New results on Neutrino Magnetic Moments and on Democratic Neutrinos

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Neutrino Magnetic Moment

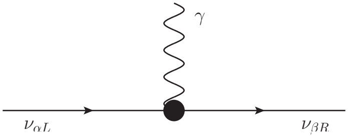
Neutrino magnetic moment $\mu_{\alpha\beta}$ can be defined by the Hermitian form factor $f^M_{\alpha\beta}(0) \equiv \mu_{\alpha\beta}$ of the term [for review see 1207.3980]

$$-f^M_{\alpha\beta}(q^2)\ \bar{\nu}_\beta(p_2)\, i\sigma_{\mu\nu} q^\nu \nu_\alpha(p_1)$$

in the effective neutrino electromagnetic current

$$\langle \nu_\beta(p_2)| j^{\rm eff}_\mu(0) |\nu_\alpha\rangle = \bar{\nu}_\beta(p_2)\, \Lambda_\mu(p_2, p_1) \nu_\alpha(p_1)$$

where $\alpha, \beta = e, \mu, \tau$ are flavor indices, $q = p_2 - p_1$, and Λ_μ is a general matrix in spinor space.



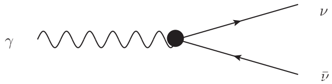
In the SM, minimally extended to include neutrino masses, NMM is suppressed by small masses m_i of observable neutrinos due to the left-handed nature of weak interaction. The diagonal and transition magnetic moments are calculated in the SM to be

$$\mu_{ii}^{\mathrm{SM}} \approx 3.2 \times 10^{-20} \left(\frac{m_i}{0.1\ \mathrm{eV}}\right) \mu_B$$

and

$$\mu_{ij}^{\mathrm{SM}} \approx -4 \times 10^{-24} \left(\frac{m_i + m_j}{0.1\ \mathrm{eV}}\right) \sum_{\ell=e,\mu,\tau} \left(\frac{m_\ell}{m_\tau}\right)^2 U_{\ell i}^* U_{\ell j}\ \mu_B,$$

respectively, where $\mu_B = e/(2m_e) = 5.788 \times 10^{-5}$ eV T^{-1} is the Bohr magneton, and $U_{\ell i}$ is the leptonic mixing matrix.



The strongest present experimental bound on NMM [Raffelt, 1999]

$$\mu_\nu < 3 \times 10^{-12}\ \mu_B$$

has been obtained from the constraint on energy loss from globular cluster red giants, which can be cooled faster by the plasmon decays due to NMM, which delays the helium ignition. This bound can be applied to all diagonal and transition NMMs.

The best present terrestrial laboratory constraint on NMM

$$\mu_{\bar{\nu}_e} < 2.9 \times 10^{-11}\ \mu_B \qquad (90\%\ \mathrm{C.L.})$$

was derived in $\bar{\nu}_e$–e elastic scattering experiment GEMMA [Beda *et al.*, 2012].

Nonstandard Neutrino Interactions

NSIs of $\nu\nu ff$ type can be generically parametrized as

$$-\mathcal{L}_{\text{eff}} = \sum_a \frac{\epsilon^{fa}_{\alpha\beta}}{M^2}(\bar{\nu}_\beta \Gamma_a \nu_\alpha)(\bar{f}\Gamma_a f) + \text{H.c.},$$

where $\epsilon^{fa}_{\alpha\beta}$ are NSI couplings, M is the scale of new physics, f denotes the component of any weak fermionic doublet (often an electron or a quark field for studies of νNSIs in matter), $\Gamma_a = \{I, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}\}$, and $a = \{S, P, V, A, T\}$.

Typically only left-handed ν are considered in this Lagrangian

$$-\mathcal{L}_{\text{eff}} = \sum_a \frac{\epsilon^{fa}_{\alpha\beta}}{M^2}(\bar{\nu}_\beta \Gamma_a \nu_\alpha)(\bar{f}\Gamma_a f) + \mathrm{H.c.},$$

which allows the study of NSIs' impact on neutrino oscillations, and on neutrino-nucleus scattering. This chirality constraint that allows $\nu\nu ff$ interaction only of $V \pm A$ types cannot describe important neutrino phenomena, such as NMM.
However a tensor term can be generated by Fierz transformation of

$$\sum_a \frac{\tilde{\epsilon}^{fa}_{\alpha\beta}}{M^2}(\bar{\nu}_\beta \Gamma_a f)(\bar{f}\Gamma_a \nu_\alpha) + \mathrm{H.c.}$$

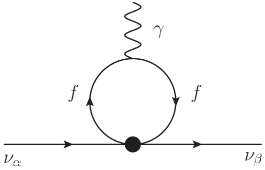
with a scalar part, which is presented in many models with leptoquarks, R-parity-violating SUSY, etc.

