Vacuum Stability & Symmetry Breaking in Left-Right Symmetric Model

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A natural extension of the SM is Left-Right Symmetric model (LRSM), which treats left & right chiralities on equal footing at high-energies. Pati, Salam (PRD ’74) Mohapatra, Pati (PRD ’75) Senjanovic, Mohapatra (PRD ’75)

It also features heavy right-handed Majorana neutrinos, and thus explains small masses of neutrinos via see-saw mechanism.
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• For stability of the scalar potential in the SM, positivity of the Higgs quartic coupling $\lambda_h$ is required.
• $\lambda_h$ becomes negative at a scale of around $10^{10}$ GeV, making the SM vacuum unstable. Isidori, Ridolfi, Strumia (Nucl.Phys ’01)
• This motivates us to ensure the stability of the scalar Higgs potential in LRSM as a candidate for beyond SM.
Particle Content of LRSM

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\[
V = -\mu_1^2 \text{Tr}[\phi^\dagger \phi] - \mu_2^2 \left( \text{Tr}[\bar{\phi}^\dagger \phi] + \text{Tr}[\bar{\phi}^\dagger \phi] \right) - \mu_3^2 \left( \text{Tr}[\Delta_L \Delta_L^\dagger] + \text{Tr}[\Delta_R \Delta_R^\dagger] \right) + \lambda_1 \text{Tr}[\phi^\dagger \phi]^2 \\
+ \lambda_2 \left( \text{Tr}[\bar{\phi}^\dagger \phi]^2 + \text{Tr}[\bar{\phi}^\dagger \phi]^2 \right) + \lambda_3 \text{Tr}[\bar{\phi}^\dagger \phi] \text{Tr}[\bar{\phi}^\dagger \phi] + \lambda_4 \text{Tr}[\phi^\dagger \phi] \left( \text{Tr}[\bar{\phi}^\dagger \phi] + \text{Tr}[\bar{\phi}^\dagger \phi] \right) \\
+ \rho_1 \left( \text{Tr}[\Delta_L \Delta_L^\dagger]^2 + \text{Tr}[\Delta_R \Delta_R^\dagger]^2 \right) + \rho_2 \left( \text{Tr}[\Delta_L \Delta_L] \text{Tr}[\Delta_L^\dagger \Delta_L^\dagger] + \text{Tr}[\Delta_R \Delta_R] \text{Tr}[\Delta_R^\dagger \Delta_R^\dagger] \right) \\
+ \rho_3 \text{Tr}[\Delta_L \Delta_L^\dagger] \text{Tr}[\Delta_R \Delta_R^\dagger] + \rho_4 \left( \text{Tr}[\Delta_L \Delta_L] \text{Tr}[\Delta_R^\dagger \Delta_R^\dagger] + \text{Tr}[\Delta_L^\dagger \Delta_L^\dagger] \text{Tr}[\Delta_R \Delta_R] \right) \\
+ \alpha_1 \text{Tr}[\phi^\dagger \phi] \left( \text{Tr}[\Delta_L \Delta_L^\dagger] + \text{Tr}[\Delta_R \Delta_R^\dagger] \right) + \alpha_3 \left( \text{Tr}[\bar{\phi}^\dagger \Delta_L \Delta_L^\dagger] + \text{Tr}[\phi^\dagger \phi \Delta_R \Delta_R^\dagger] \right) \\
+ \alpha_2 \left( \text{Tr}[\Delta_L \Delta_L^\dagger] \text{Tr}[\bar{\phi}^\dagger \phi] + \text{Tr}[\Delta_R \Delta_R^\dagger] \text{Tr}[\bar{\phi}^\dagger \phi] + \text{H.c.} \right) \\
+ \beta_1 \left( \text{Tr}[\phi \Delta_R \phi^\dagger \Delta_L^\dagger] + \text{Tr}[\phi^\dagger \Delta_L \phi \Delta_R^\dagger] \right) + \beta_2 \left( \text{Tr}[\bar{\phi} \Delta_R \phi^\dagger \Delta_L^\dagger] + \text{Tr}[\phi^\dagger \Delta_L \phi \Delta_R^\dagger] \right) \\
+ \beta_3 \left( \text{Tr}[\phi \Delta_R \phi^\dagger \Delta_L^\dagger] + \text{Tr}[\phi^\dagger \Delta_L \phi \Delta_R^\dagger] \right),
\]

