Dipole Moment Bounds on Dark Matter Annihilation

arXiv:1307.7120

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Introduction

Dipole Moment Bounds on Dark Matter Annihilation

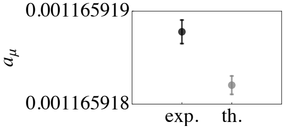
1. Dipole moments bound new physics
2. A simplified model has CP and $\not{C}\not{P}$ terms
3. Only the $\not{C}\not{P}$ muon channel is observable

Dipole moments bound new physics

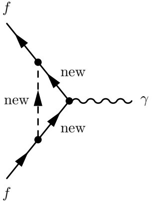
The fermion photon vertex correction is $i\mathcal{M} = -ie\bar{f}\Gamma^\mu f \tilde{A}_\mu$, where

$$\Gamma^\mu = \gamma^\mu F_1 + \frac{\imath\sigma^{\mu\nu}q_\nu}{2m} F_2 + \frac{\imath\sigma^{\mu\nu}q_\nu\gamma^5}{2m} \boxed{F_3} + (\gamma^\mu q^2 - 2mq^\mu)\gamma^5 F_A$$

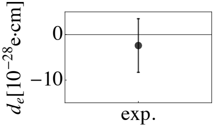
$$F_2(0) = a$$



$\Delta a_\mu = 2.87 \times 10^{-9}$
exp. BNL E821 2006
th. PDG 2013



$$\boxed{F_3(0) = 2m_f \frac{d}{|e|}}$$

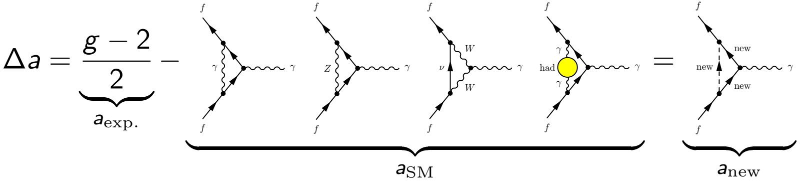


$2m_e \left|\frac{d_e}{e}\right| < 5.5 \times 10^{-17}$
Hudson et al 2011

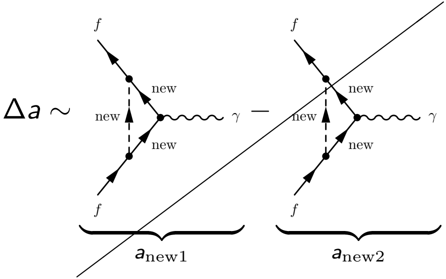
The form factors are related to the magnetic and electric dipole moments a and d. The discrepancy can be described by,

- calculation methods
- new particles running in the loop

Dipole moments bound new physics

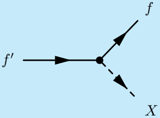


- Use new physics to fill in the anomaly gap
- assume there is no large cancellation within the new physics

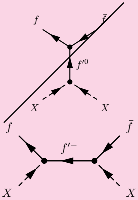


Dipole moments bound new physics

A simplified scalar annihilation model as new physics



$m_{f'} > m_X$
Symmetry stablized



Mediator
EM charged

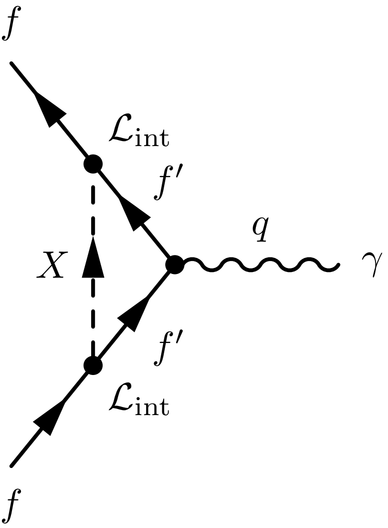
Leptons
tightest

Renormalizable

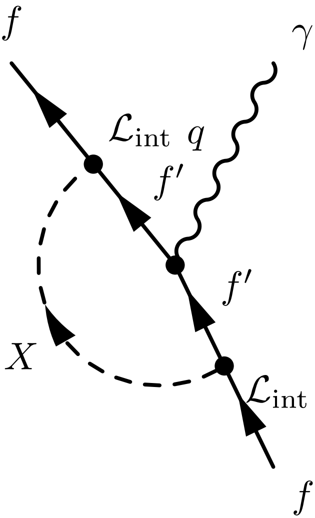
$$\mathcal{L}_{\rm int} = X^* \bar{f}' \begin{bmatrix} \lambda_L & 0 \\ 0 & \lambda_R \end{bmatrix} f + \text{h.c.}$$