Theoretical results

Among all possible $\nu_\alpha \nu_\beta ff$ interactions, the lowest-order contribution to NMM can be generated by the tensor dimension-6 operator

$$\frac{\epsilon^{fT}_{\alpha\beta}}{M^2}(\bar{\nu}_\beta \sigma_{\mu\nu} \nu_\alpha)(\bar{f} \sigma^{\mu\nu} f),$$

where for Majorana neutrinos $\bar{\nu}_\beta \equiv \bar{\nu}^c_\beta$, through the diagram



Interactions of νs with quarks q and leptons ℓ via the operators

$$\frac{\epsilon^{q}_{\alpha\beta}}{M^2}(\bar{\nu}_\beta\sigma_{\mu\nu}\nu_\alpha)(\bar{q}\sigma^{\mu\nu}q), \qquad \frac{\epsilon^{\ell}_{\alpha\beta}}{M^2}(\bar{\nu}_\beta\sigma_{\mu\nu}\nu_\alpha)(\bar{\ell}\sigma^{\mu\nu}\ell)$$

where $\epsilon^{q}_{\alpha\beta} \equiv \epsilon^{qT}_{\alpha\beta}$ and $\epsilon^{\ell}_{\alpha\beta} \equiv \epsilon^{\ell T}_{\alpha\beta}$ are real, generate NMMs

$$\mu_{\alpha\beta} = \mu^0_{\alpha\beta} - \sum_q \epsilon^q_{\alpha\beta}\frac{N_c Q_q}{\pi^2}\frac{m_e m_q}{M^2}\ln\left(\frac{M^2}{m_q^2}\right)\mu_B,$$

$$\mu_{\alpha\beta} = \mu^0_{\alpha\beta} + \sum_\ell \frac{\epsilon^\ell_{\alpha\beta}}{\pi^2}\frac{m_e m_\ell}{M^2}\ln\left(\frac{M^2}{m_\ell^2}\right)\mu_B,$$

respectively, where $N_c = 3$ is the number of colors, Q_q (m_q) is the electric charge (mass) of the quark, and $\mu^0_{\alpha\beta}$ denotes the subleading part that is not enhanced by the large logarithm.

The result for ν-quark interaction reproduces the leading order in the exact result, which can be derived in model with scalar LQs.

We note that the dominant logarithmic terms may not contribute to NMM in certain models, e.g., in the SM, due to a mutual compensation between the relevant diagrams.

Constraints on NSIs

For the new physics scale $M = 1$ TeV, using $\mu_\nu < 3 \times 10^{-12}\ \mu_B$, and taking one nonzero $\epsilon^f_{\alpha\beta}$ at a time, we obtain the following upper bounds

$\lvert\epsilon^e_{\alpha\beta}\rvert$	3.9	$\lvert\epsilon^d_{\alpha\beta}\rvert$	0.25	$\lvert\epsilon^u_{\alpha\beta}\rvert$	0.49
$\lvert\epsilon^\mu_{\alpha\beta}\rvert$	3.0×10^{-2}	$\lvert\epsilon^s_{\alpha\beta}\rvert$	1.6×10^{-2}	$\lvert\epsilon^c_{\alpha\beta}\rvert$	1.7×10^{-3}
$\lvert\epsilon^\tau_{\alpha\beta}\rvert$	2.6×10^{-3}	$\lvert\epsilon^b_{\alpha\beta}\rvert$	5.8×10^{-4}	$\lvert\epsilon^t_{\alpha\beta}\rvert$	4.8×10^{-5}

$\bar{\nu}$-e elastic scattering

Besides the limits on NMM, neutrino-electron and neutrino-nucleus scattering constrain the tensorial NSIs [Barranco *et al.*, 2012].