Deshpande, Gunion, Kayser, Olness (PRD ’91)
Maiezza, Senjanovic, Vasquez (PRD ’17)
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Vacuum Stability

- For the stability, the potential should be bounded in all field directions.
- In the large-field limit, terms with dimension $d < 4$ can be ignored in comparison to the quartic terms.
- Requiring $V_4(\phi_i) > 0$ as field values $\phi_i \rightarrow \infty$ is a strong condition for boundedness. (BFB)
- For applying BFB criterion, concepts of copositivity criteria and gauge orbit spaces can help simplify the analysis.

Kim (JMP '84), Kannike (EPJC '12)
Consider the scalar potential of a theory with two Higgs fields $\phi$ and $\pi$ charged under $G$ and $G'$ respectively:

\[
V(\phi, \pi) = -\mu_1^2 (\phi_i^* \phi_i) - \mu_2^2 (\pi_i^* \pi_i) + \lambda_1 (\phi_i^* \phi_i)^2 + \lambda_2 f_{ijkl} \phi_i^* \phi_j \phi_k^* \phi_l \\
+ \rho_1 (\pi_i^* \pi_i)^2 + \rho_2 g_{ijkl} \pi_i^* \pi_j \pi_k^* \pi_l + \cdots \\
+ \alpha_1 (\phi_i^* \phi_i)(\pi_j^* \pi_j) + \cdots (\text{other terms coupling } (\phi, \pi))
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$$+ \rho_1(\pi_i^* \pi_i)^2 + \rho_2 g_{ijkl} \pi_i^* \pi_j \pi_k^* \pi_l + \cdots$$

$$+ \alpha_1(\phi_i^* \phi_i)(\pi_j^* \pi_j) + \cdots \text{(other terms coupling } (\phi, \pi))$$

• The dimensionless ratios of invariants called orbit space parameters are defined:

$$A_n(\hat{\phi}) = \frac{f_{ijkl} \phi_i^* \phi_j \phi_k^* \phi_l}{(\phi_i^* \phi_i)^2} \quad B_n(\hat{\pi}) = \frac{g_{ijkl} \pi_i^* \pi_j \pi_k^* \pi_l}{(\pi_j^* \pi_j)^2}$$
Gauge Orbit Spaces

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$$+ \rho_1(\pi_i^* \pi_i)^2 + \rho_2 g_{ijkl} \pi_i^* \pi_j^* \pi_k^* \pi_l + \cdots$$

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- Similarly, for coupled terms $C_n(\hat{\phi}, \hat{\pi})$ can be defined but normalized by $\phi_i^* \phi_i \pi_j^* \pi_j$. 
• The potential can be written as:

\[
V(\phi, \pi) = -\mu_1^2|\phi|^2 - \mu_2^2|\pi|^2 + |\phi|^4A(\lambda, \phi) + |\pi|^4B(\rho, \hat{\pi})
+ |\phi|^2|\pi|^2C(\alpha, \hat{\phi}, \hat{\pi})
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Gauge Orbit Spaces

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\]

- where,

\[
|\phi|^2 = \phi_i^*\phi_i, \quad |\pi|^2 = \pi_i^*\pi_i, \quad \hat{\phi} = \frac{\phi}{|\phi|}, \quad \hat{\pi} = \frac{\pi}{|\pi|}
\]

\[
A(\lambda, \hat{\phi}) = \lambda_1 + \lambda_2A_1(\hat{\phi}) + \lambda_3A_2(\hat{\phi}) + \cdots
\]

\[
B(\rho, \hat{\pi}) = \rho_1 + \rho_2B_1(\hat{\pi}) + \rho_3B_2(\hat{\pi}) + \cdots
\]

\[
C(\alpha, \hat{\phi}, \hat{\pi}) = \alpha_1 + \alpha_2C_1(\hat{\phi}, \hat{\pi}) + \cdots
\]
Copositivity

