Review of recent developments in QCD theory

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DPF 2013 meeting, August 16, UC Santa Cruz
Outline

• Why do we care about QCD at higher orders
• Progress in NLO calculations: W+5jets, H+3jets, ttbar+large ETmiss from Top-quark partners
• QCD at NNLO and beyond:
  - H+jet, dijet, ttbar, inclusive Higgs production at N^3LO
  - Resummation for H+0/1 jet and jet vetos
• Conclusions

This is just a selection of some recent highlights, apologies in advance for possible omissions!
Why do we care about QCD

Telling us what the background is, so we can see any excess

Teaching us how to reduce the background, sharpen the signal

Constraining any discoveries: mass couplings etc.

And as input to nearly all measurements

G. Salam, ICHEP 2010
Why do we care about QCD

No real understanding of LHC physics is possible without sophisticated QCD calculations!

G. Salam
Collisions at Hadron Colliders

How do we make a prediction for such an event?

Multiple physics effects living at widely varying scales

• Factorization: separate hard and soft scales

\[
\sigma_{h_1 h_2 \to X} = \int dx_1 dx_2 \left[ f_{h_1 / i} (x_1; \mu_F^2) f_{h_2 / j} (x_2; \mu_F^2) \right] \sigma_{ij \to X} (x_1, x_2, \mu_F^2, \{q_k\}) + O \left( \frac{\Lambda_{QCD}}{Q} \right)^n
\]

Non-perturbative but universal; measure in deep-inelastic scattering, fixed-target, apply to Tevatron, LHC

Process dependent but calculable in pQCD

Small for sufficiently inclusive observables

• Focus of this talk is a precise understanding of \( \sigma_{ij \to X} (x_1, x_2, \mu_F^2, q_k) \)
Partonic cross section calculable in pQCD as an expansion in $\alpha_s$

- **LO**: known for all processes of interest, has large renormalization and factorization scale dependence
- **NLO**: first reliable predictions (correct shape and normalization, accounts for effects of extra radiation, smaller scale dependence)
- **NNLO**: required for precise theoretical description of few observables needed in the precise extraction of PDFs, masses and $\alpha_s\,$ or when perturbative corrections are large

Sometimes fixed order results are not sufficient, in particular when jet veto cuts are imposed to reduce the background. These could introduce large logarithms that need to be resummed.

Higgs production as an example
• NLO calculations become difficult for $2 \rightarrow 3$ and beyond
• Need both virtual corrections and real emission matrix elements (ME) in order to cancel infrared (IR) singularities
• Extracting implicit IR poles from real radiation ME is well understood at NLO with various methods (dipole subtraction, FKS, antenna subtraction)

\[
\sigma_{(m)}^{NLO} = \int_{\Phi_m} \left[ d\sigma^{Born} + d\sigma^V + \int_{\Phi_1} d\sigma^S \right] + \int_{\Phi_{m+1}} \left[ d\sigma^R - d\sigma^S \right]
\]

• Developments in unitarity based methods turned the calculation of virtual corrections into a possible task even for high multiplicity processes
• Theoretical breakthroughs ideas allowed an incredibly fast progress for fixed order NLO results with complicated final states. **Key idea: obtain one-loop amplitudes using tree amplitudes**

  • The OPP method: Ossola, Papadopoulos, Pittau (2006)
  • Rational parts of one-loop amplitudes from tree-amplitudes in multiple dimensions: Giele, Kunszt, Melnikov (2008)

• Feynmann diagramatic approach still provides competitive results:
  Bredenstein, Denner, Dittmaier, Kallweit, Pozzorini

• Ideas applied by several groups with an amazing outcome! two major directions:

  🔄 more processes: towards full automation of NLO calculations with codes like Helac, GoSam, MadLoop and OpenLoops
  🔄 more legs: e.g. Blackhat focuses on pure n jets or W/Z+n jets, pushing the frontier of n (currently n=5)
Impressive list of results:

- multiple jets (up to 4)
- gauge boson and up to 5 jets
- two gauge bosons with up to 2 jets
- top quarks with jets (up to 2) or a gauge boson
- Higgs and up to 3 jets
• First 2 → 6 NLO calculation at a hadron collider using Blackhat + Sherpa

