Unifying the Background with Perturbations in Chaplygin-gas Cosmology II

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1. The background solutions
2. Cosmological perturbations
3. Perturbations
4. Matter-Chaplygin gas System
5. Conclusion

Motivation

The aim of this project is to find a suitable model that could possibly solve the dark matter and dark energy problems, and make it possible to predict the future of the Universe.

The Friedmann Equation

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}, \tag{1}$$

The Fluid Equation

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0, \tag{2}$$

Curvature and... stuff

Barotropic Equation of State for perfect fluids: relates pressure p and energy density ρ by

$$p = \omega\rho, \tag{3}$$

where ω is the Equation of State Parameter given by

$$\omega = \begin{cases} 0 & \text{for dark and baryonic matter (dust),} \\ 1/3 & \text{for radiation, and} \\ -1 & \text{for dark energy, etc...} \end{cases} \tag{4}$$

Solutions to the Fluid Equation

The solution to the fluid equation (when substituting in different ω) produces energy densities:

$$\rho_d(a) = \rho_{0_d} a^{-3} \tag{5}$$

$$\rho_r(a) = \rho_{0_r} a^{-4} \tag{6}$$

$$\rho_\Lambda(a) = \rho_{0_\Lambda} \tag{7}$$

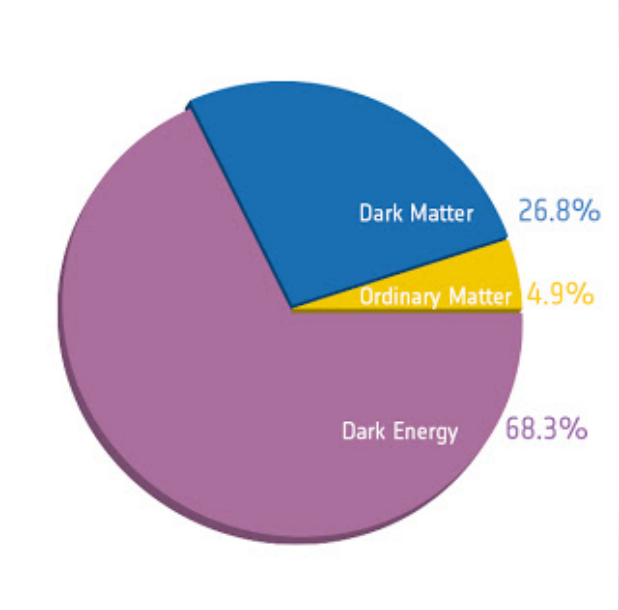


Figure: The contents of the Universe according to recent Planck data. Credit: Katie Mack.

The Chaplygin-Gas

- Chaplygin Gas: a hypothetical fluid that behaves like dark matter and dark energy combined
- Equation of State for generalized CG:

$$p = -\frac{A}{\rho^{\alpha}} \tag{8}$$

- Substitute (8) into fluid equation to produce energy density solution:

$$\rho_{ch}(a) = \left(A + \frac{B}{a^{3(1+\alpha)}}\right)^{\frac{1}{1+\alpha}}, \tag{9}$$

where A and B are positive constants, and $0 < \alpha \leq 1$.

CG behaves like DM and DE

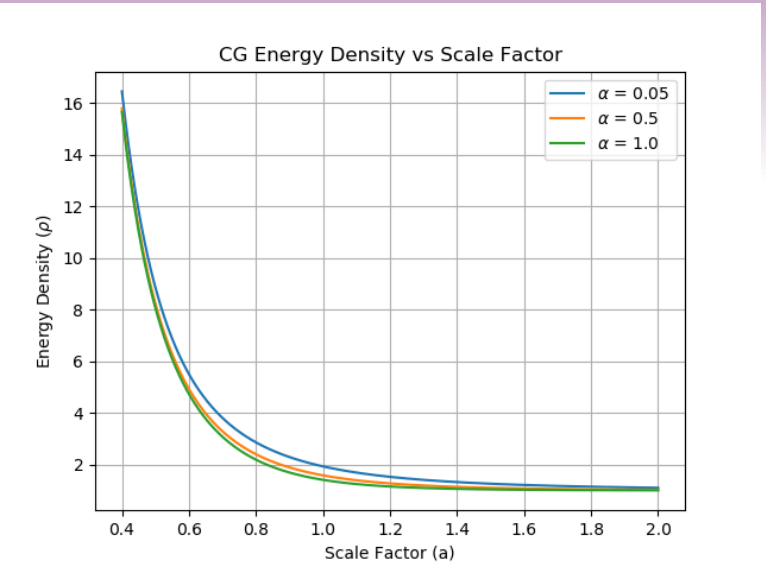
Let us take $\alpha = 1$ for simplicity (original CG):