- *Dark Matter X* is a scalar
- *Mediator f'* is a fermion

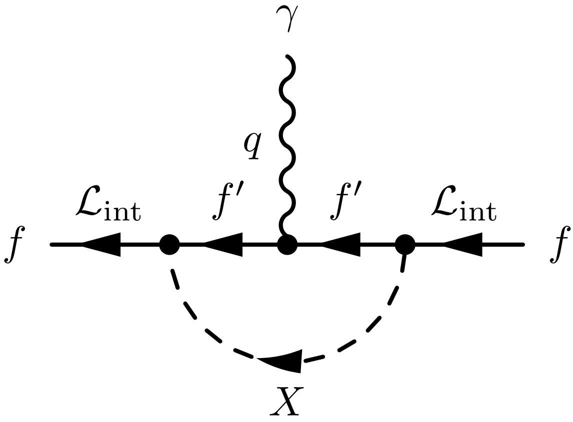
A simplified model has CP and $\not{CP}$ terms



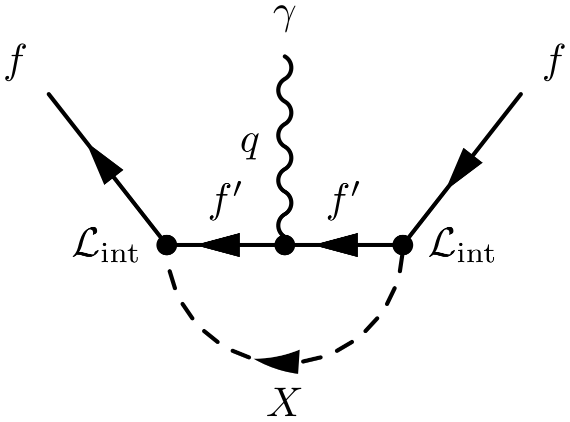
A simplified model has CP and $\not{C}\not{P}$ terms



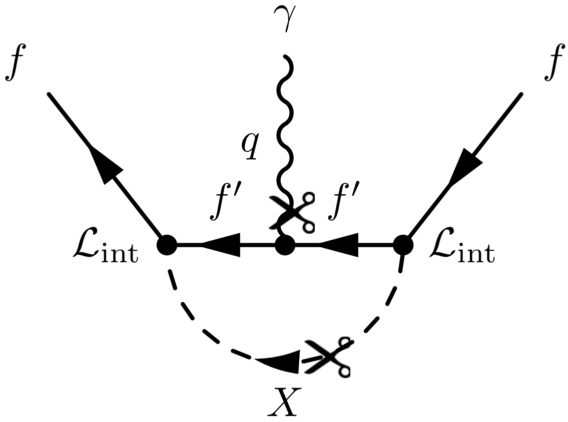
A simplified model has CP and $\not{C}\not{P}$ terms



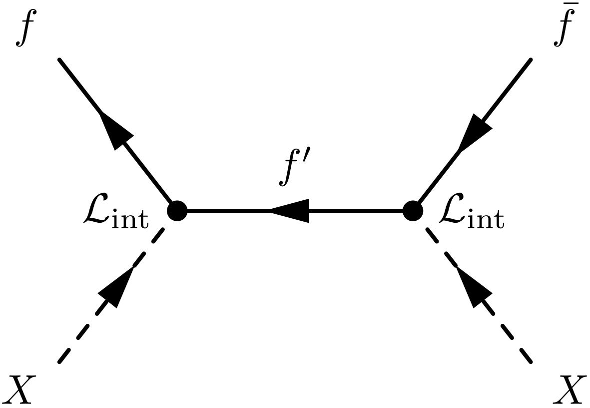
A simplified model has CP and $\not{C}\not{P}$ terms



