Theoretical Conservation of Lepton Family and Representation Theories

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Introduction

- Lorentz Transformation
- Poincare Group
- Representation of Poincare as $SU(2) \otimes SU(2)$
- Left and Right Spinner
- Lorentz Scaler
- Boson as supper partner of fermion and vice versa
- Conservation of Lepontic family

Lorentz Transformation

$$x'^{\alpha} = \Lambda^{\alpha}_{;\beta} x^{\beta} + a^{\alpha}$$

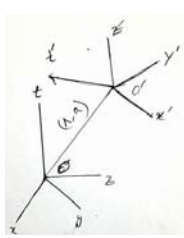
$$\| x \|^2 = \| x' \|^2 = \langle x, x \rangle = \langle x', x' \rangle,$$

$$x^{;\alpha} x_{\alpha} = x^{;\alpha} g_{\alpha\beta} x^{\beta} = x'_{;\beta} x'^{\beta}$$

Where $g_{\alpha\beta} = \begin{pmatrix} 1 & \underline{0}_{1\times3} \\ \underline{0}_{3\times1} & -\underline{I}_{3\times3} \end{pmatrix}$

with (+,-,-,-) signature

$$g^{bc} = \Lambda^{b}_{;\beta} g^{\beta\tau} \Lambda^{;c}_{\tau} \qquad \left.\begin{aligned} \partial'^{\alpha} &= \Lambda^{\alpha}_{;\beta} \partial^{\beta} \\ \partial'_{\alpha} &= \Lambda^{;\beta}_{\alpha} \partial_{\beta} \end{aligned}\right\}$$



Lorentz transformation as Lie Group

For Lie Symmetric Group

$$x \xrightarrow{\text{LT}} x' \xrightarrow{\text{LT}} x''$$

$$L'_T L''_T = (\Lambda' \Lambda, \Lambda' a + a')$$

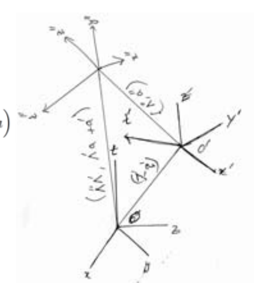
$$L_T{}^{-1} = (\Lambda^{-1}, -\Lambda^{-1} a) = \left(g^T \Lambda g, -(g^T \Lambda g) a \right)$$

For near to identity

$$\Lambda^{\mu}_{;\nu} = \delta^{\mu}_{;\nu}; \qquad a^{\mu} = 0$$

For Infinitesimal Transformation

$$\Lambda^{\mu}_{;\nu} = \delta^{\mu}_{;\nu} + \omega^{\mu}_{;\nu} \qquad a^{\mu} = \varepsilon^{\mu}$$



infinite representation as Unitary

$$\Lambda^{\mu}_{;\nu} = (\exp^{\omega})^{\mu}_{;\nu}$$

$$U(I + W, \xi) = I + \frac{i}{2!} \underbrace{w_{\propto\beta}}_{\Lambda^{\propto}_{;\beta}} J^{\propto\beta} - i \underbrace{\xi_{\propto}}_{a^{\propto}} P^{\propto} + O(w) + 0(\xi)$$

Therefore $J^{\dagger} = J$, $K^{\dagger} = k$

$$U(\Lambda, a) J^{\alpha\beta} U^{-1}(\Lambda, a) = \Lambda^{\alpha}_{a} \Lambda^{\beta}_{b} (J^{ab} - a^{a} p^{b} + a^{b} p^{a})$$

$$U(\Lambda, a) P^{\alpha} U^{-1}(\Lambda, a) = \Lambda^{\alpha}_{;a} P^{a}$$

For Spin Transformation (Rotational)

J^{ab} transform as Tensor P^{a} transform as 4-vector

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Translational Transformation

$$\Lambda^{\alpha}_{;\beta} = \delta^{\alpha}_{;\beta}$$

P is invariant but J in invariant with only origin Transformation

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For $J_i = \frac{1}{2} \varepsilon_{ijk} J^{jk}$ and $K_i = \varepsilon_{ijk} J^{i0}$

$$[J_i, J_j] = i\varepsilon^{k}_{ij} J_k$$

$$[J_i, K_j] = i\varepsilon^{k}_{ij} K_k$$

$$[K_i, K_j] = -i\varepsilon^{k}_{ij} J_k$$

- Angular momentum
- K (boost) Transform as three vector under rotation
- two boost gives rotation

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Representation as SU②⊗SU ②

By defining

$$N^{i} = \frac{1}{2}(J^{i} + iK^{i})$$

$$N^{i\dagger} = \frac{1}{2}(J^{i} - iK^{i})$$

We have

$$[N_i, N_j] = i\varepsilon^{k}_{ij} N_k$$

$$[N_i{}^{\dagger}, N_j{}^{\dagger}] = i\varepsilon^{k}_{ij} N_K{}^{\dagger}$$

$$[N_i, N_j{}^{\dagger}] = 0$$

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(L, R) - Representation

$$J^i = \frac{1}{2}\left[N^i + N^{\dagger i}\right] \qquad K^i = -\frac{i}{2}\left[N^i - N^{\dagger i}\right]$$

with $J^i = |n + n^\dagger| \;, |n + n^\dagger - 1| \;, |n + n^\dagger - 2| \,, \ldots\ldots\ldots, 0, \ldots\ldots .\, |n - n^\dagger|$

