lambda_3 A_2(\hat{\phi}) + \cdots$$

$$B(\rho,\hat{\pi}) = \rho_1 + \rho_2 B_1(\hat{\pi}) + \rho_3 B_2(\hat{\pi}) + \cdots$$

$$C(\alpha,\hat{\phi},\hat{\pi}) = \alpha_1 + \alpha_2 C_1(\hat{\phi},\hat{\pi}) + \cdots$$

Copositivity

- Requiring boundedness $\forall A(\lambda, \hat{\phi}), B(\rho, \hat{\pi}), C(\alpha, \hat{\phi}, \hat{\pi})$:

$$|\phi|^4 A(\lambda, \hat{\phi}) + |\pi|^4 B(\rho, \hat{\pi}) + |\phi|^2 |\pi|^2 C(\alpha, \hat{\phi}, \hat{\pi}) > 0$$

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$$ax^2 + bx + c > 0 \quad x \in \mathbb{R}^+$$

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- $$\implies A > 0,\ B > 0,\ C + 2\sqrt{AB} > 0$$

Scalar Potential: λ terms

$$
\begin{aligned}
V \quad \supset \quad & -\mu_1^2 \mathrm{Tr}[\phi^\dagger \phi] - \mu_2^2 \left(\mathrm{Tr}[\tilde{\phi}\phi^\dagger] + \mathrm{Tr}[\tilde{\phi}^\dagger \phi] \right) \\
& + \lambda_1 \mathrm{Tr}[\phi^\dagger \phi]^2 + \lambda_2 \left(\mathrm{Tr}[\tilde{\phi}\phi^\dagger]^2 + \mathrm{Tr}[\tilde{\phi}^\dagger \phi]^2 \right) + \lambda_3 \mathrm{Tr}[\tilde{\phi}\phi^\dagger] \mathrm{Tr}[\tilde{\phi}^\dagger \phi] \\
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\end{aligned}
$$

- we parametrize V_4^λ as follows:

$$
\begin{aligned}
\mathrm{Tr}[\Phi^\dagger \Phi] &\equiv r^2 \\
\mathrm{Tr}[\tilde{\Phi}\Phi^\dagger]/\mathrm{Tr}[\Phi^\dagger \Phi] &\equiv \xi e^{i\omega} \\
\mathrm{Tr}[\tilde{\Phi}^\dagger \Phi]/\mathrm{Tr}[\Phi^\dagger \Phi] &\equiv \xi e^{-i\omega}
\end{aligned}
$$

where $r > 0$, $\xi \in [0, 1]$ and $\omega \in [0, 2\pi]$.

Stability : λ terms

- Using parametrization,

$$V_4^\lambda = r^4 \left(\lambda_1 + 2\lambda_2 \xi^2 \cos 2\omega + \lambda_3 \xi^2 + 2\lambda_4 \, \xi \cos \omega\right) \equiv r^4 f(\lambda, \xi, \omega)$$

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- By analyzing the boundedness of λ sector of the potential,