$$\rho_{ch}(a) = \sqrt{A + \frac{B}{a^6}} \tag{10}$$

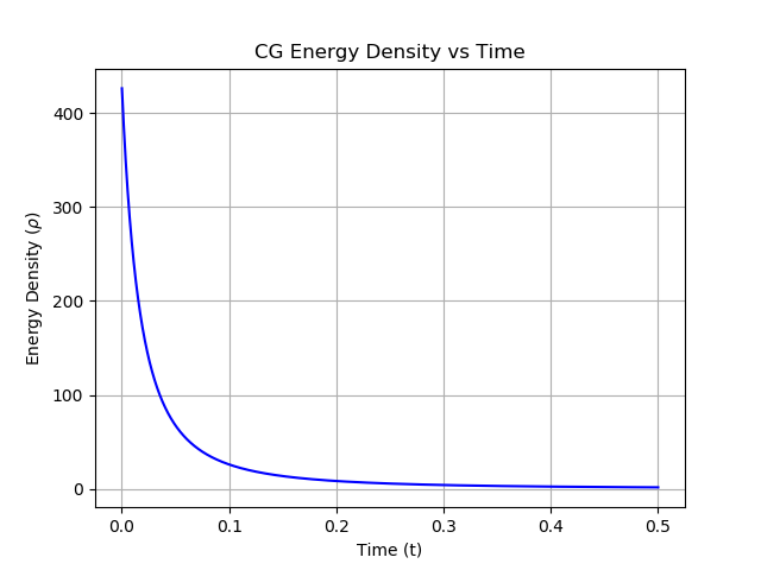
- When the scale factor a is very small (early Universe), then $\rho_{ch}(a) \simeq \frac{\sqrt{B}}{a^3} \longrightarrow$ similar to dust/dark matter ($\rho \sim a^{-3}$).
- When the scale factor a is very large (late Universe), then $\rho_{ch}(a) \simeq \sqrt{A} \longrightarrow$ similar to dark energy ($\rho \sim constant$).
- A universe consisting of Chaplygin gas, radiation and baryonic matter:

$$\rho_{total}(a) = \left[A + \frac{B}{a^{3(1+\alpha)}}\right]^{\frac{1}{(1+\alpha)}} + \rho_{0r}a^{-4} + \rho_{0b}a^{-3} \tag{11}$$

The Chaplygin gas solutions



(a) CG density versus scale factor for different values of α ($\alpha = 1$ is known as the original CG model).



(b) ρ vs. t for the CG system.

Figure

Accelerated Expansion?

The deceleration parameter

$$q \equiv -\frac{\ddot{a}}{a}\frac{1}{H^2} \tag{12}$$

describes the rate of the expansion of the Universe. If $q < 0$, the expansion of the Universe is accelerating, and $q > 0$ means that the expansion of the Universe is decelerating.

Accelerated Expansion?

