Field localization and mass generation in an alternative 5-dimensional brane model

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Field localization and Nambu-Jona-Lasinio mass generation mechanism in an alternative five-dimensional brane model

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Formally related metrics

Randall-Sundrum model

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

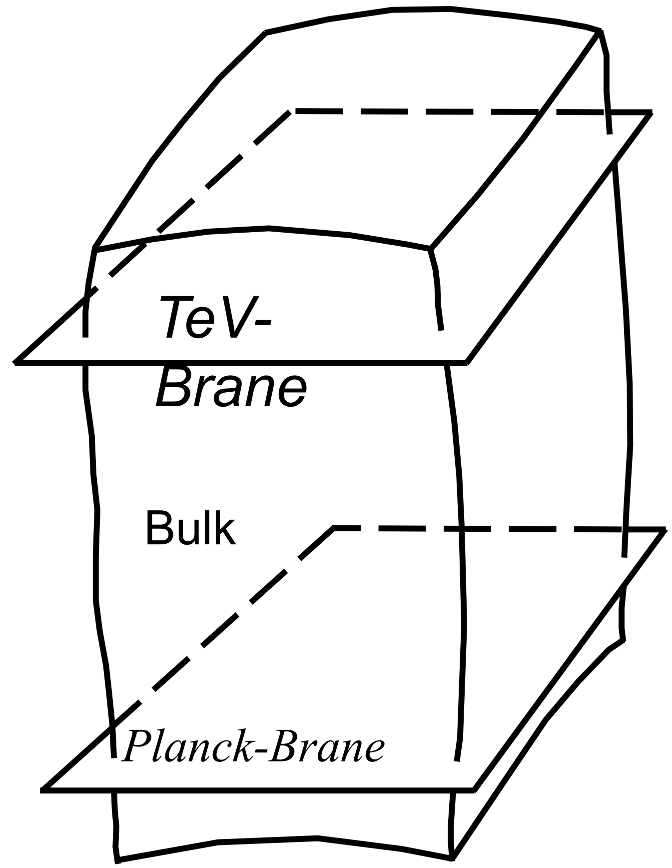
R-S alternate form

$$ds^2 = e^{-2A(z)}\eta_{AB}dx^A dx^B, \quad e^{-A(z)} = \frac{1}{1-2k|z|}$$

Warped extra dimension r-metric

$$ds^2 = e^{-2A(r)}\left(\eta_{\mu\nu}dX^\mu dX^\nu - dr^2\right)$$

$$A(r) = k|r|$$



Coordinate transformations

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

$$ds^2 = e^{-2k|r|}\left(\eta_{\mu\nu}dX^\mu dX^\nu - dr^2\right)$$

$$dX^\mu = \frac{e^{-k|y|}}{1-k|y|}dx^\mu$$

$$e^{-k|r|} = 1-k|y|$$

$$dX^\mu = A\left(x^\mu, y\right)dx^\mu + B\left(x^\mu, y\right)dy$$

$$\partial_y A \neq 0$$

$$y \to r \quad exact\ iff \quad y = constant$$

Foliations

Einstein equations

Randall-Sundrum fine tuned cosmological constant T_{55}=0

$$\eta_{\mu\nu} e^{-2k|y|}\left(6k^2 + \lambda_{[y]}\right) - 6k\eta_{\mu\nu}\delta(y) = \kappa^2 T_{\mu\nu}$$

$$-\left(6k^2 + \lambda_{[y]}\right) = \kappa^2 T_{55}$$

r-metric and λ=0

$$\eta_{\mu\nu}\left(3k^2 + e^{-2k|r|}\lambda_{[r]}\right) - 6k\eta_{\mu\nu}\delta(r) = \kappa^2 T_{\mu\nu}$$

$$-6k^2 - e^{-2k|r|}\lambda_{[r]} = \kappa^2 T_{55}$$

Bulk energy-momentum source

Energy-momentum $T^{\phi}_{AB} = \frac{\partial \mathcal{L}_{5D}}{\partial \phi^{,A}} \phi_{,B} - g_{AB} \mathcal{L}_{5D}$