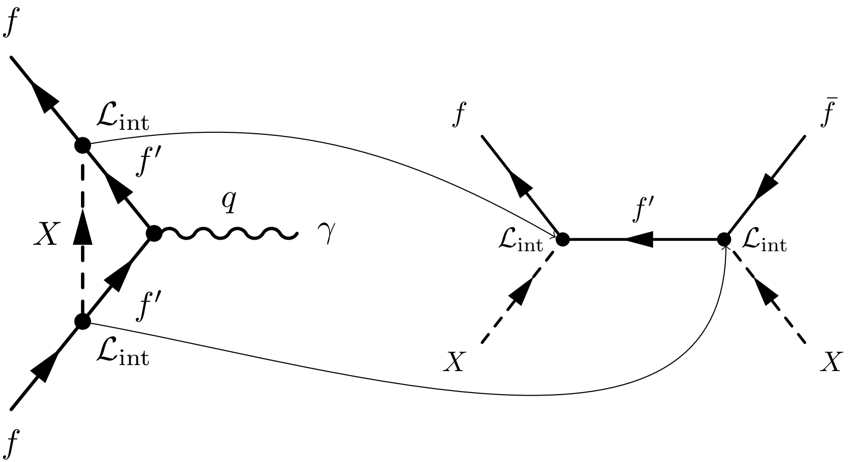
A simplified model has CP and $\not{C}\not{P}$ terms



A simplified model has CP and $\not{\text{CP}}$ terms



A simplified model has CP and $\cancel{CP}$ terms



Both diagrams has the same two vertexes. So the matrix element is given by the same interaction terms. $i\mathcal{M} \sim \langle \int d^4x\mathcal{L}_{\rm int} \int d^4y\mathcal{L}_{\rm int} \ldots \rangle$

A simplified model has CP and $\not{C}\not{P}$ terms

The matrix element reduces to the following terms In the limit $m_f \ll m_X \ll m_{f'}$ and $|\lambda_{L,R}|, \sin\alpha, \cos\alpha \not\approx 0$:

$$\mathcal{O} \sim \boxed{\frac{Re(\lambda_L \lambda_R^*)}{m_{f'}} \overbrace{(X^* X)}^{\text{CPeven}} \underbrace{(\bar{f} f)}_{\text{CPeven}}} + \boxed{\frac{Im(\lambda_L \lambda_R^*)}{m_{f'}} \overbrace{(X^* X)}^{\text{CPeven}} \underbrace{(-i\bar{f}\gamma^5 f)}_{\text{CPodd}}}$$

- coefficient of CP and $\not{C}\not{P}$ terms are real and imaginary parts of $\lambda_L \lambda_R^*$ respectively

$\boxed{\text{no interference}}$ $\boxed{\text{no suppressions}}$

A simplified model has CP and $\not{C}\not{P}$ terms

$$\Gamma^{\mu}, \sigma v = \mathrm{CP} + \boxed{\not{C}\not{P}}$$

As a consequence, both calculated diagrams produce CP and $\not{C}\not{P}$ terms which is related to the real and imaginary parts of the coupling respectively.

A simplified model has CP and $\not{CP}$ terms

The calculated diagrams are the following:

$$i\mathcal{M} = -ie\bar{u}\Gamma^{\mu}u\tilde{A}_{\mu}$$

$$\simeq -ie\Big[\bar{u}\frac{i\sigma^{\mu\nu}q_{\nu}}{2m}Re(\lambda_L\lambda_R^*)\frac{m_f}{m_{f'}}u + \boxed{\bar{u}\frac{\imath\sigma^{\mu\nu}q_{\nu}}{2m}\gamma^5 Im(\lambda_L\lambda_R^*)\frac{m_f}{m_{f'}}u}\Big]\tilde{A}_{\mu}$$

$$\sigma v \simeq \Big[Re(\lambda_L\lambda_R^*)^2 + \boxed{Im(\lambda_L\lambda_R^*)^2}\Big]\frac{1}{4\pi m_{f'}^2}$$

($\times 4$ for real scalar)

As shown, both calculated diagrams produce CP and $\not{CP}$ terms which is related to the real and imaginary parts of the coupling respectively.