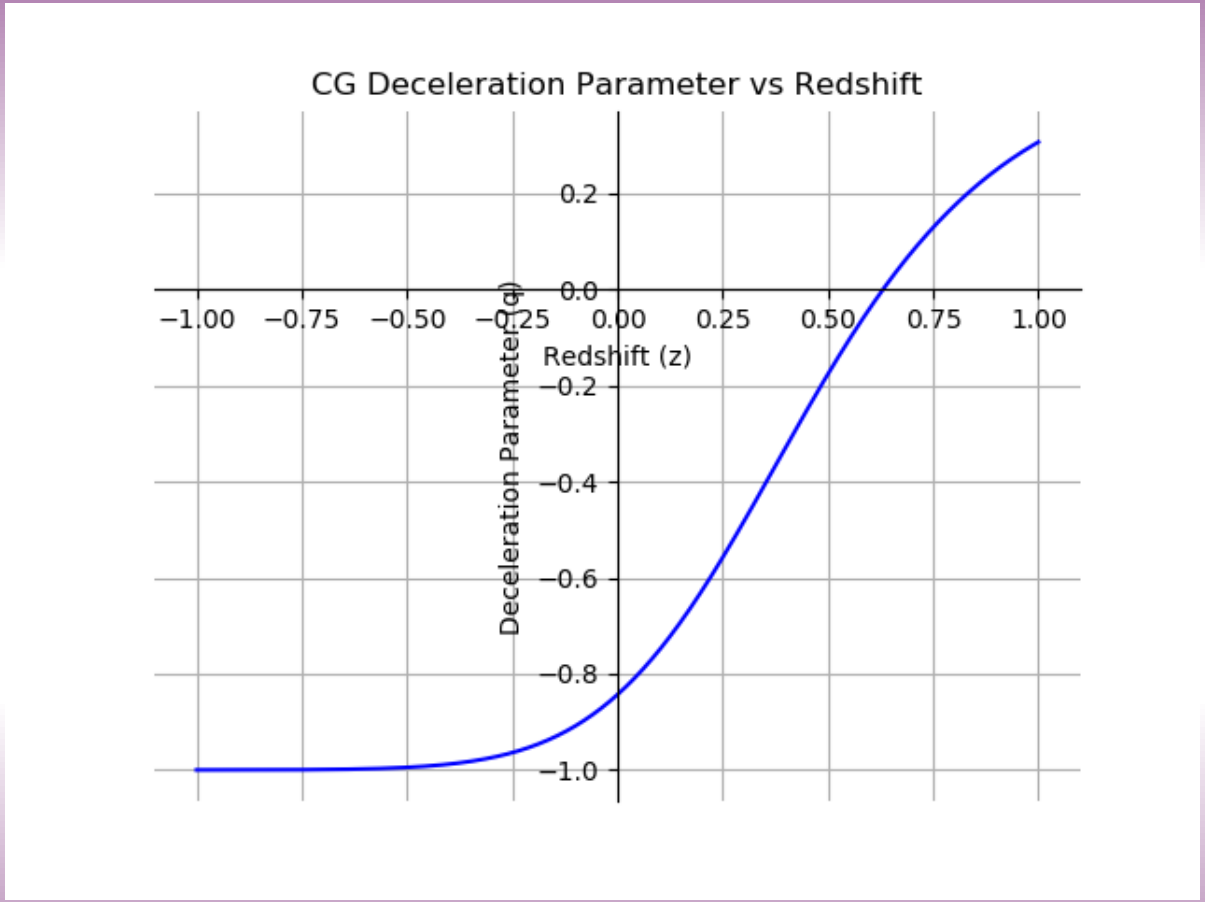


Figure : The deceleration parameter versus the redshift in a radiation-baryonic matter-CG system.

Cosmological perturbations

- The metric approach.

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- The metric approach.
  It is a non-local, linear and coordinate-dependent approach
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Cosmological perturbations

- The metric approach.
  It is a non-local, linear and coordinate-dependent approach
- The covariant approach.
  It is local and valid in all coordinate systems.