• Dynamical scale choice:
  \[ \mu_R = \mu_F = \frac{\hat{H}_T}{2} \]
  \[ \hat{H}_T = \sum_m p_T^m + E_T^W \]

• scale variation: \( \mu/2 \ldots 2\mu \)
• reduced scale dependence at NLO
• Ratio of NLO/LO constant over full kinematic range

NLO helps to motivate the scale choice
NLO highlights: $H+3$jets

- Can be used to improve the theoretical prediction of the $H+2$jet bin
  \[ \sigma_2 = \sigma_{\geq 2} - \sigma_{\geq 3} \]

- NLO corrections affect the shape of the $P_{T,H}$ distribution

- NLO corrections improve the scale dependence

Cullen, van Deurzen, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Tramontano (2013)
NLO highlights: ttbar+large $E_{T\text{miss}}$ from top-quark partners

- Search for ttbar+large missing energy signature at ATLAS; from pair production of two top partners $TT$, which decay as $T \rightarrow t\chi$
- How does the inclusion of NLO QCD, and all spin correlations through production and decay affect the robustness of the exclusion limits?

**R.B., Schulze, PRL 2013**

- Large K-factors with significant variation over phase space: can’t use an inclusive K-factor!
- LO+parton shower, as used in experimental study, is not a good framework for new physics searches. LO+PS acceptance can be very wrong.
- Scale dependence of inclusive cross section typically smaller than that of cross section after cuts; must use the second as the theoretical systematic error.

<table>
<thead>
<tr>
<th></th>
<th>LO</th>
<th>NLO</th>
<th>MG+Pythia</th>
<th>MG+PS merged</th>
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<tbody>
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<td>acceptance</td>
<td>0.19$^{+0}_{-0}$</td>
<td>0.27$^{+0.3}_{-0.2}$</td>
<td>0.46</td>
<td>0.27</td>
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</table>
How to improve NLO predictions

- Merging with parton showers
  - Add multiple radiation from parton shower to NLO prediction for a specific hard process
  - Challenge: avoid double counting
  - Two established methods:
    - MC@NLO (Frixione, Webber)
    - POWHEG (Nason, Frixione, Oleari)
  - Combines NLO accuracy for hard radiation with multiple soft emissions
    - High $P_T$ described by NLO
    - Low $P_T$ described by parton shower
- Many recent results of NLO predictions combined with parton shower (see Alioli’s talk)

Top-quark pair production at Tevatron: $P_T$-distribution (Frixione, Nason, Webber)
Without NNLO predictions, wouldn’t have even realized we were probing the SM Higgs at the Tevatron!

Motivation for NNLO

- Precision QCD played a crucial role in the hunt for the Higgs boson

Higgs predictions receive famously large perturbative corrections

**Harlander, Kilgore; Anastasiou, Melnikov; Ravindran, Smith, van Neerven 2002-2003**

Without NNLO predictions, wouldn’t have even realized we were probing the SM Higgs at the Tevatron!

**Harlander**
First three years of the LHC, Mainz, 2013
Need the following ingredients for NNLO cross sections

- IR singularities cancel in the sum of real and virtual corrections and mass factorization counterterms but only after phase space integration for real radiations.

- Virtual corrections have explicit IR poles, whereas real corrections have implicit IR poles that need to be extracted.

- A generic procedure to extract IR singularities from RR and RV was unknown until very recently.
After more than a decade of research we finally know how to generically handle NNLO QCD corrections to processes with both colored initial and final states.

- First NNLO QCD results to processes with both colored initial and final states

Based on Antenna subtraction scheme

Based on sector-improved subtraction scheme
After more than a decade of research we finally know how to generically handle NNLO QCD corrections to processes with both colored initial and final states.

First NNLO QCD results to processes with both colored initial and final states

- Gehrmann-de Ridder, Gehrmann, Glover, Pires (2013)
- R.B., Caola, Melnikov, Petriello, Schulze (2013)
- Czakon, Fiedler, Mitov (2013)

- For a long time, only color singlet final states available at full NNLO, mostly $2 \rightarrow 1$ at Born level: $H, W, Z, \gamma\gamma$
- 2013 will be remembered as the year of $2 \rightarrow 2$ at NNLO

Lance Dixon, LoopFest 2013
Higgs in association with jets

- Higgs cross-sections in $pp \rightarrow H \rightarrow WW$ are binned according to the jet multiplicity to beat the background.
- The measured value of $pp \rightarrow H \rightarrow WW$ production cross section results from combining 0 jet, 1 jet and 2 jet cross sections. Each of them has its own uncertainty.
- What we knew so far: $H+0j$ @ NNLO, $H+1j$ and $H+2j$ @ NLO.