With possible value of $J = 0, 1\,, {}^1/_2\,, {}^3/_2 \ldots . etc.$

For J=1/2 possible combination are

$$\left(\overbrace{{}^1/_2}^{n}\,,\overbrace{{}^1/_2}^{n^\dagger}\right) or \left(\overbrace{{}^1/_2}^{n}\,,\overbrace{0}^{n^\dagger}\right) or \left(\overbrace{0}^{n}\,,\overbrace{{}^1/_2}^{n^\dagger}\right) \ldots . . etc.$$

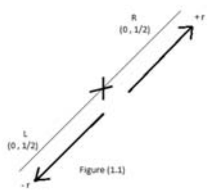

Figure (1.1)

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(0, 0) - Representation (Spin Zero)

$$U(\Lambda)\Phi(x)U^{-1}(\Lambda) = \Phi(\Lambda^{-1}x)$$

$$U(\Lambda)\partial^\alpha\Phi(x)U^{-1}(\Lambda) = \Lambda_a\partial^a\Phi(\Lambda^{-1}x)$$

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Transformation of L(1/2, 0) and R(0, 1/2)

$$U_{L_R}(w)\,\Big|\Psi_{L_R}\rangle = e^{\frac{1}{2}[i\theta \pm \phi]\,\widehat{n_k}\,\sigma^k}\Big|\Psi_{L_R}\rangle$$

For Rotation $U_{L_R}(R) = I\cos\left(\frac{\theta}{2}\right) + i\,\sigma_k \sin\left(\frac{\theta}{2}\right)$

For boost $U_{L_R}(B) = \pm\left[I\cos h\left(\frac{\phi}{2}\right) + i\,\sigma_k\,\sin\mathrm{h}\left(\frac{\phi}{2}\right)\right]$

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Transformation of left as right and vice-versa

By using the magic property of Pauli-spin matrices $\sigma_2\,\sigma_k\,\sigma_2 = -\sigma_k^*$

$$\sigma_2\left(U_{L_R}(\Lambda)\right)\sigma_2 = U_{R_L}^*(\Lambda)$$

$$\sigma_2\,U_{L_R}^\dagger(\Lambda)\sigma_2 = \left[U_{L_R}^{-1}(\Lambda)\right]^*$$

$$\sigma^2|\,\Psi_{L_R}\rangle^* \rightarrow U_{R_L}(\Lambda)\,\sigma^2|\,\Psi_{L_R}\rangle^*$$

$\sigma^2|\,\Psi_{L_R}\rangle^*$ will transform as $|\,\Psi_{R_L}\rangle$

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Boson as super partner of fermion and vice versa

$$(|X_L\rangle \otimes |\Psi_L\rangle)_A$$

$$|X_L\rangle = |\Psi_L\rangle$$

$$|\Psi_L\rangle^T \sigma^2 |\Psi_L\rangle$$

$$= [\Psi_{L1}, \Psi_{L2}] \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{bmatrix} \Psi_{L1} \\ \Psi_{L2} \end{bmatrix}$$

$= i(\Psi_{L1}\Psi_{L2} - \Psi_{L2}\Psi_{L1})$ =0 Not exisit

But if consider Grossman number then exist

Conservation of Lepontic family

$$\Psi_L = \sigma^2 \Psi_R^*$$

$i(\sigma^2\Psi_R^*)^T\sigma^2\Psi_L = -i\Psi_R^\dagger\Psi_L$ will invariant

Will give

$$\delta_R(\Psi_L^\dagger\Psi_L) = 0$$

$$\delta_R(\Psi_L^\dagger\sigma^i\Psi_L) = \xi_j^i\sigma^j$$

$$\delta_B(\Psi_L^\dagger\sigma^i\Psi_L) = \xi_0^i\Psi_L^\dagger\sigma^0\Psi_L$$

$$\delta_B(\Psi_L^\dagger\Psi_L) = \xi_0^i\Psi_L^\dagger\sigma^i\Psi_L$$

therefore $\delta(\Psi_L^\dagger\sigma^\mu\Psi_L) = \xi_\alpha^\mu\Psi_L^\dagger\sigma^\alpha\Psi_L$ can only be vector
$\xi_L^\dagger\sigma^\mu\Psi_L$ can never be transform as vector

Conservation of Lepontic family

for parity, we have to consider parity partner so

$\Psi_R^\dagger\Psi_L + \Psi_L^\dagger\Psi_R = \overline{\Psi}\Psi$ Will invariant

$J^\mu = \overline{\Psi}\gamma^\mu\Psi$ Will only choice to be vector

Lepton length should be null

$$A^\mu \to A = \begin{pmatrix} A^0 + A^3 & A^1 + iA^2 \\ A^1 - iA^2 & A^0 - A^3 \end{pmatrix} = A^\mu\sigma_\mu$$

$$detA = 0 \text{ for } \Psi = \begin{pmatrix} a \\ b \end{pmatrix}$$

Thank You

Question and Comments

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