- Requiring boundedness $\forall A(\lambda, \phi), B(\rho, \pi), C(\alpha, \phi, \hat{\phi})$:

$$|\phi|^4 A(\lambda, \phi) + |\pi|^4 B(\rho, \pi) + |\phi|^2 |\pi|^2 C(\alpha, \phi, \hat{\phi}) > 0$$
Copositivity

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- Given a condition of this form, is termed as copositive:
  \[ ax^2 + bx + c > 0 \quad x \in \mathbb{R}^+ \]
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  \[ a > 0, \ c > 0, \ b + 2\sqrt{ac} > 0 \]
Copositivity

- Requiring boundedness $\forall A(\lambda, \hat{\phi}), B(\rho, \hat{\pi}), C(\alpha, \hat{\phi}, \hat{\pi})$:

  $$\left|\phi\right|^4A(\lambda, \hat{\phi}) + \left|\pi\right|^4B(\rho, \hat{\pi}) + \left|\phi\right|^2\left|\pi\right|^2C(\alpha, \hat{\phi}, \hat{\pi}) > 0$$

- Given a condition of this form, is termed as copositive:

  $$ax^2 + bx + c > 0 \quad x \in \mathbb{R}^+$$

- The conditions for copositivity are:

  $$a > 0, c > 0, b + 2\sqrt{ac} > 0$$

  $$\implies A > 0, B > 0, C + 2\sqrt{AB} > 0$$
Scalar Potential: \( \lambda \) terms

\[
V \supset -\mu_1^2 \text{Tr}[\phi^\dagger \phi] - \mu_2^2 \left( \text{Tr}[\tilde{\phi} \phi^\dagger] + \text{Tr}[\tilde{\phi}^\dagger \phi] \right) \\
+ \lambda_1 \text{Tr}[\phi^\dagger \phi]^2 + \lambda_2 \left( \text{Tr}[\tilde{\phi} \phi^\dagger]^2 + \text{Tr}[\tilde{\phi}^\dagger \phi]^2 \right) + \lambda_3 \text{Tr}[\tilde{\phi} \phi^\dagger] \text{Tr}[\tilde{\phi}^\dagger \phi] \\
+ \lambda_4 \text{Tr}[\phi^\dagger \phi] \left( \text{Tr}[\tilde{\phi} \phi^\dagger] + \text{Tr}[\tilde{\phi}^\dagger \phi] \right)
\]
Scalar Potential: $\lambda$ terms

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$$+ \lambda_1 \text{Tr}[\phi^\dagger \phi]^2 + \lambda_2 \left( \text{Tr}[\tilde{\phi} \phi^\dagger]^2 + \text{Tr}[\tilde{\phi}^\dagger \phi]^2 \right) + \lambda_3 \text{Tr}[\tilde{\phi} \phi^\dagger] \text{Tr}[\tilde{\phi}^\dagger \phi]$$
$$+ \lambda_4 \text{Tr}[\phi^\dagger \phi] \left( \text{Tr}[\tilde{\phi} \phi^\dagger] + \text{Tr}[\tilde{\phi}^\dagger \phi] \right)$$

- we parametrize $V_4^\lambda$ as follows:

$$\text{Tr}[\Phi^\dagger \Phi] \equiv r^2$$
$$\text{Tr}[\tilde{\Phi} \Phi^\dagger]/\text{Tr}[\Phi^\dagger \Phi] \equiv \xi e^{i\omega}$$
$$\text{Tr}[\tilde{\Phi}^\dagger \Phi]/\text{Tr}[\Phi^\dagger \Phi] \equiv \xi e^{-i\omega}$$

where $r > 0$, $\xi \in [0, 1]$ and $\omega \in [0, 2\pi]$. 
Stability: $\lambda$ terms