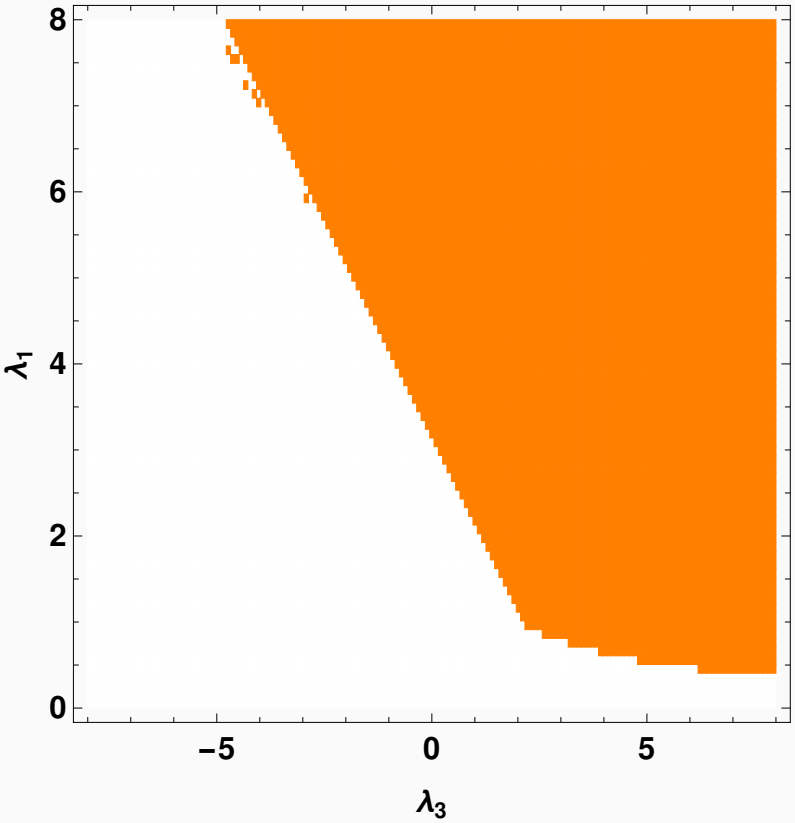
$$\lambda_1 > 0 \tag{1}$$

$$\lambda_1 - \frac{\lambda_4^2}{2\lambda_2 + \lambda_3} > 0 \tag{2}$$

$$\lambda_1 + \lambda_3 - 2\lambda_2 - \frac{\lambda_4^2}{4\lambda_2} > 0 \tag{3}$$

$$\lambda_1 + \lambda_3 + 2(\lambda_2 - |\lambda_4|) > 0 \tag{4}$$

Sample case: λ's



Values of set λ's are:

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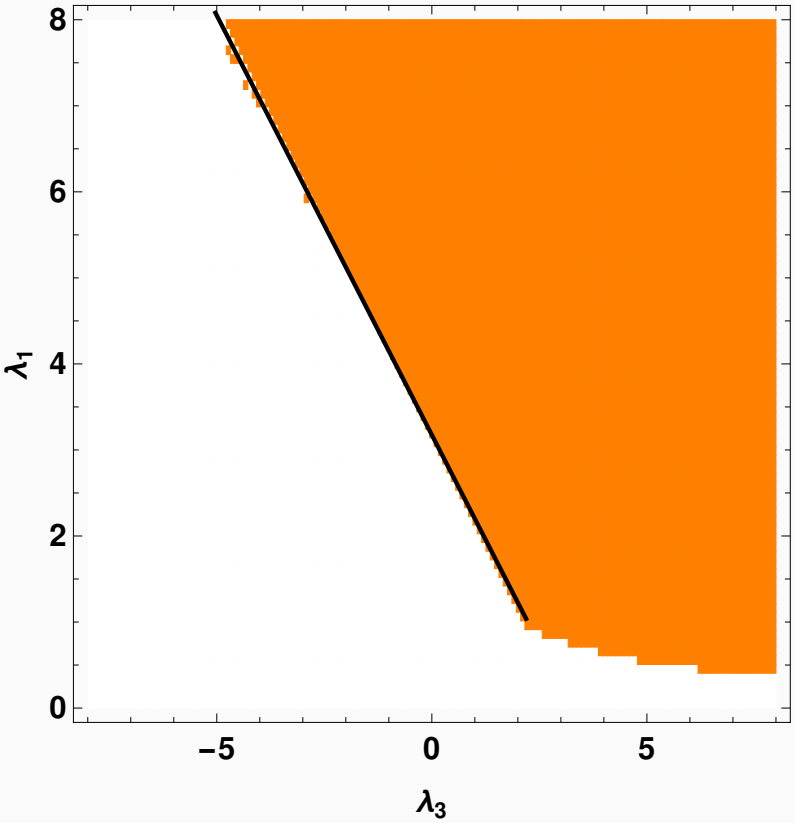