Theory uncertainties becoming a limiting factor in many analyses, especially $H \rightarrow WW$.

Precise exclusive results are needed, also to separate between gg and VBF...
Higgs + jet @ NNLO (gg only)

R.B., Caola, Melnikov, Petriello, Schulze (2013)

\[ \frac{\sigma_{NNLO}}{\sigma_{NLO}} = 1.3 \]

\[ \frac{\sigma_{NLO}}{\sigma_{LO}} = 1.6 \]

170 different subtraction terms had to be implemented for gg → H g!

Significantly reduced scale dependence O(4%) 

Large K-factor

\( \mu = m_H \)
• gg-channel is the dominant one for phenomenological studies: at NLO gg (70%), qg(30%)
• quark channels necessary for achieving the relevant precision: ongoing work

R.B., Caola, Melnikov, Petriello, Schulze
First results at NNLO available

- $gg \rightarrow gg$ subprocess at Leading Color (Gehrmann-de Ridder, Gehrmann, Glover, Pires 2013)

- Using antenna subtraction to extract IR singularities (analytic cancellation of poles)

Inclusive jet PT distribution

- NNLO/NLO differential K-factor flat over the whole PT range
- Dynamical scale choice: leading jet PT
- Stabilization of scale dependence at NNLO
ttbar @ NNLO

- Large production cross section at LHC: ~ 250 pb at 8 TeV
  - Expected experimental error ~5%
  - NLO+NLL predictions yield an uncertainty of ~10%

- Need NNLO precision for theory

- Results available for the complete total cross section
  (Czakon, Fiedler, Mitov 2013)

- Based on sector-improved subtraction scheme for IR singularities
- Comparable theoretical and experimental uncertainties
- Differential distributions in progress
- Impact on determination of parton distributions
  - Top production at LHC mainly from $qg$ and $gg$ processes
  - Total cross section sensitive to gluon distributions
  - NNLO cross section included into NNLO PDF fits
(Czakon, Mangano, Mitov, Rojo 2013)
  - Uncertainty on gluons reduced at large $x$
Approximate N^3LO results for the inclusive cross section

NLO in EFT:

\[
\Delta \sigma = \sigma_0 \frac{\alpha_s}{\pi} \left\{ \left( \frac{11}{2} + \pi^2 \right) \delta(1 - z) + 12 \left[ \frac{\ln(1 - z)}{1 - z} \right]_+ - 12z(-z + z^2 + 2)\ln(1 - z) \right\} - 6 \frac{(z^2 + 1 - z)^2}{1 - z} \ln(z) - \frac{11}{2} (1 - z)^3
\]

• What we knew so far for approximate N^3LO: the soft gluon threshold (Moch, Vogt 2005)

• New improvements by Ball, Bonvini, Forte, Marzani, Ridolfi, (2013):

1. exact phase space limits for soft gluon emission in threshold logs:

   \[
   \left( \frac{\ln(1 - z)}{1 - z} \right)_+ \rightarrow \left( \frac{\ln \frac{1-z}{\sqrt{z}}}{1 - z} \right)_+
   \]

2. they include the leading collinear gluon emissions, which are normally dropped

3. they make the perturbative expansion consistent with BFKL resummation

\[ z = \frac{m_H^2}{x_1 x_2 s} \]
Approximate $N^3$LO results for the inclusive cross section

- First attempts for directly calculating $N^3$LO contributions (Hoeschele et al. 2012; Anastasiou et al. 2013)

$\mu = m_H/2$ corrections

- New: Ball et al. 2013
- Moch, Vogt, 2005
- Catani et al. 2003
• With $b\bar{b}$ decay of Higgs, most important low-mass mode at Tevatron
• At LHC, boosted analysis possible

Butterworth, Davison, Rubin, Salam 2008

• Inclusive NLO QCD: +30% (Han, Wllenbrock 1990), NLO EW: +5-10% (Ciccolini, Dittmaier, Denner 2003)