- Using parametrization,

$$V_4^\lambda = r^4 \left( \lambda_1 + 2\lambda_2 \xi^2 \cos 2\omega + \lambda_3 \xi^2 + 2\lambda_4 \xi \cos \omega \right) \equiv r^4 f(\lambda, \xi, \omega)$$
Stability : $\lambda$ terms

- Using parametrization,

$$V^\lambda_4 = r^4 \left( \lambda_1 + 2\lambda_2 \xi^2 \cos 2\omega + \lambda_3 \xi^2 + 2\lambda_4 \xi \cos \omega \right) \equiv r^4 f(\lambda, \xi, \omega)$$

- By analyzing the boundedness of $\lambda$ sector of the potential,

$$\lambda_1 > 0 \quad (1)$$

$$\lambda_1 - \frac{\lambda_4^2}{2\lambda_2 + \lambda_3} > 0 \quad (2)$$

$$\lambda_1 + \lambda_3 - 2\lambda_2 - \frac{\lambda_4^2}{4\lambda_2} > 0 \quad (3)$$

$$\lambda_1 + \lambda_3 + 2(\lambda_2 - |\lambda_4|) > 0 \quad (4)$$
Sample case: $\lambda$'s

Values of set $\lambda$'s are:

\[ \lambda_2 = 1 \]
\[ \lambda_4 = -2 \]
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\[
\lambda_2 = 1 \\
\lambda_4 = -2
\]

\[
\lambda_1 > 0 \\
\lambda_1 - \frac{4}{2 + \lambda_3} > 0 \\
\lambda_1 + \lambda_3 > 3 \\
\lambda_1 + \lambda_3 > 2
\]
Sample case: $\lambda$’s

Values of set $\lambda$’s are:

$\lambda_2 = 1$

$\lambda_4 = -2$

$\lambda_1 > 0$

$\lambda_1 - \frac{4}{2 + \lambda_3} > 0$

$\lambda_1 + \lambda_3 > 3$

$\lambda_1 + \lambda_3 > 2$
Dreaded case: $\alpha_{1,3} \neq 0$

\[
V_4 = \lambda_1 \text{Tr} [\Phi^\dagger \Phi]^2 + \lambda_2 \left( \text{Tr} [\Phi \Phi^\dagger]^2 + \text{Tr} [\Phi^\dagger \Phi]^2 \right) + \lambda_3 \text{Tr} [\Phi \Phi^\dagger] \text{Tr} [\Phi \Phi^\dagger] + \lambda_4 \text{Tr} [\Phi^\dagger \Phi] \left( \text{Tr} [\Phi \Phi^\dagger] + \text{Tr} [\Phi^\dagger \Phi] \right) \\
+ \rho_1 \left( \text{Tr} [\Delta_L \Delta_L^\dagger]^2 + \text{Tr} [\Delta_R \Delta_R^\dagger]^2 \right) + \rho_2 \left( \text{Tr} [\Delta_L \Delta_L] \text{Tr} [\Delta_L^\dagger \Delta_L^\dagger] + \text{Tr} [\Delta_R \Delta_R] \text{Tr} [\Delta_R^\dagger \Delta_R^\dagger] \right) \\
+ \rho_3 \text{Tr} [\Delta_L \Delta_L^\dagger] \text{Tr} [\Delta_R \Delta_R^\dagger] + \rho_4 \left( \text{Tr} [\Delta_L \Delta_L] \text{Tr} [\Delta_R^\dagger \Delta_R^\dagger] + \text{Tr} [\Delta_L^\dagger \Delta_L^\dagger] \text{Tr} [\Delta_R \Delta_R] \right) \\
+ \alpha_1 \text{Tr} [\Phi^\dagger \Phi] \left( \text{Tr} [\Delta_L \Delta_L^\dagger] + \text{Tr} [\Delta_R \Delta_R^\dagger] \right) + \alpha_3 \left( \text{Tr} [\Phi \Phi^\dagger \Delta_L \Delta_L^\dagger] + \text{Tr} [\Phi^\dagger \Phi \Delta_R \Delta_R^\dagger] \right)
\]
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V_4 = \lambda_1 \text{Tr}[\Phi^\dagger \Phi]^2 + \lambda_2 \left( \text{Tr}[\Phi^\dagger \Phi]^2 + \text{Tr}[\Phi^\dagger \Phi]^2 \right) + \lambda_3 \text{Tr}[\Phi^\dagger \Phi] \text{Tr}[\Phi^\dagger \Phi] + \lambda_4 \text{Tr}[\Phi^\dagger \Phi] \left( \text{Tr}[\Phi^\dagger \Phi] + \text{Tr}[\Phi^\dagger \Phi] \right) \\
+ \rho_1 \left( \text{Tr}[\Delta_L \Delta_L^\dagger]^2 + \text{Tr}[\Delta_R \Delta_R^\dagger]^2 \right) + \rho_2 \left( \text{Tr}[\Delta_L \Delta_L] \text{Tr}[\Delta_L^\dagger \Delta_L^\dagger] + \text{Tr}[\Delta_R \Delta_R] \text{Tr}[\Delta_R^\dagger \Delta_R^\dagger] \right) \\
+ \rho_3 \text{Tr}[\Delta_L \Delta_L^\dagger] \text{Tr}[\Delta_R \Delta_R^\dagger] + \rho_4 \left( \text{Tr}[\Delta_L \Delta_L] \text{Tr}[\Delta_R^\dagger \Delta_R^\dagger] + \text{Tr}[\Delta_L^\dagger \Delta_L^\dagger] \text{Tr}[\Delta_R \Delta_R] \right) \\
+ \alpha_1 \text{Tr}[\Phi^\dagger \Phi] \left( \text{Tr}[\Delta_L \Delta_L^\dagger] + \text{Tr}[\Delta_R \Delta_R^\dagger] \right) + \alpha_3 \left( \text{Tr}[\Phi^\dagger \Phi \Delta_L \Delta_L^\dagger] + \text{Tr}[\Phi^\dagger \Phi \Delta_R \Delta_R^\dagger] \right)
\]

