Hadron Production in $e^+e^- \text{ Annihilation at } B_{\Lambda BAR}$ and Implications for the Muon Anomalous Magnetic Moment

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$e^+ e^- \rightarrow \text{hadrons}$ and $g_\mu - 2$ in $BABAR$

Outline

- Introduction
- Muon anomalous magnetic moment
- ISR measurement of $e^+ e^- \rightarrow \text{hadrons}$
- Analysis
- Results
- Conclusions
\textit{BABar} is a high luminosity ($10^{34} \text{ cm}^{-2}\text{s}^{-1}$) $e^+e^-$ experiment.

In processes involving initial state radiation, this enables precise measurement of $\sigma(e^+e^- \rightarrow \text{hadrons})$ as a function of CM energy from threshold to several GeV.

- Hadron form factors ($\pi, K, p$)
- Light hadron spectroscopy
- Hadronic vacuum polarization (HVP) contribution to $(g - 2)_\mu$
**Muon $g - 2$**

Lepton magnetic moment anomaly, sensitivity to new physics $\sim m^2_\ell$

$$\vec{\mu}_\ell = -\frac{g_\ell e}{2m_\ell} \vec{S}; \quad a_\ell \equiv \frac{(g - 2)\ell}{2}$$

$$a_\mu(\text{measured}) = 116592089 \pm 63 \times 10^{-11}$$

(BNL E821 – Bennett 2006 PRD 73, 072003; RPP 2013)

$$a_\mu(\text{SM}) = 116591802 \pm 49 \times 10^{-11}$$

Measured value is $3.6\sigma$ larger than SM prediction!

Standard Model calculation:

$$a_\mu(\text{SM}) = a_\mu(\text{QED}) + a_\mu(\text{weak}) + a_\mu(\text{had})$$

$$a_\mu(\text{QED}) = 116584718.10 \pm 0.15 \times 10^{-11}$$

$$a_\mu(\text{weak}) = 154 \pm 2 \times 10^{-11}$$

$$a_\mu(\text{had}) = 6930 \pm 49 \times 10^{-11}$$

$a_\mu(\text{had})$

- $a_\mu(\text{had})$ is largest term after $a_\mu(\text{QED})$
- Contributions from hadronic vacuum polarization and hadronic light-by-light scattering
- Largest contribution to uncertainty ($\pm 42 \times 10^{-11}$) is hadronic vacuum polarization, $a_\mu(\text{HVP})$
- Not possible to compute $a_\mu(\text{HVP})$ perturbatively
- Instead, measure $\sigma(e^+e^- \rightarrow \text{hadrons})$ as a function of CM energy and use dispersion relation
Use of dispersion relation for $a_\mu(\text{had})$

Dispersion relation

$$a_\mu(\text{had}) = \frac{\alpha^2}{3\pi^2} \int_{\text{threshold}}^{\infty} R(s) \frac{K(s)}{s} ds$$

where

$$R(s) = \frac{\sigma^0(e^+e^- \rightarrow \text{hadrons}(\gamma))}{\sigma_{pt}}$$

and

$$K(s) \sim 1/s$$

Hence, emphasis is from the low-energy portion of hadron spectrum

Dominant contribution is from $\pi^+\pi^-$ (but other channels can not be neglected)

We need to measure $\sigma^0$, the bare cross section including FSR, as a function of $s$. 
The ISR method

- **BABAR**: $e^+e^-$ collisions at $\sqrt{s} = 10.6$ GeV
- With ISR, effective $e^+e^- \rightarrow \gamma^*$ energy is $\sqrt{s'} = \sqrt{s}(1-x)$, where $x = 2E_\gamma^*/\sqrt{s}$ in CM frame
- Select events with a high energy ISR photon ($E_\gamma^* > 3$ GeV) at large angle
- ISR photon is opposite hadrons in CM. High acceptance for boosted hadrons even from threshold
- **ISR luminosity determined with** $e^+e^- \rightarrow \gamma_{\text{ISR}}\mu^+\mu^-$ ($\pi\pi$, $KK$)
- Additional ISR and FSR accounted for
- Measurement from threshold to 3–5 GeV in single dataset, reduces systematics
**BaBar’s ISR measurements**