• NNLO QCD: 1-2% in bulk of phase space (Ferrera, Grazzini, Tramontano 2011)

• Original boosted analysis vetoes additional jets to remove $t\bar{t}b\bar{b}$ background

• Negative impact on stability of expansion (jet vetoes are theoretically dangerous!)
The jet veto in the WW channel

- Required in WW channel due to background composition
- 25-30 GeV jet cut used; restriction of radiation leads to large logs

- Theory uncertainty becoming a limiting systematic in the 0-jet and 1-jet bins

ATLAS Preliminary

![Event distribution in WW channel](image)

<table>
<thead>
<tr>
<th>Source</th>
<th>Signal processes (%)</th>
<th>Background processes (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N_{\text{jet}} = 0$</td>
<td>$N_{\text{jet}} = 1$</td>
</tr>
<tr>
<td>Theoretical uncertainties</td>
<td></td>
<td></td>
</tr>
<tr>
<td>QCD scale for ggF signal for $N_{\text{jet}} \geq 0$</td>
<td>13</td>
<td>-</td>
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<tr>
<td>QCD scale for ggF signal for $N_{\text{jet}} \geq 1$</td>
<td>10</td>
<td>27</td>
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<tr>
<td>QCD scale for ggF signal for $N_{\text{jet}} \geq 2$</td>
<td>-</td>
<td>15</td>
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<td>QCD scale for ggF signal for $N_{\text{jet}} \geq 3$</td>
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<td>-</td>
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<tr>
<td>Parton shower and UE model (signal only)</td>
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<tr>
<td>PDF model</td>
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<td>7</td>
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<tr>
<td>$H \rightarrow WW$ branching ratio</td>
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<tr>
<td>QCD scale (acceptance)</td>
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<td>4</td>
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<tr>
<td>WW normalisation</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Experimental uncertainties

| Jet energy scale and resolution | 5 | 2 | 6 | 2 | 3 | 7 |
| $b$-tagging efficiency | - | - | - | - | 7 | 2 |
| $f_{\text{neut}}$ efficiency | 1 | 1 | - | 4 | 2 | - |
Why are jet vetoes dangerous?

- Illustrate with simple example of $e^+e^-\rightarrow$jets
- Infrared safety: must sum both virtual and real corrections

Virtual corrections: $-1/\varepsilon_{IR}^2$

Real corrections: $1/\varepsilon_{IR}^2-a\times\ln^2(Q/p_{T,cut})$

- Incomplete cancellation of IR divergences in presence of final state restrictions gives large logarithms of restricted kinematic variable

- Relevant log term for gluon-fusion Higgs searches: $6(\alpha_S/\pi)\ln^2(M_H/p_{T,veto})\sim 1/2$
  ⇒ potentially a large correction
• Use the H+0-jet cross section to illustrate the problem with estimating theory uncertainties on vetoed cross sections by direct scale variation in the exclusive 0-jet bin.

\[
\sigma_0 = \sigma_{\text{total}} - \sigma_{\geq 1}
\]

Large threshold corrections

\[
\sigma_{\text{total}} = (3.32 \text{ pb}) \left[ 1 + 9.5 \alpha_s + 35 \alpha_s^2 + \mathcal{O}(\alpha_s^3) \right]
\]

Stewart, Tackmann 2011

Large jet-veto log corrections

\[
\sigma_{\geq 1}(p_T^{\text{jet}} \geq 25 \text{ GeV}) = (3.32 \text{ pb}) \left[ 6.0 \alpha_s + 32 \alpha_s^2 + \mathcal{O}(\alpha_s^3) \right]
\]

• Strong cancellation between two independent series, which are sensitive to different scales. Uncertainty estimate sensitive to exactly how scales are varied.
Fixed-order scale variation

- The cancellation leads to a pinch point in the scale variation when $\sigma_{\text{total}}$ and $\sigma_{>1}$ are varied together.
- Very likely an underestimate of higher-order corrections; why should the two independent series exhibit the same terms at each order?
- ST prescription: vary the scales separately then combine in quadrature $\implies$ works well for Higgs but not other processes.
- Best solution is to resum the large logarithms.
Zero-jet resummation