Using the cross section for the $\bar{\nu}_e$–e scattering published by the TEXONO Collab. [Deniz *et al.*, 2010] and taking $M = 1$ TeV, the bound $|\epsilon^e_{e\beta}| < 6.6$ at 90% C.L. can be obtained, and for the GEMMA sensitivity it gives

$$|\epsilon^e_{e\beta}| < 2.7 \qquad (90\% \text{ C.L.}),$$

which slightly improves the respective bound from NMM.

The planned $\bar{\nu}_e$–nucleus coherent scattering experiments, e.g., part of TEXONO low-energy neutrino program, can reach the sensitivity of $|\epsilon^{u,d}_{e\beta}| < 0.2\,(M/1\,\mathrm{TeV})^2$ at 90% C.L. [Barranco *et al.*, 2012].

Introduction to Democratic Neutrinos

Consider the mass term for three left-handed Majorana neutrinos

$$\mathcal{L}_m^\nu = -\frac{1}{2}\sum_{\alpha\beta} \bar{\nu}^c_{\alpha L} M_{\alpha\beta}\nu_{\beta L} + \mathrm{H.c.},$$

where $\alpha, \beta = e, \mu, \tau$ are the flavor indices, and

$$M = m \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

is a "democratic" mass matrix, which is invariant under the permutation group of three elements S_3 [Harari *et al.*, 1978; Fritzsch and Xing, 1995].

The eigenvalues of M result in the neutrino mass spectrum

$$\{m, m, 2m\}, \tag{1}$$

and the eigenvectors form the mixing matrix of tri-bimaximal type

$$\begin{aligned} U &= R_{12}(\theta_{12}) \times R_{23}(\theta_{23}) \\ &= \begin{pmatrix} c_{12} & s_{12}c_{23} & s_{12}s_{23} \\ -s_{12} & c_{12}c_{23} & c_{12}s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix} \end{aligned} \tag{2}$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$,
$\theta_{12} = 45^\circ$, $\theta_{23} = \pi/2 - \arctan(1/\sqrt{2}) \approx 54.7^\circ$.

Atmospheric Neutrinos: Oscillations

In the limits $L \ll L_{ij}^{\rm coh}$ and $\sigma_x \ll L_{ij}^{\rm osc}$ the neutrino oscillation probability can be written as

$$P^{(L,E)}_{\nu_\alpha\to\nu_\beta} = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2 + 2\sum_{i>j} |U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}| \cos\left(\phi_{\rm osc} - \phi\right)$$

with $\phi_{\rm osc} = \frac{\Delta m^2_{ij}}{2E} L = 2\pi L / L_{ij}^{\rm osc}$ and $\phi = \arg(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j})$.
For the neutrino masses in Eq. (1) and mixing in Eq. (2) we have

$$P^{(L,E)}_{\nu_e\to\nu_\tau} = P^{(L,E)}_{\nu_\mu\to\nu_\tau} = 4 s^2_{12} c^2_{23} s^2_{23} \sin^2\left(\frac{\Delta m^2 L}{4E}\right) = \frac{4}{9}\sin^2\left(\frac{\Delta m^2 L}{4E}\right), \quad (3)$$

$$P^{(L,E)}_{\nu_e\to\nu_\mu} = 4 c^2_{12} s^2_{12} s^4_{23} \sin^2\left(\frac{\Delta m^2 L}{4E}\right) = \frac{4}{9}\sin^2\left(\frac{\Delta m^2 L}{4E}\right), \quad (4)$$

where $\Delta m^2 \equiv m_3^2 - m_{i<3}^2 = 3m^2$.

Using the atmospheric neutrino mass splitting [PDG2012]

$$\Delta m_a^2 = (2.06 - 2.67) \times 10^{-3}\ \mathrm{eV}^2 (at\ 99.73\%\ \mathrm{CL}),$$

we have $0.026\ \mathrm{eV} < m < 0.030\ \mathrm{eV}$. Neutrino masses are determined!

How to explain the difference between the e-like and μ-like event distributions in the Super-Kamiokande experiment [SK Collab.: Ashie *et al.*, 2005]?

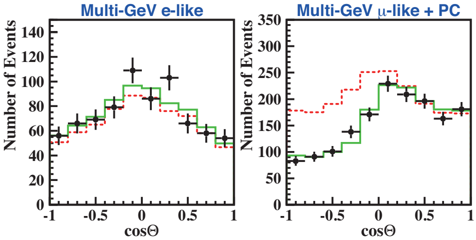


Fig. is taken from [PGD Collab.: Beringer *et al.*, 2012].