The $1+3$ covariant formalism

The space-time is split into space and time; it is divided into perpendicular 4-velocity field vector u^a

$$u^a = \frac{dx^a}{d\tau}, \qquad u^a u_a = -1$$

In this approach the metric g_{ab} is decomposed into the projected tensor h_{ab} as follows;

$$g_{ab} = h_{ab} - u_a u_b \tag{13}$$

with

$$h^a_c h^c_b = h^a_b,\ h^a_a = 3,\ h_{ab}u^b = 0$$

Kinematic and dynamic quantities

$$\nabla_a u_b = \tilde{\nabla}_a u_b - u_a \dot{u}_b = \frac{1}{3}\Theta h_{ab} + \sigma_{ab} + \omega_{ab} - u_a \dot{u}_b \tag{14}$$

The relativistic acceleration vector is given as

$$\dot{u}^a = u^b \nabla_b u^a \tag{15}$$

The vector gradient variables are defined as:

$$Z_a = a\tilde{\nabla}_a\theta \tag{16}$$

$$D_a^m = \frac{a}{\rho_m}\tilde{\nabla}_a\rho_m \tag{17}$$

$$D_a^{ch} = \frac{a\tilde{\nabla}_a\rho_{ch}}{\rho_{ch}} \tag{18}$$

Conservation equations for non−interacting fluids

$$\dot{\rho}_t = -\theta(\rho_t + p_t) + (\rho_t + p_t)\tilde{\nabla}^a \Psi_a \tag{19}$$

$$(\rho_t + p_t)\dot{u}_a + \tilde{\nabla}_a p_t + \dot{\Psi}_a - (3c_s^2 - 1)\frac{\theta}{3}\Psi_a + \Pi_a = 0 \tag{20}$$

where $\Psi_a = q_a/(\rho_t + p_t)$ is the heat flux of the fluids and $\Pi_a = \tilde{\nabla}^a \pi_{ab}$ is anisotropic stress.

Perturbations

For totall non-interacting fluids, the time derivative of the four-velocity vector

$$\dot{u}_a = \frac{\alpha\omega_{ch}\rho_{ch}D_a^{ch}}{a(\rho_t + p_t)} - \frac{\omega_m\rho_m D_a^m}{a(\rho_t + p_t)} \tag{21}$$

The another key equation for general fluid so called Raychaudhuri equation and it can be expressed as

$$\dot{\Theta} = -\frac{1}{3}\Theta^2 - \frac{1}{2}(\rho_t + 3p_t) - \tilde{\nabla}^a \dot{u}_a \ . \tag{22}$$

Linear evolution equations

$$\dot{Z}_a = -\frac{2\theta}{3} Z_a - \left(\frac{1}{2} + \frac{3\omega_m}{2} + \frac{\theta^2 \omega_m}{3(\rho_t + p_t)} + \frac{\omega_m(\rho_t + 3p_t)}{2(\rho_t + p_t)}\right)\rho_m \; D_a^m \tag{23}$$

$$- \left(\frac{1}{2} - \frac{3\alpha w_{ch}}{2} - \frac{\alpha\theta^2 w_{ch}}{3(\rho_t + p_t)} - \frac{\alpha w_{ch}(\rho_t + 3p_t)}{2(\rho_t + p_t)}\right) \rho_c h \; D_a^{ch} - a\tilde{\nabla}^2 \dot{u}_a$$

$$\dot{D}_a^m = -(1 + \omega_m) Z_a + \omega_m \theta D_a^m \tag{24}$$

$$\dot{D}_a^{ch} = -(1 + \omega_{ch}) Z_a + \omega_{ch} \theta D_a^{ch} \tag{25}$$

$$\dot{D}_a^t = \theta(1 + \omega_t) D_a^t - (1 + \omega_t) Z_a - \frac{\theta \rho_m}{\rho_t} D_a^m - \frac{\theta \rho_{ch}}{\rho_t} D_a^{ch} \tag{26}$$