Bulk $\begin{matrix} T^{bulk}_{\mu\nu} = \eta_{\mu\nu} 3 \frac{k^2}{\kappa^2}{}_{\mu\nu} \\ T^{bulk}_{55} = -6 \frac{k^2}{\kappa^2} \end{matrix} \rightarrow \quad T^{bulk}_{AB} = \frac{3k^2}{\kappa^2} diag[1,-1,-1,-1,-2]$

Scalar field lagrangian $\mathcal{L}^{*}_{5D} = \frac{1}{2}(\partial_A \phi)(\partial^A \phi) - V(\phi)$

This is not possible!

Bulk source ghost scalar field

Ghost peppers

Ghost field

$$\mathcal{L}_{5D} = -\frac{1}{2}(\partial_A \phi)(\partial^A \phi) - V(\phi)$$

Source scalar field $\phi = \sqrt{3}\,\frac{k}{\kappa}\,r$

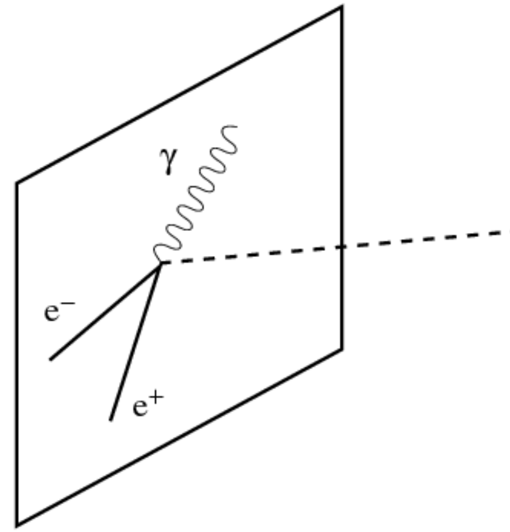
Self-interaction $V(\phi) = \frac{9k^2}{2\kappa^2} e^{\frac{2}{\sqrt{3}}\kappa\phi}$

Energy-momentum $T^{bulk}_{AB} = \frac{3k^2}{\kappa^2} diag[1,-1,-1,-1,-2]$

Localization of particles

Wick rotated propagator

$$\int Dx\, e^{-S} \rightarrow S = finite$$



Spin dependent confinement

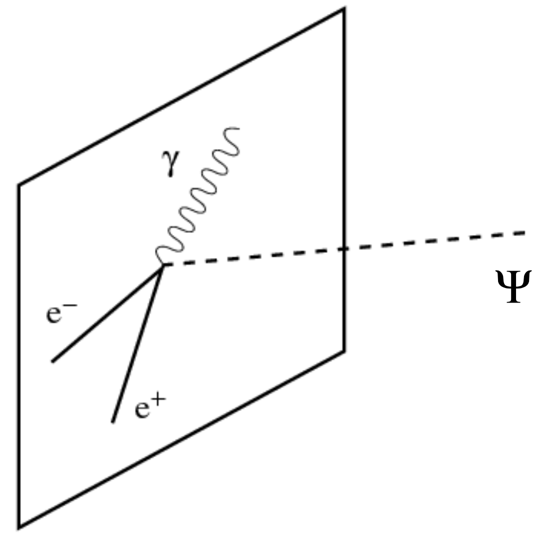
$Spin$	$"RS\ model"$	$r - metric$
0	$-2k$	$-2k \rightarrow 2m < \sqrt{5}k$ $+2k \rightarrow 2m < 3k$
1	$none$	$-2k$
$\frac{1}{2}$	$+2k$	$none$

Spinor field and r-metric

Action $S_\psi = \int d^5x\sqrt{g}\,\bar{\Psi} i\Gamma^M D_M \Psi$

Separated 5D spinor

$$\Psi = \begin{pmatrix} \psi_R(x_\mu)\, p_R(r) \\ \psi_L(x_\mu)\, p_L(r) \end{pmatrix}$$