\[
\text{Tr}[\Phi^\dagger \Phi] + \text{Tr}[\Delta_L \Delta_L^\dagger] + \text{Tr}[\Delta_R \Delta_R^\dagger] = r^2 \\
\text{Tr}[\Phi^\dagger \Phi] = r^2 \cos^2 \theta \\
\text{Tr}[\Delta_L \Delta_L^\dagger] = r^2 \sin^2 \gamma \sin^2 \theta \\
\text{Tr}[\Delta_R \Delta_R^\dagger] = r^2 \cos^2 \gamma \sin^2 \theta \\
\text{Tr}[\Phi^\dagger \Phi]/\text{Tr}[\Phi^\dagger \Phi] = \xi e^{i\omega} \\
\text{Tr}[\Phi^\dagger \Phi]/\text{Tr}[\Phi^\dagger \Phi] = \xi e^{-i\omega} \\
\text{Tr}[\Delta_L \Delta_L^\dagger]/\text{Tr}[\Delta_L \Delta_L^\dagger] = \eta_1 e^{i\theta_1} \\
\text{Tr}[\Delta_L^\dagger \Delta_L^\dagger]/\text{Tr}[\Delta_L \Delta_L^\dagger] = \eta_1 e^{-i\theta_1} \\
\text{Tr}[\Delta_R \Delta_R^\dagger]/\text{Tr}[\Delta_R \Delta_R^\dagger] = \eta_2 e^{i\theta_2} \\
\text{Tr}[\Delta_R^\dagger \Delta_R^\dagger]/\text{Tr}[\Delta_R \Delta_R^\dagger] = \eta_2 e^{-i\theta_2} \\
\text{Tr}[\Phi^\dagger \Phi \Delta_L \Delta_L^\dagger]/\text{Tr}[\Phi^\dagger \Phi \Delta_L \Delta_L^\dagger] = \zeta_1 \\
\text{Tr}[\Phi^\dagger \Phi \Delta_R \Delta_R^\dagger]/\text{Tr}[\Phi^\dagger \Phi \Delta_R \Delta_R^\dagger] = \zeta_2
\]

with $r > 0$, $|\xi| \leq 1$, $\theta \in [0, \frac{\pi}{2}]$, $\gamma \in [0, \frac{\pi}{2}]$, $\eta_1, \eta_2 \in [0, 1]$, $\theta_1, \theta_2 \in [0, 2\pi]$ and $\zeta_1, \zeta_2 \in [0, 1]$. 
Analytic Conditions for Vacuum Stability in LRSM