Only the $\not{CP}$ muon channel is observable

"What are the allowed values of $\lambda_L \lambda_R^*$ and m_X?"

$$\Gamma^\mu \in \quad \Delta a_f \gtrsim Re(\lambda_L \lambda_R^*) \frac{m_f}{m_{f'}} \qquad \boxed{2m_f \frac{d_f}{|e|} \gtrsim Im(\lambda_L \lambda_R^*) \frac{m_f}{m_{f'}}}$$

$$\sigma v \simeq \left[Re(\lambda_L \lambda_R^*)^2 + \boxed{Im(\lambda_L \lambda_R^*)^2} \right] \frac{1}{4\pi m_{f'}^2}$$

"What are the allowed values of σv and m_X?"

constraints squared

m_f^2 relative factor

Only the $\not{C}\not{P}$ muon channel is observable

$$7.7 \times 10^{11}[\mathrm{pb}] \left\{ (\Delta a_f)^2 + \left(2 m_f \frac{d_f}{|e|}\right)^2 \right\} \left(\frac{\mathrm{GeV}}{m_f}\right)^2 \gtrsim \sigma v$$

Upper bound on an annihilation cross section to ...

- Electrons e

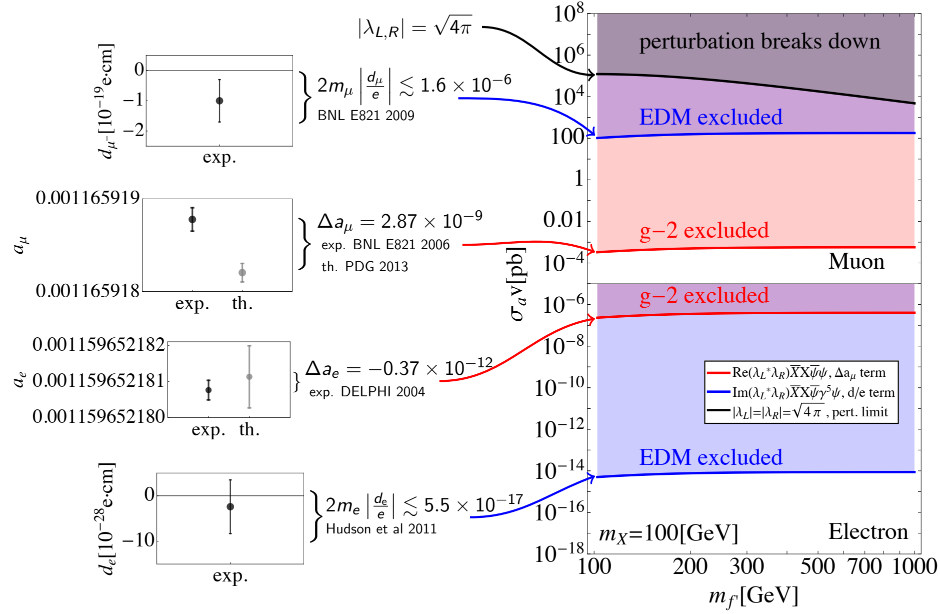
$$4.0 \times 10^{-7}[\mathrm{pb}] + 8.8 \times 10^{-15}[\mathrm{pb}] \gtrsim \sigma v$$

- Muons μ

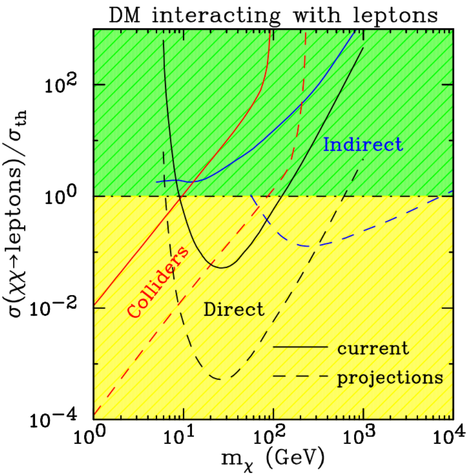
$$5.6 \times 10^{-4}[\mathrm{pb}] + 180[\mathrm{pb}] \gtrsim \sigma v$$

- Taus τ is above the perturbative $|\lambda_{L,R}| = \sqrt{4\pi}$ limit of the photon vertex correction