This difference between the e-like and μ-like event distributions can be explained by the matter effect on νs which travel through Earth. Using the Mikheyev-Smirnov-Wolfenstein theory, for the mean electron number density in the Earth core $\bar{N}_e^c \approx 5.4\ \mathrm{cm}^{-3}\ \mathrm{N_A}$ we have

$$P^{\mathrm{m}}_{\nu_e \to \nu_x} \approx 0.05\ \sin^2\left(2.8\frac{\Delta m^2 L}{4E}\right), \qquad x = \mu, \tau,$$

which is significantly suppressed with respect to $P_{\nu_\mu \to \nu_\tau}$ in Eq. (3).

($\nu_\mu \leftrightarrow \nu_\tau$ oscillations in the matter of the Earth proceed practically as in vacuum.)

Solar Neutrinos: Incoherence

Solar νs are detecting using charged-current (CC) and neutral current (NC) reactions

$$\nu_e + d \to e^- + p + p,$$
$$\nu_\ell + d \to \nu_\ell + p + n.$$

Ratio of the neutrino fluxes measured by Sudbury Neutrino Observatory (SNO) with CC and NC events is

$$\frac{\Phi^{CC}_{SNO}}{\Phi^{NC}_{SNO}} = \frac{1.68 \pm 0.06^{+0.08}_{-0.09}}{4.94 \pm 0.21^{+0.38}_{-0.34}} = 0.340^{+0.074}_{-0.063}. \tag{5}$$

For solar νs with the energies $E \lesssim 10$ MeV the oscillations due to Δm_a^2 proceed in the matter of the Sun as in vacuum. In the limit

$$L_{ij}^{\text{coh}} = \frac{4\sqrt{2}E^2}{|\Delta m_{ij}^2|}\sigma_x \ll L \approx 1.5 \times 10^8 \text{ km}$$

the oscillation probability takes a simple incoherent form

$$P_{\nu_\alpha \to \nu_\beta}^{\text{incoh}} = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2.$$

Using Eq. (2), we have

$$\frac{\Phi_{\text{sol}}^{\text{CC}}}{\Phi_{\text{sol}}^{\text{NC}}} = \frac{P_{\nu_e \to \nu_e}^{\text{incoh}}}{\sum_\beta P_{\nu_e \to \nu_\beta}^{\text{incoh}}} = \sum_i |U_{ei}|^4 = \frac{7}{18} \simeq 0.39,$$

which is in good agreement with the experimental data in Eq. (5).

Predictions of Theory with Democratic Neutrinos

- **Low energy β decays**
  $\langle m_\beta \rangle \equiv \sqrt{\sum_i m_i^2 |U_{ei}|^2} = m\sqrt{2} \approx 0.04$ eV,
  which is below KATRIN sensitivity of 0.2 eV, but can be probed by next sub-eV experiments (MARE, ECHO, Project8).
- **$0\nu 2\beta$ decay**
  $\langle m \rangle \equiv \sum_i s_i m_i U_{ei}^2 = 0$, where s_i are sign factors.
- **Neutrino (Transition) Magnetic Moment**
  Using the SM result for Majorana neutrinos
  $$\mu_{ij}^{\rm SM} \approx -4 \times 10^{-24} \left(\frac{m_i + m_j}{0.1 \text{ eV}}\right) \sum_{\ell=e,\mu,\tau} \left(\frac{m_\ell}{m_\tau}\right)^2 U_{\ell i}^* U_{\ell j}\ \mu_B,$$
  and Eqs. (1) and (2), we have
  $\mu_{23} \approx 1.7 \times 10^{-23}\ \mu_B \gg \mu_{12}, \mu_{13}$.

Conclusion

- First, we constrained generic nonstandard neutrino interactions, using existing limits on neutrino transition magnetic moments, and derived new bounds on tensorial couplings of neutrinos to charged fermions.
- Second, I introduced a simple theory of democratic neutrinos with only one mass splitting, which elegantly explains the established neutrino oscillation data, using the effect of incoherence. And I discussed the predictions of this model.

  (For more detailed discussion see coming update of DZ, arXiv:1304.4870.)