Theoretical Conservation of Lepton Family and Representation Theories

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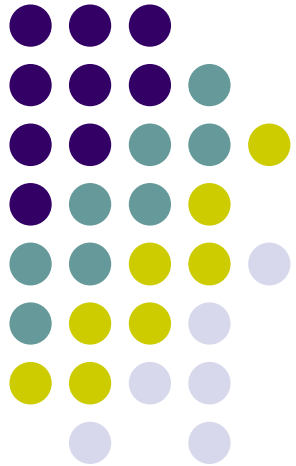


Introduction

- Lorentz Transformation
- Poincare Group
- Representation of Poincare as $SU(2) \otimes SU(2)$
- Left and Right Spinner
- Lorentz Scaler
- Boson as supper partner of fermion and vice versa
- Conservation of Lepontic family

Lorentz Transformation

$$x'^{\alpha} = \Lambda^{\alpha}_{;\beta} x^{\beta} + a^{\alpha}$$

$$\| x \|^2 = \| x' \|^2 = \langle x, x \rangle = \langle x', x' \rangle,$$

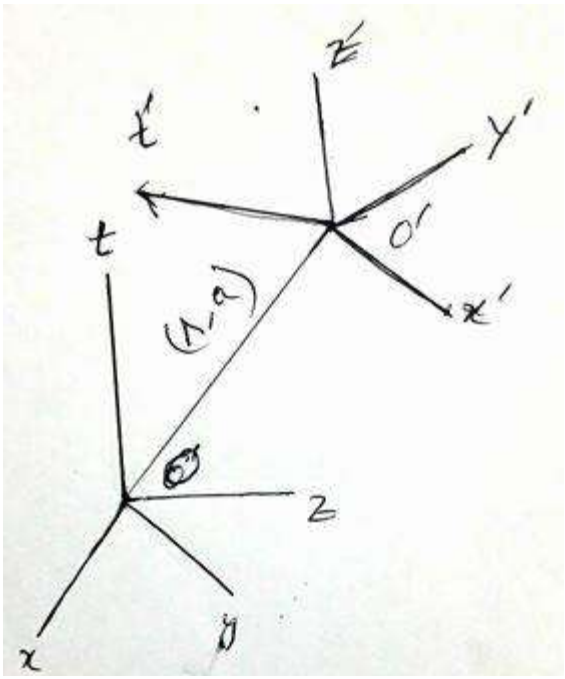
$$x^{;\alpha} x_{\alpha} = x^{;\alpha} g_{\alpha\beta} x^{\beta} = x'_{;\beta} x'^{\beta}$$

Where $g_{\alpha\beta} = \begin{pmatrix} 1 & \underline{0}_{1\times 3} \\ \underline{0}_{3\times 1} & -\underline{I}_{3\times 3} \end{pmatrix}$

with (+,-,-,-) signature

$$g^{bc} = \Lambda^{b}_{;\beta} g^{\beta\tau} \Lambda^{;c}_{\tau}$$

$$\left.\begin{aligned} \partial'^{\alpha} &= \Lambda^{\alpha}_{;\beta} \partial^{\beta} \\ \partial'_{\alpha} &= \Lambda^{;\beta}_{\alpha} \partial_{\beta} \end{aligned}\right\}$$



Lorentz transformation as Lie Group

For Lie Symmetric Group

$$x \xrightarrow{\text{LT}} x' \xrightarrow{\text{LT}} x''$$

$$L_T' L_T'' = (\Lambda'\Lambda, \Lambda' a + a')$$

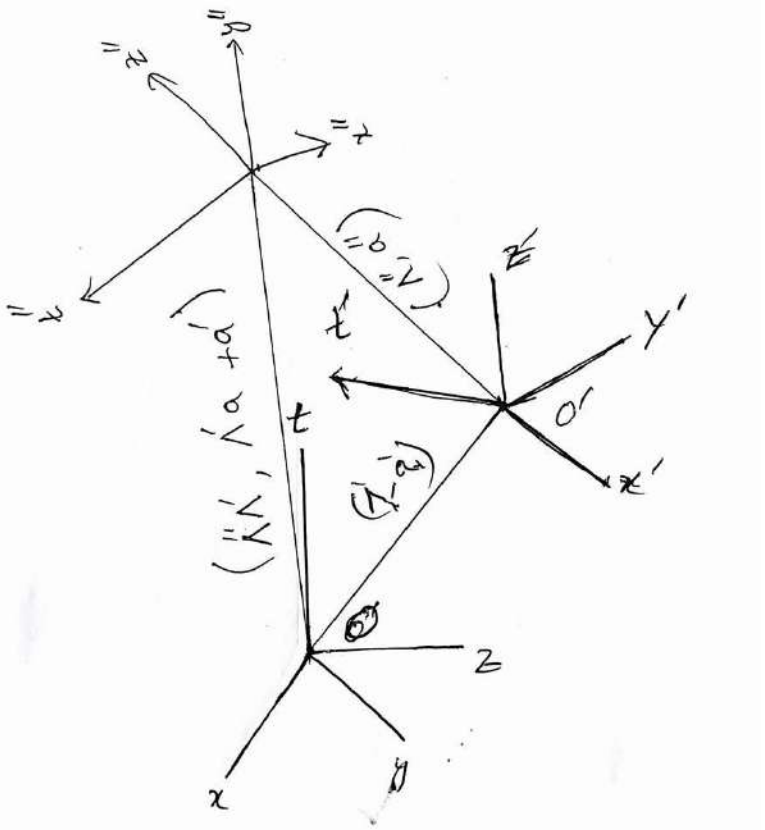
$$L_T{}^{-1} = (\Lambda^{-1}, -\Lambda^{-1}a) = \left(g^T \Lambda g, -(g^T \Lambda g)a\right)$$