- $f > 0:\ \begin{cases} \lambda_1 \\ (\lambda_1 - \frac{\lambda_4^2}{2\lambda_2 + \lambda_3}) \iff 2\lambda_2 + \lambda_3 > |\lambda_4| \\ (\lambda_1 + \lambda_3 + 2(\lambda_2 - |\lambda_4|)) \\ (\lambda_1 + \lambda_3 - 2\lambda_2 - \frac{\lambda_4^2}{4\lambda_2}) \iff |4\lambda_2| > |\lambda_4| \end{cases}$
Analytic Conditions for Vacuum Stability in LRSM

- $f > 0$:
  \[
  \begin{aligned}
  \lambda_1 & \\
  \left( \lambda_1 - \frac{\lambda_4^2}{2\lambda_2 + \lambda_3} \right) & \iff 2\lambda_2 + \lambda_3 > |\lambda_4| \\
  (\lambda_1 + \lambda_3 + 2(\lambda_2 - |\lambda_4|)) & \\
  \left( \lambda_1 + \lambda_3 - 2\lambda_2 - \frac{\lambda_4^2}{4\lambda_2} \right) & \iff |4\lambda_2| > |\lambda_4|
  \end{aligned}
  \]

- $g > 0$:
  \[
  \left\{ \rho_1, \rho_1 + \rho_2, \frac{\rho_3 + 2\rho_1}{4}, \frac{\rho_3 - 2|\rho_4| + 2(\rho_1 + \rho_2)}{4} \right\}
  \]
Analytic Conditions for Vacuum Stability in LRSM

- $f > 0$:
  \[
  \begin{align*}
  \lambda_1 & \quad \left( \lambda_1 - \frac{\lambda_4^2}{2\lambda_2 + \lambda_3} \right) \iff 2\lambda_2 + \lambda_3 > |\lambda_4| \\
  (\lambda_1 + \lambda_3 + 2(\lambda_2 - |\lambda_4|)) & \quad \left( \lambda_1 + \lambda_3 - 2\lambda_2 - \frac{\lambda_4^2}{4\lambda_2} \right) \iff |4\lambda_2| > |\lambda_4|
  \end{align*}
  \]

- $g > 0$:
  \[
  \rho_1, \quad \rho_1 + \rho_2, \quad \frac{\rho_3 + 2\rho_1}{4}, \quad \frac{\rho_3 - 2|\rho_4| + 2(\rho_1 + \rho_2)}{4}
  \]

- \[
  \alpha_1 + 2\sqrt{\text{Min}(f) \text{ Min}(g)} > 0
  \]

- \[
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Symmetry Breaking in LRSM

- A BFB potential does not necessarily lead to correct symmetry breaking. *Dev, Mohapatra, Rodejohann, Xu (JHEP ’19)*
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- The desired VEV structure for LRSM vacuum is

\[
\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 e^{i\theta_2} \end{pmatrix}, \quad \Delta_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_L e^{i\theta_L} & 0 \end{pmatrix},
\]

\[
\Delta_R = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}
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\[ \Delta_R = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \]

- We can generalize the gauge-independent conditions for correct vacuum in the LRSM as:

\[ \text{Tr}[\langle \Phi \rangle \langle \Phi \rangle] \neq 0 \]

\[ \text{Tr}[\langle \Delta_L \rangle \langle \Delta_L \rangle] = \text{Tr}[\langle \Delta_R \rangle \langle \Delta_R \rangle] = 0 \]

\[ \text{Tr}[\langle \Delta_L \rangle \langle \Delta_L^\dagger \rangle] < \text{Tr}[\langle \Delta_R \rangle \langle \Delta_R^\dagger \rangle] \]
Symmetry Breaking in LRSM