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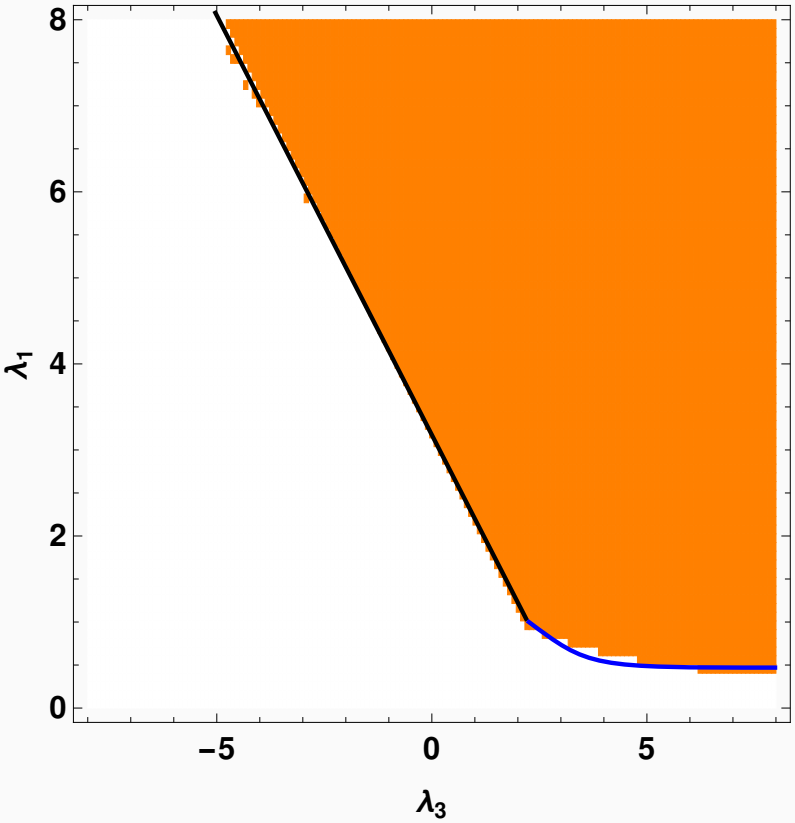

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Dreaded case: $\alpha_{1,3} \neq 0$