Only the $\cancel{CP}$ muon channel is observable

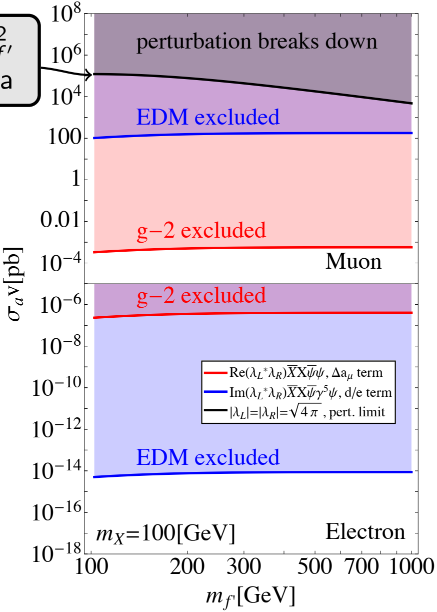


Only the $\not{CP}$ muon channel is observable



DM Complimentarity, arXiv:1305.1605

$\times m_f^2/m_{f'}^2$
Majorana



Conclusion

	CP g-2 bounds	$\not{C}\not{P}$ EDM bounds
Experimental bound	$\Delta a \gtrsim Re(\lambda_L \lambda_R^*) \frac{m_f}{m_{f'}}$	$2m\frac{d}{e} \gtrsim Im(\lambda_L \lambda_R^*) \frac{m_f}{m_f}$
Annihilation cross section to...	$Re(\lambda_L \lambda_R^*)^2 \frac{1}{4\pi m_{f'}^2}$	$Im(\lambda_L \lambda_R^*)^2 \frac{1}{4\pi m_{f'}^2}$
Electrons e	4.0×10^{-7}[pb]	8.8×10^{-15} [pb]
Muons μ	5.6×10^{-4}[pb]	180 [pb]
Taus τ	?	?
Quarks q	?	?

1. Dipole moments bound new physics
2. A simplified model has CP and $\not{C}\not{P}$ terms
3. Only the $\not{C}\not{P}$ muon channel is observable

Thank You very much for listening!

$m_f \ll m_X, m_{f'}$	$\lvert\lambda_{L,R}\rvert \not\approx 0$	Same with mixing	t-ch	$\times 4$ real scalar

Proof of F_2 and MDM

$$
\begin{aligned}
& -ie\bar{u}(p')\frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(0)u(p)\tilde{A}^{cl}_\mu(q) \\
= \quad & -ie\bar{u}(p')\frac{-i\frac{i}{2}\gamma^i\gamma^j q_\nu}{2m}F_2(0)u(p)i\tilde{F}^{cl}_{\mu\nu}(q) \\
= \quad & -i(2m)e\xi^\dagger\left[\frac{1}{2m}\sigma^k F_2(0)\right]\xi\frac{1}{2}\epsilon^{ijk}\tilde{F}^{cl}_{\mu\nu}(q) \\
= \quad & -i(2m)e\xi^\dagger\left[\frac{1}{2m}\sigma^k F_2(0)\right]\xi\tilde{B}^k_{cl}(q) \\
= \quad & -i(2m)\left[-\frac{e}{m}F_2(0)\vec{S}\cdot\tilde{\vec{B}}^{cl}(q)\right] \\
= \quad & -i(2m)\left[-\langle\vec{\mu}\rangle\cdot\tilde{\vec{B}}^{cl}(q)\right] \\
= \quad & -i(2m)\left[-\langle\vec{\mu}\rangle\cdot\tilde{\vec{B}}^{cl}(q)\right]
\end{aligned}
$$

Proof of F_3 and EDM

$$
\begin{aligned}
& -ie\bar{u}(p')\frac{\imath\sigma^{\mu\nu}q_\nu}{2m}\gamma^5 F_3(0)u(p)\tilde{A}^{cl}_\mu(q) \\
= \quad & -ie\bar{u}(p')\frac{-i\frac{i}{2}2\gamma^k\gamma^0 q_\nu}{2m}\gamma^5 F_3(0)u(p)i\tilde{F}^{cl}_{k0}(q) \\
= \quad & -ie\bar{u}(p')\frac{-i\frac{i}{2}\epsilon^{ijk}\gamma^i\gamma^j q_\nu}{2m}\gamma^5 F_3(0)u(p)i\tilde{F}^{cl}_{k0}(q) \\
= \quad & -i(2m)e\xi^\dagger\left[\frac{1}{2m}\sigma^l(-iF_3(0))\right]\xi\frac{1}{2}\epsilon^{ijk}\epsilon^{ijl}\tilde{F}^{k0}_{cl}(q) \\
= \quad & -i(2m)e\xi^\dagger\left[\frac{1}{2m}\sigma^k(-iF_3(0))\right]\xi\tilde{F}^{k0}_{cl}(q) \\
= \quad & -i(2m)e\xi^\dagger\left[\frac{1}{2m}\sigma^k(-iF_3(0))\right]\xi\tilde{E}^{k}_{cl}(q) \\
= \quad & -i(2m)\left[-\frac{e}{m}(-iF_3(0))\vec{S}\cdot\tilde{\vec{E}}_{cl}(q)\right] \\
= \quad & -i(2m)\left[-\langle\vec{d}\rangle\cdot\tilde{\vec{E}}_{cl}(q)\right]
\end{aligned}
$$