- Plugging the VEV structure in the scalar potential, we get:

\[
V = -\frac{(\kappa_1^2 + \kappa_2^2)}{2} \mu_1^2 - 2\kappa_1\kappa_2\mu_2^2 \cos(\theta_2) - \mu_3^2 (v_L^2 + v_R^2) \\
+ \frac{(\kappa_1^2 + \kappa_2^2)^2}{4} \lambda_1 + 2\kappa_1^2\kappa_2^2 \lambda_2 \cos(2\theta_2) + \\
\kappa_1\kappa_2 (\kappa_1^2 + \kappa_2^2) \lambda_4 \cos(\theta_2) + \kappa_1^2\kappa_2^2 \lambda_3 + \rho_1 (v_L^4 + v_R^4) + \rho_3 v_L^2 v_R^2 \\
+ \alpha_1 \frac{(\kappa_1^2 + \kappa_2^2)}{2} (v_L^2 + v_R^2) + \alpha_3 \frac{\kappa_2^2}{2} (v_L^2 + v_R^2)
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\]

- Using earlier parametrization:

\[
V_4 \equiv r^4 \left( f_{SSB} \cos^4 \theta + g_{SSB} \sin^4 \theta + h_{SSB} \cos^2 \theta \sin^2 \theta \right)
\]
• Plugging the VEV structure in the scalar potential, we get:

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\]

• Requiring a deeper minima for \(V_{SSB}\)

\[
f \geq f_{SSB}, \quad g > g_{SSB} \\
h + 2\sqrt{fg} > h_{SSB} + 2\sqrt{f_{SSB} g_{SSB}}
\]
Analytic Conditions for Symmetry Breaking to Correct Vacuum

\[ f_{SSB} > 0 : \begin{cases} 
  \lambda_1 > 0, \quad \sigma = 0, \\
  \left( \lambda_1 - \frac{\lambda_4^2}{2\lambda_2 + \lambda_3} \right) > 0 \iff 2\lambda_2 + \lambda_3 > |\lambda_4|, \quad \sigma = -\frac{\lambda_4}{2\lambda_2 + \lambda_3}, \\
  (\lambda_1 + \lambda_3 + 2(\lambda_2 - |\lambda_4|)) > 0, \quad \sigma = -\text{sgn}(\lambda_4), \\
  (\lambda_1 + \lambda_3 - 2\lambda_2 - \frac{\lambda_4^2}{4\lambda_2}) > 0 \iff |4\lambda_2| > |\lambda_4|, \quad \sigma = -\frac{\lambda_4}{4\lambda_2}, 
\end{cases} \]

\[ \rho_1 > 0, \quad \rho_2 > 0, \quad \rho_3 > 2\rho_1, \quad |\rho_4| < \frac{\rho_3 - 2\rho_1}{2} + \rho_2 \]

\[ \alpha_1 + 2\sqrt{\text{Min}[f_{SSB}]\rho_1} > 0 \]

\[ \alpha_1 + \alpha_3 + 2\sqrt{\text{Min}[f_{SSB}]\rho_1} > 0 \]

\[ \bar{\mu}_1^2 = \mu_1^2 + 2\sigma \mu_2^2 \]

\[ 2\sqrt{\text{Min}[f_{SSB}]\rho_1} - ||\text{Min}[\alpha_1, \alpha_1 + \alpha_3]|| > 0 \]

\[ 2\rho_1\bar{\mu}_1^2 - \text{Min}[\alpha_1, \alpha_1 + \alpha_3]\mu_3^2 > 0 \]
Numerical Minimization
The scalar mass spectrum for LRSM:

\[ M_{H_0}^2 = 2 \left( \lambda_1 - \frac{\alpha_1^2}{4\rho_1} \right) \kappa_+^2, \]

\[ M_{H_0}^2 \sim M_{A_0}^2 \sim M_{H_1}^2 = \frac{1}{2} \alpha_3 v_R^2, \]

\[ M_{H_2}^2 = 2\rho_1 v_R^2, \]

\[ M_{H_1}^2 \sim M_{H_1}^2 \sim M_{A_2}^2 = M_{H_3}^2 = \frac{1}{2} (\rho_3 - 2\rho_1) v_R^2, \]

\[ M_{H_2}^2 = 2\rho_2 v_R^2 + \frac{1}{2} \alpha_3 \kappa_+^2. \]

