Review of recent developments in QCD theory

Radja Boughezal



DPF 2013 meeting, August 16, UC Santa Cruz



- Why do we care about QCD at higher orders
- Progress in NLO calculations: W+5jets, H+3jets, ttbar+large ETmiss

from Top-quark partners

- QCD at NNLO and beyond:
 - H+jet, dijet, ttbar, inclusive Higgs production at N³LO
 - Resummation for H+0/1 jet and jet vetos
- Conclusions

This is just a selection of some recent highlights, apologies in advance for possible omissions!





Telling us what the background is, so we can see any excess

Teaching us how to reduce the background, sharpen the signal

measurements

G. Salam, ICHEP 2010



No real understanding of LHC physics is possible without sophisticated QCD calculations!



• Factorization: separate hard and soft scales



• Focus of this talk is a precise understanding of $\sigma_{ij \to X}(x_1, x_2, \mu_F^2, q_k)$



Partonic cross section calculable in pQCD as an expansion in α_s



- LO: known for all processes of interest, has large renormalization and factorization scale dependence
- NLO: first reliable predictions (correct shape and normalization, accounts for effects of extra radiation, smaller scale dependence)
- NNLO: required for precise theoretical description of few observables needed in the precise extraction of PDFs, masses and α_s or when perturbative corrections are large

Sometimes fixed order results are not sufficient, in particular when jet veto cuts are imposed to reduce the background. These could introduce large logarithms that need to be resummed.



Higgs production as an example



- NLO calculations become difficult for $2 \rightarrow 3$ and beyond
- Need both virtual corrections and real emission matrix elements (ME) in order to cancel infrared (IR) singularities
- Extracting implicit IR poles from real radiation ME is well understood at NLO with various methods (dipole subtraction, FKS, antenna subtraction)

$$\sigma_{(m)}^{NLO} = \int_{\Phi_m} \left[\mathrm{d}\sigma^{Born} + \mathrm{d}\sigma^V + \int_{\Phi_1} \mathrm{d}\sigma^S \right] + \int_{\Phi_{m+1}} \left[\mathrm{d}\sigma^R - \mathrm{d}\sigma^S \right]$$

• Developments in unitarity based methods turned the calculation of virtual corrections into a possible task even for high multiplicity processes



• Theoretical breakthroughs ideas allowed an incredibly fast progress for fixed order NLO results with complicated final states. Key idea: obtain one-loop amplitudes using tree amplitudes

- Generalized unitarity: Bern, Dixon, Dunbar, Kosower (1994); Britto, Cachazo, Feng (2004)
- The OPP method: Ossola, Papadopoulos, Pittau (2006)
- Rational parts of one-loop amplitudes from tree-amplitudes in multiple dimensions: Giele, Kunszt, Melnikov (2008)
- Feynmann diagramatic approach still provides competitive results: Bredenstein, Denner, Dittmaier, Kallweit, Pozzorini
- Ideas applied by several groups with an amazing outcome! two major directions:

more processes: towards full automation of NLO calculations with codes like Helac, GoSam, MadLoop and OpenLoops
 more legs: e.g. Blackhat focuses on pure n jets or W/Z+n jets, pushing the frontier of n (currently n=5)

Next-to-Leading Order



Impressive list of results:

- **M** multiple jets (up to 4)
- gauge boson and up to 5 jets
- **M** two gauge bosons with up to 2 jets
- or a gauge boson
- If Higgs and up to 3 jets