Stationary solution

$$p_R = c\, e^{(2k-m)r}, \quad p_L = d\, e^{(2k+m)r}$$

Action for spinor field

Two kinetic terms

$$c^2 \int_0^\infty dr\, e^{-2mr} \int d^4x\, \bar{\psi}_R i\gamma^\mu \partial_\mu \psi_R \rightarrow finite$$

$$d^2 \int_0^\infty dr\, e^{2mr} \int d^4x\, \bar{\psi}_L i\gamma^\mu \partial_\mu \psi_L \rightarrow \infty$$

Gamma five terms

$$c\, d \int_0^\infty dr \int d^4x \left(\psi_L^\dagger \psi_R + \psi_R^\dagger \psi_L \right) \rightarrow \infty$$

Other mechanisms for localization?

Vector fields and r-metric

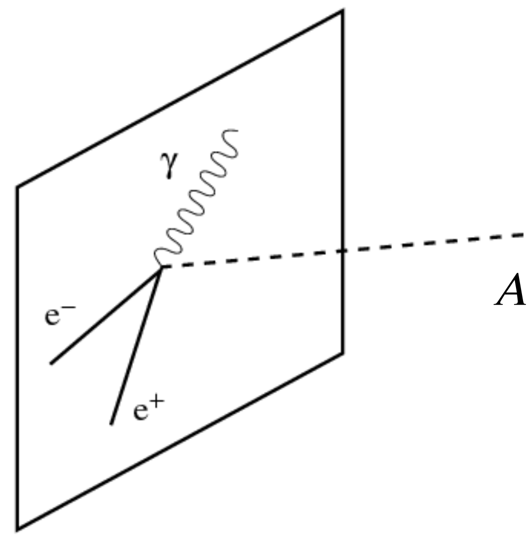
Action

$$S_A = -\frac{1}{4}\int d^5x\sqrt{g}\,g^{MN}g^{RS}\left(\partial_M A_N - \partial_N A_M\right)\left(\partial_R A_S - \partial_S A_R\right)$$

Ansatz

$$A_r = constant$$

$$A_\mu\left(x^M\right) = a_\mu\left(x^\nu\right)c(r)$$



Stationary solutions

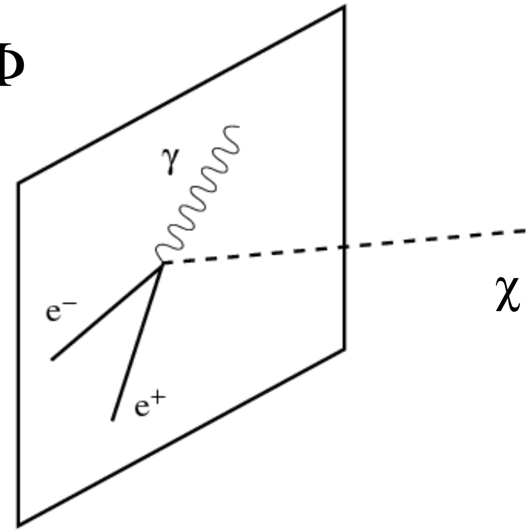
$$c(r) = c_1, \quad c(r) = c_1 e^{k|r|}$$

Localized

$$S_A = -\frac{c^2}{4}\int_0^\infty dr\, e^{-kr}\int dx^4\left(\partial^\mu a_\mu - \partial^\nu a_\nu\right)\left(\partial^\sigma a_\sigma - \partial^\rho a_\rho\right)$$

Scalar field and r-metric

Action $S_\Phi = \int d^5x \sqrt{g}\, g^{MN} \partial_M \Phi^* \partial_N \Phi$



Separation $\Phi = \varphi\left(X^\mu\right)\chi(r)$

Stationary solution

$$\chi(r) = ce^{\frac{3}{2}kr} e^{-\frac{1}{2}r\sqrt{9k^2-4m^2}} + de^{\frac{3}{2}kr} e^{\frac{1}{2}r\sqrt{9k^2-4m^2}}$$