- Current status with anti-k\_T algorithm:
  - Banfi, Monni, Salam, Zanderighi: NNLL+NNLO 1203.5573, 1206.4998
  - Becher, Neubert NNLL+NNLO 1205.3806, partial N^3LL+NNLO 1307.0025
  - Stewart, Tackmann, Walsh, Zuberi NNLL’+NNLO 1307.1808

<table>
<thead>
<tr>
<th>Counting in the log of the cross section</th>
</tr>
</thead>
<tbody>
<tr>
<td>LL</td>
</tr>
<tr>
<td>NLL</td>
</tr>
<tr>
<td>NLL’</td>
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<tr>
<td>NNLL</td>
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<tr>
<td>NNNLL</td>
</tr>
<tr>
<td>$\alpha_s L^2$</td>
</tr>
<tr>
<td>$\alpha_s L$</td>
</tr>
<tr>
<td>$\alpha_s$</td>
</tr>
<tr>
<td>$\alpha_s^2 L^3$</td>
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<tr>
<td>$\alpha_s^2 L^2$</td>
</tr>
<tr>
<td>$\alpha_s^2 L$</td>
</tr>
<tr>
<td>$\alpha_s^2$</td>
</tr>
<tr>
<td>$\alpha_s^3 L^4$</td>
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<tr>
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<td>$\alpha_s^3 L^2$</td>
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<tr>
<td>$\alpha_s^3 L$</td>
</tr>
<tr>
<td>$\alpha_s^3$</td>
</tr>
</tbody>
</table>

$L = \ln \left( \frac{p_T^{\text{cut}}}{m_H} \right)$

taken from J. Walsh
NNLL’+NNLO resummation for $P_{Tj}$

- green: NLL$_{pT}$
- blue: NLL’$_{pT}$ + NLO
- orange: NNLL’$_{pT}$ + NNLO

Including resummation and fixed-order uncertainties

Stewart, Tackmann, Walsh, Zuberi
Numerical results for zero jets

- Central value: scheme (a) with
  \[ \mu_R = \mu_F = Q = M/2 \]

- \( \mu_R \) and \( \mu_F \) variations
  \[ \frac{M}{4} \leq \mu_R, \mu_F \leq M \quad \frac{1}{2} \leq \frac{\mu_R}{\mu_F} \leq 2 \]

- Resummation scale (Q) variation
  i.e.
  \[ \ln \frac{M}{p_{T,\text{veto}}} \to \ln \frac{Q}{p_{T,\text{veto}}} \]
  \[ \frac{M}{4} \leq Q \leq M \quad \mu_{R,F} = M/2 \]

- Scheme (b) and (c) with
  \[ \mu_R = \mu_F = Q = M/2 \]

- Total uncertainty \( \leftrightarrow \) envelope

From P. Monni

Banfi, Monni, Salam, Zanderighi
One-jet resummation: numerical results

- Integration over entire $p_T$ range used in the ATLAS measurement
- Large uncertainty from the high-$p_T$ region makes this resummation very effective in reducing errors
- Very conservative (turn off resummation at $p_{T,j}=m_H/2$, use ST below this value). Error on 1-jet bin result is decreased by 25%

<table>
<thead>
<tr>
<th>$m_H$ (GeV)</th>
<th>$p_T^{veto}$ (GeV)</th>
<th>$\sigma_{NLO}$ (pb)</th>
<th>$\sigma_{NLL'+NLO}$ (pb)</th>
<th>$f_{NLO}^{1j}$</th>
<th>$f_{NLL'+NLO}^{1j}$</th>
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<tr>
<td>124</td>
<td>25</td>
<td>5.92+35% -46%</td>
<td>5.62+29% -30%</td>
<td>0.299+38% -49%</td>
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<td>126</td>
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<td>4.72+30% -29%</td>
<td>0.266+35% -43%</td>
<td>0.246+33% -32%</td>
</tr>
</tbody>
</table>

Liu, Petriello 2013
Remarkable progress was achieved in higher order calculations within just a few years.

At NLO, the goals of automation and high multiplicity processes were achieved.

New results for di-jet, Higgs+jet and ttbar at NNLO in QCD. Extremely challenging calculations and the first NNLO QCD results for two-to-two scattering processes at LHC.

Issues can appear in the interplay of experimental cuts with QCD. Significant progress has been made in resumming jet-veto logarithms, and these should propagate into the experimental analysis.