*BaBar* has extensive program to measure $e^+e^- \rightarrow$ hadrons as a function of energy using ISR method (channels include possible additional FSR photon)

<table>
<thead>
<tr>
<th>Final state(s)</th>
<th>Publication</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+\pi^-$</td>
<td>PRD 86 032013 (2012)</td>
</tr>
<tr>
<td>$\pi^+\pi^-\pi^0$</td>
<td>PRD 70 072004 (2004)</td>
</tr>
<tr>
<td>$K^+K^-\eta$, $K^+K^-\pi^0$, $K_S^0K^{\pm}\pi^\mp$</td>
<td>PRD 77 092002 (2008)</td>
</tr>
<tr>
<td>$\pi^+\pi^-\pi^+\pi^-$</td>
<td>PRD 85 112009 (2012)</td>
</tr>
<tr>
<td>$K^+K^-\pi^+\pi^-$, $K^+K^-\pi^0\pi^0$, $2(K^+K^-)$</td>
<td>PRD 86 012008 (2012)</td>
</tr>
<tr>
<td>$\Lambda\bar{\Lambda}$, $\Lambda\bar{\Sigma}^0$, $\Sigma\bar{\Sigma}^0$</td>
<td>PRD 76 092006 (2007)</td>
</tr>
<tr>
<td>$2(\pi^+\pi^-)\pi^0$, $2(\pi^+\pi^-)\eta$, $K^+K^-\pi^+\pi^-\pi^0$, $K^+K^-\pi^+\pi^-\eta$</td>
<td>PRD 76 0922005 (2007)</td>
</tr>
<tr>
<td>$\phi\eta$</td>
<td>PRD RC 74 111103 (2006)</td>
</tr>
<tr>
<td>$3(\pi^+\pi^-)$, $2(\pi^+\pi^-\pi^0)$, $K^+K^-2(\pi^+\pi^-)$</td>
<td>PRD 73 052003 (2006)</td>
</tr>
<tr>
<td>$p\bar{p}$, see C. Cartaro talk</td>
<td>PRD 87 092005 (2013)</td>
</tr>
<tr>
<td>$K_S^0K_L^0$, $K_S^0K_L^0\pi^+\pi^-$, $K_S^0K^{\pm}\pi^\mp\pi^0$, $K_S^0K^{\pm}\pi^\mp\eta$, $\pi^+\pi^-2\pi^0$</td>
<td>in progress</td>
</tr>
</tbody>
</table>
Analysis (e.g., $K^+K^-$)

- Measured $K^+K^-(\gamma)$ yield in ISR production
- Effective luminosity from measured $\mu^+\mu^- (\gamma)$ rate (for $\pi\pi$, $KK$)
- Efficiency from data-corrected simulations
- Gives the result for the cross section

$$\frac{dN_{K^+K^-}(\gamma)_{\text{ISR}}}{d\sqrt{s'}} = \frac{dL_{\text{ISR}}^{\text{eff}}}{d\sqrt{s'}} \epsilon_{KK\gamma_{\text{ISR}}} (\sqrt{s'}) \sigma^0_{KK(\gamma)} (\sqrt{s'})$$

“Bare” cross section $\sigma^0$ includes Final State Radiation (FSR), but no leptonic or hadronic vacuum polarization effects. These have been removed by using the normalization based on the measured $\mu^+\mu^- (\gamma)$ rate.
Efficiency and Background

(Many details skipped! See publications)

Monte Carlo efficiency corrected for MC/data differences, using in situ efficiency measurements:

- **Trigger:** Corrections $\sim \text{few} \times 10^{-4}$
- **Tracking:** Systematic uncertainties $< \text{few} \times 10^{-3}$
- **Particle identification:** Systematic uncertainties typically few $\times 10^{-3}$
- **Kinematic fit selection errors from uncertainty in modeling of additional ISR/FSR:** Systematic uncertainties $< \text{few} \times 10^{-3}$

Backgrounds mainly cross-feed from other ISR processes; systematic uncertainty typically few $\times 10^{-3}$ or less depending on channel, but tends higher at extremes of $\sqrt{s'}$. 
\( K^+K^- \) results: Cross section

Based on 232 fb\(^{-1} \) dataset

J\( \psi \), \( \psi(2S) \) subtracted

**BaBar**

![Cross section plots](image)

For purposes of measuring the \( \phi \) resonance parameters and providing an empirical parametrization of the overall systematic uncertainty is 7.