For near to identity

$$\Lambda^{\mu}_{;\nu} = \delta^{\mu}_{;\nu}; \qquad a^{\mu} = 0$$

For Infinitesimal Transformation

$$\Lambda^{\mu}_{;\nu} = \delta^{\mu}_{;\nu} + \omega^{\mu}_{;\nu} \qquad a^{\mu} = \varepsilon^{\mu}$$



infinite representation as Unitary

$$\Lambda^{\mu}_{;\nu} = (\exp^{\omega})^{\mu}_{;\nu}$$

$$U(I + W, \xi) = I + \frac{i}{2!} \underbrace{w_{\propto\beta}}_{\Lambda^{\propto}_{;\beta}} J^{\propto\beta} - i \underbrace{\xi_{\propto}}_{a^{\propto}} P^{\propto} + O(w) + 0(\xi)$$

Therefore $\mathrm{J}^{\dagger} = \mathrm{J}$, $\mathrm{K}^{\dagger} = \mathrm{k}$

$$U(\Lambda, a) J^{\alpha\beta} U^{-1}(\Lambda, a) = \Lambda^{\alpha}_{a} \Lambda^{\beta}_{b} (J^{ab} - a^{a} p^{b} + a^{b} p^{a})$$

$$U(\Lambda, a) P^{\alpha} U^{-1}(\Lambda, a) = \Lambda^{\alpha}_{;a} P^{a}$$

For Spin Transformation (Rotational)

J^{ab} transform as Tensor P^{a} transform as 4-vector

Translational Transformation

$$\Lambda^{\alpha}_{;\beta} = \delta^{\alpha}_{;\beta}$$

P is invariant but J in invariant with only origin Transformation

For $J_i = \frac{1}{2}\varepsilon_{ijk}J^{jk}$ and $K_i = \varepsilon_{ijk}J^{i0}$

$$[J_i, J_j] = i\varepsilon_{ij}^{k}J_k$$

$$[J_i, K_j] = i\varepsilon_{ij}^{k}K_k$$

$$[K_i, K_j] = -i\varepsilon_{ij}^{k}J_k$$

- Angular momentum
- K (boost) Transform as three vector under rotation
- two boost gives rotation

Representation as SU(2)⊗SU(2)

By defining

$$N^i = \frac{1}{2}(J^i + iK^i)$$

$$N^{i\dagger} = \frac{1}{2}(J^i - iK^i)$$

We have

$$[N_i, N_j] = i\varepsilon_{ij}^k N_k$$

$$[N_i{}^\dagger, N_j{}^\dagger] = i\varepsilon_{ij}^k N_K{}^\dagger$$

$$[N_i, N_j{}^\dagger] = 0$$

(L, R) - Representation

$$J^i = \frac{1}{2}\left[N^i + N^{\dagger i}\right] \qquad K^i = -\frac{i}{2}\left[N^i - N^{\dagger i}\right]$$

with $J^i = |n + n^\dagger|\ , |n + n^\dagger - 1|\ , |n + n^\dagger - 2|\ , \ldots\ldots\ldots, 0, \ldots\ldots |n - n^\dagger|$

With possible value of $J = 0, 1\ , {}^1\!/_2\ , {}^3\!/_2\ \ldots . etc.$

For J=1/2 possible combination are

$$\left(\overbrace{{}^1\!/_2}^{n}\ , \overbrace{{}^1\!/_2}^{n^\dagger}\right) or \left(\overbrace{{}^1\!/_2}^{n}\ , \overbrace{\tilde{0}}^{n^\dagger}\right) or \left(\overbrace{\tilde{0}}^{n}\ , \overbrace{{}^1\!/_2}^{n^\dagger}\right) \ldots . . etc.$$

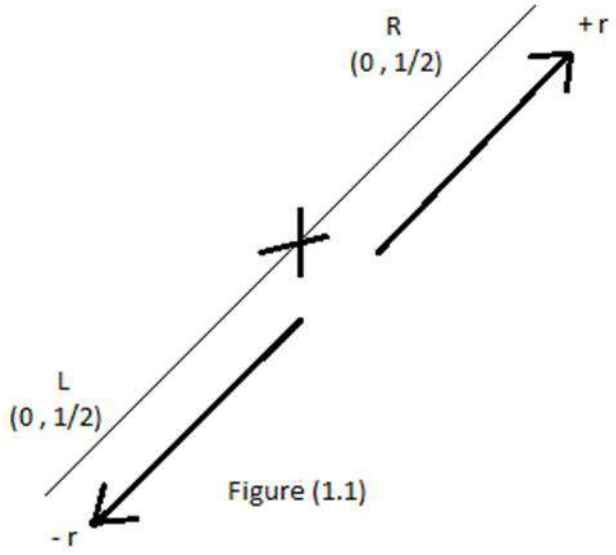


Figure (1.1)