The scalar decomposition

$$a\tilde{\nabla} a X_b = X_{ab} = \frac{1}{3} h_{ab} X + \Sigma^X_{ab} + X_{[ab]} \tag{27}$$

where $\Sigma^X_{ab} = X_{(ab)} - 3h_{ab}X$

We extract the scalar parts of the gradient variables as:

$$\Delta_m = a\tilde{\nabla}^a D^m_a \tag{28}$$

$$Z = a\tilde{\nabla}^a Z_a \tag{29}$$

$$\Delta_{ch} = a\tilde{\nabla}^a D^{ch}_a \tag{30}$$

Harmonic decomposition

$$\ddot{X} + A\dot{X} + BX = C(Y, \dot{Y}) \,, \tag{31}$$

$$X = \sum_k X^k Q_k(\vec{x}) \,, \qquad Y = \sum_k Y^k(t) Q_k(\vec{x}) \,, \tag{32}$$

where $Q_k(x)$ are the eigenfunctions of the covariantly defined spatial Laplace-Beltrami operator such that

$$\tilde{\nabla}^2 Q = -\frac{k^2}{a^2} Q \,. \tag{33}$$

The order of the harmonic (wave number) is given by

$$k = \frac{2\pi a}{\lambda} \,, \tag{34}$$

where λ is the physical wavelength of the mode.

Second−order evolution equations

The harmonic decomposition equations become

$$\Delta_m'' = \frac{1}{1+z}(\frac{1}{2} - 4\omega_m)\Delta_m' + \left\{ \frac{(1+\omega_m)}{(1+z)^2} \left[\frac{1}{2}(1+3\omega_m) \right.\right.$$
$$+ \frac{\omega_m}{(\Omega_{ch}+\Omega_m)(1+\omega_t)} + \frac{\omega_m(1+3\omega_t)}{2(1+\omega_t)} + \left. \frac{9\pi^2(1+\omega_m)^2\omega_m}{3\lambda^2(1+z)^{3(1+\omega_m)}(\Omega_{ch}+\Omega_m)} \right] 3\Omega_m$$
$$+ \left[3\omega_m - \frac{3}{2}\omega_m(\Omega_{ch}+\Omega_m)(1+\omega_t) \right\} \Delta_m^k$$
$$+ \frac{(1+\omega_m)}{(1+z)^2} \left[\frac{1}{2} - \frac{3\alpha\omega_{ch}}{2} - \frac{\alpha\omega_{ch}}{(\Omega_{ch}+\Omega_m)(1+\omega_t)} - \frac{\alpha\omega_{ch}(1+3\omega_t)}{2(1+\omega_t)} + \right.$$
$$\left. \frac{9\pi^2(1+\omega_m)^2\alpha\omega_{ch}}{3\lambda^2(1+z)^{3(1+\omega_m)}(\Omega_{ch}+\Omega_m)} \right] 3\Omega_{ch}\Delta_{ch}^k \ , \tag{35}$$

Second-order evolution equations (continued)

$$
\begin{aligned}
\Delta''_{ch} = {} & \frac{3\sqrt{4}-6}{\sqrt{4}(1+z)}\left(\omega_{ch}(1+\alpha)-\omega_{ch}+\frac{2}{3}\right)\Delta'_{ch} \\
& + \frac{(1+\omega_{ch})}{(1+z)^2}\left[\frac{1}{2}(1+3\omega_m)+\frac{\omega_m}{3(\Omega_m+\Omega_{ch})(1+\omega_t)}+\frac{\omega_m(1+3\omega_t)}{2(1+\omega_t)}\right. \\
& \left. + \frac{9\pi^2(1+\omega_m)^2\omega_m}{3\lambda^2(1+z)^{3(1+\omega_m)}(\Omega_{ch}+\Omega_m)}\right]3\Omega_m\Delta^k_m \\
& + \frac{1}{(1+z)^2}\left\{(1+\omega_{ch})\left[\frac{1}{2}-\frac{3\alpha\omega_{ch}}{2}-\frac{\alpha\omega_{ch}}{(\Omega_m+\Omega_{ch})(1+\omega_t)}\right.\right. \\
& \left. - \frac{\alpha\omega_{ch}(1+3\omega_t)}{2(1+\omega_t)}+\frac{9\pi^2(1+\omega_m)^2\alpha\omega_{ch}}{\lambda^2(A+B(1+z)^{3(1+\alpha)})^{\frac{1}{1+\alpha}}(\Omega_{ch}+\Omega_m)}\right]3\Omega_{ch} \\
& \left. -\omega_{ch}\left(3+\frac{3}{2}(1+3\omega_t)(\Omega_m+\Omega_{ch})\right)+9\omega_{ch}\left[(1+\alpha)(1+2\omega_{ch})3\Omega_{ch}+\frac{2}{3}\right]\right\}\Delta^k_{ch}\,, \qquad (36)
\end{aligned}
$$

Matter-Chaplygin gas System

Here we consider two non-interacting fluids namely radiation-Chaplygin gas and dust-Chaplygin system for further analysis.