Process $(V \in \{Z, W, \gamma\})$	Comments				
Calculations completed since Les Houches 2005					
1. $pp \rightarrow VV$ jet	WW jet completed by Dittmaien/Kallweit/Uwer [27, 28 Campbell/Ellis/Zanderighi [29].				
2. $pp \rightarrow Higgs+2jets$	Z Zjet completed by Binoth/Gleisberg/Karg/Kauer/Sanguinetti [30] NLO QCD to the gg channel convoluted by Convolvell/Ellis/Zanderiabi [31]:				
3. $pp \rightarrow VVV$	NLO QCD+EW to the VBF channel completed by Ciccolini/Denner/Dittmaier [32, 33] Interference QCD-EW in VBF channel [34, 35] ZZZ completed by Lazopoulos/Melnikov/Petriello [36]				
	and W W Z by Hansele/Zeppenheid [37], see also Binoth/Ossola/Papadopoulos/Pittau [38] VBFNLO [39, 40] meanwhile also contains WWW, ZZW, WWγ, ZZγ, WZγ, Wγγ, Zγγ, γγγ WZj, Wγj, γjj, Wγγj				
4. $pp \rightarrow t\bar{t} b\bar{b}$	relevant for ttH, computed by Browner, Denner/Dittmaier/Pozzorini [41, 42]				
5. $pp \rightarrow V+3jets$	a / Bevilaet u/Czakon/Papadopoulos/Pittan/Worek [43] 1+3jets cale laters, the Blackhat/Sherpa [44] an Rocket [6] a llaboration Z+3jets ov Blackh (Sherja [5])				
Calculations remaining from Les Houches 2005	UKA				
6. $pp \rightarrow t\bar{t}$ +2jets	relevant for tTH, compact by Bevilacqua/Czakon/Papadopos s/Wore! [47, 48]				
7. $pp \rightarrow VV bb$, 8. $pp \rightarrow VV+2jets$	Pozzocini et al.[25],Bevilacqua et al.[25] W+W++2jets [49], W+W-+2jets [50], VBF contributions calculated by Contributions calculated by				
NLO calculations added to list in 2007	(BOZZU)JagenOlean/Zeppenteia [51, 52, 53]				
9. $pp \rightarrow b\bar{b}b\bar{b}$	Binoth et al. [54, 55]				
NLO calculations added to list in 2009					
10. $pp \rightarrow V + 4$ jets	top pair production, various new physics signatures Blackhat/Sherpa: W+4jets [22], Z+4jets [20] see also HEI [50] for W + niets				
11. $pp \rightarrow Wb\bar{b}j$ 12. $pp \rightarrow t\bar{t}t\bar{t}$	top, new physics signatures, Reina/Schutzmeier [11] various new physics signatures				
aler m - A liste	Rfackhat/Sherna [10]				

NLO wish list



• First $2 \rightarrow 6$ NLO calculation at a hadron collider using Blackhat + Sherpa

• Dynamical scale choice:

$$\mu_R = \mu_F = \hat{H}'_T/2$$
$$\hat{H}'_T \equiv \sum_m p_T^m + E_T^W$$

- ullet scale variation: $\mu/2\dots 2\,\mu$
- reduced scale dependence at NLO
- Ratio of NLO/LO constant over full kinematic range

NLO helps to motivate the scale choice





Cullen, van Deurzen, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Tramontano (2013)

• Can be used to improve the theoretical prediction of the H+2jet bin

 $\sigma_2 = \sigma_{\geq 2} - \sigma_{\geq 3}$

- NLO corrections affect the shape of the P_{T,Higgs} distribution
- NLO corrections improve the scale dependence

NLO highlights: ttbar+large E_{Tmiss} from top-quark partners

• Search for ttbar+large missing energy signature at ATLAS; from pair production of two top partners TT, which decay as $T \rightarrow t\chi$

• How does the inclusion of NLO QCD, and all spin correlations through production and decay affect the robustness of the exclusion limits?



 Large K-factors with significant variation over phase space: can't use an inclusive K-factor!

 LO+parton shower, as used in experimental study, is not a good framework for new physics searches.
 LO+PS acceptance can be very wrong.

• Scale dependence of inclusive cross section typically smaller than that of cross section after cuts; must use the second as the theoretical systematic error.