Zero mass $\chi(r) = c + de^{3kr}$

Mass dependent localization

$$S_\Phi = N\left(g^{\mu\nu}\right)\int d^4x\,\partial_\mu\varphi\,\partial_\nu\varphi + M^2\int d^4x\,\varphi^2$$

$$N = \int_0^\infty dr\sqrt{g}\,\chi^2 g^{\mu\nu} = \begin{cases} 1 & if \quad 3|k| > 2m \\ \infty & if \quad 3|k| < 2m \end{cases}$$

Bulk mass

$$M^2 = \int_0^\infty dr\sqrt{g}\,g^{rr}\partial_r\chi^*\partial_r\chi = \begin{cases} M(m,k) \;\; if \; \sqrt{5}\,|k| > 2m \\ \infty \quad if \quad \sqrt{5}\,|k| < 2m \end{cases}$$

Nambu Jona-Lasinio mass

On the brane

$$\partial^{\nu}\partial_{\nu}\varphi = -m^2\varphi$$

$k\sqrt{\frac{11+\sqrt{13}}{8}}$ – – – –

Brane and bulk equal

$$M^2(m,k) = m^2$$

$k\frac{\sqrt{5}}{2}$ ———

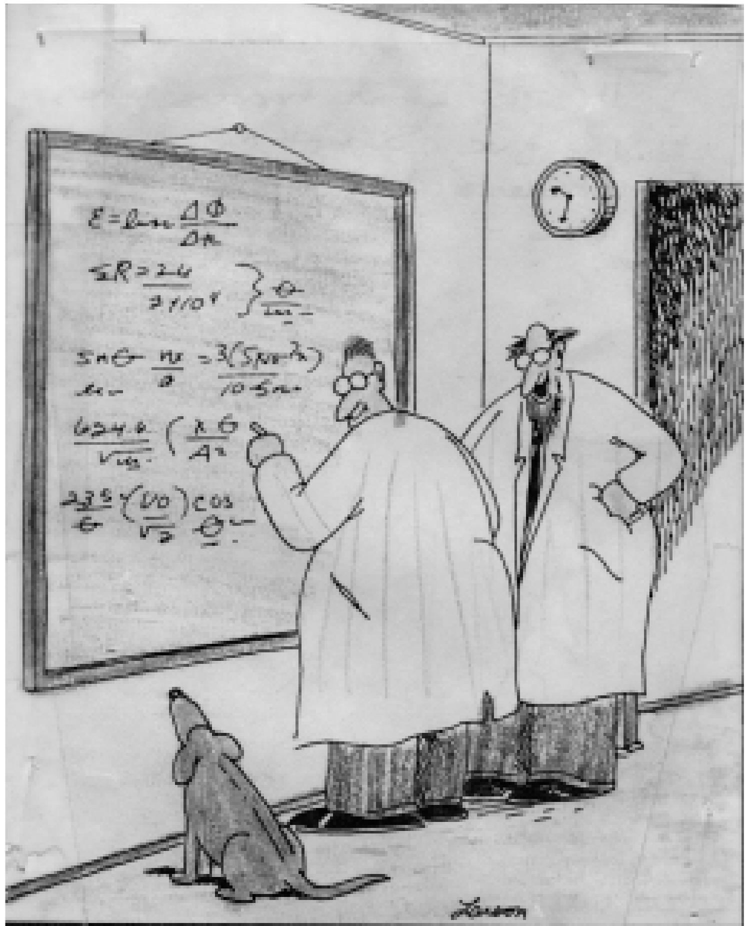
NJL type mass (-*2k*)

$k\sqrt{\frac{11-\sqrt{13}}{8}}$ – – – –

$$m^2 = k^2\,\frac{11\pm\sqrt{13}}{8}, \quad m^2 < \frac{5}{4}k^2$$

Mass

Questions?



"Ohhhhhh…Look at that, Schuster…Dogs are so cute when they try to comprehend quantum mechanics."