Radiation-Chaplygin dominated Universe

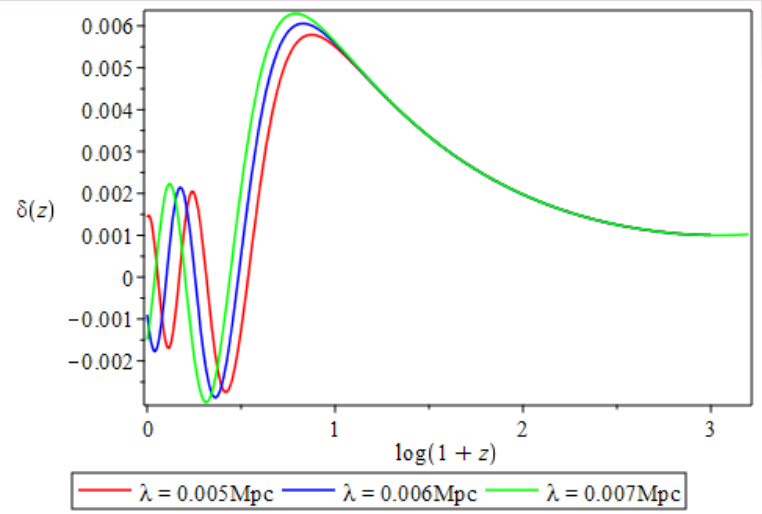
We consider the growth of the radiation fluctuations as a background of CG fluid, and $\Delta_r \gg \Delta_{ch}$, i.e., $\Delta_{ch} \approx 0$. Then solutions of our leading equation Eq. 35 or short- wavelength range becomes

$$\Delta(z) = C_1(1+z)^{\frac{1}{12}}\,\mathrm{BesselJ}\left(\frac{\Sigma}{24(\Omega_{ch}+\Omega_r)}, \frac{4\pi\sqrt{\Omega_r}}{3\lambda\sqrt{\Omega_{ch}+\Omega_r}(1+z)^2}\right) + C_2(1+z)^{\frac{1}{12}}\,\mathrm{BesselY}\left(\frac{\Sigma}{24(\Omega_{ch}+\Omega_r)}, \frac{4\pi\sqrt{\Omega_r}}{3\lambda\sqrt{\Omega_{ch}+\Omega_r}(1+z)^2}\right), \quad (37)$$

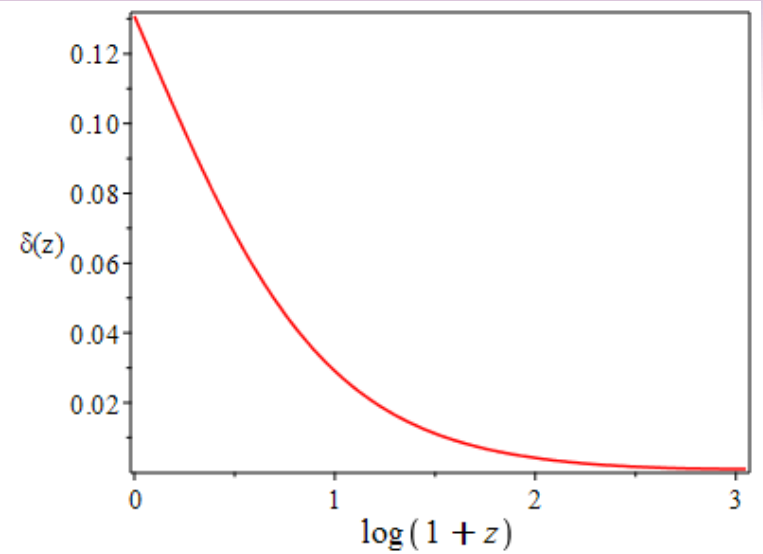
for long-wavelength range is given as

$$\Delta(z) = C_1(1+z)^{\Omega_{ch}+\Omega_r+\psi} + C_2(1+z)^{-(\Omega_{ch}+\Omega_r-\psi)}, \quad (38)$$