How to improve NLO predictions

Merging with parton showers

• Add multiple radiation from parton shower to NLO prediction for a specific hard process

- Challenge: avoid double counting
- Two established methods:
 - MC@NLO (Frixione, Webber)
 - POWHEG (Nason, Frixione, Oleari)
- Combines NLO accuracy for hard radiation with multiple soft emissions
 - High P_T described by NLO
 - Low P_{T} described by parton shower

Top-quark pair production at Tevatron: P_T-distribution (Frixione, Nason, Webber)

• Many recent results of NLO predictions combined with parton shower (see Alioli's talk)





Precision QCD played a crucial role in the hunt for the Higgs boson



Higgs predictions receive famously large perturbative corrections

Harlander, Kilgore; Anastasiou, Melnikov; Ravindran, Smith, van Neerven 2002-2003



Without NNLO predictions, wouldn't have even realized we were probing the SM Higgs at the Tevatron!

> Harlander First three years of the LHC, Mainz, 2013



Need the following ingredients for NNLO cross sections



• IR singularities cancel in the sum of real and virtual corrections and mass factorization counterterms but only after phase space integration for real radiations

• Virtual corrections have explicit IR poles, whereas real corrections have implicit IR poles that need to be extracted.

• A generic procedure to extract IR singularities from RR and RV was unknown until very recently

First NNLO QCD results to processes with both colored initial and final states

• After more than a decade of research we finally know how to generically handle NNLO QCD corrections to processes with both colored initial and final states



First NNLO QCD results to processes with both colored initial and final states

• After more than a decade of research we finally know how to generically handle NNLO QCD corrections to processes with both colored initial and



- For a long time, only color singlet final states available at full NNLO, mostly 2 → 1 at Born level: H, W, Z, γγ
- 2013 will be remembered as the year of $2 \rightarrow 2$ at NNLO Lance Dixon, LoopFest 2013

Higgs in association with jets

• Higgs cross-sections in $pp \rightarrow H \rightarrow WW$ are binned according to the jet multiplicity to beat the background

• The measured value of $pp \rightarrow H \rightarrow WW$ production cross section results from combining

0 jet, 1 jet and 2 jet cross sections. Each of them has its own uncertainty

What we knew so far: H+0j @ NNLO, H+1j and H+2j @ NLO



The H+1 jet bin: large NLO K-factor and large theoretical uncertainty

- Theory uncertainties becoming a limiting factor in many analyses, especially $H \rightarrow WW$
- Precise exclusive results are needed, also to separate between gg and VBF...

Higgs + jet @ NNLO (gg only)

R.B., Caola, Melnikov, Petriello, Schulze (2013)



170 different subtraction terms had to be implemented for $gg \rightarrow H g!$

Significantly reduced scale dependence O(4%)

 $\sigma_{\text{NLO}}/\sigma_{\text{LO}} = 1.6$ $\sigma_{\text{NNLO}}/\sigma_{\text{NLO}} = 1.3$ Large K-factor

Higgs + jet @ NNLO (gg only)



gg-channel is the dominant one for phenomenological studies: at NLO gg (70%), qg(30%)
quark channels necessary for achieving the relevant precision: ongoing work R.B., Caola, Melnikov, Petrielo, Schulze



- First results at NNLO available
 - gg → gg subprocess at Leading Color (Gehrmann-de Ridder, Gehrmann, Glover, Pires 2013)
 - Using antenna subtraction to extract IR singularities (analytic cancellation of poles)
- Inclusive jet PT distribution
 - NNLO/NLO differential K-factor flat over the whole PT range
 - Dynamical scale choice: leading jet PT
 - Stabilization of scale dependence at NNLO





- Large production cross section at LHC: ~ 250 pb at 8 TeV
 - Expected experimental error ~5%
 - NLO+NLL predictions yield an uncertainty of ~10%
- Need NNLO precision for theory
- Results available for the complete total cross section (Czakon, Fiedler, Mitov 2013)

Based on sector-improved

subtraction scheme for IR singularities

- Comparable theoretical and experimental uncertainties
- Differential distributions in progress





- Impact on determination of parton distributions
 - Top production at LHC mainly from qg and gg processes
 - Total cross section sensitive to gluon distributions
 - NNLO cross section included into NNLO PDF fits