(0, 0) - Representation (Spin Zero)

$$U(\Lambda)\Phi(x)U^{-1}(\Lambda) = \Phi(\Lambda^{-1}x)$$

$$U(\Lambda)\partial^{\alpha}\Phi(x)U^{-1}(\Lambda) = \Lambda_{a}\partial^{a}\Phi(\Lambda^{-1}x)$$

Transformation of L(1/2, 0) and R(0, 1/2)

$$U_{L_R}(w) \left| \Psi_{L_R} \right\rangle = e^{\frac{1}{2}[i\theta \pm \phi]\widehat{n_k}\ \sigma^k} \left| \Psi_{L_R} \right\rangle$$

For Rotation $$U_{L_R}(R) = I\cos\left(\frac{\theta}{2}\right) + i\ \sigma_k \sin\left(\frac{\theta}{2}\right)$$

For boost $$U_{L_R}(B) = \pm\left[I\cos h\left(\frac{\phi}{2}\right) + i\ \sigma_k \sin h\left(\frac{\phi}{2}\right)\right]$$

Transformation of left as right and vice-versa

By using the magic property of Pauli-spin matrices

$$\sigma_2 \, \sigma_k \, \sigma_2 = -\sigma_k^*$$

$$\sigma_2 \left(U_{L_R}(\Lambda) \right) \sigma_2 = U_{R_L}^*(\Lambda)$$

$$\sigma_2 \, U_{L_R}^{\dagger}(\Lambda)\sigma_2 = \left[U_{L_R}^{-1}(\Lambda) \right]^*$$

$$\sigma^2 | \, \Psi_{L_R} \, \rangle^* \rightarrow U_{R_L}(\Lambda) \, \sigma^2 | \, \Psi_{L_R} \, \rangle^*$$

$\sigma^2 | \, \Psi_{L_R} \, \rangle^*$ will transform as $| \, \Psi_{R_L} \, \rangle$

Boson as super partner of fermion and vice versa

$$(|X_L\rangle \otimes |\Psi_L\rangle)_A$$

$$|X_L\rangle = |\Psi_L\rangle$$

$$|\Psi_L\rangle^T \sigma^2 |\Psi_L\rangle$$

$$= [\Psi_{L1}, \Psi_{L2}] \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{bmatrix} \Psi_{L1} \\ \Psi_{L2} \end{bmatrix}$$

$$= i(\Psi_{L1}\Psi_{L2} - \Psi_{L2}\Psi_{L1}) \quad = 0$$

Not exisit

But if consider Grossman number then exist

Conservation of Lepontic family

$$\Psi_L = \sigma^2 \Psi_R^*$$

$$i(\sigma^2 \Psi_R^*)^T \sigma^2 \Psi_L = -i\Psi_R^\dagger \Psi_L \text{ will invariant}$$

$$\delta_R(\Psi_L^\dagger \Psi_L) = 0$$

Will give

$$\delta_R(\Psi_L^\dagger \sigma^i \Psi_L) = \xi_j^i \sigma^j$$

$$\delta_B(\Psi_L^\dagger \sigma^i \Psi_L) = \xi_0^i \Psi_L^\dagger \sigma^0 \Psi_L$$

$$\delta_B(\Psi_L^\dagger \Psi_L) = \xi_0^i \Psi_L^\dagger \sigma^i \Psi_L$$

therefore $\delta(\Psi_L^\dagger \sigma^\mu \Psi_L) = \xi_\alpha^\mu \Psi_L^\dagger \sigma^\alpha \Psi_L$ can only be vector

$\xi_L^\dagger \sigma^\mu \Psi_L$ can never be transform as vector

Conservation of Lepontic family

for parity, we have to consider parity partner so

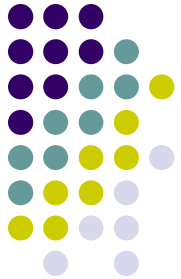
$$\Psi_R^\dagger \Psi_L + \Psi_L^\dagger \Psi_R = \overline{\Psi}\Psi$$ Will invariant

$$J^\mu = \overline{\Psi}\gamma^\mu\Psi$$ Will only choice to be vector

Lepton length should be null

$$A^{\mu} \rightarrow A = \begin{pmatrix} A^0 + A^3 & A^1 + iA^2 \\ A^1 - iA^2 & A^0 - A^3 \end{pmatrix} = A^{\mu}\sigma_{\mu}$$

$$detA = 0 \text{ for } \Psi = \begin{pmatrix} a \\ b \end{pmatrix}$$



Thank You

Question and Comments