Radiation-Chaplygin dominated Universe



(a) $\delta(z)$ with z for short-wavelength, for $\Omega_r = 1 - \Omega_{ch}$ and for different wavelengths.



(b) $\delta(z)$ with z for long-wavelength and for $\Omega_r = 1 - \Omega_{ch}$. We use $\Omega_r = 4.48x10^{-5}$

Figure

Dust-Chaplygin dominated Universe

The solutions of our leading evolution equations for the density perturbations Eq.(35) dust-CG system becomes

$$\Delta(z) = C_1(1+z)^{\frac{3}{4}+\frac{1}{4}\sqrt{9+24\Omega_d}} + C_2(1+z)^{\frac{3}{4}-\frac{1}{4}\sqrt{9+24\Omega_d}} \,. \tag{39}$$

Dust-Chaplygin dominated Universe

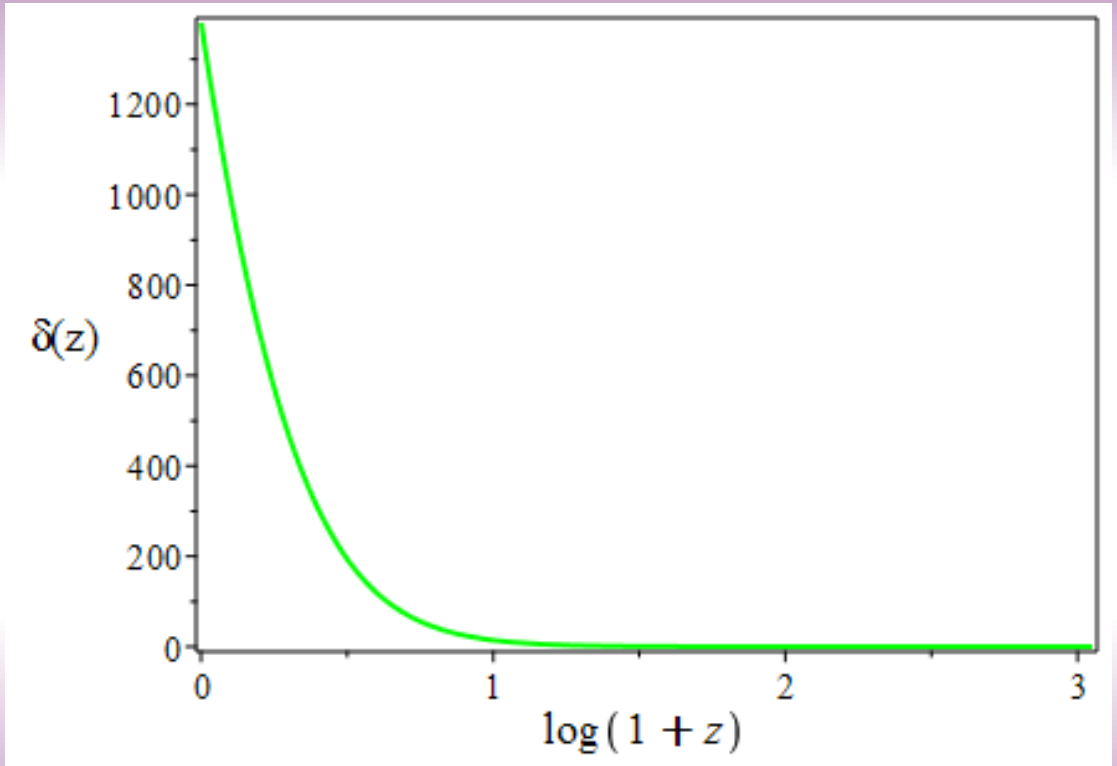


Figure : $\delta(z)$ with z for dust and for $\Omega_d = 1 - \Omega_{ch}$. We use $\Omega_d = 0.32$.