$$
\begin{aligned}
V_4 \;=\; & \lambda_1 \mathrm{Tr}[\Phi^\dagger\Phi]^2 + \lambda_2\left(\mathrm{Tr}[\tilde{\Phi}\Phi^\dagger]^2 + \mathrm{Tr}[\tilde{\Phi}^\dagger\Phi]^2\right) + \lambda_3 \mathrm{Tr}[\tilde{\Phi}\Phi^\dagger]\mathrm{Tr}[\tilde{\Phi}^\dagger\Phi] + \lambda_4 \mathrm{Tr}[\Phi^\dagger\Phi]\left(\mathrm{Tr}[\tilde{\Phi}\Phi^\dagger] + \mathrm{Tr}[\tilde{\Phi}^\dagger\Phi]\right) \\
& + \rho_1\left(\mathrm{Tr}[\Delta_L\Delta_L^\dagger]^2 + \mathrm{Tr}[\Delta_R\Delta_R^\dagger]^2\right) + \rho_2\left(\mathrm{Tr}[\Delta_L\Delta_L]\mathrm{Tr}[\Delta_L^\dagger\Delta_L^\dagger] + \mathrm{Tr}[\Delta_R\Delta_R]\mathrm{Tr}[\Delta_R^\dagger\Delta_R^\dagger]\right) \\
& + \rho_3 \mathrm{Tr}[\Delta_L\Delta_L^\dagger]\mathrm{Tr}[\Delta_R\Delta_R^\dagger] + \rho_4\left(\mathrm{Tr}[\Delta_L\Delta_L]\mathrm{Tr}[\Delta_R^\dagger\Delta_R^\dagger] + \mathrm{Tr}[\Delta_L^\dagger\Delta_L^\dagger]\mathrm{Tr}[\Delta_R\Delta_R]\right) \\
& + \alpha_1 \mathrm{Tr}[\Phi^\dagger\Phi]\left(\mathrm{Tr}[\Delta_L\Delta_L^\dagger] + \mathrm{Tr}[\Delta_R\Delta_R^\dagger]\right) + \alpha_3\left(\mathrm{Tr}[\Phi\Phi^\dagger\Delta_L\Delta_L^\dagger] + \mathrm{Tr}[\Phi^\dagger\Phi\Delta_R\Delta_R^\dagger]\right)
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&+\rho_3 \mathrm{Tr}[\Delta_L\Delta_L^\dagger]\mathrm{Tr}[\Delta_R\Delta_R^\dagger] + \rho_4\left(\mathrm{Tr}[\Delta_L\Delta_L]\mathrm{Tr}[\Delta_R^\dagger\Delta_R^\dagger] + \mathrm{Tr}[\Delta_L^\dagger\Delta_L^\dagger]\mathrm{Tr}[\Delta_R\Delta_R]\right) \\
&+\alpha_1 \mathrm{Tr}[\Phi^\dagger\Phi]\left(\mathrm{Tr}[\Delta_L\Delta_L^\dagger] + \mathrm{Tr}[\Delta_R\Delta_R^\dagger]\right) + \alpha_3\left(\mathrm{Tr}[\Phi\Phi^\dagger\Delta_L\Delta_L^\dagger] + \mathrm{Tr}[\Phi^\dagger\Phi\Delta_R\Delta_R^\dagger]\right)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Tr}[\Phi^\dagger\Phi] + \mathrm{Tr}[\Delta_L\Delta_L^\dagger] + \mathrm{Tr}[\Delta_R\Delta_R^\dagger] &\equiv r^2 \\
\mathrm{Tr}[\Phi^\dagger\Phi] &\equiv r^2\cos^2\theta \\
\mathrm{Tr}[\Delta_L\Delta_L^\dagger] &\equiv r^2\sin^2\gamma\sin^2\theta \\
\mathrm{Tr}[\Delta_R\Delta_R^\dagger] &\equiv r^2\cos^2\gamma\sin^2\theta \\
\mathrm{Tr}[\tilde{\Phi}\Phi^\dagger]/\mathrm{Tr}[\Phi^\dagger\Phi] &\equiv \xi e^{i\omega} \\
\mathrm{Tr}[\tilde{\Phi}^\dagger\Phi]/\mathrm{Tr}[\Phi^\dagger\Phi] &\equiv \xi e^{-i\omega} \\
\mathrm{Tr}[\Delta_L\Delta_L]/\mathrm{Tr}[\Delta_L\Delta_L^\dagger] &\equiv \eta_1 e^{i\theta_1} \\
\mathrm{Tr}[\Delta_L^\dagger\Delta_L^\dagger]/\mathrm{Tr}[\Delta_L\Delta_L^\dagger] &\equiv \eta_1 e^{-i\theta_1} \\
\mathrm{Tr}[\Delta_R\Delta_R]/\mathrm{Tr}[\Delta_R\Delta_R^\dagger] &\equiv \eta_2 e^{i\theta_2} \\
\mathrm{Tr}[\Delta_R^\dagger\Delta_R^\dagger]/\mathrm{Tr}[\Delta_R\Delta_R^\dagger] &\equiv \eta_2 e^{-i\theta_2} \\
\mathrm{Tr}[\Phi\Phi^\dagger\Delta_L\Delta_L^\dagger]/\mathrm{Tr}[\Phi^\dagger\Phi]\mathrm{Tr}[\Delta_L\Delta_L^\dagger] &\equiv \zeta_1 \\
\mathrm{Tr}[\Phi^\dagger\Phi\Delta_R\Delta_R^\dagger]/\mathrm{Tr}[\Phi^\dagger\Phi]\mathrm{Tr}[\Delta_R\Delta_R^\dagger] &\equiv \zeta_2
\end{aligned}
$$