(Czakon, Mangano, Mitov, Rojo 2013)

• Uncertainty on gluons reduced at large x





- What we knew so far for approximate $N^{3}LO$: the soft gluon threshold (Moch, Vogt 2005)
- New improvements by Ball, Bonvini, Forte, Marzani, Ridolfi, (2013) :
- I. exact phase space limits for soft gluon emission in threshold logs: $\left(\frac{\ln(1-z)}{1-z}\right)_{\perp} \longrightarrow \left(\frac{\ln\frac{1-z}{\sqrt{z}}}{1-z}\right)$

- 2. they include the leading collinear gluon emissions, which are normally dropped
- 3. they make the perturbative expansion consistent with BFKL resummation





• First attempts for directly calculating N³LO contributions (Hoeschele et al 2012; Anastasiou et al 2013)

Associated VH production



- With bbar decay of Higgs, most important low-mass mode at Tevatron
- At LHC, boosted analysis possible

Butterworth, Davison, Rubin, Salam 2008

- Inclusive NLO QCD: +30% (Han, Wllenbrock 1990), NLO EW: +5-10% (Ciccolini, Dittmaier, Denner 2003)
- NNLO QCD: I-2% in bulk of phase space (Ferrera, Grazzini, Tramontano 2011)



- Original boosted analysis vetoes additional jets to remove ttbar background
- Negative impact on stability of expansion (jet vetoes are theoretically dangerous!)

The jet veto in the WW channel



 Required in WW channel due to background composition

 25-30 GeV jet cut used; restriction of radiation leads to large logs

	Signal processes (%)			Background processes (%)					
Source	$N_{\rm jet} = 0$	$N_{\rm jet} = 1$	$N_{\rm jet} \ge 2$	$N_{jet} = 0$	$N_{\rm jet} = 1$	$N_{\rm jet} \geq 2$			
Theoretical uncertainties									
QCD scale for ggF signal for $N_{jet} \ge 0$	13	-	-	-	-	-			
QCD scale for ggF signal for $N_{jet} \ge 1$	10	27	-	-	-	-			
QCD scale for ggF signal for $N_{jet} \ge 2$	-	15	4	-	-	-			
QCD scale for ggF signal for $N_{jet} \ge 3$	-	-	4	-	-	-			
Parton shower and UE model (signal only)	3	10	5	-	-	-			
PDF model	8	7	3	1	1	1			
$H \rightarrow WW$ branching ratio	4	4	4	-	-	-			
QCD scale (acceptance)	4	4	3	-	-	-			
WW normalisation	-	-	-	1	2	4			
Experimental uncertainties									
Jet energy scale and resolution	5	2	6	2	3	7			
b-tagging efficiency	-	-	-	-	7	2			
frecoil efficiency	1	1	-	4	2	-			
ATLAS									

 Theory uncertainty becoming a limiting systematic in the 0-jet and 1-jet bins



- Illustrate with simple example of e⁺e⁻→jets
- Infrared safety: must sum both virtual and real corrections



Virtual corrections: $-1/\epsilon_{IR}^2$ Real corrections: $1/\epsilon_{IR}^2$ -a×ln²(Q/p_{T,cut})

 Incomplete cancellation of IR divergences in presence of final state restrictions gives large logarithms of restricted kinematic variable

• Relevant log term for gluon-fusion Higgs searches: $6(\alpha_s/\pi)\ln^2(M_H/p_{T,veto}) \sim 1/2$ \Rightarrow potentially a large correction



• Use the H+0-jet cross section to illustrate the problem with estimating theory uncertainties on vetoed cross sections by direct scale variation in the exclusive 0-jet bin



Large jet-veto log corrections

• Strong cancellation between two independent series, which are sensitive to different scales. Uncertainty estimate sensitive to exactly how scales are varied.

Fixed-order scale variation



Stewart, Tackmann 2011

• The cancellation leads to a pinch point in the scale variation when σ_{total} and $\sigma_{\geq 1}$ are varied together

 Very likely an underestimate of higher-order corrections; why should the two independent series exhibit the same terms at each order?