Photon Vertex Correction

The exact photon vertex correction $\bar{u}\Gamma u$ and the relevant terms are given by the following:

$$\gamma^\mu F_1(q^2) + \frac{\imath\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2) + \frac{\imath\sigma^{\mu\nu}q_\nu\gamma^5}{2m}F_3(q^2) + (\gamma^\mu q^2 - 2mq^\mu)\gamma^5 F_A(q^2) + \dots$$

where,

$$F_1(0) = \frac{1}{(4\pi)^2}\int_0^1 dz \frac{1}{(1-z)(m_{f'}^2 - zm_f^2) + zm_X^2}\Big\{(\lambda_L^*\lambda_R + \lambda_R^*\lambda_L)z(1-z)m_f m_{f'} + (|\lambda_L|^2 + |\lambda_R|^2)z^2(1-z)m_f^2 + (|\lambda_L|^2 + |\lambda_R|^2)(1-z)m_{f'}^2\Big\}$$

$$F_2(0) = \frac{1}{(4\pi)^2}\int_0^1 dz \frac{-(|\lambda_L|^2 + |\lambda_R|^2)\frac{1}{2}z(1-z)^2 m_f + (\lambda_L\lambda_R^* + \lambda_R\lambda_L^*)(1-z)^2 m_{f'}}{(1-z)(m_{f'}^2 - zm_f^2) + zm_X^2}$$

$$F_3(0) = \frac{1}{(4\pi)^2}\int_0^1 dz \frac{(\lambda_L\lambda_R^* - \lambda_R\lambda_L^*)(1-z)^2 m_{f'}}{(1-z)(m_{f'}^2 - zm_f^2) + zm_X^2}$$

Total annihilation cross section

The exact annihilation cross section in the non-relativistic limit is given by the following:

$$
\begin{aligned}
(\sigma|v_A - v_B|)_{CM} = \quad & -\frac{\sqrt{1-\frac{m_f^2}{m_X^2}}}{64\pi m_X^2(-m_f^2+m_{f'}^2+m_X^2)^2} \\
& \times\Big\{(\lambda_L\lambda_L^* + \lambda_R\lambda_R^*)^2\Big[m_f^2(m_f-m_X)(m_f+m_X)\Big] \\
& +(\lambda_L^*\lambda_R + \lambda_L\lambda_R^*)(\lambda_L\lambda_L^* + \lambda_R\lambda_R^*)\Big[2m_f(m_f-m_X)(m_f+m_X)m_{f'}\Big] \\
& +(\lambda_L\lambda_L^*\lambda_R\lambda_R^*)\Big[2m_{f'}^2(m_f^2-2m_X^2)\Big] \\
& +(\lambda_L^{*2}\lambda_R^2 + \lambda_L^2\lambda_R^{*2})\Big[m_{f'}^2 m_f^2\Big]\Big\}
\end{aligned}
$$

Conversion to Fermions

One can convert dark matter to Majorana fermions through a factor of

$$Re(\lambda_L \lambda_R^*) \frac{m_f}{m_{f'}} \lesssim a \rightarrow Re(\lambda_L \lambda_R^*) \frac{m_f m_X}{m_{f'}^2} \times \left[\frac{m_f}{m_X} \text{or} \sin\alpha \right] \lesssim a$$

Which comes out to an overall extra factor of $\sim \frac{m_f}{m_{f'}}$. In the annihilation cross section, this factor is cancelled by the chirality suppression factor. Therefore,

1. the bound is unchanged
2. σv, $\sqrt{4\pi}$ perturbative limit comes down tighter with an factor of $\frac{m_f^2}{m_{f'}^2}$

So, if $m_e \sim$1MeV, $m_{f'} \sim$1TeV, then σv, $\sqrt{4\pi}$ perturbative limit comes down with an factor of 10^{-12}

Reference of equations: arXiv 0904.4352 Cheung, Kong and Lee