with $r > 0$, $|\xi| \leq 1$, $\theta \in [0, \frac{\pi}{2}]$, $\gamma \in [0, \frac{\pi}{2}]$, $\eta_1, \eta_2 \in [0, 1]$, $\theta_1, \theta_2 \in [0, 2\pi]$ and $\zeta_1, \zeta_2 \in [0, 1]$.

Analytic Conditions for Vacuum Stability in LRSM

- $\mathrm{f} > 0 : \begin{cases} \lambda_1 \\ \left(\lambda_1 - \frac{\lambda_4^2}{2\lambda_2+\lambda_3}\right) & \Longleftarrow \; 2\lambda_2 + \lambda_3 > |\lambda_4| \\ (\lambda_1 + \lambda_3 + 2(\lambda_2 - |\lambda_4|)) \\ \left(\lambda_1 + \lambda_3 - 2\lambda_2 - \frac{\lambda_4^2}{4\lambda_2}\right) & \Longleftarrow \; |4\lambda_2| > |\lambda_4| \end{cases}$

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- $$g > 0 : \left\{\rho_1,\, \rho_1 + \rho_2,\, \frac{\rho_3 + 2\rho_1}{4},\, \frac{\rho_3 - 2|\rho_4| + 2(\rho_1 + \rho_2)}{4}\right\}$$

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- $$g > 0 : \left\{\rho_1,\, \rho_1 + \rho_2,\, \frac{\rho_3 + 2\rho_1}{4},\, \frac{\rho_3 - 2|\rho_4| + 2(\rho_1 + \rho_2)}{4}\right\}$$
- $$\alpha_1 + 2\sqrt{\mathsf{Min}(f)\,\mathsf{Min}(g)} > 0$$
$$\alpha_1 + \alpha_3 + 2\sqrt{\mathsf{Min}(f)\,\mathsf{Min}(g)} > 0$$

Symmetry Breaking in LRSM

- A BFB potential does not necessarily leads to correct symmetry breaking. Dev, Mohapatra, Rodejohann, Xu (JHEP '19)

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- The desired VEV structure for LRSM vacuum is

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- We can generalize the gauge-independent conditions for correct vacuum in the LRSM as:

$$\mathrm{Tr}[\langle\Phi\rangle\langle\Phi\rangle] \neq 0$$

$$\mathrm{Tr}[\langle\Delta_L\rangle\langle\Delta_L\rangle] = \mathrm{Tr}[\langle\Delta_R\rangle\langle\Delta_R\rangle] = 0$$

$$\mathrm{Tr}[\langle\Delta_L\rangle\langle\Delta_L^\dagger\rangle] < \mathrm{Tr}[\langle\Delta_R\rangle\langle\Delta_R^\dagger\rangle]$$

Symmetry Breaking in LRSM

- Plugging the VEV structure in the scalar potential, we get:

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\begin{aligned}
V \quad = \quad & -\frac{\left(\kappa_1^2+\kappa_2^2\right)}{2}\mu_1^2 - 2\kappa_1\kappa_2\mu_2^2\cos(\theta_2) - \mu_3^2\left(v_L^2+v_R^2\right) \\
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- Using earlier parametrization :