 ST prescription: vary the scales separately then combine in quadrature ⇒ works well for Higgs but not other processes

 Best solution is to resum the large logarithms



- Current status with anti-k_T algorithm:
 - * Banfi, Monni, Salam, Zanderighi: NNLL+NNLO 1203.5573, 1206.4998
 - * Becher, Neubert NNLL+NNLO 1205.3806, partial N³LL+NNLO 1307.0025
 - Stewart, Tackmann, Walsh, Zuberi NNLL'+NNLO 1307.1808





green: NLL_{p_T} blue: $NLL'_{p_T} + NLO$ orange: $NNLL'_{p_T} + NNLO$

Including resummation and fixed-order uncertainties



Stewart, Tackmann, Walsh, Zuberi



Central value: scheme (a) with

$$\mu_R = \mu_F = Q = M/2$$

- $\stackrel{@}{=} \mu_R \text{ and } \mu_F \text{ variations}$ $\frac{M}{4} \leq \mu_R, \mu_F \leq M \qquad \frac{1}{2} \leq \frac{\mu_R}{\mu_F} \leq 2$
- Resummation scale (Q) variation i.e.

$$\frac{\ln \frac{M}{p_{t,veto}} \rightarrow \ln \frac{Q}{p_{t,veto}}}{\frac{M}{4} \le Q \le M} \qquad \mu_{R,F} = M/2$$

- Scheme (b) and (c) with $\mu_R = \mu_F = Q = M/2$
- \bigcirc Total uncertainty \longleftrightarrow envelope



Banfi, Monni, Salam, Zanderighi



• Integration over entire p_T range used in the ATLAS measurement



• Large uncertainty from the high-p_T region makes this resummation very effective in reducing errors

• Very conservative (turn off resummation at p_{T,J}=m_H/2, use ST below this value). Error on I-jet bin result is decreased by 25%

$m_H ~({\rm GeV})$	p_T^{veto} (GeV)	$\sigma_{\rm NLO}~({\rm pb})$	$\sigma_{\rm NLL'+NLO}~(\rm pb)$	$f_{ m NLO}^{1j}$	$f_{\rm NLL'+NLO}^{1j}$
124	25	$5.92^{+35\%}_{-46\%}$	$5.62^{+29\%}_{-30\%}$	$0.299^{+38\%}_{-49\%}$	$0.283^{+33\%}_{-34\%}$
125	25	$5.85^{+34\%}_{-46\%}$	$5.55^{+29\%}_{-30\%}$	$0.300^{+37\%}_{-49\%}$	$0.284^{+33\%}_{-33\%}$
126	25	$5.75^{+35\%}_{-46\%}$	$5.47^{+30\%}_{-30\%}$	$0.300^{+38\%}_{-49\%}$	$0.284^{+34\%}_{-33\%}$
124	30	$5.25^{+31\%}_{-41\%}$	$4.83^{+29\%}_{-29\%}$	$0.265^{+35\%}_{-43\%}$	$0.244^{+33\%}_{-33\%}$
125	30	$5.19^{+32\%}_{-41\%}$	$4.77^{+30\%}_{-29\%}$	$0.266^{+35\%}_{-43\%}$	$0.244^{+33\%}_{-33\%}$
126	30	$5.12^{+32\%}_{-41\%}$	$4.72^{+30\%}_{-29\%}$	$0.266^{+35\%}_{-43\%}$	$0.246^{+33\%}_{-32\%}$

Liu, Petriello 2013



Remarkable progress was achieved in higher order calculations within just few years

At NLO, the goals of automation and high multiplicity processes was achieved

New results for di-jet, Higgs+jet and ttbar at NNLO in QCD. Extremely challenging calculations and the first NNLO QCD results for two-to-two scattering processes at LHC

Issues can appear in the interplay of experimental cuts with QCD. Significant progress has been made in resumming jet-veto logarithms, and these should propagate into the experimental analysis