**FIG. 25:** Relative variations of the energy interval. Data from previous measurements are luminosity (Sec. VII A). The calibration and resolution correlated in all mass bins, except for the ones from the cross section. Each systematic error is treated as fully 1.03\( \text{GeV mass range, but significantly larger outside the mass range. The form factor values and kaon velocity, and state correction [27–29]. At the calculation). The error on the vacuum polarization correction integral (Sec. VII H).**

The systematic uncertainties affecting the bare cross section, \( \Delta \sigma / \sigma \), are shown in the histograms. Systematic and statistical uncertainties are decreasing at higher masses. The form factor values and kaon velocity, and state correction [27–29]. At the calculation). The error on the vacuum polarization correction integral (Sec. VII H).
Confirm discrepancy with QCD prediction for normalization ($\sim \times 4$); shape is consistent.
$\pi^+\pi^-$ cross section results

Based on 232 fb$^{-1}$ dataset

Cross section (nb)

$\sqrt{s'}$ (GeV)

$0.5$  $1$  $1.5$  $2$  $2.5$  $3$

$10^{-3}$  $10^{-2}$  $10^{-1}$  $1$  $10$  $10^2$  $10^3$
$K^+K^-\pi\pi$ cross section results

Based on 454 fb$^{-1}$ dataset (statistical uncertainties shown)
Dressed cross section, including vacuum polarization

$K^+K^-\pi^+\pi^-$

$K^+K^-\pi^0\pi^0$

BaBar
Based on 454 fb⁻¹ dataset (statistical uncertainties shown)
Dressed cross section, including vacuum polarization

\[ \pi^+\pi^-\pi^+\pi^- \] cross section results

J. P. LEES et al. PHYSICAL REVIEW D 85, 112009 (2012)
112009-10

\[ \sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-) \text{ (mb)} \]

\[ E_{\text{CM}} (\text{GeV}) \]

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\[ E_{\text{CM}} (\text{MeV}) \]
## Conclusions

<table>
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<tr>
<th>Channel</th>
<th>$a_\mu$(had) ($10^{-11}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi\pi(\gamma)$</td>
<td>$5141 \pm 22 \pm 31$</td>
</tr>
<tr>
<td>$\pi^+\pi^-\pi^+\pi^-$</td>
<td>$136.4 \pm 0.3 \pm 3.6$</td>
</tr>
<tr>
<td>$K^+K^-$</td>
<td>$229.3 \pm 1.8 \pm 2.2$</td>
</tr>
</tbody>
</table>

- Precision on $\pi^+\pi^-$ comparable with previous WA
- Precision on $4\pi$ factor 2.6 better than previous WA
- Precision on $K^+K^-$ factor 3 better than previous WA

- FNAL E989 goal: reduce error bar on measured $a_\mu$ from $63 \times 10^{-11}$ to $16 \times 10^{-11}$
  - Expect lattice calculations to eventually provide most precise SM predictions for HVP, but:
  - “few percent error on timescale of Muon $g-2$ experiment”

- Dominant $\pi\pi$ channel result is on half of the $BABAR$ dataset.
  Possibly could use other half as well to help on E989 timescale.

- Analysis of $K^+_S K^-_L$, $K^+_S K^-_L \pi^+ \pi^-$, $K^+_S K^{\pm} \pi^\mp \pi^0$ in progress
BACKUP
$K^+K^-$ comparison with other experiments

$\phi$ mass consistent within calibration uncertainties with CMD2 and SND. However, normalization is not:

$$\frac{\text{norm}(\text{BABAR})}{\text{norm( CMD2, SND) }} - 1$$

$$= \begin{cases} 
0.051 \pm 0.003_{\text{CMD2}} \pm 0.006_{\text{BABAR}} & \text{BABAR vs CMD2} \\
0.096 \pm 0.009_{\text{SND}} \pm 0.006_{\text{BABAR}} & \text{BABAR vs SND}
\end{cases}$$
Substructure in $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$

$M_{3\pi}$

$M_{2\pi}$

$M_{2\pi}$ against $\rho$

Red: Data
Black: MC