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V_4 \quad \equiv \quad r^4\left(f_{SSB}\cos^4\theta + g_{SSB}\sin^4\theta + h_{SSB}\cos^2\theta\sin^2\theta\right)
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- Requiring a deeper minima for V_{SSB}

$$
f \geq f_{SSB}, \quad g > g_{SSB}
$$

$$
h + 2\sqrt{f\,g} > h_{SSB} + 2\sqrt{f_{SSB}\,g_{SSB}}
$$

Analytic Conditions for Symmetry Breaking to Correct Vacuum

$$\mathrm{f_{SSB}} > 0 : \begin{cases} \lambda_1 > 0, \quad \sigma = 0\,, \\ \left(\lambda_1 - \frac{\lambda_4^2}{2\lambda_2+\lambda_3}\right) > 0 \implies 2\lambda_2 + \lambda_3 > |\lambda_4|, \quad \sigma = -\frac{\lambda_4}{2\lambda_2+\lambda_3}, \\ (\lambda_1 + \lambda_3 + 2(\lambda_2 - |\lambda_4|)) > 0, \quad \sigma = -\mathrm{sgn}(\lambda_4)\,, \\ \left(\lambda_1 + \lambda_3 - 2\lambda_2 - \frac{\lambda_4^2}{4\lambda_2}\right) > 0 \implies |4\lambda_2| > |\lambda_4|, \quad \sigma = -\frac{\lambda_4}{4\lambda_2}, \end{cases}$$

$$\rho_1 > 0, \quad \rho_2 > 0, \quad \rho_3 > 2\rho_1, \quad |\rho_4| < \frac{\rho_3 - 2\rho_1}{2} + \rho_2$$

$$\alpha_1 + 2\sqrt{\mathsf{Min}[f_{SSB}]\rho_1} > 0$$

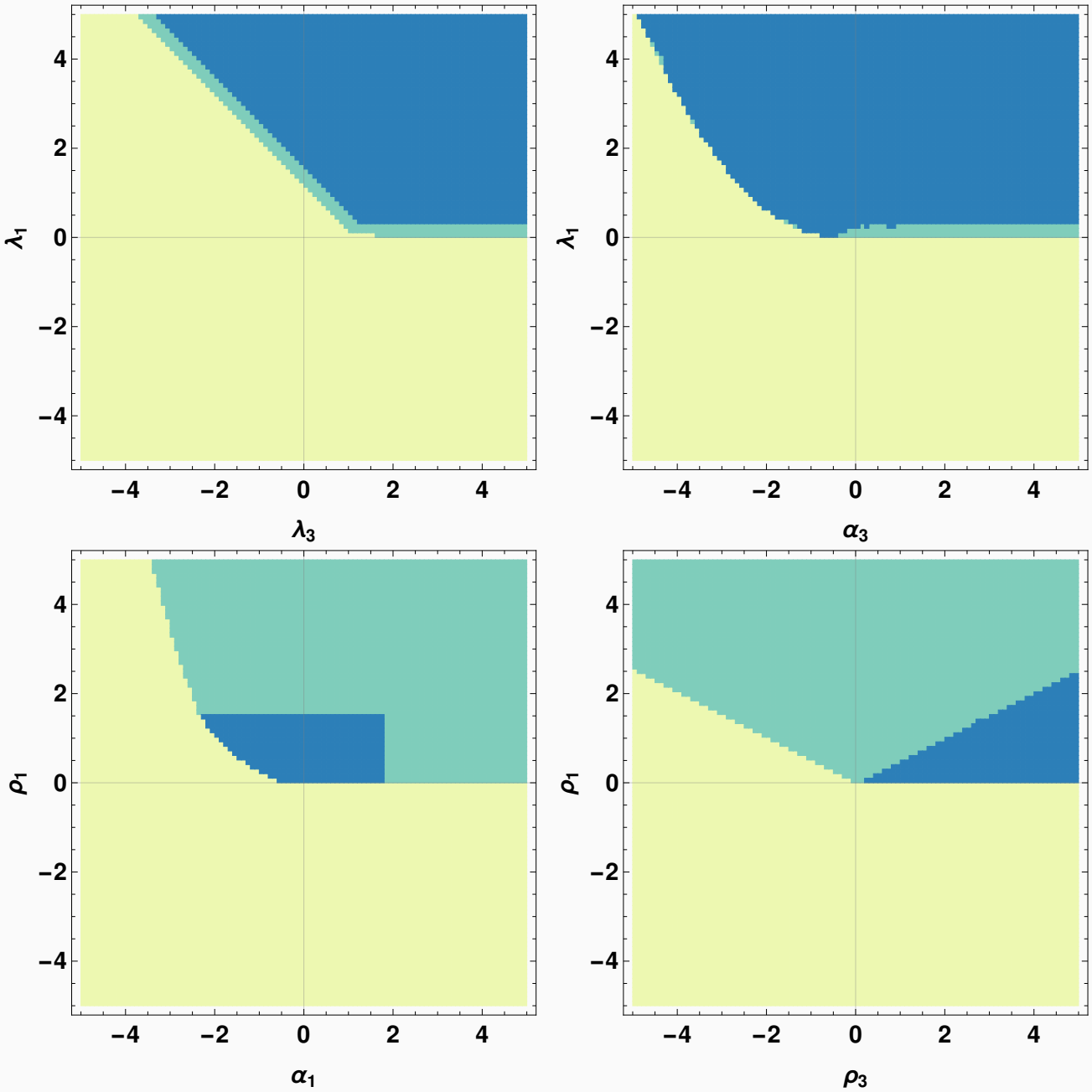
$$\alpha_1 + \alpha_3 + 2\sqrt{\mathsf{Min}[f_{SSB}]\rho_1} > 0$$