Renormalization Group Analysis

- The scalar mass spectrum for LRSM:

\[
M_{H_0}^2 = 2 \left( \lambda_1 - \frac{\alpha_1^2}{4\rho_1} \right) \kappa_+^2,
\]

\[
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\]

\[
M_{H_2}^2 = 2\rho_1 v_R^2,
\]

\[
M_{H_{1\pm}}^2 \sim M_{H_1}^2 \sim M_{A_2}^2 = M_{H_3}^2 = \frac{1}{2} (\rho_3 - 2\rho_1) v_R^2,
\]

\[
M_{H_{2\pm}}^2 = 2\rho_2 v_R^2 + \frac{1}{2} \alpha_3 \kappa_+^2.
\]


- We have taken the best fit value of \(M_{H_0} = m_h = 125 \text{ GeV}\).
• There are strong experimental bounds on most scalar masses in LRSM. GC,Dev,Mohapatra,Zhang (JHEP ’19)
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\[ M_{H^0}, A^0_1 > 15 \text{ TeV} \]
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• Stringent limits on the heavy neutral scalars masses from the FCNC constraints : Zhang,An,Ji,Mohapatra (Nucl.Phys. ’08)

\[ M_{H_1^0,A_1^0} > 15 \text{ TeV} \]

• The current bounds on doubly charged Higgs masses are from LHC 13 TeV run data : ATLAS, CMS

\[ M_{H_1^{\pm \pm}} \gtrsim (770 - 870) \text{ GeV} \quad M_{H_2^{\pm \pm}} \gtrsim (660 - 760) \text{ GeV} \]
This sample benchmark is in complete agreement with the current experimental bounds on the scalar masses.

\[
\begin{align*}
\mu_1^2, \mu_2^2, \mu_3^2 & \equiv (8.48^2, 0, (11.99)^2) \text{ TeV}^2 \\
\lambda_1, \lambda_2, \lambda_3, \lambda_4 & \equiv (0.0625, 0, 0, 0) \\
\rho_1, \rho_2, \rho_3, \rho_4 & \equiv (0.01, 0.0005, 0.0226, 0) \\
\alpha_1, \alpha_2, \alpha_3 & \equiv (0.01, 0, 0.64) \\
\beta_1, \beta_2, \beta_3 & \equiv (0, 0, 0)
\end{align*}
\]
Renormalization Group Analysis

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\[
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\]

• The VEVs for the Higgses are:

\[
\kappa_+ = \sqrt{\kappa_1^2 + \kappa_2^2} = 246 \text{ GeV}, \quad v_L = 0 \text{ TeV}, \quad v_R = 26.8 \text{ TeV}
\]
Renormalization Group Analysis

- **Quartic couplings**
  - $\lambda_1$, $\lambda_2$, $\lambda_3$, $\lambda_4$
  - $\rho_1$, $\rho_2$, $\rho_3$, $\rho_4$

- **Mass**
  - $\mu_1$, $\mu_2$, $\mu_3$

Graphs show the evolution of quartic couplings and mass with respect to the scale $\mu$ (GeV) and $\mu$ (TeV), highlighting the perturbative limit.
Conclusions

- We obtained necessary and sufficient conditions for the stability of LRSM potential using copositivity and gauge orbit spaces.

- Only requiring vacuum stability does not ensure SSB to a vacuum which reproduces SM at low-energies.

- Extended the vacuum stability analysis to yield necessary and sufficient conditions to achieve SSB to the correct vacuum which should be charge conserving and also parity violating at low-energies.

- These analytic techniques can be extended to analyze metastability of the vacuum and one-loop effective potential.
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Thank you !!