Chaplygin gas dominated Universe

In this case $\Delta_{ch} \gg \Delta_m$ and $\Delta_m \approx 0$, then the solution of our leading equation (36) read as

$$\Delta(z) = C_1 \log(1+z) \sin(\Omega_r+\Omega_d+\Omega_{ch}+5)+C_2 \log(1+z) \cos(\Omega_r+\Omega_d+\Omega_{ch}+5) \,. \tag{40}$$

Chaplygin gas dominated Universe

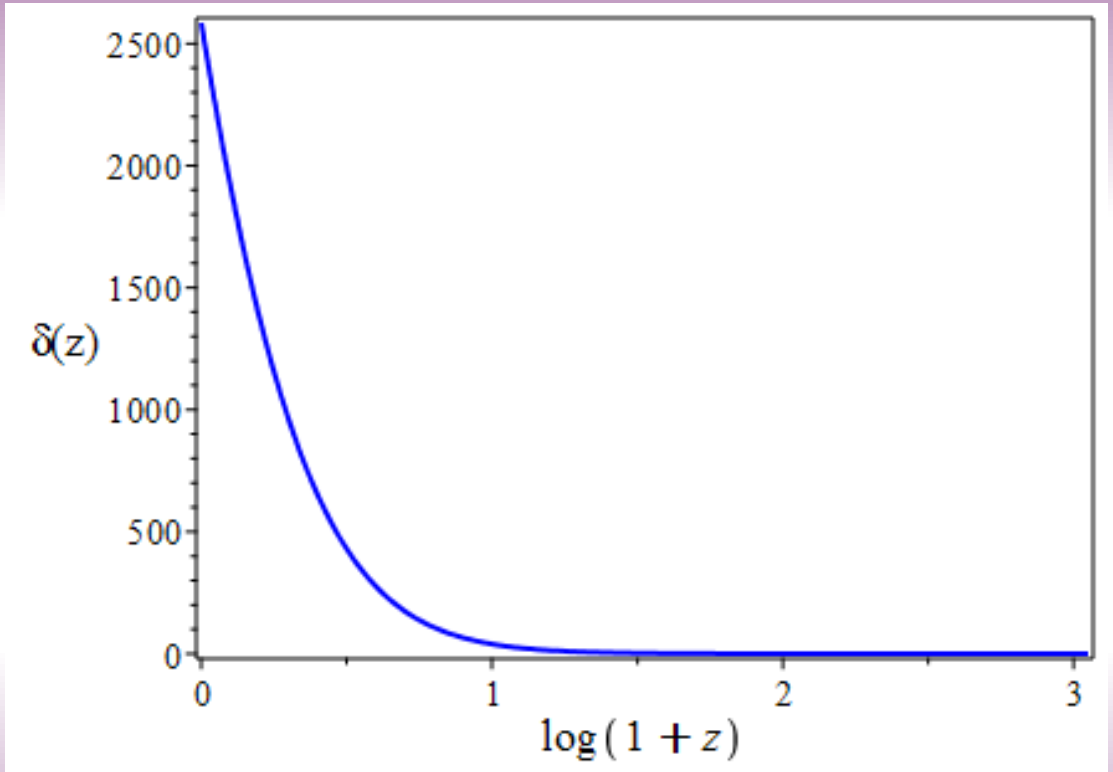


Figure : $\delta(z)$ with z for CG and for $1 - \Omega_r + \Omega_d = \Omega_{ch}$.

Conclusion

- The CG complies to the behaviour of dark energy and dark matter.
- It explains well the background expansion history.
- It has a significant role in the formation of the large-scale structure.
- A detailed exploration of cosmological perturbations of growth density with red-shift by unified the dark energy and dark matter in Chaplygin gas.

The Future

- More needs to be done to constrain the CG model with the cosmological observations.