$$\bar{\mu}_1^2 = \mu_1^2 + 2\sigma\mu_2^2$$

$$2\sqrt{\mathsf{Min}[f_{SSB}]\,\rho_1} - ||\mathsf{Min}[\alpha_1, \alpha_1 + \alpha_3]|| > 0$$

$$2\rho_1\bar{\mu}_1^2 - \mathsf{Min}[\alpha_1, \alpha_1 + \alpha_3]\mu_3^2 > 0$$

Numerical Minimization



Renormalization Group Analysis

- The scalar mass spectrum for LRSM :

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\begin{aligned}
M^2_{H^0_0} &= 2\left(\lambda_1 - \frac{\alpha_1^2}{4\rho_1}\right)\kappa_+^2, \\
M^2_{H^\pm_2} \simeq M^2_{A^0_1} \simeq M^2_{H^0_1} &= \frac{1}{2}\alpha_3 v_R^2, \\
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- We have taken the best fit value of $M_{H^0_0} = m_h = 125$ GeV.

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- The current bounds on doubly charged Higgs masses are from LHC 13 TeV run data : ATLAS, CMS

$$M_{H_1^{\pm\pm}} \gtrsim (770 - 870) \text{ GeV} \qquad M_{H_2^{\pm\pm}} \gtrsim (660 - 760) \text{ GeV}$$

Renormalization Group Analysis

- This sample benchmark is in complete agreement with the current experimental bounds on the scalar masses.

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\begin{aligned}
\mu_1^2,\ \mu_2^2,\ \mu_3^2 &\equiv ((8.48)^2, 0, (11.99)^2)\ \text{TeV}^2 \\
\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4 &\equiv (0.0625, 0, 0, 0) \\
\rho_1,\ \rho_2,\ \rho_3,\ \rho_4 &\equiv (0.01, 0.0005, 0.0226, 0) \\
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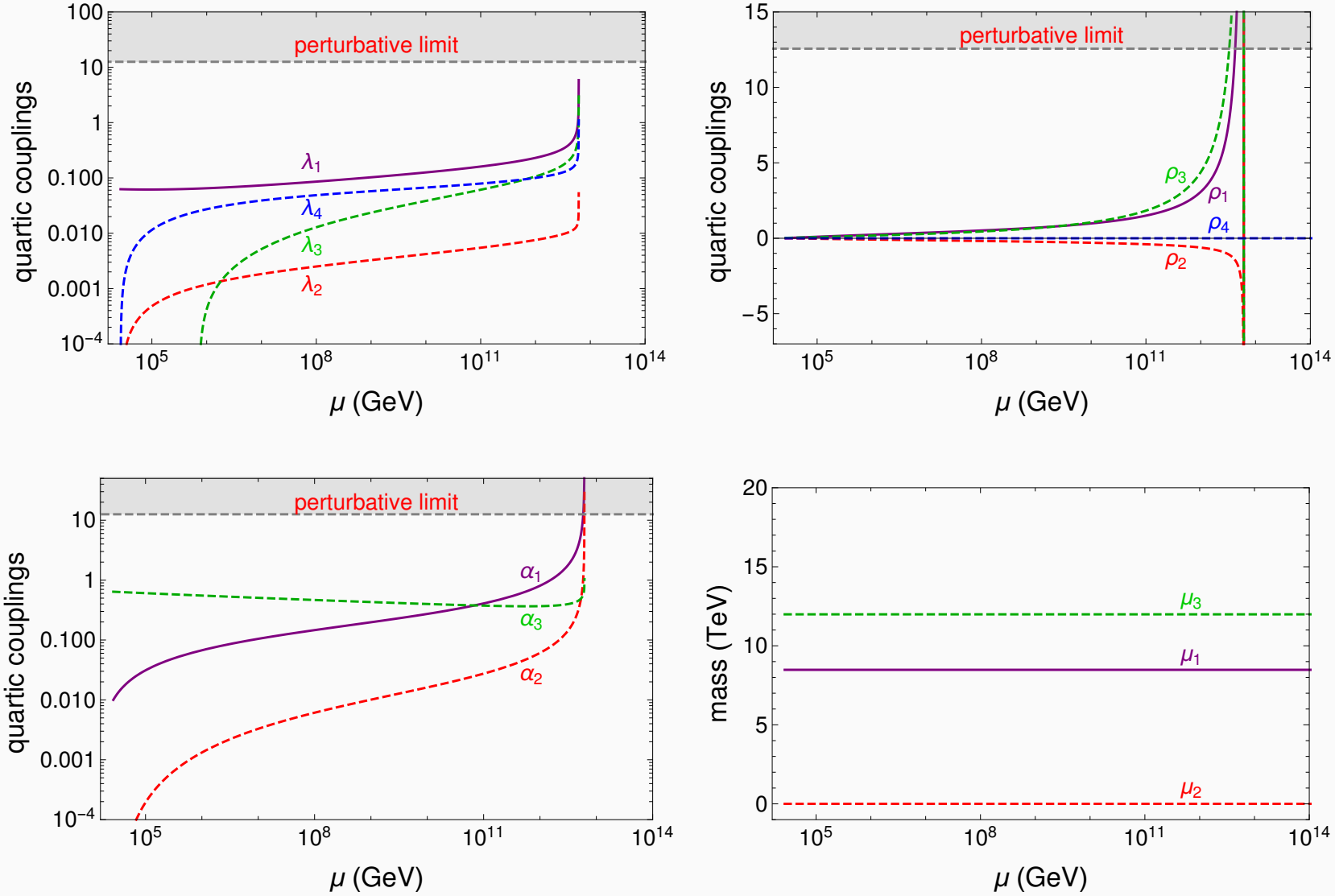
- The VEVs for the Higgses are:

$$\kappa_+ = \sqrt{\kappa_1^2 + \kappa_2^2} = 246\ \text{GeV},\ v_L = 0\ \text{TeV},\ v_R = 26.8\ \text{TeV}$$

Renormalization Group Analysis



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- These analytic techniques can be extended to analyze metastability of the vacuum and one-loop effective potential.

Thank you !!